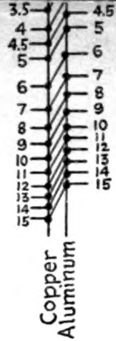


60 C

$$\text{Reg'n. Volts} = \frac{1000\,K.V.A. \times l\,V}{E}$$

$$\%\text{ Line Drop (percent of } E) = \%\text{ Reg'n.} - 100K\left(\frac{l}{1000}\right)^2$$

$$\text{Line Drop} = \text{Reg'n. Volts} - EK\left(\frac{l}{1000}\right)^2$$

where

$K = 2.16$ for 60 Cycles
$K = .375$ for 25 Cycles
E = Line Voltage at Load
K.V.A. = K.V.A. of Load
l = Length of Line in Miles

Equivalent Spacing of 3 Phase Line

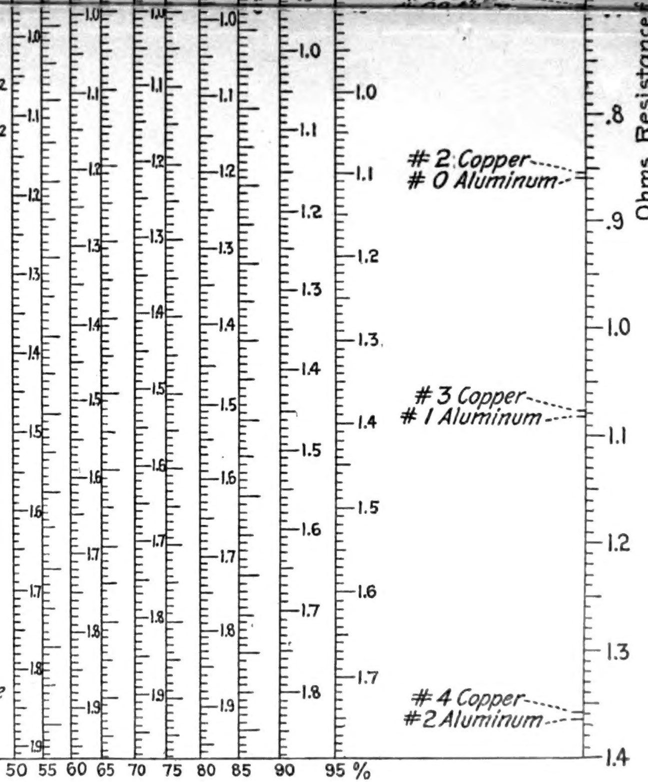
$$s = \sqrt[3]{abc}$$

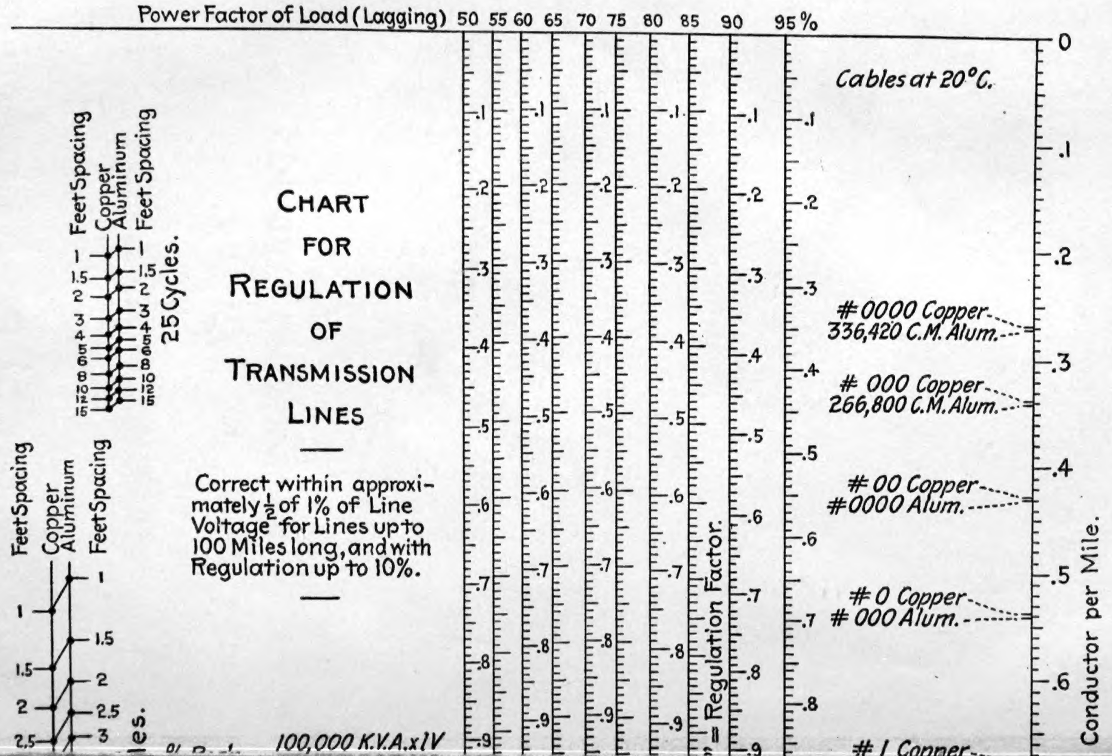
$$s = 1.26\,a \quad \text{for Flat Spacing}$$

To determine the Size of Conductor required for a given % Regulation, use the following :-

$$V = \frac{\%\text{ Reg'n.} \times E^2}{100{,}000\,K.V.A. \times l}$$

The above Equations are for 2 and 3 Phase Lines. For single Phase use 2 V in place of V.





Power Factor of Load (Lagging) 50 55 60 65 70 75 80 85 90 95 %
Cables at 20°C.
Feet Spacing
Copper
Aluminum
Feet Spacing
25 Cycles.
CHART FOR REGULATION OF TRANSMISSION LINES
Correct within approximately ½ of 1% of Line Voltage for Lines up to 100 Miles long, and with Regulation up to 10%.
#0000 Copper
336,420 C.M. Alum.
000 Copper
266,800 C.M. Alum.
00 Copper
#0000 Alum.
0 Copper
000 Alum.
= Regulation Factor.
Conductor per Mile.
100,000 K.V.A.x1V
les.

TRANSMISSION LINE FORMULAS

FOR

ELECTRICAL ENGINEERS

AND

ENGINEERING STUDENTS

BY

HERBERT BRISTOL DWIGHT, B. Sc.

ASSOCIATE OF THE AMERICAN INSTITUTE OF
ELECTRICAL ENGINEERS

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PREFACE.

The object of this book is to compile a set of instructions for engineers, which will enable them to make electrical calculations for transmission lines with the least possible amount of work.

The chart and working formulas have for the most part been developed independently by the author. Where the same or similar methods have been previously published, the fact is generally stated in the footnotes, but it has not been found possible to make these references absolutely complete.

The second part of the book is for reference and contains the derivation of the principal formulas used in connection with transmission lines. As many recent articles on transmission lines make use of formulas which are only roughly approximate, or are even incorrect, a reliable collection of formulas, with the method of obtaining them, should be found valuable.

It should not be presumed, because the second part of the book requires the use of the integral calculus, that the working formulas will require a knowledge of higher mathematics. The first five or six chapters are complete in themselves, and are planned for the use of those who have an ordinary acquaintance with alternating-current calculations.

H. B. DWIGHT.

HAMILTON, CANADA,
August, 1912.

CONTENTS.

PART I. — WORKING FORMULAS.

PART II. — THEORY.

PART III. — TABLES.

Transmission Line Formulas

PART I.

WORKING FORMULAS.

CHAPTER I.

INTRODUCTION.

THE determination of the electrical characteristics of transmission lines is a problem of considerable practical importance to engineers. It occurs frequently in electrical engineering work, and various methods have been proposed for carrying out the calculations with greater or less degrees of accuracy. Unfortunately, most of the methods so far presented have been such as to require special mathematical skill in using unfamiliar forms such as hyperbolic sines, etc. Even the approximate methods, whose results are not intended to be reliable for lines of considerable length, are often too cumbersome to be used by an engineer who has not time to make himself thoroughly familiar with them. The result has been that engineers have often been satisfied with calculations for practical cases which were not nearly as correct as they might easily have been.

Working methods have been developed, and are presented in the following chapters, for solving problems in connection with actual transmission lines. As these working methods involve comparatively simple operations in algebra and arithmetic only, they should be found useful by all electrical engineering students and engineers. Groups of problems with answers are added to provide practice in the use of the formulas. The working formulas can be immediately used by electrical engineers without the delay caused by working out the true meaning and correct operation of long and intricate systems of calculation. They are also arranged to require the minimum amount of labor for routine work where many lines are to be calculated.

The first method, which is described in Chapter III, is in the form of a chart which gives the regulation or voltage drop of a line, and which also shows directly the required size of conductor for given conditions.

In Chapter IV are given formulas for distribution lines and transmission lines only a few miles long. These are extended in Chapter V by means of the constant K to apply to transmission lines of any length in ordinary practice.

For purposes of checking different formulas, and for the calculation of extremely long and unusual lines, the fundamental formulas of transmission lines are expressed by rapidly converging series in Chapter VI. While these series require much more arithmetical work than the K formulas, they will give the exact results to any degree of accuracy desired. The method of convergent series involves the use of complex numbers, that is, numbers in which "j" terms appear. They are easier to handle, however, than logarithms, sines and cosines of angles, or hyperbolic functions, and therefore the use of these other mathematical functions has been avoided.

Each of the above groups of working formulas is printed in a table, ready for practical use. The tables will be found in the collection at the back of the book, as well as in the separate chapters describing them.

When any formula is given which uses approximations, the limits of its accuracy should be clearly stated so that one can tell at a glance whether the method is sufficiently accurate for the purpose in hand, or whether a longer method giving greater accuracy is desirable. This is especially necessary in the calculation of transmission lines, because approximate formulas are quite permissible for lines only a few miles long, but become very untrustworthy when the length is increased to one hundred miles or more. For this reason, each table of formulas has its percentage and range of accuracy printed in a prominent position, so that the most suitable method for any case may be instantly chosen.

CHAPTER II.

ELEMENTS OF A TRANSMISSION LINE.

The essential elements of a transmission line have been described many times, but a short discussion of them, with an explanation of some of the terms used in connection with the subject, may be useful before proceeding with the actual calculations.

A transmission line consists of two or more conductors insulated from each other so that they can carry energy by electric currents to some more or less distant point.

The conductors may be solid copper wires, copper cables, or aluminum cables. The diameters and resistances of various standard conductors are given in Tables VII and VIII, pages 118 to 120. It will be noted that the exact value of the resistance of a conductor differs slightly when a direct current, and an alternating current of 25 or 60 cycles, is flowing. This is due to the "skin effect," by which an alternating current tends to flow near the surface of a conductor, as explained in Chapter X. The drop in voltage due to resistance is proportional to the current and is in phase with it when the current is alternating.

Only overhead lines, carrying alternating currents, will be considered in this book. Such lines are supported by poles or steel towers at a considerable height above the ground. The conductors are separated from each other by a distance which may be several inches or several feet. This distance is called the "spacing" of the conductors and it has an important bearing on the electrical characteristics of the line.

An alternating magnetic field is formed around, and inside of, conductors carrying alternating currents. This field generates a voltage along the conductors which is proportional to the current, like the voltage drop due to resistance, but which is 90° out of phase with the current. This voltage is called the reactance drop. Tables of reactance of transmission lines will be found in Part III.

Since the voltage drop in a transmission line is due to resistance and reactance, a simple line may be considered to be made up of the elements shown in Fig. 1. If R is

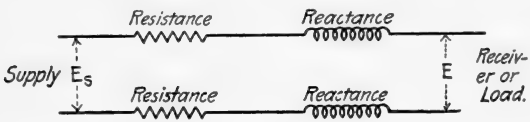

Fig. 1.

the total resistance, the voltage drop in phase with the current I will be IR, and if X is the total reactance, the voltage drop in quadrature with the current will be IX.

The vector diagram of the above quantities will be as in Fig. 2. The current is in general not in phase with the voltage

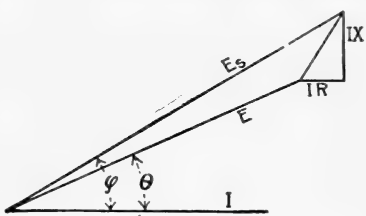

Fig. 2.

E, but lags behind it by an angle θ, according to the power factor, $\cos \theta$, of the load. The resistance drop IR will therefore not be added directly to E, but must be added vectorially, along with the reactance drop IX, as in

Fig. 2. It is evident that the voltages E and E_s, and the power factors $\cos \theta$ and $\cos \phi$, at the two ends of the line, are not the same in value.

A long transmission line acts as a condenser and this fact also must be taken into account. A condenser consists of two electrical conductors placed close together but insulated from each other so that a direct current cannot pass between them. However, if an alternating voltage be applied between them, a charge of electricity proportional to the electrostatic capacity of the condenser will flow into and out of the conductors. The result is that an alternating current will appear to flow between them, proportional to the capacity susceptance of the condenser. This current, called the charging current, will be 90° out of phase with the voltage, and, unlike most currents in ordinary practice, it will lead the voltage in phase, instead of lagging behind it. The amount of the charging current may be determined by means of the tables of capacity susceptance of transmission lines, in Part III.

A current in phase with the voltage will flow between the conductors, but it is only noticeable at very high voltages. Part of it is a leakage current flowing over the insulators, and part is a discharge through the air, and produces the glow called corona, on high-voltage conductors.

The elements of a transmission line accounting for the leakage current and charging current are shown in Fig. 3, in which resistances and condensers are shunted across the line all along its length.

Considering for the present that the voltage of the line is the same at all parts and is equal to E, the current in phase with E flowing across from one conductor to the other will be EG, where G is the total conductance between the wires. So also, if B is the capacity susceptance of the

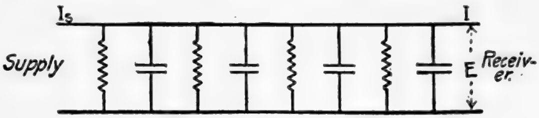

Fig. 3.

line considered as a condenser, EB will be the value of the shunted current in quadrature with E.

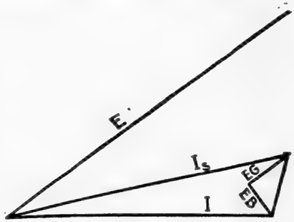

Fig. 4.

The vector diagram for the line indicated in Fig. 3 (neglecting the voltage drop in the conductors) is shown in Fig. 4. It is seen that the current I_s at the supply end is different in magnitude and phase from the current I at the receiver.

In order to calculate the combined effect of the above phenomena, formulas must be used which will take into account the fact that the resistance, capacity, etc., are uniformly distributed along the line, and that the line current and voltage are different at all parts of the line.

CHAPTER III.

REGULATION CHART.

THE characteristic of a transmission line which limits the load it may carry is its regulation, or the variation in voltage which occurs when the load is thrown on and off. This is especially true when the load has a low power factor, which is the case in most instances at the present time.

For estimating the regulation of a line, or the size of conductor required, the regulation chart which forms the frontispiece of the book may be used, and it will give the required result much quicker than any method of calculation. An extra copy of the chart is inserted at the back of the book; it may be found useful for cutting out and mounting on cardboard.

The chart is accurate within approximately ½ of 1% of full line voltage, when the regulation is less than 10% and the line is not more than 100 miles long.

In using the chart,* one places a straightedge across it from the point on the left corresponding to the spacing of the transmission line, to the point on the right, corresponding to the resistance of the conductor per mile. The regulation voltage, V, per total ampere per mile of line, is then read directly from the chart for the power factor of load considered. The regulation is taken as the change in

* The process of using the chart is similar to that used with the transformer regulation and efficiency charts published by J. F. Peters, *Electric Journal*, December, 1911.

load voltage when the load is thrown on or off, assuming constant supply voltage.

The total regulation is quickly figured on the slide rule from the following formula for two-phase (four wire) or three-phase lines:

$$\text{Regulation Volts} = \frac{1000 \text{ K.V.A.} \times lV}{E},$$

where K.V.A. = Kilovolt-amperes of load, at the receiver end.

E = Line voltage at the load, or receiver end.

l = Length of line in miles.

For single-phase lines use $2V$ instead of V, making the formula as follows:

$$\text{Regulation Volts} = \frac{1000 \text{ K.V.A.} \times l \times 2\,V}{E}.$$

The regulation volts may be expressed as a percentage of E to give the per cent regulation, and a formula is given on the chart for obtaining this result directly.

The line drop, or difference in voltage between the supply end and the receiver end of the line, is the same as the regulation for lines less than about 20 miles long, but for longer lines the effect of the charging current must be taken into account by the formula

$$\text{Line Drop} = \text{Regulation Volts} - EK\left(\frac{l}{1000}\right)^2,$$

where $K = 2.16$ for 60 cycles,
and $K = .375$ for 25 cycles.

It is seen that the voltage due to the charging current is proportional to the line voltage E, and to the square of the number of miles, but is independent of the size or spacing of the conductors, within the assigned limit of accuracy. The constant K does not need to be used in the formula for regulation, since the charging current is present at both no load and full load.

In selecting the spacing point on the chart, one notes whether the frequency is 25 or 60 cycles, and whether the conductor is of copper or aluminum. The spacing points are the same for both solid wire and cable. When the wires of a three-phase line are not spaced at the corners of an equilateral triangle, but are at irregular distances a, b, and c from each other, as in Figs. 5 and 6, the equivalent spacing

$$s = \sqrt[3]{abc}$$

should be used.

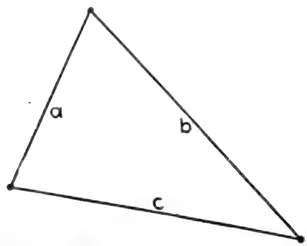

Fig. 5. Irregular Triangular Spacing.

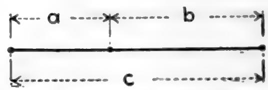

Fig. 6. Irregular Flat Spacing.

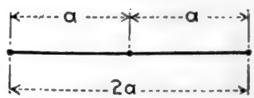

Fig. 7. Regular Flat Spacing.

With regular flat spacing, as in Figs. 7 and 8, the equation for the equivalent spacing becomes simply

$$s = 1.26\,a.$$

It makes no difference whether the plane of the wires with flat spacing is horizontal, vertical or inclined.

The spacing of a two-phase line is the average distance between wires of the same phase. The distance between wires of different phases is not considered.

The points marked on the resistance scale at the right of the chart are for cables at 20° C., assuming hard-drawn copper of a conductivity equal to 97.3% of the Annealed

Copper Standard, and hard-drawn aluminum of 60.86% conductivity, and allowing an increase of 1% in resistance for the effect of spiralling of the wires in the cable. However, these resistance points are placed on the chart for convenience only, and are not essential. If other assumptions are made, or if other sizes of conductor are used, all that is needed is to find the resistance of the conductor per mile, and use the corresponding resistance point on the chart to find "V."

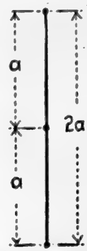

Fig. 8. Regular Flat Spacing.

One of the most common problems in estimating new projects is to determine the size of wire needed for any given value of regulation, and the chart will be found especially applicable to this work. "V" is first found from the equation,

$$V = \frac{\%\ \text{Reg'n} \times E^2}{100{,}000\ \text{K.V.A.} \times l}.$$

Then lay a straightedge through "V" and the point for the spacing to be used, and the nearest size of conductor can be seen, at a glance, on the resistance scale at the right.

The chart is quite as useful for finding the voltage drop, or required size of conductor, for distribution lines a few hundred feet long as it is for transmission lines many miles long.

PROBLEM A.

Find, by means of the chart, the regulation and line drop for the following set of conditions:

Length of line	100 miles.
Spacing	8 feet.
Conductor	No. 3 copper cable.

Load (measured at receiver end), 3000 K.V.A., 66,000 volts, 90% P.F., three phase, 60 cycles.

Lay a straightedge from the 8-foot spacing point (60 cycles, copper conductor) to the point on the resistance scale for No. 3 copper cable. It is found to cross the 90% P.F. line at the reading 1.344. Then, by the formula on the chart,

$$\text{Per cent Regulation} = \frac{100{,}000 \times 3000 \times 100 \times 1.344}{66{,}000 \times 66{,}000}$$

$$= 9.26\%, \text{ or approximately } 9.3\%.$$

The calculated value of the regulation of this line is 9.40% (Chap. VI, Prob. 2), so that the error involved in using the chart is less than $\frac{1}{5}$ of 1% of line voltage.

The per cent line drop, according to the chart, is

$$9.26 - 100 \times 2.16 \times \tfrac{1}{100} = 7.10\%.$$

As the calculated value is 7.08% (Chap. VI, Prob. 2), the error from the chart is less than $\frac{1}{10}$ of 1% of the line voltage.

Problem B.

To find the size of copper required to give approximately 10% voltage drop in the following case:

Length of line .... 3 miles.
Flat spacing as in Fig. 7. Wires 2 feet apart.
Load (measured at receiver end), 250 K.V.A.,
2200 volts, 85% P.F., three phase, 60 cycles.

First, find V from the formula on the chart.

$$V = \frac{10 \times 2200 \times 2200}{100{,}000 \times 250 \times 3} = 0.64.$$

The equivalent spacing is 1.26×2, or 2.52 feet. The proper spacing point will therefore be just below the spacing point for $2\frac{1}{2}$ feet, copper conductor, 60 cycles. Lay a straightedge from this point to the reading 0.64 on the line for 85% P.F. and it cuts the resistance scale at 0.36 ohm per mile. The nearest size of copper is seen to be No. 000.

Problem C.

Find the voltage drop of the following two-phase line:

Length of line .... 80 miles.
Spacing .... 10 feet.
Conductor .... No. 00 aluminum cable.
Load (measured at receiver end), 15,000 K.V.A.,
100,000 volts, 95% P.F., two phase, 25 cycles.

Laying a straightedge across the chart from the 10-foot spacing point for 25 cycles and aluminum conductor, to the resistance point for No. 00 aluminum, the value of V for 95% P.F. is found to be 0.750. Then the line drop, in volts, is equal to

$$\frac{1000 \times 15{,}000 \times 80 \times 0.750}{100{,}000} - 100{,}000 \times 0.375 \times 0.08 \times 0.08$$

$$= 9000 - 240$$

$$= 8760 \text{ volts.}$$

The calculated value is 8810 volts (Chap. VI, Prob. 1). The error from the chart is 0.05% of line voltage.

PROBLEM D.

Find the regulation of the following single-phase line:

Length of line . . . . 15 miles.
Spacing . . . . 3 feet.
Conductor . . . . No. 0 copper wire.
Load (at receiver end), 300 K.V.A., 50% P.F.,
11,000 volts, single phase, 60 cycles.

From the chart, $V = 0.851$.

$$\text{Therefore Regulation} = \frac{100{,}000 \times 300 \times 15 \times 2 \times 0.851}{11{,}000 \times 11{,}000} = 6.33\%.$$

[Calculated value, 6.40% (Chap. IV, Prob. A). Error from chart, 0.07% of line voltage.]

PROBLEM E.

Find the K.V.A. which can be delivered at the end of the following line, with 8% regulation:

Length of line . . . . 75 miles.
Spacing . . . . 8 feet, regular flat spacing.
Conductor . . . . No. 00 aluminum cable.
Character of load (at receiver end),
88,000 volts, 85% P.F., three phase, 25 cycles.

Equivalent spacing $s = 8 \times 1.26 = 10.08$ feet.

$$V = 0.755.$$

$$\text{K.V.A.} = \frac{8 \times 88{,}000 \times 88{,}000}{100{,}000 \times 0.755 \times 75}$$

$$= 10{,}900.$$

PROBLEMS, CHAPTER III.

(Regulation Chart.)

1. Find the size of copper cable which is needed to deliver 200 K.V.A. at a distance of 3 miles with 10% drop or less.

Spacing of line.................................. 2 feet.

Character of load (at receiver end), 2200 volts, 80% P.F., three phase, 60 cycles.

[*Ans.* No. 0.]

2. Assuming No. 0 copper cable for the previous problem, find the volts drop in the line.

[*Ans.* 219 volts. Calculated, 221 volts (Chap. IV., Prob. 1). Error 0.1% of line voltage.]

3. Find the size of copper required for a drop of 6% or less in the following case:

Length of line.......................... 5000 feet.
Spacing................................. 18 inches.
Load (at receiver end), 75 Kw. (79 K.V.A.), 95% P.F., 2000 volts, single phase, 60 cycles.

[*Ans.* No. 4 copper.]

4. Assuming No. 4 copper wire of 1.312 ohms per mile, find the per cent line drop and the supply voltage. (See *Elec. Journal*, Apr., 1907, p. 231.)

[*Ans.* 5.43%, 2110 volts, calc. 5.43%, 2109 volts (Chap. IV., Prob. 3).]

5. Find the required size of copper for 9% regulation in the following case:

Length of line.......................... 25 miles.
Spacing................................. 3 feet.
Load (at receiver end), 2500 K.V.A., 20,000 volts, 60% P.F., three phase, 25 cycles.

[*Ans.* No. 0 copper.]

6. Assuming No. 0 copper wire of 0.520 ohm per mile, find the per cent regulation. (See *Elec. Journal*, Apr., 1907, p. 231.)

[*Ans.* 8.42%, calc. 8.51% (Chap. IV, Prob. 5). Error 0.09% of line voltage.]

7. Find the per cent regulation and the voltage drop of the following line:

Length of line .... 75 miles.
Spacing, 8 feet, regular flat spacing.
Conductor .... No. 00 aluminum cable.
Load (at receiver end), 10,000 K.V.A.,
88,000 volts, 85% P.F., three phase, 25 cycles.
[*Ans.* 7.36% Reg'n., 7.15% line drop, calc. 7.35% Reg'n., 7.13% line drop (Chap. V, Prob. 3).]

8. Find the K.V.A. which can be delivered at the end of the following line, with 10% regulation, at 75% and at 90% P.F.:

Length of line .... 100 miles.
Spacing .... 10 feet.
Conductor .... No. 0000 aluminum cable.
Receiver voltage .... 110,000 volts, three phase, 60 cycles.

[*Ans.* 14,300 K.V.A., 16,300 K.V.A.]

CHAPTER IV.

FORMULAS FOR SHORT LINES.

THE effect of capacity is inappreciable with short lines as it amounts to only $\frac{1}{10}$ of 1% for a line about 20 miles long. Thus distribution lines and many short transmission lines can be quite accurately calculated without considering the line capacity at all. The formulas in Tables I and II, pp. 18 and 20, enable one to solve many problems connected with such lines.

The formulas are divided into two groups, those in Table I being used when all the particulars describing the load, such as K.V.A., voltage and power factor, are specified at the receiver end. Table II is used when these particulars are specified at the supply end.

One first finds the quantities P and Q or P_s and Q_s. These are the values of in-phase current, and reactive or quadrature current, at the point where the conditions are specified. It is to be noted that only values of current expressed in total amperes are to be used in connection with the formulas in this book. The number of amperes per wire is never used in the calculations (except with single phase lines), but if it is desired to be known, it may be determined from the formulas at the bottom of the tables.

The next step is to find the quantities A and B, or F and G. One is then ready to find the value of any of the ten quantities, whose formulas are given in the tables. It should be remembered that each of these ten quantities may be determined independently of all the others. Thus it is not necessary to work out the first six equations in order to obtain the value of the seventh, since the seventh, like any of the others, may be calculated directly.

TABLE I. — FORMULAS FOR SHORT LINES.

Conditions given at Receiver End.

These formulas are exact when the line is short. When the line is 20 miles long, they are correct within approximately $\frac{1}{10}$ of 1% of line voltage.

Conditions given:

K.V.A. = K.V.A. at receiver end.

E = Full load voltage at receiver end.

$\cos\theta$ = Power factor at receiver end.

K.W. = K.V.A. $\cos\theta$.

r = Resistance of conductor per mile. (From Tables VII–VIII.)

x = Reactance of conductor per mile. (From Tables IX–XII.)

l = Length of line in miles.

Then $P = \dfrac{1000 \text{ K.V.A. } \cos\theta}{E}$ = In-phase current at receiver end (in total amps.).

$Q = \dfrac{1000 \text{ K.V.A. } \sin\theta}{E}$ = Reactive current at receiver end (in total amps.) when current is lagging

$= -\dfrac{1000 \text{ K.V.A. } \sin\theta}{E}$ when current is leading.

Find the following quantities:

Three phase or two phase.	*Single phase.*
$A = E + Prl + Qxl.$	$A = E + 2\,Prl + 2\,Qxl.$
$B = Pxl - Qrl.$	$B = 2\,Pxl - 2\,Qrl.$

Formulas (capacity neglected):

(1) Voltage at supply end $= A + \dfrac{B^2}{2A}\cdot$

(2) Regulation of line $= A + \dfrac{B^2}{2A} - E.$ (Same as line drop.)

(3) Per cent regulation of line $= \dfrac{100\left(A + \dfrac{B^2}{2A} - E\right)}{E}$ per cent.
(Same as per cent line drop.)

(4) K.V.A. at supply end $= \dfrac{A + \dfrac{B^2}{2A}}{E} \times$ K.V.A.

(5) K.W. at supply end $= \dfrac{1}{1000}(AP - BQ).$

(6) Power factor at supply end $= \dfrac{1}{1000}\dfrac{(AP - BQ)\,E}{\left(A + \dfrac{B^2}{2A}\right) \times \text{K.V.A.}}$

(in decimals).

(7) In-phase current at supply end $= \dfrac{AP - BQ}{A + \dfrac{B^2}{2A}}$ in total amperes.*

(8) Reactive or quadrature current at supply end $= \dfrac{BP + AQ}{A + \dfrac{B^2}{2A}}$ in total amperes.*

When this quantity is positive, the current is lagging.
When this quantity is negative, the current is leading.

(9) K.W. loss in line $= \dfrac{1}{1000}(AP - BQ - EP)$.

(10) Per cent efficiency of line $= \dfrac{100\,EP}{AP - BQ}$ per cent.

* Amperes per wire, three phase $= \dfrac{\text{Total amps.}}{\sqrt{3}}$.

Amperes per wire, two phase $= \dfrac{\text{Total amps.}}{2}$.

TABLE II. — FORMULAS FOR SHORT LINES.

CONDITIONS GIVEN AT SUPPLY END.

These formulas are exact when the line is short. When the line is 20 miles long, they are correct within approximately $\frac{1}{10}$ of 1% of line voltage.

Conditions given:

K.V.A. = K.V.A. at supply end.
E_s = Full load voltage at supply end.
$\cos\theta$ = Power factor at supply end.
K.W. = K.V.A. $\cos\theta$.
r = Resistance of conductor per mile. (From Tables VII–VIII.)
x = Reactance of conductor per mile. (From Tables IX–XII.)
l = Length of line in miles.

Then $P_s = \dfrac{1000 \text{ K.V.A.} \cos\theta}{E_s}$ = In-phase current at supply end (in total amps.).

$Q_s = \dfrac{1000 \text{ K.V.A.} \sin\theta}{E_s}$ = Reactive current at supply end (in total amperes) when current is lagging.

$= -\dfrac{1000 \text{ K.V.A.} \sin\theta}{E_s}$ when current is leading.

Find the following quantities:

Three Phase or Two Phase.

$$F = E_s - P_srl - Q_sxl$$
$$G = Q_srl - P_sxl$$

Single Phase.

$$F = E_s - 2\,P_srl - 2\,Q_sxl$$
$$G = 2\,Q_srl - 2\,P_sxl.$$

Formulas (capacity neglected):

(1) Voltage at receiver end $= F + \dfrac{G^2}{2F}$.

(2) Regulation of line $= E_s - F - \dfrac{G^2}{2F}$. (Same as line drop.)

(3) Per cent regulation of line $= \dfrac{100\left(E_s - F - \dfrac{G^2}{2F}\right)}{F + \dfrac{G^2}{2F}}$ per cent.

(Same as per cent line drop.)

(4) K.V.A. at receiver end $= \dfrac{F + \dfrac{G^2}{2F}}{E_s} \times$ K.V.A.

(5) K.W. at receiver end $= \frac{1}{1000}(FP_s - GQ_s)$.

(6) Power factor at receiver end $= \frac{1}{1000} \frac{(FP_s - GQ_s)\,E_s}{\left(F + \frac{G^2}{2F}\right) \times \text{K.V.A.}}$

(in decimals).

(7) In-phase current at receiver end $= \frac{FP_s - GQ_s}{F + \frac{G^2}{2F}}$ in total amperes.*

(8) Reactive or quadrature current at receiver end $= \frac{GP_s + FQ_s}{F + \frac{G_2}{2F}}$

in total amperes.*

When this quantity is positive, the current is lagging.
When this quantity is negative, the current is leading.

(9) K.W. loss in line $= \frac{1}{1000}(E_sP_s - FP_s + GQ_s)$.

(10) Per cent efficiency of line $= \frac{100\,(FP_s - GQ_s)}{E_sP_s}$ per cent.

* Amperes per wire, three phase, $= \frac{\text{Total amps.}}{\sqrt{3}}$.

Amperes per wire, two phase, $= \frac{\text{Total amps.}}{2}$.

Problem A.

Find the regulation of the following single-phase line:

Length of line........................ 15 miles.
Spacing............................... 3 feet.
Conductor............................. No. 0 copper wire.
Load (at receiver end), 300 K.V.A., 11,000 volts, 50% P.F., single phase, 60 cycles. (Same as Prob. D, Chap. III.)

From the tables in Part III,

$$r = 0.5331.$$
$$x = 0.686.$$
$$l = 15.$$

$$P = \frac{1000 \times 300 \times 0.50}{11,000} = 13.64.$$

$$Q = \frac{1000 \times 300 \times 0.866}{11,000} = 23.62.$$

$$\begin{aligned} A &= 11,000 + 2 \times 13.64 \times 0.5331 \times 15 \\ &\quad + 2 \times 23.62 \times 0.686 \times 15 \\ &= 11,704. \end{aligned}$$

$$\begin{aligned} B &= 2 \times 13.64 \times 0.686 \times 15 - 2 \times 23.62 \times 0.5331 \times 15 \\ &= -97. \end{aligned}$$

$$\begin{aligned} \text{Per cent regulation} &= \frac{100}{11,000}\left(11,704 + \frac{97 \times 97}{2 \times 11,704} - 11,000\right) \\ &= 6.40\%. \end{aligned}$$

Problem B.

Calculate the volts drop for the following case, where all the conditions are specified at the supply, or generator, end of the line:

Length of line........................ 10 miles.
Spacing............................... 3 feet.
Conductor............................. No. 2 copper wire.
Quantities measured at supply end: 600 K.V.A., 6600 volts, 80% P.F., three phase, 60 cycles.

From the tables in Part III,

$$r = 0.8469.$$
$$x = 0.714.$$
$$l = 10.$$

$$P_s = \frac{1000 \times 600 \times 0.80}{6600} = 72.73.$$

$$Q_s = \frac{1000 \times 600 \times 0.60}{6600} = 54.55.$$

$$\begin{aligned} F &= 6600 - 72.73 \times 8.47 - 54.55 \times 7.14 \\ &= 5594.5. \\ G &= 54.55 \times 8.47 - 72.73 \times 7.14 \\ &= -57. \end{aligned}$$

Voltage at receiver end

$$\begin{aligned} &= 5594.5 + \frac{57 \times 57}{2 \times 5594.5} \\ &= 5594.5 + 0.3 \\ &= 5595 \text{ approximately.} \end{aligned}$$

Drop in volts $= 6600 - 5595$

$= 1005$ volts.

Per cent drop $= \dfrac{100 \times 1005}{5595}$

$= 17.96\%$ of receiver voltage.

PROBLEMS, CHAP. IV.

(Formulas for Short Lines.)

1. Find the voltage drop in the following case:

Length of line .... 3 miles.
Spacing .... 2 feet.
Conductor .... No. 0 copper cable.
Load (at receiver end), 200 K.V.A., 2200 volts, 80% P.F., three phase, 60 cycles. (Prob. 2, Chap. III.)

[*Ans.* 221 volts.]

2. Find (*a*) the P.F. at the supply end,
(*b*) the per cent efficiency of the line, for the case in Prob. 1.

[*Ans.* (*a*) 78.7% P.F. (*b*) 92.3% efficiency.]

3. Find the supply voltage in the following case:

Length of line .... 5000 feet.
Spacing .... 18 inches.
Conductor, No. 4 copper wire of 1.312 ohms per mile.
Load (at receiver end), 75 Kw., 2000 volts, 95% P.F., single phase, 60 cycles. (Prob. 4, Chap. III.)

[*Ans.* 2109 volts.]

4. Find the volts drop and the watts loss, in the following line:

Length of line........................ 20 miles.
Spacing........................... 5 feet.
Conductor........................ No. 1 aluminum cable.
Two phase, 25 cycles.
K.V.A. at supply end................ 10,000.
Volts at supply end................. 35,000.
P.F. at supply end.................. 80%.

[*Ans.* 5950 volts, 1770 Kw.]

5. Find the per cent regulation of the following line:

Length of line.............................. 25 miles.
Spacing.................................. 3 feet.
Conductor, No. 0 copper wire of 0.520 ohm per mile.
Load (at receiver end), 2500 K.V.A., 20,000 volts, 60% P.F., three phase, 25 cycles. (Prob. 6, Chap III.)

[*Ans.* 8.51%.]

CHAPTER V.

K FORMULAS.

WHEN a transmission line is more than 20 miles long, the formulas for short lines given in Chapter IV are no longer accurate, and other formulas must be used, which will take into account the capacity of the line. Such formulas, called *K* formulas, will be found in Tables III and IV, pp. 28 to 35. The same tables will be found in Part III at the end of the book.

The *K* formulas will be found very similar to those of the last chapter, and while they require more arithmetical work, they should not be found any more difficult to understand. No more values of line constants need to be looked up for the *K* formulas than for the "Short Line" formulas. The capacity of the line does not enter into the calculations, since its effect is allowed for by means of the constant *K* which is the same, at any one frequency, for all values of line capacity.

The formulas of this chapter assume that the leakage current is zero; that is, that no power is lost from leakage over the insulators or from corona. This is a correct assumption to make for all voltages except the very highest in use. If it is desired to make allowance for corona loss, the formulas of Chapter VI should be used.

The accuracy of the *K* formulas is given as approximately $\frac{1}{10}$ of 1% of line voltage for lines up to 100 miles long and with regulation up to 20%, and as $\frac{1}{2}$ of 1% for lines up to 200 miles long, and with the same regulation.

These limits are close enough for commercial work, so that the K formulas can be recommended for all ordinary engineering calculations of the performance of long power transmission lines under steady conditions, where the corona loss is small. The accuracy of the electrical calculations will be better than the accuracy with which the resistance and the physical dimensions of the line are generally known.

The K formulas are well adapted to the solution of long transmission lines which have substations at intermediate points between the ends. In such cases each section of the line between substations must be calculated separately, beginning with the end where conditions are known. The first step is to find the voltage, in-phase current and quadrature current at the first substation. The load taken by the substation, expressed as in-phase current and quadrature current, must be added to, or subtracted from, the above values of current. When conditions are given at the receiver end and one is proceeding toward the supply end, the substation load must be added to the line load. When conditions are given at the supply end, the substation load must be subtracted from the line load, since one is proceeding away from the supply. Having thus found complete conditions at one end of the second section of the line, the calculation of this section may be taken up in the same way as for the first section. In this manner the entire line may be calculated and the voltage and current at the unknown end may be determined.

Examples are worked out, which will give a clear idea of the manner in which the K formulas are used. Many other such examples have been calculated and carefully compared with the fundamental formulas. As these examples have covered the range of practicable transmission lines, a sound basis is afforded for the estimate of the accuracy of the *K* formulas and for the statement that they are sufficiently reliable for all ordinary engineering purposes in the calculation of electric power transmission lines.

TABLE III. — K FORMULAS FOR TRANSMISSION LINES.

Conditions given at Receiver End.

Accurate within approximately $\frac{1}{10}$ of 1% of line voltage up to 100 miles, and $\frac{1}{2}$ of 1% up to 200 miles, for lines with regulation up to 20%.

$K = 6\,\frac{(\text{cycles})^2}{10{,}000}$. $K = 2.16$ for 60 cycles. $K = 0.375$ for 25 cycles.

Conditions given:

K.V.A. = K.V.A. at receiver end.

E = Full load voltage at receiver end.

$\cos\theta$ = Power factor at receiver end.

K.W. = K.V.A. $\cos\theta$.

r = Resistance of conductor per mile. (From Tables VII–VIII.)

x = Reactance of conductor per mile. (From Tables IX–XII.)

l = Length of transmission line in miles.

Then $P = \frac{1000\ \text{K.V.A.}\cos\theta}{E}$ = In-phase current at receiver end (in total amps.).

$Q = \frac{1000\ \text{K.V.A.}\sin\theta}{E}$ = Reactive current at receiver end (in total amps.), when current is lagging.

$= -\frac{1000\ \text{K.V.A.}\sin\theta}{E}$, when current is leading.

Find the following quantities:

Full Load.

$$A = E\left\{1 - K\left(\frac{l}{1000}\right)^2\right\} + Prl\left\{1 - \frac{2}{3}K\left(\frac{l}{1000}\right)^2\right\} + Qxl\left\{1 - \frac{1}{6}K\left(\frac{l}{1000}\right)^2\right\}$$

$$B = E\frac{rK}{x}\left(\frac{l}{1000}\right)^2 + Pxl\left\{1 - \frac{1}{6}K\left(\frac{l}{1000}\right)^2\right\} - Qrl\left\{1 - \frac{2}{3}K\left(\frac{l}{1000}\right)^2\right\}.$$

$$C = P\left\{1 - K\left(\frac{l}{1000}\right)^2\right\} + Q\frac{rK}{x}\left(\frac{l}{1000}\right)^2 - \frac{2}{3}E\frac{rK^2}{lx^2}\left(\frac{l}{1000}\right)^4.$$

$$D = P\frac{rK}{x}\left(\frac{l}{1000}\right)^2 - Q\left\{1 - K\left(\frac{l}{1000}\right)^2\right\} + 2E\frac{K}{lx}\left(\frac{l}{1000}\right)^2\left\{1 - \frac{1}{3}K\left(\frac{l}{1000}\right)^2\right\}.$$

No Load.

$$A_0 = E\left\{1 - K\left(\frac{l}{1000}\right)^2\right\}.$$

$$B_0 = E\frac{rK}{x}\left(\frac{l}{1000}\right)^2.$$

$$C_0 = -\frac{2}{3}E\frac{rK^2}{lx^2}\left(\frac{l}{1000}\right)^4.$$

$$D_0 = 2E\frac{K}{lx}\left(\frac{l}{1000}\right)^2\left\{1 - \frac{1}{3}K\left(\frac{l}{1000}\right)^2\right\}.$$

Note. — The above are for two- and three-phase lines. For single-phase lines use $2r$ and $2x$ in place of r and x.

TABLE III. *(Continued.)*

CONDITIONS GIVEN AT RECEIVER END.

Formulas:

Full Load. *No Load.*

Voltage at receiver end.

(1) E.

(2) $E_0 = \dfrac{A + \dfrac{B^2}{2A}}{A_0 + \dfrac{B_0^2}{2A_0}} E$

(for constant supply voltage).

Regulation at receiver end in volts, for constant supply voltage.

(3) $\dfrac{A + \dfrac{B^2}{2A}}{A_0 + \dfrac{B_0^2}{2A_0}} E - E.$

N.B. The regulation at receiver end may be expressed as a percentage of E.

Voltage at supply end.

(4) $E_s = A + \dfrac{B^2}{2A}$.

(5) $E_{0s} = A_0 + \dfrac{B_0^2}{2A_0}$

(for constant receiver voltage).

Regulation at supply end in volts, for constant receiver voltage.

(6) $A + \dfrac{B^2}{2A} - A_0 - \dfrac{B_0^2}{2A_0}$.

N.B. The regulation at supply end may be expressed as a percentage of E_s.

*Current at supply end in total amperes.**

(7) $\sqrt{C^2 + D^2}$. (8) $\sqrt{C_0^2 + D_0^2}$.

K.V.A. at supply end.

(9) $\frac{1}{1000}\left(A + \frac{B^2}{2A}\right)\sqrt{C^2 + D^2}$. (10) $\frac{1}{1000}\left(A_0 + \frac{B_0^2}{2A_0}\right)\sqrt{C_0^2 + D_0^2}$.

K.W. at supply end.

(11) $\frac{1}{1000}(AC + BD)$. (12) $\frac{1}{1000}(A_0C_0 + B_0D_0)$.

Power factor at supply end, in decimals.

(13) $\dfrac{AC + BD}{\left(A + \frac{B^2}{2A}\right)\sqrt{C_2 + D^2}}$. (14) $\dfrac{A_0C_0 + B_0D_0}{\left(A_0 + \frac{B_0^2}{2A_0}\right)\sqrt{C_0^2 + D_0^2}}$.

*In-phase current at supply end in total amperes.**

(15) $\dfrac{AC + BD}{\left(A + \frac{B^2}{2A}\right)}$. (16) $\dfrac{A_0C_0 + B_0D_0}{A_0 + \frac{B_0^2}{2A_0}}$.

*Reactive current at supply end in total amperes.**

(17) $\dfrac{BC - AD}{A + \frac{B^2}{2A}}$. (18) $\dfrac{B_0C_0 - A_0D_0}{A_0 + \frac{B_0^2}{2A_0}}$.

When this quantity is positive, the current is lagging.
When this quantity is negative, the current is leading.

K.W. loss in line.

(19) $\frac{1}{1000}(AC + BD - EP)$. (20) $\frac{1}{1000}(A_0C_0 + B_0D_0)$ [same as No. 12].

Per cent efficiency of line.

(21) $\dfrac{100\,EP}{AC + BD}$ per cent.

* Amperes per wire, three-phase, $= \dfrac{\text{Total amps.}}{\sqrt{3}}$.

Amperes per wire, two phase, $= \dfrac{\text{Total amps.}}{2}$.

TABLE IV.—K FORMULAS FOR TRANSMISSION LINES.

Conditions given at Supply End.

Accurate within approximately $\frac{1}{10}$ of 1% of line voltage up to 100 miles and $\frac{1}{2}$ of 1% up to 200 miles, for lines with regulation up to 20%.

$K = \frac{6\,(\text{cycles})^2}{10{,}000}$. $K = 2.16$ for 60 cycles. $K = 0.375$ for 25 cycles.

Conditions given:

K.V.A. = K.V.A. at supply end.

E_s = Full load voltage at supply end.

$\cos\theta$ = Power factor at supply end.

K.W. = K.V.A. $\cos\theta$.

r = Resistance of conductor per mile. (From Tables VII–VIII.)

x = Reactance of conductor per mile. (From Tables IX–XII.)

l = Length of transmission line in miles.

Then $P_s = \frac{1000 \text{ K.V.A. } \cos\theta}{E_s}$ = In-phase current at supply end (in total amps.).

$Q_s = \frac{1000 \text{ K.V.A. } \sin\theta}{E_s}$ = Reactive current at supply end (in total amps.), when current is lagging

$= -\frac{1000 \text{ K.V.A. } \sin\theta}{E_s}$, when current is leading.

Find the following quantities:

Full Load.

$$F = E_s\left\{1 - K\left(\frac{l}{1000}\right)^2\right\} - P_s rl\left\{1 - \frac{2}{3}K\left(\frac{l}{1000}\right)^2\right\}$$
$$- Q_s xl\left\{1 - \frac{1}{6}K\left(\frac{l}{1000}\right)^2\right\}.$$

$$G = E_s\frac{rK}{x}\left(\frac{l}{1000}\right)^2 - P_s xl\left\{1 - \frac{1}{6}K\left(\frac{l}{1000}\right)^2\right\} + Q_s rl\left\{1 - \frac{2}{3}K\left(\frac{l}{1000}\right)^2\right\}.$$

$$M = P_s\left\{1 - K\left(\frac{l}{1000}\right)^2\right\} + Q_s\frac{rK}{x}\left(\frac{l}{1000}\right)^2 + \frac{2}{3}E_s\frac{rK^2}{lx^2}\left(\frac{l}{1000}\right)^4.$$

$$N = P_s\frac{rK}{x}\left(\frac{l}{1000}\right)^2 - Q_s\left\{1 - K\left(\frac{l}{1000}\right)^2\right\}$$
$$- 2E_s\frac{K}{lx}\left(\frac{l}{1000}\right)^2\left\{1 - \frac{1}{3}K\left(\frac{l}{1000}\right)^2\right\}.$$

No Load.

$$F_0 = E_s\left\{1 + K\left(\frac{l}{1000}\right)^2\right\}.$$

$$G_0 = -E_s\frac{rK}{x}\left(\frac{l}{1000}\right)^2.$$

$$M_0 = \frac{4}{3}E_s\frac{rK^2}{lx^2}\left(\frac{l}{1000}\right)^4.$$

$$N_0 = 2E_s\frac{K}{lx}\left(\frac{l}{1000}\right)^2\left\{1 + \frac{2}{3}K\left(\frac{l}{1000}\right)^2\right\}.$$

Note. — The above are for two- and three-phase lines. For single-phase lines use $2r$ and $2x$ in place of r and x.

TABLE IV. (*Continued.*)

CONDITIONS GIVEN AT SUPPLY END.

Formulas:

Full Load. *No Load.*

Voltage at receiver end.

(1) $E = F + \frac{G^2}{2F}$. (2) $E_0 = F_0 + \frac{G_0^2}{2F_0}$

(for constant supply voltage).

Regulation at receiver end in volts, for constant supply voltage.

(3) $F_0 + \frac{G_0^2}{2F_0} - F - \frac{G^2}{2F}$.

N.B. The regulation at receiver end may be expressed as a percentage of E.

Voltage at supply end.

(4) E_s. (5) $E_{0s} = \frac{F + \frac{G^2}{2F}}{F_0 + \frac{G_0^2}{2F_0}} E_s$

(for constant receiver voltage).

Regulation at supply end, in volts, for constant receiver voltage.

(6) $E_s - \frac{F + \frac{G^2}{2F}}{F_0 + \frac{G_0^2}{2F_0}} E_s$.

N.B. The regulation at supply end may be expressed as a percentage of E_s.

*Current in total amperes.**

(7) $\sqrt{M^2 + N^2}$ at receiver end. (8) $\sqrt{M_0^2 + N_0^2}$ at supply end.

K.V.A.

(9) $\frac{1}{1000}\left(F + \frac{G_2}{2F}\right)\sqrt{M^2 + N^2}$ at receiver end.

(10) $\frac{1}{1000} E_s \sqrt{M_0^2 + N_0^2}$ at supply end.

K.W.

(11) $\frac{1}{1000}(FM + GN)$ at receiver end. (12) $\frac{1}{1000} E_s M_0$ at supply end.

Power factor, in decimals.

(13) $\dfrac{FM + GN}{\left(F + \frac{G^2}{2F}\right)\sqrt{M^2 + N^2}}$ at receiver end.

(14) $\dfrac{M_0}{\sqrt{M_0^2 + N_0^2}}$ at supply end.

*In-phase current in total amperes.**

(15) $\dfrac{FM + GN}{F + \frac{G^2}{2F}}$ at receiver end. (16) M_0 at supply end.

*Reactive current in total amperes.**

(17) $\dfrac{GM - FN}{F + \frac{G_2}{2F}}$ at receiver end. (18) N_0 at supply end.

When this quantity is positive, the current is lagging.
When this quantity is negative, the current is leading.

K.W. loss in line.

(19) $\frac{1}{1000}(E_s P_s - FM - GN)$. (20) $\frac{1}{1000} E_s M_0$ (same as No. 12).

Per cent efficiency of line.

(21) $\dfrac{100\,(FM + GN)}{E_s P_s}$ per cent.

* Amperes per wire, three phase, $= \dfrac{\text{Total amps.}}{\sqrt{3}}$.

Amperes per wire, two phase, $= \dfrac{\text{Total amps.}}{2}$.

Problem A.

Find by the K formulas, the line drop in the following case:

Length of line .... 100 miles.
Spacing .... 8 feet.
Conductor .... No. 3 copper cable.
Load (at receiver end), 3000 K.V.A., 66,000 volts, 90% P.F., three phase, 60 cycles. (Prob. A, Chap. III.)

From the tables, $r = 1.078$, $x = 0.840$.

$$P = \frac{1000 \times 3000 \times 0.90}{66{,}000} = 40.91.$$

$$Q = \frac{1000 \times 3000 \times 0.4359}{66{,}000} = 19.81.$$

$$A = 66{,}000 - 66{,}000 \times \frac{2.16}{100}$$
$$+ 40.91 \times 107.8\left(1 - \frac{2}{3} \times \frac{2.16}{100}\right)$$
$$+ 19.81 \times 84\left(1 - \frac{1}{6} \times \frac{2.16}{100}\right)$$
$$= 70{,}580 \text{ volts.}$$

$$B = 66{,}000 \times \frac{1.078}{0.840} \times \frac{2.16}{100}$$
$$+ 40.91 \times 84\left(1 - \frac{1}{6} \times \frac{2.16}{100}\right) - 19.81 \times 107.8\left(1 - \frac{2}{3} \times \frac{2.16}{100}\right)$$
$$= 3150 \text{ volts.}$$

$$\text{Supply voltage} = 70{,}580 + \frac{3150 \times 3150}{2 \times 70{,}580} = 70{,}650.$$

$$\text{Line drop} = 4650 \text{ volts} = 7.05\%.$$

By the fundamental formulas, using the same line constants, the line drop is 7.08% (Prob. 2, Chap. VI). The discrepancy is 0.03% of line voltage.

Problem B.

Find by the K formulas the voltage at the supply end of the following line:

Total length of line .... 300 miles.
Spacing .... 12 feet.

Conductor, 266,800 c.m. aluminum cable.

Load at receiver end of line, 9000 K.V.A., 80% P.F. (lagging), 100,000 volts, three phase, 60 cycles.

Load taken by a substation at the middle of the line, 150 miles from either end, 2000 K.V.A. at the line voltage and at 70% P.F. (lagging).

Solution of first section of line.

$$r = 0.3410 \quad x = 0.791 \quad l = 150,$$
$$P = 72 \qquad Q = 54.$$

From the *K* formulas,

$$\begin{aligned} A &= 100{,}000 - 4860 + 3560 + 6360 \\ &= 105{,}060 \text{ volts.} \\ B &= 2100 + 8470 - 2670 \\ &= 7900 \text{ volts.} \\ C &= 72 - 3.50 + 1.13 - 0.57 \\ &= 69.06 \text{ amperes.} \\ D &= 1.51 - 54 + 2.63 + 80.59 \\ &= 30.73 \text{ amperes.} \end{aligned}$$

$$A + \frac{B^2}{2\,A} = E_1 = 105{,}360 \text{ volts.}$$

$$\text{In-phase current} = \frac{AC + BD}{A + \dfrac{B^2}{2\,A}}$$
$$= 71.15 \text{ amps}$$

$$\text{Reactive current} = \frac{BC - AD}{A + \dfrac{B^2}{2\,A}}$$
$$= -\,25.46 \text{ amps}$$

Solution of second section of line.

Conditions at middle of line:

$$E_1 = 105{,}360.$$

In-phase current of substation load

$$= \frac{1000 \times 2000 \times 0.70}{105{,}360} = 13.29 \text{ amps.}$$

Reactive current of substation load

$$= \frac{1000 \times 2000 \times 0.7141}{105{,}360} = 13.56 \text{ amps.}$$

$$P_1 = 71.15 + 13.29 = 84.44 \text{ amps.}$$
$$Q_1 = -25.46 + 13.56 = -11.90 \text{ amps.}$$

Then by the K formulas,

$$A_1 = 105{,}360 - 5120 + 4180 - 1410$$
$$= 103{,}010 \text{ volts.}$$
$$B_1 = 2200 + 9940 + 590$$
$$= 12{,}730 \text{ volts.}$$
$$A_1 + \frac{B_1^2}{2A_1} = 103{,}800 \text{ volts}$$
$$= \text{voltage at the supply end of the line.}$$

[By the fundamental formulas, supply voltage = 103,900 volts (Prob. B, Chap. VI).]

PROBLEMS, CHAP. V.

(K FORMULAS.)

1. Find, by means of the K formulas, the voltage drop from the supply end to the receiver end (the line drop) of the following line:

Length of line	200 miles.
Spacing	9 feet.
Conductor	No. 000 aluminum cable.

Load (at receiver end), 4500 K.V.A., 66,000 volts, 80% P.F., three phase, 60 cycles.

Ans. 6650 volts.

[By the fundamental formulas, 6700 volts. (Chap. VI, Prob. A).]

2. Find the regulation of the line in Prob. A, Chap. V. [See Prob. A, Chap. III.]

Ans. 9.37%.

[By the fundamental formulas 9.40%. (Chap. VI, Prob. 2.) Error 0.03% of line voltage.]

3. Find the per cent regulation and voltage drop of the following line:

Length of line	75 miles.

Spacing, 8 feet, regular flat spacing.

Conductor	No. 00 aluminum cable.

Load (at receiver end), 10,000 K.V.A., 88,000 volts, 85% P.F., three phase, 25 cycles. (Prob. 7, Chap. III.)

Ans. 7.35% reg'n, 7.13% drop.

4. Find, by the K formulas, the per cent voltage drop, the per cent loss, and the power factor at the supply end of the following line:

Length of line .... 100 miles.
Spacing .... 6 feet.
Conductor .... No. 0000 copper wire.
Take $r = 0.267$, $x = 0.727$, $b = 6.03 \times 10^{-6}$.
Load (at receiver end), 100 amperes per wire, 60,000 volts, 95% P.F., three phase, 60 cycles. [Problem of Pender and Thomson, *Proc. A. I. E. E.*, July, 1911.]

Ans. 13.09% drop, 7.61% loss, 96.58% P.F.

[Calc. by series, 13.03% drop, 7.60% loss, 96.66% P.F. (Prob. 4, Chap. VI).]

5. Find, by the K formulas, the K.V.A. and voltage at the supply end, and the efficiency of the following line:

Length of line 250 Km. = 155.34 miles.
Spacing .... 6 feet.
Conductor .... No. 000 copper wire.
Total resistance of one conductor .... 51.5 ohms.
Total reactance of one conductor .... 48.0 ohms.
Load (at receiver end), 15,000 K.V.A., 86,600 volts, 80% P.F., three phase, 25 cycles.

[See page 91, "Application of Hyperbolic Functions," by A. E. Kennelly, University of London Press, 1912.]

Ans. 15,130 K.V.A., 97,920 volts, 89.68%.

[By series, 15,153 K.V.A., 97,934 volts, 89.71%.]

6. Find (*a*) star voltage at supply end at full load,
(*b*) star voltage at supply end at no load,
(*c*) regulation volts (star) at the supply end,
(*d*) amperes per wire at supply end at full load,
(*e*) power factor at supply end at full load,
(*f*) loss in line at full load,
(*g*) efficiency of the transmission line,
(*h*) amperes per wire at supply end at no load (*i.e.*, the "charging current"),
(*i*) power factor at supply end at no load,
(*j*) loss in line at no load,

for the following line:

Length of line .... 300 miles.
Spacing.................................... 10 feet.
Conductor, No. 000 copper cable of 0.330 ohms per mile.
Load (at receiver end), 18,000 K.V.A., 104,000 line volts, 90% P.F., three phase, 60 cycles.

[Prob. A, page 2, *G. E. Review Supplement*, May, 1910.]

Answers:

	By *K* Formulas.	By Fundamental Formulas.	Error in per cent of full voltage or current.
(*a*)	69,820	69,670 volts	0.3%
(*b*)	48,610	48,950 volts	0.6%
(*c*)	21,210	20,720 volts	0.9%
(*d*)	97.0	96.59 amps.	0.5%
(*e*)	92.2	92.35%	0.2%
(*f*)	2530	2440 Kw.	0.6%
(*g*)	86.5	86.90%	0.5%
(*h*)	91.3	90.97 amps.	0.5%
(*i*)	7.1	6.47%	0.7%
(*j*)	940	860 Kw.	0.5%

7. Find, by the *K* formulas, the voltage at the supply end of the following line:

Total length of line.................... 400 miles.
Spacing............................. 15 feet.
Conductor .... No. 0000 copper cable.
Load at receiver end of line, 5000 K.V.A., 85% P.F. (lagging) 110,000 volts, three phase, 60 cycles.

Load taken by a substation at the middle of the line, 200 miles from either end, 2500 K.V.A., at the line voltage and at 90% P.F. (lagging).

Ans. 89,720 volts.

[By the convergent series, 90,190 volts (Prob. 6, Chap. VI). Error 0.5%.]

A method of operation of transmission lines is coming into prominence, by which the voltage of the line is kept constant at all points, and the inconveniences due to poor regulation are obviated. Synchronous machinery, consisting of either synchronous motors or generators, is installed in the stations throughout the transmission system in sufficient quantity to hold the voltage at a constant value by controlling the amount of leading or lagging current supplied to the line. This method will probably come into considerable favor, for there seems to be practically no limit to the extent of a transmission system operated at constant voltage.

The following problem outlines the method of calculation for such cases.

A line 400 miles long has a substation at the middle, 200 miles from either end. A load of 10,000 Kw. is taken at the receiver end of the line, and 8000 Kw. at the substation. Find the power factor which is required for these loads, in order that the voltage at the generator, substation, and receiver end may be 110,000 volts, the following data being given:

Conductor, 250,000 c.m. copper cable, 14-foot spacing, 3-phase, 60 cycles,

$$r = 0.2284,\ x = 0.813.$$

1st section of line, $l = 200$ miles.

By the *K* formulas,

$$P = \frac{1000 \times 10{,}000}{110{,}000} = 90.91;$$

Q is unknown;

$$\begin{aligned} A &= 110{,}000 - 9500 + 3910 + 160.3\ Q \\ &= 104{,}410 + 160.3\ Q; \end{aligned}$$

$$\begin{aligned} B &= 2670 + 14{,}570 - 43.0\, Q \\ &= 17{,}240 - 43.0\, Q. \end{aligned}$$

Now the voltage at the substation end of the 1st section of the line is 110,000; that is,

$$A + \frac{B^2}{2A} = 110{,}000;$$

squaring both sides, $A^2 + B^2 = 121 \times 10^8$.

This gives a quadratic equation in Q,

$$2.754\, Q^2 + 3198\, Q - 90{,}150 = 0,$$

from which $Q = 27.53$ total amps.;

$$\sqrt{P^2 + Q^2} = 94.99;$$

$$\text{Power factor} = \frac{90.91}{94.99} = 95.7\%,$$

and, as Q is positive, this is a lagging power factor.

The power factor obtained by the hyperbolic formulas is 95.9%.

Using the above value of Q, we obtain

$$\begin{aligned} C &= +\,82.78, \\ D &= +\,90.59. \end{aligned}$$

In-phase current at substation end of 1st section $= +\,95.11$.

Reactive current at substation end of 1st section $= -\,77.53$.

2nd section of line, $l = 200$ miles.

In-phase current in 1st section = + 95.11.
In-phase current of substation load

$$= \frac{1000 \times 8000}{110{,}000} = +\ 72.73.$$

In-phase current at substation end of 2nd section = 167.84.
Reactive current at substation end of 2nd section

$$= Q_x - 77.53,$$

where Q_x is the unknown reactive current of the substation load.

By the *K* formulas, as before,

$$A_1 = 95{,}300 + 160.3\ Q_x,$$
$$B_1 = 32{,}910 - \ \ 43.0\ Q_x,$$

and voltage at generator end of line

$$= A_1 + \frac{B_1{}^2}{2\ A_1} = 110{,}000.$$

From the above we obtain as before a quadratic equation in Q_x, which gives

$$Q_x = +\ 65.59.$$

Power factor of substation load = 74.3%, lagging.

The power factor obtained by the hyperbolic formulas is 74.6%.

CHAPTER VI.

CONVERGENT SERIES.

THE mathematical expression for finding the operating characteristics of a transmission line, in which exact account is taken of all the electrical properties of the line, has been published many times. It involves the use of hyperbolic sines and cosines, as well as of complex quantities,* and, without some special arrangement, cannot be directly applied to the calculation of a particular case. For this reason, most of the systems so far published for calculating transmission lines have used approximate formulas which have been based on the hyperbolic formulas. In a few cases, an attempt has been made to devise a system of working which would give the exact results of the fundamental hyperbolic formulas, but generally the labor required in using the systems is so great as almost to prohibit obtaining the exact result, or else the accuracy of the work is seriously impaired by the necessity of interpolating values from tables of hyperbolic functions which have been recently prepared for this purpose and are not as large and complete as they should be for good working.

The original hyperbolic formulas can be expressed in the form of convergent series.† In this form they do not

* The hyperbolic formulas are given in Chap. XV.

† Prof. T. R. Rosebrugh, *Applied Science Magazine*, University of Toronto, March, 1909; Prof. T. R. Rosebrugh, *Proc. A. I. E. E.*, Nov., 1909, p. 1460; J. F. H. Douglas, *Electrical World*, April 28, 1910; Dr. C. P. Steinmetz, *Electrical World*, June 23, 1910; Dr. C. P. Steinmetz, "Engineering Mathematics," Chap. V, 1911.

involve hyperbolic or trigonometrical functions, and so do not require any mathematical tables, the only operations being multiplication and addition. The series can be carried to any accuracy desired by merely using enough of the terms, which diminish very rapidly when commercial frequencies are involved.

The fundamental formulas as expressed by convergent series have been rearranged, and some new convergent series have been added, to make the formulas as tabulated in this chapter directly applicable to the exact solution of all the problems treated by the K formulas. Exactly the same final formulas in A, B, C, D, etc., are used with the convergent series as with the K formulas.

Unlike the K formulas, which are expressed in the simplest algebraical form, the convergent series involve the use of complex numbers, that is, numbers containing the well-known "j" terms. No difficulty should be experienced on this account, however, as the rules for using complex quantities are quite straightforward, and even one who has never worked with them should be able to make use of the formulas described in this chapter by closely following the instructions.

Each of the complex quantities, $(A + jB)$, $(P - jQ)$, $Z = (r + jx)\,l$,* $Y = (g + jb)\,l$, etc., is composed of two parts, the first, a so-called "real" term, and the second, a j term. In adding complex numbers, the j terms must be kept separate from the others. Thus

$$4 + j\,5 \text{ added to } 7 + j\,3 = 11 + j\,8.$$

In multiplying two complex quantities, the simple rules

* The notation $Z = (r + jx)\,l$, etc., is used in accordance with the resolution adopted by the International Electrotechnical Commission in Turin, Sept., 1911.

of ordinary algebra are followed, and it must be remembered that

$$j \times j = j^2$$
$$= -1,$$

and, therefore,

$$-j \times j = +1$$
$$j^3 = -j$$
$$j^4 = +1$$
$$j^5 = +j, \text{ etc.}$$

Thus $(4 + j\,5) \times (7 + j\,3)$ is worked out as follows:

$$\begin{array}{r} 4 + j\,5 \\ 7 + j\,3 \\ \hline +28 + j\,35 \\ -15 + j\,12 \\ \hline +13 + j\,47. \end{array}$$

In using the convergent series, E, P, and Q are the same as used with the K formulas, E being expressed as a real number without any j term. Z is equal to $(r + jx)\,l$, where r and x are taken from Tables VII to XII, Part III, for resistance and reactance per mile. Y is equal to $(g + jb)\,l$. The leakage conductance, g, per mile, should be estimated from the most suitable test data available, giving insulator leakage and corona loss under conditions similar to those of the line considered. The capacity susceptance, b, per mile, will be found in Tables XIII to XVI, Part III.

After Y and Z have been written down in the form of complex numbers, the product YZ should be found, as described above for the multiplication of complex quantities. From this is obtained

$$\frac{YZ}{2}, \quad \frac{YZ}{4}, \text{ and } \frac{YZ}{6},$$

each expressed as a complex number of a single real term and a single j term. Multiplying the last two together gives $\frac{Y^2Z^2}{2 \cdot 3 \cdot 4}$, from which $\frac{Y^2Z^2}{2 \cdot 3 \cdot 4 \cdot 5}$ may be written down. In most cases no more terms need to be calculated, even for very accurate work, but this is to be determined while doing the work, as one usually figures out the terms of these series until they become too small to be considered when added to $\frac{YZ}{2}$.

By addition of terms obtained above, the values of

$$\frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \text{etc.} \quad \text{and} \quad \frac{YZ}{2 \cdot 3} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4 \cdot 5} + \text{etc.}$$

are obtained, each as a complex number of two terms.

Multiply E by the value found for $\left(\frac{YZ}{2} + \text{etc.}\right)$ and add it to E. Multiply $(P - jQ)$ by Z, or $(r + jx)\,l$, and by the value of $\left(\frac{YZ}{2 \cdot 3} + \text{etc.}\right)$ and add it to $(P - jQ)\,Z$. The above quantities are added together to give $A + jB$, the sum of all the real parts being equal to A, and the sum of all the j terms being equal to B.

Similarly, $C + jD$ is found by adding

$$(P - jQ),\ (P - jQ)\left(\frac{YZ}{2} + \text{etc.}\right),\ EY, \text{ and } EY\left(\frac{YZ}{2 \cdot 3} + \text{etc.}\right).$$

These values of A, B, etc., are inserted in equations 1 to 21 given with the K formulas, in exactly the same way as the values of A, B, etc., found according to the second page of Table III. Each step of the above procedure is shown in the examples in this chapter.

The use of Table VI is the same as that of Table V described above.

TABLE V. — CONVERGENT SERIES FOR TRANSMISSION LINES.

Conditions given at Receiver End.

The convergent series give the results of the fundamental formulas as accurately as desired, if a sufficient number of terms is used.

When conditions are given at the receiver end, the same as with the K formulas, find the quantities:

Full Load.

$$A + jB = E\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2\cdot 3\cdot 4} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \text{etc.}\right)$$
$$+ (P - jQ)\,Z\left(1 + \frac{YZ}{2\cdot 3} + \frac{Y^2Z^2}{2\cdot 3\cdot 4\cdot 5} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \text{etc.}\right).$$

$$C + jD = (P - jQ)\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2\cdot 3\cdot 4} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \text{etc.}\right)$$
$$+ EY\left(1 + \frac{YZ}{2\cdot 3} + \frac{Y^2Z^2}{2\cdot 3\cdot 4\cdot 5} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \text{etc.}\right).$$

No Load.

$$A_0 + jB_0 = E\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2\cdot 3\cdot 4} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \text{etc.}\right),$$

$$C_0 + jD_0 = EY\left(1 + \frac{YZ}{2\cdot 3} + \frac{Y^2Z^2}{2\cdot 3\cdot 4\cdot 5} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \text{etc.}\right),$$

where $Z = (r + jx)\,l.$
r = resistance of conductor per mile.
x = reactance of conductor per mile.
l = length of transmission line in miles.
$Y = (g + jb)\,l.$
g = leakage conductance of conductor per mile.
b = capacity susceptance of conductor per mile.

Use A, B, C, D, etc., with the equations in the third and fourth pages of Table III to solve transmission line problems.

Note 1. — In the formulas, $A + \dfrac{B^2}{2A}$ is used instead of $\sqrt{A^2 + B^2}$. This approximation may be used for very accurate work, as it is correct within approximately $\frac{2}{100}$ of 1% when the regulation is not more than 20%.

Note 2. — The above are for two- and three-phase lines. For single-phase lines use $2r$ and $2x$ in place of r and x, and use $\frac{1}{2}g$ and $\frac{1}{2}b$ in place of g and b.

TABLE VI. — CONVERGENT SERIES FOR TRANSMISSION LINES.

Conditions given at the Supply End.

The convergent series give the results of the fundamental formulas as accurately as desired, if a sufficient number of terms is used.

When conditions are given at the supply end, the same as with the K formulas, find the quantities:

Full Load.

$$F + jG = E_s\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2\cdot 3\cdot 4} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \text{etc.}\right)$$
$$- (P_s - jQ_s)\, Z\left(1 + \frac{YZ}{2\cdot 3} + \frac{Y^2Z^2}{2\cdot 3\cdot 4\cdot 5} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \text{etc.}\right).$$

$$M + jN = (P_s - jQ_s)\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2\cdot 3\cdot 4} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \text{etc.}\right)$$
$$- E_sY\left(1 + \frac{YZ}{2\cdot 3} + \frac{Y^2Z^2}{2\cdot 3\cdot 4\cdot 5} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \text{etc.}\right).$$

No Load.

$$F_0 + jG_0 = E_s\left(1 - \tfrac{1}{2}YZ + \frac{5}{24}Y^2Z^2 - \frac{61}{720}Y^3Z^3 + \frac{277}{8064}Y^4Z^4 - \text{etc.}\right),$$

$$M_0 + jN_0 = E_sY\left(1 - \tfrac{1}{3}YZ + \frac{2}{15}Y^2Z^2 - \frac{17}{315}Y^3Z^3 + \frac{62}{2835}Y^4Z^4 - \text{etc.}\right),$$

where $Z = (r + jx)\,l.$
r = resistance of conductor per mile.
x = reactance of conductor per mile.
l = length of transmission line in miles.
$Y = (g + jb)\,l.$
g = leakage conductance of conductor per mile.
b = capacity susceptance of conductor per mile.

Use F, G, M, N, etc., with the equations in the third and fourth pages of Table IV to solve transmission line problems.

Note 1. — In the formulas, $F + \dfrac{G^2}{2F}$ is used instead of $\sqrt{F^2 + G^2}$. This approximation may be used for very accurate work, as it is correct within approximately $\frac{2}{100}$ of 1% when the regulation is not more than 20%.

Note 2. — The above are for two- and three-phase lines. For single-phase lines use $2\,r$ and $2\,x$ in place of r and x, and use $\frac{1}{2}\,g$ and $\frac{1}{2}\,b$ in place of g and b.

Problem A.

Find the line drop, by means of the convergent series, for the following line:

Length of line .... 200 miles.
Spacing .... 9 feet.
Conductor .... No. 000 aluminum cable.
Load (at receiver end), 4500 K.V.A., 66,000 volts, 80% P.F., three phase, 60 cycles.

From the tables,

$$r = 0.5412, \quad x = 0.784, \quad b = 5.49 \times 10^{-6}.$$

Then

$$P = \frac{1000 \times 4500 \times 0.8}{66{,}000} = 54.55.$$

$$Q = \frac{1000 \times 4500 \times 0.6}{66{,}000} = 40.91.$$

$$Z = 108.24 + j\,156.8.$$

$$Y = \qquad + j\,0.001098.$$

$$YZ = -0.17216 + j\,0.11885.$$

$$\frac{YZ}{2} = -0.08608 + j\,0.05942.$$

$$\frac{YZ}{4} = -\,0.04304 + j\,0.02971.$$

$$\frac{YZ}{6} = -\,0.02869 + j\,0.01981.$$

$$-\,0.00059 - j\,0.00085.$$

$$+\,0.00124 - j\,0.00085.$$

$$\frac{Y^2Z^2}{2 \cdot 3 \cdot 4} = +\,0.00065 - j\,0.00170.$$

$$\frac{YZ}{2} = -\,0.08608 + j\,0.05942.$$

$$\left(\frac{YZ}{2} + \text{etc.}\right) = -\,0.08543 + j\,0.05772.$$

$$\frac{Y^2Z^2}{2 \cdot 3 \cdot 4 \cdot 5} = +\,0.00013 - j\,0.00034.$$

$$\frac{YZ}{2 \cdot 3} = -\,0.02869 + j\,0.01981.$$

$$\left(\frac{YZ}{2 \cdot 3} + \text{etc.}\right) = -\,0.02856 + j\,0.01947.$$

Now
$$E = 66{,}000.$$
$$\left(\frac{YZ}{2} + \text{etc.}\right) = -0.08543 + j\,0.05772.$$
$$E\left(\frac{YZ}{2} + \text{etc.}\right) = -5640 + j\,3810.$$
$$P - jQ = 54.55 - j\;40.91.$$
$$Z = 108.24 + j\,156.8.$$
$$\begin{aligned} &+5900 - j\,4430. \\ &+6420 + j\,8550. \end{aligned}$$
$$(P - jQ)\,Z = +12320 + j\,4120.$$
$$\left(\frac{YZ}{2 \cdot 3} + \text{etc.}\right) = -0.02856 + j\,0.01947.$$
$$\begin{aligned} &-80 + j\,240. \\ &-350 - j\,120. \end{aligned}$$
$$(P - jQ)\,Z\left(\frac{YZ}{2 \cdot 3} + \text{etc.}\right) = -430 + j\,120.$$
$$E = 66{,}000.$$
$$E\left(\frac{YZ}{2} + \text{etc.}\right) = -5640 + j\,3810.$$
$$(P - jQ)\,Z = 12320 + j\,4120.$$
$$(P - jQ)\,Z\left(\frac{YZ}{2 \cdot 3} + \text{etc.}\right) = -430 + j\,120.$$
$$A + jB = 78{,}320 - 6070 + j\,8050$$
$$= 72{,}250 + j\,8050.$$
$$A + \frac{B^2}{2\,A} = 72{,}700 \text{ volts.}$$
$$\text{Line drop} = 6700 \text{ volts.}$$

Problem B.

Find, by the convergent series, the voltage at the supply end of the following line:

Total length of line........................... 300 miles.
Spacing.. 12 feet.
Conductor, 266,800 c.m. aluminum cable.
Load at receiver end of line, 9000 K.V.A., 80% P.F. (lagging), 100,000 volts, three phase, 60 cycles.

Load taken by a substation at the middle of the line, 150 miles from either end, 2000 K.V.A., at the line voltage and at 70% P.F. (lagging). (Prob. B, Chap. V.)

Solution of first section of line:

$$r = 0.3410, \quad x = 0.791, \quad b = 5.44 \times 10^{-6}, \quad l = 150.$$

$$Z = 51.15 + j\,118.65.$$

$$Y = +j\,0.000816.$$

$$YZ = -0.09682 + j\,0.04174.$$

$$\left(\frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \text{etc.}\right) = -0.04809 + j\,0.02053.$$

$$\left(\frac{YZ}{2 \cdot 3} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4 \cdot 5} + \text{etc.}\right) = -0.01607 + j\,0.00689.$$

$$E = +100{,}000.$$

$$E\left(\frac{YZ}{2} + \text{etc.}\right) = \quad -4810 + j\,2050.$$

$$(P - jQ)\,Z = \quad 10{,}090 \quad + j\,5780.$$

$$(P - jQ)\,Z\left(\frac{YZ}{2 \cdot 3} + \text{etc.}\right) = \quad -200 \quad -j\,20.$$

$$A + jB = \quad 105{,}080 \quad + j\,7810.$$

$$A + \frac{B^2}{2A} = E_1 = 105{,}370 \text{ volts.}$$

In a similar manner, it is found that

$$C + jD = 69.08 + j\,30.36 \text{ amps.}$$

In-phase current, $\dfrac{AC + BD}{A + \dfrac{B^2}{2A}} = 71.15$ amps.

Reactive current, $\dfrac{BC - AD}{A + \dfrac{B^2}{2A}} = -25.15$ amps.

Solution of second section of line:

Conditions at middle of line,

$$E_1 = 105{,}370 \text{ volts.}$$

In-phase current of substation load

$$= \frac{1000 \times 2000 \times 0.70}{105{,}370} = 13.29 \text{ amps.}$$

Reactive current of substation load

$$= \frac{1000 \times 2000 \times 0.7141}{105{,}370} = 13.55 \text{ amps.}$$

Current of substation load $= 13.29 - j\,13.55.$
Current of first section $= 71.15 + j\,25.15.$

$$P_1 - jQ_1 = 84.44 + j\,11.60.$$
$$E_1 = +\,105{,}370.$$
$$E_1\left(\frac{YZ}{2} + \text{etc.}\right) = -\,5070 + j\,2{,}160.$$
$$(P_1 - jQ_1)\,Z = 2{,}940 + j\,10{,}610.$$
$$(P_1 - jQ_1)\,Z\left(\frac{YZ}{2 \cdot 3} + \text{etc.}\right) = -\,120 - j\,150.$$
$$A_1 + jB_1 = 103{,}120 + j\,12{,}620 \text{ volts.}$$
$$A_1 + \frac{B_1^2}{2A_1} = 103{,}900 \text{ volts}$$
$$= \text{voltage at the supply end of the line.}$$

PROBLEMS, CHAP. VI.

(CONVERGENT SERIES.)

1. Find, by the convergent series, the voltage drop of the following line:

Length of line .... 80 miles.
Spacing .... 10 feet.
Conductor .... No. 00 aluminum cable.
Load (at receiver end), 15,000 K.V.A., 100,000 volts, 95% P.F., two phase, 25 cycles. (Prob. C, Chap. III.)

Ans. 8810 volts.

2. Find, by the convergent series, the per cent line drop and the per cent regulation of the following line:

Length of line .... 100 miles.
Spacing .... 8 feet.
Conductor .... No. 3 copper cable.
Load (at receiver end), 3000 K.V.A., 66,000 volts, 90% P.F., three phase, 60 cycles. (See Prob. A., Chap. III, and Prob. A, Chap. V.)

Ans. 7.08% drop, 9.40% reg'n.

3. Find, by the convergent series, the K.V.A. and voltage at the supply end, and the efficiency of the following line:

Length of line .... 250 Km. = 155.34 miles.
Spacing.......................... 6 feet.
Conductor .... No. 000 copper wire.
Total resistance of one conductor, 51.5 ohms.
Total reactance of one conductor, 48.0 ohms.
Total susceptance of one conductor, 3.724×10^{-4} mhos.
Load (at receiver end), 15,000 K.V.A., 86,600 volts, 80% P.F., three phase, 25 cycles. (Prob. 5, Chap. V.)

Ans. 15,153 K.V.A., 97,934 volts, line; 56,542 volts, star; 89.71%.

4. Find, by the convergent series, the per cent voltage drop, the per cent loss, and the power factor at the supply end of the following line:

Length of line .... 100 miles.
Spacing............................ 6 feet.
Conductor .... No. 0000 copper wire.
Take $r = 0.267$, $x = 0.727$, $b = 6.03 \times 10^{-6}$.
Load (at receiver end), 100 amperes per wire, 60,000 volts, 95% P.F., three phase, 60 cycles. [Prob. 4, Chap. V.]

Ans. 13.03% drop, 7.60% loss, 96.66% P.F.

5. Find, by the convergent series,
   (*a*) star voltage at supply end at full load,
   (*b*) star voltage at supply end at no load,
   (*c*) regulation volts (star), at the supply end,
   (*d*) amperes per wire at supply end at full load,
   (*e*) power factor at supply end at full load,
   (*f*) loss in line at full load,
   (*g*) efficiency of the transmission line,
   (*h*) amperes per wire at supply end at no load (*i.e.*, the "charging current"),
   (*i*) power factor at supply end at no load,
   (*j*) loss in line at no load, for the following line:

Length of line .... 300 miles.
Spacing.................................. 10 feet.
Conductor, No. 000 copper cable of 0.330 ohm per mile.
Load (at receiver end), 18,000 K.V.A., 104,000 volts, 90% P.F., three phase, 60 cycles. (Prob. 6, Chap. V.)

Ans. (*a*) 69,670 volts, (*b*) 48,950 volts, (*c*) 20,720 volts, (*d*) 96.59 amps., (*e*) 92.35%, (*f*) 2440 Kw., (*g*) 86.90%, (*h*) 90.97 amps., (*i*) 6.47%, (*j*) 860 Kw.

6. Find, by the convergent series, the voltage at the supply end of the following line:

Total length of line................ 400 miles.
Spacing.......................... 15 feet.
Conductor........................ No. 0000 copper cable.
Load at receiver end of line, 5000 K.V.A., 85% P.F. (lagging), 110,000 volts, three phase, 60 cycles.

Load taken by a substation at the middle of the line, 200 miles from either end, 2500 K.V.A. at the line voltage and at 90% P.F. (lagging). (Prob. 7, Chap. V.)

Ans. 90,190 volts.

PART II.

THEORY.

CHAPTER VII.

CONDUCTORS.

THREE main classes of conductors are used for overhead lines for the transmission of electric power; namely, copper wires, copper cables and aluminum cables. The cables used are generally strands of seven wires; that is, they consist of a central straight wire with six wires wound spirally around it, as indicated by the cross section in Fig. 9.

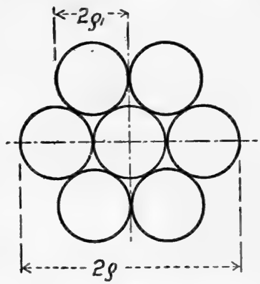

Fig. 9. 7-Wire Strand.

From this figure it is seen that the maximum diameter of a 7-wire strand is equal to 3 times the diameter of one of the wires.

The outside wires do not follow a straight path parallel to the central wire and the axis of the cable, but lie in a spiral around it, as mentioned above. As there is always a slight insulating film of oxide on any wire, the current flowing in the cable tends to stay in the individual wires, and so follows the longer path. Thus, the resistance of a cable is greater than that of a solid wire of the same area of cross section. The amount of the difference depends on the number of wires in the cable and the pitch of the spiralling, but an average value of 1% is assumed in making up the tables in Part III. The cross section of the cable is assumed to be equal to the sum of the cross sections of the individual wires. The weight per unit length of the cable calculated from this cross section must be increased by the same percentage as the above increase in resistance, due to the extra length of the outside wires. Since the cross section in Fig. 9 does not cut the outside wires exactly at right angles, their sections as shown in the figure are really ellipses, and the diameter of the cable is slightly greater than 6 ρ_1. However, this difference is small and has been neglected in the figures for diameter of cable tabulated in Part III.

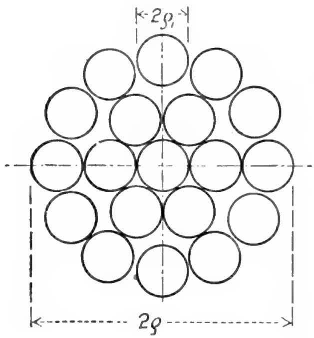

Fig. 10. 19-Wire Strand.

The number of wires in a strand varies in practice according to the degree of flexibility and mechanical strength desired by the user. The number of wires per strand in the tables represents average practice for overhead lines. The larger cables often have 19 or even 37 wires.

The section of a 19-wire strand is shown in Fig. 10, and it is seen that the maximum diameter is 5 times the diameter of one of the individual wires. The same increase of 1% in resistance is allowed as with a 7-wire strand.

There is only a very slight difference in the reactance and capacity of a 7-wire and a 19-wire strand of the same sectional area, so that values listed for 7 wires may be used for 19 wires, and vice versa, without very much error.

The resistances for direct current tabulated in Part III have been calculated in accordance with the recommendations of the Bureau of Standards for the preparation of wire tables.* The Standardization Rules of the American Institute of Electrical Engineers are in agreement with these recommendations. According to the Bureau of Standards and the A. I. E. E., the "Annealed Copper Standard," which is of 100% conductivity, is represented by a resistivity of 0.153022 ohm per meter-gram at 20° C. This is equivalent to 1.72128 micro-ohms per centimeter cube at 20° C., assuming a density of 8.89. This is the same as the resistivity of Matthiessen's Standard at 20° C., formerly used by the A. I. E. E. The conductivity of hard drawn copper recommended for wire tables by the Bureau of Standards is 97.3%, this value representing an average for good commercial copper. The average conductivity given by the Bureau of Standards for hard drawn aluminum on the centimeter cube basis, assuming a density of 2.699, is 60.86%. The above values have been used in preparing the tables in Part III, 1% being added to the resistance for the effect of spiralling, as already noted.

If it is desired to calculate the resistance of copper conductors for other temperatures than 20° C., the temperature coefficient, α_{20}, for hard drawn copper of 97.3% conductivity should be used in connection with the formula

$$R_t = R_{20}\left\{1 + \alpha_{20}(t - 20)\right\}$$

where t is the temperature in degrees Centigrade for which the resistance R_t is desired and where

$$\alpha_{20} = 0.00383.$$

* *Bulletin of the Bureau of Standards*, Vol. VII, pp. 71–126, Washington, 1911; *Proc. A. I. E. E.*, Dec., 1910.

For other initial temperatures and other conductivities, temperature coefficients should be used as given in the table of temperature coefficients in Part III, which is taken from Appendix E of the Standardization Rules of the A. I. E. E.

For the temperature coefficient of hard drawn aluminum, a value of

$$\alpha_{20} = 0.0039,$$

which is recommended by the Bureau of Standards, may be used.

CHAPTER VIII.

TRANSMISSION LINE PROBLEMS.

WHEN conditions are given at the receiver, or load, end of a transmission line, the convergent series of Table V give at once the voltage, $A + jB$, and the current, $C + jD$, at the other end of the line. By putting the load current equal to zero, we obtain the following expression for the no-load voltage at the supply end:

$$A_0 + jB_0 = E\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \text{etc.}\right)$$

Thus the ratio of the voltages at the two ends of the line at no load is

$$\frac{A_0 + jB_0}{E} = \left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \text{etc.}\right), \qquad (1)$$

which is independent of the voltage E, and depends only on the constants of the line.

The absolute value of a complex quantity like the voltage $A_0 + jB_0$, is its total numerical value independent of its phase relation. This is the same, in the case of the voltage $A_0 + jB_0$, as its measured value, and is equal to

$$\sqrt{A_0^2 + B_0^2}, \quad \text{or} \quad A_0 + \frac{B_0^2}{2\,A_0},$$

to a very close approximation when B_0 is smaller than A_0. Since the two complex quantities making up equation (1) are equal in all respects, their absolute values are eoual, and hence

$$\frac{A_0 + \dfrac{B_0^2}{2\,A_0}}{E} = \text{absol. value of}\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \text{etc.}\right) \qquad (2)$$

When the line is carrying full load, the measured value of the receiver voltage is E, and of the supply voltage, $A + \frac{B^2}{2A}$. If the load be thrown off and the supply voltage be kept constant at $A + \frac{B^2}{2A}$, then the receiver voltage will rise to a value E_0. The line is now at no load, and the ratio of the voltages at the two ends is, by equation (2),

$$\frac{A + \frac{B^2}{2A}}{E_0} = \text{absolute value of} \left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \text{etc.}\right)$$

$$= \frac{A_0 + \frac{B_0^2}{2A_0}}{E}.$$

Thus
$$E_0 = \frac{A + \frac{B^2}{2A}}{A_0 + \frac{B_0^2}{2A_0}} E$$

(equation 2, Table III).

We are now in a position to obtain the regulation of the line, since by the definition in the A. I. E. E. Standardization Rules,

$$\text{Per cent regulation} = \frac{E_0 - E}{E} \times 100.$$

Thus, the regulation volts at the receiver end which are to be expressed as a percentage of E, are

$$E_0 - E = \frac{A + \frac{B^2}{2A}}{A_0 + \frac{B_0^2}{2A_0}} E - E$$

as in equation 3, Table III.

It is often desirable to find the regulation of a line at the supply end, that is, the per cent change in supply voltage from full-load conditions to no-load conditions, when the receiver voltage is kept constant. If the receiver voltage is E, we have seen in the preceding paragraph that the full-load supply voltage is equal to

$$E_s = A + \frac{B^2}{2A},$$

and the no-load supply voltage is

$$E_{0s} = A_0 + \frac{B_0^2}{2A_0}.$$

The per cent regulation at the supply end is

$$\frac{E_s - E_{0s}}{E_s} \times 100,$$

and the regulation volts at the supply end are, thus

$$E_s - E_{0s} = A + \frac{B^2}{2A} - A_0 - \frac{B_0^2}{2A_0},$$

as in equation 6, Table III.

In the expression $C + jD$ for current at the supply end, the quantity C denotes the component of current which is in phase with the voltage E at the other end of the line. We can, however, find the component of supply current which is in phase with the supply voltage, by first finding the watts at the supply end.

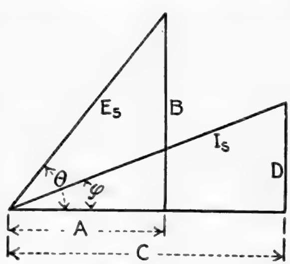

Fig. 11.

Let the supply voltage be

$$E_s = A + jB$$

and let its phase be denoted by the angle θ, Fig. 11, where

$$\tan\theta = \frac{B}{A},$$

and, therefore,
$$\sin\theta = \frac{B}{\sqrt{A^2+B^2}},$$

and
$$\cos\theta = \frac{A}{\sqrt{A^2+B^2}}.$$

Similarly, let the current at the supply end be $C + jD$, at a phase angle ϕ, where

$$\tan\phi = \frac{D}{C}$$

and, therefore
$$\sin\phi = \frac{D}{\sqrt{C^2+D^2}}$$

and
$$\cos\phi = \frac{C}{\sqrt{C^2+D^2}}.$$

The watts at the supply end are equal to the current, multiplied by the voltage, multiplied by the power factor; that is,

$$\begin{aligned}
\text{Watts} &= \text{absolute value of } I_s \times \text{absolute value of } E_s \\
&\qquad \times \cos(\theta - \phi) \\
&= \sqrt{C^2+D^2} \times \sqrt{A^2+B^2} \times (\cos\theta\cos\phi + \sin\theta\sin\phi) \\
&= \sqrt{C^2+D^2} \times \sqrt{A^2+B^2} \\
&\qquad \left\{ \frac{A}{\sqrt{A^2+B^2}} \times \frac{C}{\sqrt{C^2+D^2}} + \frac{B}{\sqrt{A^2+B^2}} \times \frac{D}{\sqrt{C^2+D^2}} \right\} \\
&= AC + BD,
\end{aligned}$$

as in equation 11, Table III.

The quadrature volt-amperes, or reactive power, are given by the following equation:

Reactive power

$$= \text{absolute value of } I_s \times \text{absolute value of } E_s \times \sin(\theta - \phi).$$
$$= \sqrt{C^2 + D^2} \times \sqrt{A^2 + B^2}\,(\sin\theta\cos\phi - \cos\theta\sin\phi)$$
$$= \sqrt{C^2 + D^2} \times \sqrt{A^2 + B^2}$$
$$\left\{\frac{B}{\sqrt{A^2+B^2}} \times \frac{C}{\sqrt{C^2+D^2}} - \frac{A}{\sqrt{A^2+B^2}} \times \frac{D}{\sqrt{C^2+D^2}}\right\}$$
$$= BC - AD.$$

When the expression $BC - AD$ has a positive value, the current at the supply end is lagging behind the supply voltage, and when the expression has a negative value, the current leads the voltage in phase.

We can now obtain the in-phase component of current, which is equal to watts divided by voltage (equations 15 and 16), and in the same way the quadrature component of current, which is equal to reactive power divided by voltage (equations 17 and 18). The power factor at the supply end is equal to watts divided by volt-amperes (equations 13 and 14). Since the power supplied is known, being $AC + BD$, and the power delivered at the receiver is also known, being equal to EP, their difference represents the loss of power in the line due to resistance of the conductors, leakage over the insulators and corona loss.

The equations in F, G, M and N are quite similar to the above equations in their derivation, and they give the solutions of similar problems when conditions are given at the supply end of the line.

CHAPTER IX.

REACTANCE OF WIRE, SINGLE-PHASE.

Effect of Flux in Air. — Let there be an alternating current, I, in the transmission line wire, A, indicated in Fig. 12.

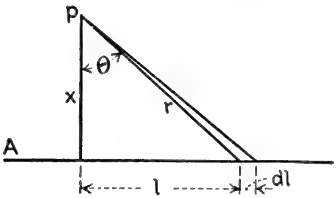

Fig. 12.

The magnetic field set up by the current at P, a distance x away from the wire, will be at right angles to the wire. The intensity of the field will be equal to the force on a unit magnetic pole at P due to the current in the wire. The force due to the current in a short length, dl, of the wire will be

$$\frac{Idl}{r^2}\cos\theta = \frac{I}{x}\cos\theta\, d\theta,$$

since $$dl\cos\theta = r\,d\theta$$

and $$r = \frac{x}{\cos\theta}.$$

The total force at P due to the current in the wire A is equal to

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{I\cos\theta\,d\theta}{x} \text{ (where } x \text{ is a constant)}$$

$$= \frac{I\sin\theta}{x}\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{2I}{x},$$

where I is measured in absolute electromagnetic units. When I is in amperes, the field at distance x is

$$\frac{2I}{10x} \text{ lines per sq. cm.} \qquad (1)$$

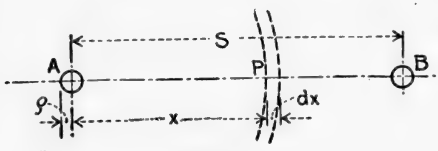

Fig. 13.

In Fig. 13 is shown the cross section of a single-phase transmission line. The lines of force in the path of thickness dx surrounding the wire A are

$$\frac{2I}{10x}dx$$

per centimeter of the transmission line. These lines cut the wire A and produce an alternating voltage in it which is 90° out of phase with the current and is equal to

$$j\omega\frac{2I}{x}dx \times 10^{-9} \text{ volts,}$$

where $\omega = 2\pi \times$ number of cycles per second, and where I is in amperes.

The voltage drop between the wires A and B, due to flux in the air produced by the current in A, is obtained by integrating the above expression from $x = \rho$ to $x = s$. The integration is not carried beyond $x = s$, since flux which cuts both A and the return wire B does not produce any voltage between them. The voltage drop is equal to

$$\int_\rho^s j\omega\frac{2I}{x}dx \times 10^{-9} = j\omega\, 2I \log_\epsilon \frac{s}{\rho} \times 10^{-9}.$$

There will be an equal drop due to the flux produced by the current in the wire B, so that the total drop due to flux in air is

$$j\omega\, 4\, I \log_\epsilon \frac{s}{\rho} \times 10^{-9}$$

volts per centimeter of line,

$$= 2\, j\omega \times 741.1 \log_{10} \frac{s}{\rho} \times 10^{-6} \qquad (2)$$

volts per ampere per mile of single-phase line.

Effect of Flux in the Conductor. — Let i be the current per unit area of section at any point in the wire shown in Fig. 14. (For the present assume that i is the same at all points of the section.)

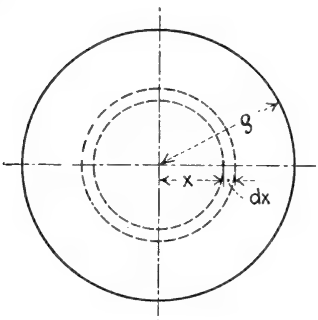

Fig. 14. Section of Wire.

The total area of section of the wire is $\pi\rho^2$ and therefore the total current in the wire is

$$I = \pi i \rho^2.$$

The total current inside the circle of radius x is

$$I_1 = \pi i x^2.$$

This is the only current forcing flux around the circular path of width dx, since currents flowing nearer the surface of the wire do not tend to produce magnetic lines in a path which does not surround them. Thus the flux density at the radius x is

$$\frac{2\, I_1}{10\, x} = \frac{2\, \pi i x^2}{10\, x}$$
$$= \frac{2\, \pi i x}{10}.$$

The total flux in the outer ring of the section is

$$\int_x^\rho \frac{2\,\pi i x\,dx}{10} = \frac{\pi i\,(\rho^2 - x^2)}{10}.$$

This cuts the element dx of the wire and produces a voltage along it equal to

$$j\omega\pi i\,(\rho^2 - x^2)\,10^{-9} \text{ volts per cm.} \tag{3}$$

This voltage leads the current by 90° in phase at all sections. It is greatest at the center and zero at the surface and so is an unbalanced voltage; it therefore causes a local quadrature current to flow along the center of the wire and return near the surface.

Let the local current at the element dx be $i_{(x)}$ per unit area of section. Then the average voltage drop along the wire due to the flux inside it and the resulting local balancing current, is equal to

$$j\omega I L_1 = j\omega\pi i\,(\rho^2 - x^2)\,10^{-9} + i_{(x)}\,r,$$

where L_1 is the self-inductance of the wire due to the above-mentioned flux inside it, and where r is the specific resistance of the metal in centimeter units, that is, the resistance of a centimeter cube of the metal. The current $i_{(x)}$ adjusts itself so that the drop is the same at all parts of the section. From the last equation, we have

$$i_{(x)} = \frac{j\omega\pi i\rho^2 L_1}{r} - \frac{j\omega\pi i\,(\rho^2 - x^2)}{r} \times 10^{-9}, \tag{4}$$

since $\quad I = \pi i \rho^2.$

As $i_{(x)}$ is a local current in the wire, and does not increase or decrease the main current I, its sum when added up all over the section must be zero, and thus

$$\int_0^\rho 2\,\pi x i_{(x)}\,dx = 0,$$

that is, $$j\frac{2\,\omega\pi^2 i}{r}\int_0^{\rho}(\rho^2 L_1 x - \rho^2 x\, 10^{-9} + x^3\, 10^{-9})\,dx = 0$$

Now L_1 is a constant, independent of x, and so

$$\rho^4 L_1 - \rho^4\, 10^{-9} + \frac{\rho^4}{2}\, 10^{-9} = 0.$$

Therefore $$L_1 = \tfrac{1}{2} \times 10^{-9} \tag{5}$$

The voltage drop between the wires A and B due to the flux inside both wires is

$$\begin{aligned} 2\,j\omega I L_1 &= 2j\omega I \times \tfrac{1}{2} \times 10^{-9} \text{ volts per cm.} \\ &= 2\,j\omega \times 80.47 \times 10^{-6} \end{aligned}$$

volts per ampere per mile. The total reactive drop between the wires is thus

$$2\,j\omega\left(80.47 + 741.1 \log_{10}\frac{s}{\rho}\right) \times 10^{-6} \tag{6}$$

volts per ampere per mile of single phase line.

This may be written in the following form which is more convenient for computation by means of logarithm tables:

$$\text{Reactance drop} = 2\,j\omega \times 741.1 \log_{10}\frac{s}{0.779\,\rho} \times 10^{-6} \tag{7}$$

volts per ampere per mile of single-phase line.

The above is the usual formula for reactance of a single-phase line. The proof is longer than that generally given, but it has the advantage of giving a correct idea of the distribution of current and magnetic flux inside the wire. As the irregular distribution of current produces the "skin effect" described in the next chapter, and necessitates slight corrections in the above formula for reactance and in the resistance, the importance of calculating the correct current distribution is evident. The above formula is sufficiently accurate, however, for calculating the tables of reactance of wire in Part III.

CHAPTER X.

SKIN EFFECT.

In the last chapter a local quadrature current $i_{(x)}$ was assumed, whose resistance drop balances up the unequal voltages produced at the center and near the surface by the flux inside the wire. This local current, $i_{(x)}$, when added up over all parts of the section of the wire, amounts to zero, and so cannot produce magnetic lines in the air outside the wire. But it can produce lines inside the wire, and the effect of these will now be calculated.

The reactive drop in one wire due to the flux inside it produced by the main current i is

$$j\omega\pi i\rho^2 L_1 = j\omega\pi i\rho^2 \times \tfrac{1}{2} \times 10^{-9} \text{ volts,}$$

where i is in amperes.

Then at a distance x from the center we have, from equation (4), Chap. IX,

$$\begin{aligned} i_{(x)} &= \frac{j\omega\pi i\rho^2}{r} \times \frac{1}{2} \times 10^{-9} - \frac{j\omega\pi i}{r}(\rho^2 - x^2) \times 10^{-9} \\ &= \frac{j\omega\pi i}{r}\left(-\frac{\rho^2}{2} + x^2\right) \times 10^{-9}. \end{aligned}$$

This is a lagging current at the center and a leading current at the surface, and it equals zero when integrated over the entire section.

The current $i_{(x)}$, integrated over the circle of radius x, is

$$\begin{aligned} I_{(x)} &= \int_0^x \frac{2j\omega\pi^2 i}{r} \times 10^{-9}\left(-\frac{\rho^2 x}{2} + x^3\right)dx \\ &= \frac{2j\omega\pi^2 i}{r} \times 10^{-9}\left(-\frac{\rho^2 x^2}{4} + \frac{x^4}{4}\right). \end{aligned}$$

The flux density at the element dx, due to the above current, is

$$\frac{2\,I_{(x)}}{10\,x} = \frac{j\omega\pi^2 i}{10\,r} \times 10^{-9}\,(-\,\rho^2 x + x^3).$$

The flux in the ring outside of the circle of radius x, due to $I_{(x)}$, is

$$\begin{aligned}\phi_{(x)} &= \frac{j\omega\pi^2 i}{10\,r} \times 10^{-9} \int_x^{\rho} (-\,\rho^2 x + x^3)\,dx \\ &= \frac{j\omega\pi^2 i\ 10^{-9}}{10\,r}\left(-\frac{\rho^4}{2} + \frac{\rho^2 x^2}{2} + \frac{\rho^4}{4} - \frac{x^4}{4}\right) \\ &= \frac{j\omega\pi^2 i\ 10^{-9}}{40\,r}(-\,\rho^4 + 2\,\rho^2 x^2 - x^4).\end{aligned}$$

This flux produces a voltage at the element dx, equal to

$$-\frac{\omega^2\pi^2 i\ 10^{-18}}{4\,r}(-\,\rho^4 + 2\,\rho^2 x^2 - x^4). \qquad (1)$$

A local current, $i_{(2x)}$, will flow in order to keep the voltage drop uniform over the section. Let the average drop due to $\phi_{(x)}$ be

$$j\omega I L_2 = j\omega\pi\rho^2 i L_2;$$

then

$$j\omega\pi\rho^2 i L_2 = \frac{\omega^2\pi^2 i\ 10^{-18}}{4\,r}(\rho^4 - 2\,\rho^2 x^2 + x^4) + i_{(2x)}\,r \qquad (2)$$

Integrate $i_{(2x)}$ over the entire surface and as it is a local current

$$\begin{aligned}\int_0^{\rho} 2\,\pi x i_{(2x)}\,dx &= 0 \\ &= \frac{2\,j\pi^2\omega i}{r}\int_0^{\rho} \rho^2 L_2 x\,dx \\ &\quad - \frac{\omega^2\pi^3 i\ 10^{-18}}{2\,r^2}\int_0^{\rho} (\rho^4 x - 2\,\rho^2 x^3 + x^5)\,dx.\end{aligned}$$

Therefore, $$jL_2\rho^4 = \frac{\omega\pi\ 10^{-18}}{2\,r}\left(\frac{\rho^6}{2} - \frac{\rho^6}{2} + \frac{\rho^6}{6}\right)$$

and $$L_2 = -j\,\frac{1}{12}\,\frac{\omega\pi\rho^2\ 10^{-18}}{r}. \tag{3}$$

Thus $$i_{(2x)} = -\frac{\omega^2\pi^2 i\ 10^{-18}}{12\,r^2}(2\,\rho^4 - 6\,\rho^2 x^2 + 3\,x^4) \tag{4}$$

This current is in phase with the main current and, as it is negative at the center and positive at the surface, it produces a stronger resultant current near the surface of the wire. This is the well-known "skin effect." The effect of the quadrature current $i_{(x)}$ is to increase the resultant current both at the center and near the surface, but its effect is not as large as that of the in-phase current $i_{(2x)}$ and so the net result is a crowding of current toward the surface.

The above process may be continued indefinitely, each step adding a smaller correction than the one before to the current at radius x and to the average drop in the wire.

Thus the expression for $i_{(2x)}$, equation (4), may be integrated over the circle of radius x, and will give the value of $I_{(2x)}$. This current produces a flux density at the radius x, and by integrating this over the outer ring of the section, the value of $\phi_{(2x)}$ is obtained. The flux $\phi_{(2x)}$ produces an unbalanced voltage which must be corrected by a local current $i_{(3x)}$, so as to give a uniform drop over the section, due to the inductance L_3. Equating the total local current to zero, as before, gives

$$L_3 = -\frac{1}{48}\,\frac{\omega^2\pi^2\rho^4\ 10^{-27}}{r^2}$$

In the same way it is found that

$$L_4 = j\,\frac{1}{180}\,\frac{\omega^3\pi^3\rho^6\ 10^{-36}}{r^3},$$

and
$$L_5 = \frac{13}{8640}\,\frac{\omega^4\pi^4\rho^8\,10^{-45}}{r^4}.$$

Let the resistance of the wire per centimeter be R, where

$$R = \frac{r}{\pi\rho^2} \text{ ohms per cm.,}$$

and let
$$m = \frac{\omega\pi\rho^2\,10^{-9}}{r}$$
$$= \frac{\omega\,10^{-9}}{R}.$$

Then the total drop in the wire is

$$IR + j\omega I\,(L + L_1 + L_2 + \cdots)$$
$$= IR + j\omega I\,10^{-9}\Bigl(2\log_\epsilon\frac{s}{\rho} + \frac{1}{2} - j\frac{1}{12}m$$
$$-\frac{1}{48}m^2 + j\,\frac{1}{180}\,m^3 + \frac{13}{8640}\,m^4 - \cdots\Bigr) \qquad (5)$$

volts per centimeter.

The total drop in phase with the current is

$$IR\left(1 + \frac{1}{12}m^2 - \frac{1}{180}m^4 + \cdots\right). \qquad (6)$$

The total copper loss due to all the currents in the wire is therefore equal to

$$I^2R\left(1 + \frac{1}{12}m^2 - \frac{1}{180}m^4 + \cdots\right).$$

This can be checked by integrating the losses due to the total in-phase and quadrature currents in all parts of the section of the wire, the above result being obtained by this method also. Thus, in every respect, both as to voltage drop and watts loss, the resistance of the wire to the alternating current is

$$R' = R\left(1 + \frac{1}{12}m^2 - \frac{1}{180}m^4 \cdots\right).$$

Values of R' for both 25 and 60 cycles are tabulated in Part III. · When taking the resistance of a conductor from the tables, R' should always be used for alternating current, and R should be used only when the conductor carries direct current.

The total drop in quadrature with the current is

$$j\omega I\ 10^{-9}\left(2\log_\epsilon\frac{s}{\rho}+\frac{1}{2}-\frac{1}{48}m^2+\frac{13}{8640}m^4-\cdots\right)$$

$$=j\omega I\ 10^{-9}\left\{2\log_\epsilon\frac{s}{\rho}+\frac{1}{2}\left(1-\frac{1}{24}m^2+\frac{13}{4320}m^4-\cdots\right)\right\} \quad (7)$$

The series $$1-\frac{1}{24}m^2+\frac{13}{4320}m^4-\cdots$$

is thus a correction factor for the term $\frac{1}{2}$ or 80.47 in the ordinary formula for reactance. Its effect is too small, however, to make any appreciable change in the tabulated values of reactance.

Proof by Infinite Series. — The above formulas for the resistance and inductance of a wire carrying alternating current are sufficiently accurate for transmission line calculations with ordinary frequencies. They may also be extended to include more terms without undue labor. However, as skin effect formulas are generally obtained and expressed by means of infinite series which can be carried out to any degree of accuracy for high-frequency work, a short outline of the derivation of the infinite series will be given. It will prove a check upon the correctness of the formulas given above, but it will probably not give as clear an idea as they do of the actual distribution of current in the wire.

Let an alternating current, I, of sine wave form and of steady value, flow in a round wire of radius ρ. (See Fig. 14, Chap. IX.) Let it take up such a distribution that the drop at all parts of the section of the wire, due to resistance and to magnetic flux, is the same. Then if i' be the current density at radius x, we may assume

$$i' = a_0 + a_1x^2 + a_2x^4 + \cdots + a_nx^{2n} + \cdots \tag{8}$$

where a_0, a_1, ... a_n, etc., are constants, independent of x. (As the same value of i' would be obtained for both $+x$ and $-x$, only even powers of x need be assumed for the series.)

The total current in the part of the section inside a circle of radius x will be

$$\begin{aligned} I' &= \int_0^x 2\,\pi x i'\,dx \\ &= 2\,\pi\left(\frac{a_0x^2}{2} + \frac{a_1x^4}{4} + \cdots + \frac{a_{n-1}x^{2n}}{2\,n} + \cdots\right). \end{aligned} \tag{9}$$

The flux density at the radius x is

$$\frac{2\,I'}{10\,x} = \frac{2\,\pi}{10}\left(a_0x + \frac{a_1x^3}{2} + \cdots + \frac{a_{n-1}x^{2n-1}}{n} + \cdots\right),$$

and the total flux in the outer ring of the section, outside the circle of radius x, is

$$\begin{aligned} \phi' &= \int_x^\rho \frac{2\,I'}{10\,x}\,dx \\ &= \frac{\pi}{10}\left(a_0\rho^2 + \frac{a_1\rho^4}{2^2} + \frac{a_2\rho^6}{3^2} + \cdots + \frac{a_{n-1}\rho^{2n}}{n^2} + \cdots\right) \\ &\quad - \frac{\pi}{10}\left(a_0x^2 + \frac{a_1x^4}{2^2} + \frac{a_2x^6}{3^2} + \cdots + \frac{a_{n-1}x^{2n}}{n^2} + \cdots\right). \end{aligned} \tag{10}$$

The drop at radius x due to the flux ϕ' is

$$j\omega\phi' \times 10^{-8}$$

and the resistance drop due to the current at the same part is $i'r$. Thus the total drop per centimeter of wire, which is the same at all parts of the section, is

$$
\begin{aligned}
V &= j\omega\phi' \, 10^{-8} + i'r \\
&= j\omega\pi \, 10^{-9}\left(a_0\rho^2 + a_1\frac{\rho^4}{2^2} + \frac{a_2\rho^6}{3^2} + \cdots + \frac{a_{n-1}\rho^{2n}}{n^2} + \cdots\right) \\
&\quad - j\omega\pi \, 10^{-9}\left(a_0x^2 + \frac{a_1x^4}{2^2} + \frac{a_2x^6}{3^2} + \cdots + \frac{a_{n-1}x^{2n}}{n^2} + \cdots\right) \\
&\quad + r\left(a_0 + a_1x^2 + a_2x^4 + \cdots + a_nx^{2n} + \cdots\right). \qquad (11)
\end{aligned}
$$

The above expression for V is the same for all values of x, and we may therefore equate each coefficient of x^2, x^4, etc., to zero. Thus, putting

$$\frac{\omega\pi\rho^2 \, 10^{-9}}{r} = m,$$

we have

$$a_1 = \frac{jma_0}{\rho^2}$$

$$a_2 = \frac{jma_1}{2^2\rho^2}$$

$$\cdots\cdots\cdots$$

$$a_n = \frac{jma_{n-1}}{n^2\rho^2}$$

$$\cdots\cdots\cdots \quad \text{etc.}$$

and

$$V = a_0r + j\omega\pi \, 10^{-9}\left(a_0\rho^2 + \frac{a_1\rho^4}{2^2} + \cdots + \frac{a_{n-1}\rho^{2n}}{n^2} + \cdots\right)$$

Substituting the values of a_1, a_2, etc., we obtain

$$
\begin{aligned}
V &= a_0r + j\pi\omega \, 10^{-9} \\
&\quad \left\{a_0\rho^2 + \frac{jma_0\rho^2}{(\lfloor 2)^2} + \frac{(jm)^2a_0\rho^2}{(\lfloor 3)^2} + \cdots + \frac{(jm)^{n-1}a_0\rho^2}{(\lfloor n)^2} + \cdots\right\} \\
&= a_0r\left\{1 + jm + \frac{(jm)^2}{(\lfloor 2)^2} + \frac{(jm)^3}{(\lfloor 3)^2} + \cdots + \frac{(jm)^n}{(\lfloor n)^2} + \cdots\right\} \qquad (12)
\end{aligned}
$$

Now by putting $x = \rho$ in the expression for I', equation (9), we obtain the value of the total current in the wire,

$$I = \pi\left(a_0\rho^2 + \frac{a_1\rho^4}{2} + \frac{a_2\rho^6}{3} + \cdots + \frac{a_{n-1}\rho^{2n}}{n} + \cdots\right)$$

$$= \pi a_0\rho^2\left\{1 + \frac{2\,jm}{(\underline{|2})^2} + \frac{3\,(jm)^2}{(\underline{|3})^2} + \cdots + \frac{n\,(jm)^{n-1}}{(\underline{|n})^2} + \cdots\right\}.$$

Therefore,

$$a_0 = \frac{I}{\pi\rho^2\left\{1 + \frac{2\,jm}{(\underline{|2})^2} + \frac{3(jm)^2}{(\underline{|3})^2} + \cdots + \frac{n\,(jm)^{n-1}}{(\underline{|n})^2} + \cdots\right\}}.$$

Substituting this value of a_0 in equation (12), and putting

$$\frac{r}{\pi\rho^2} = R,$$

the resistance per centimeter of the wire, we obtain

$$V = IR\frac{1 + jm + \frac{(jm)^2}{(\underline{|2})^2} + \cdots + \frac{(jm)^n}{(\underline{|n})^2} + \cdots}{1 + \frac{2\,jm}{(\underline{|2})^2} + \frac{3\,(jm)^2}{(\underline{|3})^2} + \cdots + \frac{n\,(jm)^{n-1}}{(\underline{|n})^2} + \cdots}. \quad (13)$$

This expression can evidently be carried to any accuracy desired. It will give the same results as were previously obtained in equation (5), by expanding the denominator as a binomial of the form $(1 + x)^{-1}$ and multiplying by the numerator. This gives

$$V = IR\{1 + \tfrac{1}{2}jm - \tfrac{1}{12}(jm)^2 + \tfrac{1}{48}(jm)^3 - \tfrac{1}{180}(jm)^4 + \tfrac{13}{8640}(jm)^5 - \cdots\},$$

or,

$$V = IR\,(1 + \tfrac{1}{12}m^2 - \tfrac{1}{180}m^4 + \cdots) + \tfrac{1}{2}j\omega I\,10^{-9}(1 - \tfrac{1}{24}m^2 + \tfrac{13}{4320}m^4 - \cdots).$$

This is the voltage drop, omitting the effect of the flux outside the wire, and is the same as the value previously obtained. (See equations 6 and 7.)

References.

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Rayleigh, *Phil. Mag.*, 1886, Vol. 21, page 381.
Kelvin, *Math. Papers*, 1889, Vol. 3, page 491.
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CHAPTER XI.

REACTANCE OF CABLE, SINGLE-PHASE.

As stranded cables are very commonly used for transmission lines, it is desirable to have a special formula for the reactance of cables. An outline will be given of the method of obtaining the formula for a seven-wire cable. This formula was used in preparing the reactance tables in Part III.

A seven-wire strand consists of a central straight wire, with six wires of the same size laid around it in a spiral. The spiralling of the wires increases the resistance of the cable by an amount which is taken as 1%. The spiralling also increases the outside diameter of the cable by about $\frac{6}{10}$ of 1%. (See Chapter VII.)

In calculating the reactance of the cable, the first step is to plot the flux density at various distances from the center of the cable. (See Fig. 15.) For points entirely outside the cable, the flux density obeys the law

$$f_{(x)} = \frac{2\,I}{10\,x},$$

where x is the distance from the center. The total voltage due to these lines which cut the entire cable is

$$2 \log_\epsilon \frac{S}{\rho} \times 10^{-9}$$

volts per ampere per centimeter. When x is less than the radius of the cable, the flux at the distance x is

$$f_{(x)} = \frac{2\,I_{(x)}}{10\,x}.$$

$I_{(x)}$ is proportional to the area of conductor inside the circle of radius x, and this must be measured from the diagram of the section of the conductor (Fig. 15).

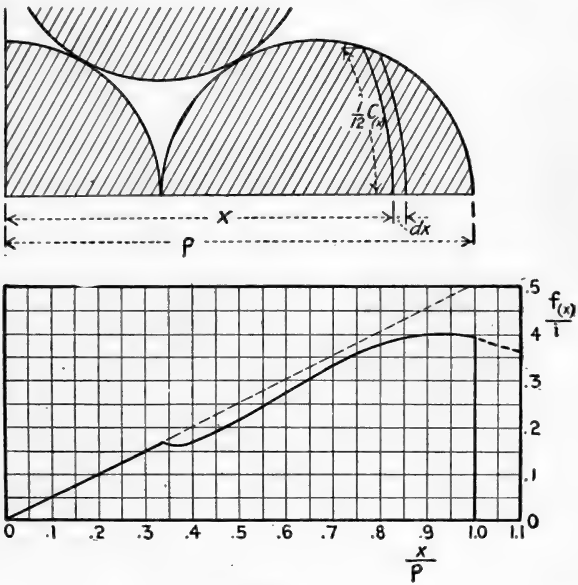


Fig. 15.

Plot a curve of flux density, $f_{(x)}$, for various values of x. Let $\phi_{(x)}$ be the area of the curve of $f_{(x)}$ between the values x and ρ, where ρ is the outside radius of the cable. Also, let $c_{(x)}$ be the length of that part of the circle $2\pi x$ which lies in the section of the wires of the cable.

As shown in Chapter IX, page 65, the voltage drop along different parts of the cable is not uniform, and must be balanced by a local quadrature current. Thus we have the conditions

$$j\omega I L_1 = j\omega\phi_{(x)}\, 10^{-8} + i_{(x)}\, r$$

at any section,

and $$\sum_0^\rho i_{(x)} c_{(x)}\, dx = 0.$$

Substituting the value of $i_{(x)}$ from the first equation, we have

$$\sum_0^\rho I L_1 c_{(x)}\, dx - \sum_0^\rho \phi_{(x)} c_{(x)}\, dx\, 10^{-8} = 0.$$

Now IL_1 is a constant, and

$$\sum_0^\rho c_{(x)}\, dx = A,$$

the area of the section of the cable.

By dividing ρ into a number of equal parts and calling each part dx, the value of $\phi_{(x)} c_{(x)}\, dx$ for each successive value of x is found. Adding these together, a close estimate of the value of

$$\sum_0^\rho \phi_{(x)} c_{(x)}\, dx$$

is obtained. This, divided by IA gives the value

$$L_1 = 0.633 \times 10^{-9}.$$

If the conductor were a solid wire of radius ρ, with a straight line curve of $f_{(x)}$, L_1 would be $\frac{1}{2} \times 10^{-9}$, which would become 80.5×10^{-6} when reduced to the ordinary formula for reactance per mile. Thus the reactance per mile of cable is

$$x = \left(80.5 \times \frac{0.633}{0.500} + 741.1 \log_{10} \frac{s}{\rho}\right) 10^{-6} \times 2\,\pi f$$

$$= \left(102 + 741.1 \log_{10} \frac{s}{\rho}\right) 10^{-6} \times 2\,\pi f$$

ohms per mile.

The same process applied to a 19-wire strand gives the formula

$$x = \left(89 + 741.1 \log_{10} \frac{s}{\rho}\right) 10^{-6} \times 2\,\pi f$$

ohms per mile.

CHAPTER XII.

REACTANCE OF TWO-PHASE AND THREE-PHASE LINES.

Reactance, Two-phase. — The reactive drop in a single-phase line, in which round wire is used, is

$$2jI \times 2\pi f\left(80.5 + 741.1 \log\frac{s}{\rho}\right) 10^{-9}$$

volts per mile of line. In a two-phase four-wire line, the drop will be the same as the above, when I is the current in one phase.

Now $\qquad 2I = \dfrac{\text{K.V.A.}}{E} = \text{the total amperes.}$

Therefore the reactive drop per mile of line, in absolute value, is

$$\frac{\text{K.V.A.}}{E} \times 2\pi f\left(80.5 + 741.1 \log\frac{s}{\rho}\right) 10^{-9}$$

$$= \frac{\text{K.V.A.}}{E} \times x,$$

wnere x is the tabulated value of reactance.

Reactance, Three-phase, Irregular Spacing. — When the conductors of a three-phase line are spaced so that they are not exactly equidistant, the voltage drop due to reactance is not the same in the different phases. It is the practice with such lines to interchange, or transpose, the conductors at intervals along the line, so that the different reactive voltages are applied to an equal extent to all three conductors. Such a line, when carrying a balanced load (equal currents in each conductor, at 120° in phase from each other), will have the voltages of the three phases equal at the end of the line.

The average reactance of an irregularly spaced three-phase line, in which the conductors are transposed at regular intervals, may be calculated as follows:

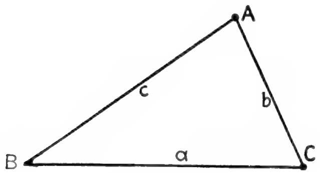


Fig. 16.

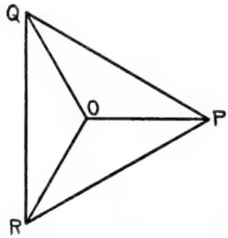


Fig. 17.

Let Fig. 16 represent the spacing of the conductors A, B, and C of a transmission line. Let the currents in the conductors be represented by the vectors OP, OQ, and OR (Fig. 17). If the power factor is 100%, these vectors may also represent the star voltages, and the line voltages will be represented by the vectors PQ, QR, and RP.

Let the current in conductor A be

$$I_A = OP = 1.00\, I \text{ amperes,}$$

and let $$I_B = OQ = (-0.50 + 0.866j)\, I \text{ amperes,}$$

and $$I_C = OR = (-0.50 - 0.866j)\, I \text{ amperes.}$$

Let also

Voltage from neutral to $A = 1.00\, V$.
Voltage from neutral to $B = (-0.50 + 0.866j)\, V$.
And voltage from neutral to $C = (-0.50 - 0.866j)\, V$.

Then the measured, or absolute, value of voltage between B and C is $1.732\, V$, where V is the star voltage of the line.

The reactive voltage on conductor A per mile is

$$1.00\left(80.5 + 741.1 \log \frac{\infty}{\rho}\right) Ij\, 2\,\pi f \times 10^{-6}$$

(due to flux from I_A)

$$+ (-0.50 + 0.866 j)\left(741.1 \log \frac{\infty}{c}\right) Ij\, 2\,\pi f \times 10^{-6}$$

(due to flux from I_B)

$$+ (-0.50 - 0.866 j)\left(741.1 \log \frac{\infty}{b}\right) Ij\, 2\,\pi f \times 10^{-6}$$

(due to flux from I_C),

where ρ is the radius of the wire.

The reactive voltage on conductor B per mile is

$$1.00\left(741.1 \log \frac{\infty}{c}\right) Ij\, 2\,\pi f \times 10^{-6}$$

(due to flux from I_A)

$$+ (-0.50 + 0.866 j)\left(80.5 + 741.1 \log \frac{\infty}{\rho}\right) Ij\, 2\pi f \times 10^{-6}$$

(due to flux from I_B)

$$+ (-0.50 - 0.866 j)\left(741.1 \log \frac{\infty}{a}\right) Ij\, 2\,\pi f \times 10^{-6}$$

(due to flux from I_C).

The reactive voltage between A and B is therefore

$$(1.50 - 0.866 j)\left(80.5 + 741.1 \log \frac{c}{\rho}\right) Ij\, 2\,\pi f \times 10^{-6}$$

$$+ (0.50 + 0.866 j)\, 741.1\left(\log \frac{b}{\rho} - \log \frac{a}{\rho}\right) Ij\, 2\pi f \times 10^{-6}.$$

Suppose at the end of one mile the line is transposed so that the above two conductors occupy the positions B

and C. Then the reactive voltage between these conductors for the next mile is

$$(1.50 - 0.866j)\left(80.5 + 741.1 \log\frac{a}{\rho}\right) Ij\, 2\,\pi f \times 10^{-6}$$

$$+ (0.50 + 0.866j)\, 741.1 \left(\log\frac{c}{\rho} - \log\frac{b}{\rho}\right) Ij\, 2\,\pi f \times 10^{-6}.$$

Let the line be transposed again. Then for the third mile the reactance voltage between these same two conductors is

$$(1.50 - 0.866j)\left(80.5 + 741.1 \log\frac{b}{\rho}\right) Ij\, 2\,\pi f \times 10^{-6}$$

$$+ (0.50 + 0.866j)\, 741.1 \left(\log\frac{a}{\rho} - \log\frac{c}{\rho}\right) Ij\, 2\,\pi f \times 10^{-6}.$$

The total reactive voltage for the three miles is

$$(1.50-0.866j)\left\{80.5\times 3+741.1\left(\log\frac{a}{\rho} + \log\frac{b}{\rho} + \log\frac{c}{\rho}\right)\right\}$$

$$\times\, Ij\, 2\,\pi f \times 10^{-6}.$$

Thus the average reactive voltage per mile, which is the same for all phases, is, in absolute value,

$$1.732\left(80.5 + 741.1 \log\frac{\sqrt[3]{abc}}{\rho}\right) I\, 2\,\pi f \times 10^{-6}.$$

Now $$1.732\, I = \text{the total amperes.}$$

$$= \frac{\text{K.V.A.}}{E}.$$

Therefore the drop in line voltage, in absolute value, is

$$\left(80.5 + 741.1 \log\frac{s}{\rho}\right) 2\,\pi f \times 10^{-6}$$

in volts per total ampere per mile of transmission line, where

$$s = \sqrt[3]{abc}.$$

Thus the total reactive drop is equal to

$$\frac{\text{K.V.A.}}{E}\left(80.5 + 741.1 \log \frac{s}{\rho}\right) 2\,\pi f \times 10^{-6}$$
$$= \frac{\text{K.V.A.}}{E} \times x,$$

where x is the tabulated value of reactance.

Reactance, Three-phase, Regular Spacing. — When the conductors are spaced at the corners of an equilateral triangle of side s, then the expression for reactance is the same as the usual formula:

$$x = \left(80.5 + 741.1 \log_{10} \frac{s}{\rho}\right) 2\,\pi f \times 10^{-6}.$$

Reactance, Three-phase, Flat Spacing. — When the three conductors lie in one plane (either horizontal or vertical), the center one being equidistant from the other two, as in Fig. 18, the reactance per mile is

Fig. 18.

$$80.5 + 741.1 \log \frac{a\sqrt[3]{1 \times 1 \times 2}}{\rho}$$
$$= 80.5 + 741.1 \log \frac{1.26\,a}{\rho}.$$

This is approximately 4% higher in an ordinary case than the reactance for spacing on an equilateral triangle of side a.

The formulas of this chapter will apply to cable as well as to wire, if the term 80.5 is changed as per Chapter XI.

CHAPTER XIII.

CAPACITY OF SINGLE-PHASE LINE.

Capacity of Two Round Wires. — The conductors of a transmission line form a condenser, the electrostatic capacity of which can be calculated from the dimensions of the line. The simplest line to calculate is a single-phase line consisting of two round wires, and this case will be investigated first.

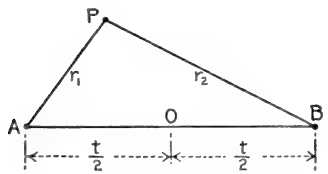


Fig. 19.

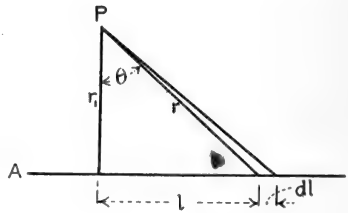


Fig. 20.

Suppose that A and B (Fig. 19) are two long parallel wires of infinitesimal section and that they are spaced a distance t centimeters apart. Let A carry a charge of $+q$ electrostatic units of electricity per centimeter, and let B carry $-q$ units per centimeter.

First, find the force exerted on a unit charge near the wire A.

From the symmetry of the arrangement it is evident that the resultant force on a unit charge at P (Fig. 20) will be a repulsion away from the wire at right angles to it, since the total effect of the right half of the wire must be equal to the total effect of the left half. The force at right angles to the wire exerted by the charge on the element dl will be

$$q\frac{dl}{r^2}\cos\theta = \frac{q}{r_1}\cos\theta\, d\theta,$$

since $$dl\cos\theta = r\,d\theta$$

and $$r = \frac{r_1}{\cos\theta}.$$

The total force exerted by the wire will be

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{q}{r_1}\cos\theta\, d\theta = \frac{2\,q}{r_1}.$$

The potential of the point O (Fig. 19) midway between the wires, will be zero, since the effect of the positive charge on A will be equal to the effect of the negative charge on B. The potential difference between P and O is the work done in moving a unit charge from one point to the other. The force due to the wire A on a unit charge at any point is $\frac{2\,q}{r}$, acting directly away from A. Therefore the work done in moving a distance dr toward A is

$$\frac{2\,q}{r}dr$$

and thus the total work in moving from P to O against the force due to A is

$$\int_{r_1}^{\frac{t}{2}}\frac{2\,q}{r}dr = 2\,q\log_\epsilon\frac{t}{2\,r_1}.$$

Similarly, the work against the force due to B is equal to

$$-\,2\,q\log_\epsilon\frac{t}{2\,r_2}.$$

Therefore the potential difference between P and O is equal to the total work, and is

$$2\,q \log_\epsilon \frac{r_2}{r_1}.$$

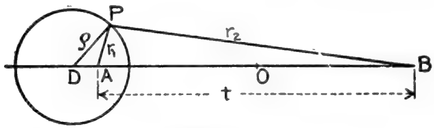


Fig. 21.

At P (Fig. 21) make the angle APD equal to the angle PBD. Then the triangle PBD is similar to the triangle APD, and therefore

$$\frac{PB}{AP} = \frac{r_2}{r_1}$$
$$= \frac{DB}{\rho}.$$

If we draw a circle of radius ρ about the fixed point D, then at any point on this circle similar triangles are formed by ρ, r_1, and r_2, as in Fig. 21, and therefore

$$\frac{r_2}{r_1} = \frac{DB}{\rho} = \text{constant},$$

where r_1 and r_2 are the distances of the point on the circle from A and B respectively. Therefore, the potential

$$2\,q \log_\epsilon \frac{r_2}{r_1}$$

will be the same at all points on this circle.

Now let a solid cylindrical conductor fill all the space inside the circle of radius ρ. All points on its surface will be at the same potential. The distribution of potential outside of the cylinder will not be altered from the previous condition when all points on the circle of radius ρ were also at the same potential. The potential of the cylinder will be

$$2\,q \log_\epsilon \frac{r_2}{r_1} = 2\,q \log_\epsilon \frac{DB}{\rho}.$$

In the same way, let the wire B be replaced by a solid cylinder of radius ρ and center E, as in Fig. 22.

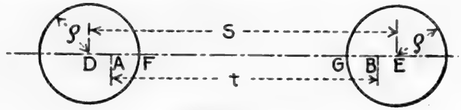


Fig. 22.

The potential of this cylinder, which carries $-\,q$ units per centimeter, will be

$$-\,2\,q \log_\epsilon \frac{AE}{\rho} = -\,2\,q \log_\epsilon \frac{DB}{\rho},$$

since the second cylinder is symmetrical with the first.

Thus the potential difference between the two cylinders is

$$4\,q \log_\epsilon \frac{DB}{\rho} = \frac{q}{C},$$

where C is the capacity per centimeter of line.

Thus
$$C = \frac{1}{4 \log_\epsilon \cdot \dfrac{DB}{\rho}}.$$

Now
$$\frac{DB}{\rho} = \frac{FB}{FA},$$

since F is a point on the circle.

Therefore
$$\frac{DB}{\rho} = \frac{DB - \rho}{DB + \rho - s},$$

where s is the interaxial distance between the two solid cylinders, and is equal to DE, Fig. 22.

Therefore,

$$DB^2 + DB \cdot \rho - DB \cdot s = DB \cdot \rho - \rho^2$$

or

$$DB^2 - DB \cdot s + \rho^2 = 0.$$

Solving this quadratic equation in DB, we have

$$DB = \frac{s + \sqrt{s^2 - 4\rho^2}}{2}.$$

[The negative value of the radical must not be used, since it would give the value of DA instead of DB.]

Therefore,

$$C = \frac{1}{4 \log_\epsilon \left(\frac{s}{2\rho} + \sqrt{\frac{s^2}{4\rho^2} - 1} \right)},$$

which may be expressed as

$$C = \frac{1}{4 \cosh^{-1}\left(\frac{s}{2\rho}\right)},$$

or it may be expanded by the Binomial Theorem to give the approximate value

$$C = \frac{1}{4 \log_\epsilon \left(\frac{s}{\rho} - \frac{\rho}{s} \right)} \text{ per centimeter,}$$

or, very nearly,

$$C = \frac{1}{4 \log_\epsilon \frac{s}{\rho}}.^*$$

* It is evident that the expression

$$C = \frac{1}{4 \log_\epsilon \frac{s - \rho}{\rho}},$$

which is sometimes published, is less accurate than the simpler expression

$$C = \frac{1}{4 \log_\epsilon \frac{s}{\rho}}.$$

Transferring to other units,

$$C = \frac{1}{\log_{10}\left(\frac{s}{\rho} - \frac{\rho}{s}\right)} \times \frac{.4343 \times 2.540 \times 12 \times 5280}{4 \times 9 \times 10^{11}}$$

$$= \frac{1}{2} \times \frac{38.83 \times 10^{-9}}{\log_{10}\left(\frac{s}{\rho} - \frac{\rho}{s}\right)}$$

farads per mile of single-phase line.

The capacity susceptance is

$$2\pi f C = 2\pi f \times \frac{1}{2} \times \frac{38.83 \times 10^{-9}}{\log_{10}\left(\frac{s}{\rho} - \frac{\rho}{s}\right)}$$

mhos per mile of single-phase line. The charging current in this line will be

$$\frac{1}{2} E \times 2\pi f \times \frac{38.83 \times 10^{-9}}{\log_{10}\left(\frac{s}{\rho} - \frac{\rho}{s}\right)} \text{ amperes,}$$

$$= \tfrac{1}{2} Eb,$$

where b is the tabulated value of capacity susceptance.

REFERENCE. — "A Treatise on the Theory of Alternating Currents," by Alexander Russell, 1904, Vol. I, page 99.

Capacity of Cable. — The formula for capacity of a line using stranded cables will be the same as the above formula for solid wires, ρ being taken as the maximum radius of the cable. All the electrostatic charge on the cable does not lie at the maximum radius from the center, but as actual cables are generally slightly larger than the calculated diameters in the tables, it will be sufficiently close to take ρ from the tables and use it in the regular formula for capacity.

Effect of the Earth on Capacity of Line. — The effect of bringing a conducting plane, such as the earth, near to two charged wires is to change their electrostatic field and increase their capacity.

Consider two long parallel wires, A and A_1 (Fig. 23), of infinitesimal section and carrying $+q$ and $-q$ units of electricity per centimeter respectively. As in Fig. 19, the point O midway between the two wires will be at zero potential. All points having the same potential must have $\frac{r_1}{r_2}$ equal to a constant. It is evident that all points at the same potential as O lie in the plane MON, perpendicular to AA_1, since for all such points

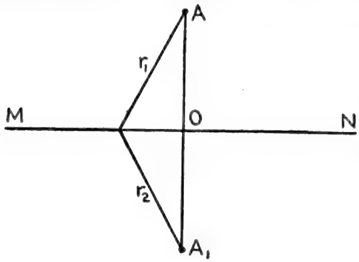


Fig. 23.

$$r_1 = r_2.$$

Therefore, the wire A_1' may be replaced by a solid conducting plane MN, which will be at zero potential. Thus, when the conducting plane is the earth, its effect is the same as that of a charged wire at a depth below the surface equal to the height of the original wire.* The assumed wire is called an image wire, since it occupies the same position as the image of the real wire, considering the surface of the ground as a mirror.

In the case of a single-phase transmission line, image wires A' and B' must be assumed for both wires A and B

* See "Elements of Electricity and Magnetism," by J. J. Thomson, page 138.

(Fig. 24) and the capacity of the entire system of four wires is then calculated as follows:

Let h be the distance of the wires from the ground and s their distance apart from center to center. Let A carry a charge of $+ q$ units per centimeter; A', $- q$ units; B, $- q$ units; and B', $+ q$ units. Let a unit charge be carried from the surface of A to that of B. Assuming that the charges are concentrated at the centers of the wires, the total work done is equal to

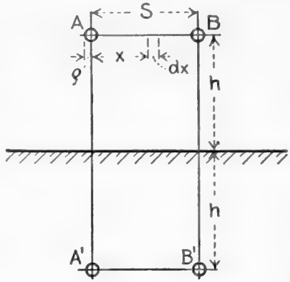


Fig. 24.

$$\int_{\rho}^{s-\rho} \frac{2\,q\,dx}{x} + \int_{\rho}^{s-\rho} \frac{2 \cdot q}{s-x}\,dx - \int_{\rho}^{s-\rho} \frac{2\,qx}{4\,h^2 + x^2}\,dx$$

$$- \int_{\rho}^{s-\rho} \frac{2\,q\,(s-x)}{4\,h^2 + (s-x)^2}\,dx.$$

The sum of the first two integrals has been shown to be approximately

$$4\,q \log_{\epsilon} \frac{s}{\rho}.$$

The sum of the last two integrals is approximately

$$- q \log_{\epsilon} \frac{4\,h^2 + s^2}{4\,h^2} + q \log_{\epsilon} \frac{4\,h^2}{4\,h^2 + s^2}$$

$$= 2\,q \log_{\epsilon} \frac{4\,h^2}{4\,h^2 + s^2}.$$

Therefore, the total work is equal to

$$4\,q \log_{\epsilon} \frac{s}{\rho} + 4\,q \log_{\epsilon} \frac{2\,h}{\sqrt{4\,h^2 + s^2}} = \frac{q}{C}.$$

Therefore, C is approximately

$$\frac{1}{4 \log_\epsilon \frac{s}{\rho} + 4 \log_\epsilon \frac{2h}{\sqrt{4h^2 + s^2}}}.$$

Taking as an average case,

$$h = 360 \text{ inches (30 feet)},$$
$$s = 120 \text{ inches (10 feet)},$$
$$\rho = 0.25 \text{ inch},$$

we have $$\frac{s}{\rho} = 480,$$

and $$\frac{2h}{s} = 6.$$

Therefore, $$\frac{2h}{\sqrt{4h^2 + s^2}} = \frac{6}{\sqrt{37}}.$$

Now, $$\log_{10} 480 = 2.681,$$

while $$\log_{10} \frac{\sqrt{37}}{6} = 0.006.$$

Thus the capacity is changed by the nearness of the ground by less than $\frac{1}{4}$ of 1%, even with the comparatively wide spacing of 10 feet.

Tests have shown that the effect of the ground in increasing the capacity is even less than the above amount, due partly to the fact that the ground is a poor conductor. As the effect of the ground is so slight, it has been neglected entirely in the calculations in this book.

REFERENCE. — For an alternative proof, see "A Treatise on the Theory of Alternating Currents," by Alexander Russell, 1904, Vol. I.

See also "The Calculation of Capacity Coefficients for Parallel Suspended Wires," by Frank F. Fowle, *Elec. World*, Aug. 19, 1911.

CHAPTER XIV.

CAPACITY OF TWO-PHASE AND THREE-PHASE LINES.

Capacity, Two-phase. — The charging current of a single-phase line was shown in Chapter XIII to be

$$\frac{1}{2}E \times \frac{38.83 \times 10^{-9}}{\log_{10}\left(\frac{s}{\rho} - \frac{\rho}{s}\right)} \times 2\pi f$$

amperes per mile of line

$$= \tfrac{1}{2} Eb,$$

where b is the tabulated value of capacity susceptance per mile.

In a two-phase, four-wire line, each phase is quite similar to a single-phase line, and so the charging current per wire is

$$\tfrac{1}{2} Eb.$$

As this amount of charging current flows in each phase, the total amperes of charging current are

$$Eb,$$

for a two-phase, four-wire line.

Capacity, Three-phase, Irregular Spacing. — When the wires of a three-phase transmission line are not spaced at the corners of an equilateral triangle, but the transposition of the conductors is carried out at regular intervals, the charging current in the wires will be a balanced, three-phase current, since each wire will have passed through the same average conditions. This is shown in an approximate manner as follows:

As when calculating the self-induction of an irregularly spaced line, consider a line three miles long which is transposed at the end of each mile.

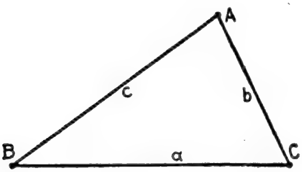


Fig. 25.

The work in carrying a unit charge from C to B (Fig. 25) assuming the charges concentrated at the centers of the wires, is approximately

$$E_a = q_B \, 2 \log_\epsilon \frac{a}{\rho} - q_C \, 2 \log_\epsilon \frac{a}{\rho} + q_A \, 2 \log_\epsilon \frac{c}{b}.$$

Now q_B is a periodic quantity, which alternates in value at the same frequency as the voltage or current.

We have
$$q_B = C_1 E$$
$$= -j \frac{I_B'}{2 \pi f},$$

where I_B' is the charging current flowing into the capacity C_1 of the wire B.

Thus

$$E_a = \frac{-j}{2 \pi f}\left(I_B' \, 2 \log_\epsilon \frac{a}{\rho} - I_C' \, 2 \log_\epsilon \frac{a}{\rho} + I_A' \, 2 \log_\epsilon \frac{c}{b}\right).$$

In the second mile the conductors are transposed into new positions. Let the currents in them remain the same. Therefore,

$$E_a = \frac{-j}{2 \pi f}\left(I_B' \, 2 \log_\epsilon \frac{b}{\rho} - I_C' \, 2 \log_\epsilon \frac{b}{\rho} + I_A' \, 2 \log_\epsilon \frac{a}{c}\right),$$

and in the third mile

$$E_a = \frac{-j}{2 \pi f}\left(I_B' \, 2 \log_\epsilon \frac{c}{\rho} - I_C' \, 2 \log_\epsilon \frac{c}{\rho} + I_A' \, 2 \log_\epsilon \frac{b}{a}\right).$$

Adding together and dividing by 3, we obtain the approximate average value per mile,

$$E_a = \frac{-j}{2\pi f}\left(I_B' - I_C'\right) 2 \log_\epsilon \frac{\sqrt[3]{abc}}{\rho};$$

that is,
$$E_a = \frac{-j}{2\pi f}\left(I_B' - I_C'\right) 2 \log_\epsilon \frac{s}{\rho},$$

where
$$s = \sqrt[3]{abc}.$$

Similarly,
$$E_b = \frac{-j}{2\pi f}\left(I_C' - I_A'\right) 2 \log_\epsilon \frac{s}{\rho},$$

and
$$E_c = \frac{-j}{2\pi f}\left(I_A' - I_B'\right) 2 \log_\epsilon \frac{s}{\rho}.$$

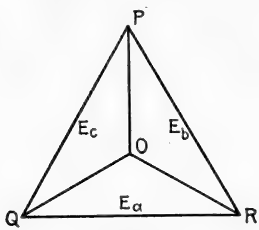


Fig. 26.

Let $E_a = -\ 1.00\, E$, as in Fig. 26,
and $E_b = (0.50 - 0.866j)\, E$,
and $E_c = (0.50 + 0.866j)\, E$.

Then
$$I_A' - I_B' = \frac{j\, 2\pi f E\, (0.50 + 0.866j)}{2 \log_\epsilon \frac{s}{\rho}}, \tag{1}$$

$$I_B' - I_C' = \frac{j\, 2\pi f E}{2 \log_\epsilon \frac{s}{\rho}} \times (-\ 1.00), \tag{2}$$

$$I_C' - I_A' = \frac{j\, 2\pi f E}{2 \log_\epsilon \frac{s}{\rho}} (0.50 - 0.866j), \tag{3}$$

also
$$I_A' + I_B' + I_C' = 0, \tag{4}$$
since they are currents flowing in a three-phase line.

From equations (1) and (3)
$$2\,I_A' - I_B' - I_C' = -\frac{j\,2\,\pi f E\,(-1.732\,j)}{2\log_\epsilon \frac{S}{\rho}}. \tag{5}$$
Adding (4) and (5), we have
$$I_A' = -\frac{1}{\sqrt{3}} \times \frac{2\,\pi f E \times 1.00}{2\log_\epsilon \frac{S}{\rho}}.$$
From this,
$$I_B' = -\frac{1}{\sqrt{3}} \times \frac{2\,\pi f E\,(-0.50 + 0.866\,j)}{2\log_\epsilon \frac{S}{\rho}}.$$
and
$$I_C' = -\frac{1}{\sqrt{3}} \times \frac{2\,\pi f E\,(-0.50 - 0.866\,j)}{2\log_\epsilon \frac{S}{\rho}}.$$

The vectors for I_A', I_B' and I_C' may now be plotted as in Fig. 27, and it is seen that the vectors are the same length and are at 120° to each other. Thus the charging current is a balanced three-phase current.

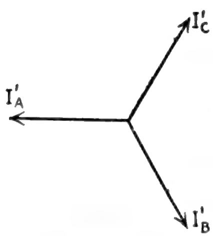


Fig. 27.

The power factor of the charging current is zero, since the current in any wire I_A' (Fig. 27) is at right angles to the direction OP (Fig. 26) of the corresponding star voltage or in-phase current.

The total amperes of charging current are
$$\sqrt{3}\,I_A' = \frac{2\,\pi f E}{2\log_\epsilon \frac{S}{\rho}} \text{ per centimeter}$$

$$= \frac{38.83 \times 2\,\pi f E}{\log_{10} \frac{s}{\rho}} \times 10^{-9} \text{ per mile}$$

$$= Eb,$$

where b is the tabulated value of capacity susceptance per mile.

Capacity, Three-phase, Regular Spacing. — When the conductors are placed an equal distance, s, from each other, the formula for b is

$$b = \frac{38.83 \times 2\,\pi f}{\log_{10} \frac{s}{\rho}} \times 10^{-9} \text{ mhos per mile.}$$

Capacity, Three-phase, Flat Spacing. — When the wires lie in one plane (either horizontal or vertical), the center one being at a distance a from the other two (see Fig. 18), and the wires being transposed at regular intervals, the formula for susceptance is

$$b = \frac{38.83 \times 2\,\pi f}{\log_{10} \frac{a\sqrt[3]{1 \times 1 \times 2}}{\rho}} \times 10^{-9}$$

$$= \frac{38.83 \times 2\,\pi f}{\log_{10} \frac{1.26\,a}{\rho}} \times 10^{-9}$$

mhos per mile of transmission line.

CHAPTER XV.

THEORY OF CONVERGENT SERIES.

THE well-known fundamental formulas for a transmission line without branches, in which the load is delivered only at the end of the line, are as follows:

$$E_s = E \cosh \sqrt{YZ} + \frac{1}{\sqrt{YZ}} IZ \sinh \sqrt{YZ},$$

and

$$I_s = I \cosh \sqrt{YZ} + \frac{1}{\sqrt{YZ}} EY \sinh \sqrt{YZ},$$

where E_s and I_s are the voltage and current at the supply end,

E and I are the voltage and current at the receiver end,

Y is the line admittance

and Z is the line impedance.

The above equations have been published at various times. They are obtained as follows:

Let r = resistance of conductor per unit length,

and x = reactance of conductor per unit length.

Then $z = r + jx$

= impedance of conductor per unit length.

Let g = leakage conductance from conductor per unit length,

and b = capacity susceptance of conductor per unit length.

Then $y = g + jb$

= admittance of conductor per unit length.

Let E_l = voltage of line at a distance l from the receiver end,

and let $I_l = P_l - jQ_l$

= current in the line at a distance l from the receiver end.

(Since I_l is usually not in phase with the voltage, it must be expressed as a complex quantity.)

Now in an element of length, dl, of the line, the voltage consumed by impedance is

$$dE_l = zI_l dl.$$

The current consumed by admittance is

$$dI_l = yE_l dl.$$

Thus
$$\frac{dE_l}{dl} = zI_l, \tag{1}$$

and
$$\frac{dI_l}{dl} = yE_l. \tag{2}$$

Differentiating (1)

$$\frac{d^2E_l}{dl^2} = z\frac{dI_l}{dl}.$$

Substituting (2) in this gives

$$\frac{d^2E_l}{dl^2} = zy\,E_l. \tag{3}$$

This is a differential equation of the second order and may be expressed in the form

$$(D^2 - yz)\,E_l = 0,$$

and we have

$$E_l = A_1\epsilon^{l\sqrt{yz}} + A_2\epsilon^{-l\sqrt{yz}} \tag{4}$$

and from (1),

$$I_l = \frac{1}{z}\frac{dE_l}{dl}$$

$$= \sqrt{\frac{y}{z}}\left(A_1\epsilon^{l\sqrt{yz}} - A_2\epsilon^{-l\sqrt{yz}}\right). \tag{5}$$

Now at the supply end,

$$yl = Y,$$

and

$$zl = Z$$

Therefore,

$$E_s = A_1\epsilon^{\sqrt{YZ}} + A_2\epsilon^{-\sqrt{YZ}} \tag{6}$$

and

$$I_s = \sqrt{\frac{Y}{Z}}(A_1\epsilon^{\sqrt{YZ}} - A_2\epsilon^{-\sqrt{YZ}}). \tag{7}$$

At the receiver end,

$$l = 0$$

and

$$E = A_1 + A_2 \tag{8}$$

and

$$I = \sqrt{\frac{Y}{Z}}(A_1 - A_2). \tag{9}$$

From (6)

$$\begin{aligned} E_s = &\tfrac{1}{2}(A_1 + A_2)(\epsilon^{\sqrt{YZ}} + \epsilon^{-\sqrt{YZ}}) \\ &+ \tfrac{1}{2}(A_1 - A_2)(\epsilon^{\sqrt{YZ}} - \epsilon^{-\sqrt{YZ}}) \end{aligned} \tag{10}$$

which, by the definition of $\cosh\theta$ and $\sinh\theta$, is

$$(A_1 + A_2)\cosh\sqrt{YZ} + (A_1 - A_2)\sinh\sqrt{YZ}.$$

Therefore, from (8) and (9),

$$E_s = E\cosh\sqrt{YZ} + \frac{1}{\sqrt{YZ}}IZ\sinh\sqrt{YZ}. \tag{11}$$

Similarly, from (7),

$$\begin{aligned} I_s = &\sqrt{\frac{Y}{Z}}\,\frac{1}{2}(A_1 - A_2)(\epsilon^{\sqrt{YZ}} + \epsilon^{-\sqrt{YZ}}) \\ &+ \sqrt{\frac{Y}{Z}}\,\frac{1}{2}(A_1 + A_2)(\epsilon^{\sqrt{YZ}} - \epsilon^{-\sqrt{YZ}}), \end{aligned} \tag{12}$$

$$= I\cosh\sqrt{YZ} + \frac{1}{\sqrt{YZ}}EY\sinh\sqrt{YZ}. \tag{13}$$

Equations (11) and (13) are the fundamental formulas of transmission lines, as generally written.

Now
$$\epsilon^{\sqrt{YZ}} + \epsilon^{-\sqrt{YZ}}$$
$$= 1 + \sqrt{YZ} + \frac{(\sqrt{YZ})^2}{2} + \frac{(\sqrt{YZ})^3}{2 \cdot 3} + \frac{(\sqrt{YZ})^4}{2 \cdot 3 \cdot 4} + \text{etc.}$$
$$+ 1 - \sqrt{YZ} + \frac{(\sqrt{YZ})^2}{2} - \frac{(\sqrt{YZ})^3}{2 \cdot 3} + \frac{(\sqrt{YZ})^4}{2 \cdot 3 \cdot 4} - \text{etc.}$$
$$= 2\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \text{etc.}\right).$$

Similarly
$$\epsilon^{\sqrt{YZ}} - \epsilon^{-\sqrt{YZ}}$$
$$= 2\sqrt{YZ}\left(1 + \frac{YZ}{2 \cdot 3} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4 \cdot 5} + \text{etc.}\right).$$

Substituting these results in (10) and (12), we can express the fundamental equations as follows:

$$E_s = E\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \frac{Y^3Z^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \text{etc.}\right)$$
$$+ IZ\left(1 + \frac{YZ}{2 \cdot 3} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{Y^3Z^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} + \text{etc.}\right) \quad (14)$$

and

$$I_s = I\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \frac{Y^3Z^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \text{etc.}\right)$$
$$+ EY\left(1 + \frac{YZ}{2 \cdot 3} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{Y^3Z^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} + \text{etc.}\right)^* \quad (15)$$

Equations (14) and (15) are the same as those tabulated in Chapter VI for obtaining E_s or $A + jB$, and I_s or $C + jD$. For no-load values, all that is necessary is to put the load current, I, equal to zero.

When conditions are given at the supply end, the same equations for full-load conditions are obtained, except that

* See references to T. R. Rosebrugh, J. F. H. Douglas, and C. P. Steinmetz, page 41.

the second half of the expressions for voltage and current is negative, since power is now flowing away from the point where the voltage is specified, instead of toward it. Thus we have the series for $F + jG$ and $M + jN$.

At no load, the conditions are really not all specified at the supply end, but the current is specified to be zero at the receiver end, and this necessitates the use of special series. From Table V we have the ratio of the voltage at the two ends of the line,

$$\frac{E_{0s}}{E} = \frac{A_0 + jB_0}{E}$$
$$= \left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \text{etc.}\right).$$

This ratio is independent of the voltage E, and depends only on the constants of the line. Thus, if E_s, the voltage at the supply end at no load, is given, we can obtain the no-load voltage at the receiver end from the equation,

$$\frac{E_s}{E_0} = \left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \text{etc.}\right)$$

or
$$E_0 = E_s\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \text{etc.}\right)^{-1},$$

which, when expanded by the binomial theorem, gives

$$E_0 = F_0 + jG_0$$
$$= E_s\left(1 - \frac{1}{2}YZ + \frac{5}{24}Y^2Z^2 - \frac{61}{720}Y^3Z^3 + \frac{277}{8064}Y^4Z^4 - \text{etc.}\right) \quad (16)$$

as in Table VI.

The no-load current at the supply end is

$$I_{0s} = E_0Y\left(1 + \frac{YZ}{2 \cdot 3} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4 \cdot 5} + \text{etc.}\right),$$

as in the equation for $C_0 + jD_0$.

Substituting the value of E_0 from equation (16), we have

$$I_{0s}=E_sY\left(1-\frac{1}{2}YZ+\frac{5}{24}Y^2Z^2-\frac{61}{720}Y^3Z^3+\frac{277}{8064}Y^4Z^4-\text{etc.}\right)$$

$$\times\left(1+\frac{YZ}{2\cdot3}+\frac{Y^2Z^2}{2\cdot3\cdot4\cdot5}+\frac{Y^3Z^3}{2\cdot3\cdot4\cdot5\cdot6\cdot7}+\frac{Y^4Z^4}{2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9}+\text{etc.}\right).$$

Multiplying the two series together by the ordinary algebraical method, we obtain

$$I_{0s}=M_0+jN_0$$

$$=E_sY\left(1-\frac{1}{3}YZ+\frac{2}{15}Y^2Z^2-\frac{17}{315}Y^3Z^3+\frac{62}{2835}Y^4Z^4-\text{etc.}\right)$$

as given in Table VI of convergent series.

PART III

TABLES.

TABLE I. — FORMULAS FOR SHORT LINES.

CONDITIONS GIVEN AT RECEIVER END.

These formulas are exact when the line is short. When the line is 20 miles long, they are correct within approximately $\frac{1}{10}$ of 1% of line voltage.

Conditions given:

K.V.A. = K.V.A. at receiver end.

E = Full load voltage at receiver end.

$\cos\theta$ = Power factor at receiver end.

K.W. = K.V.A. $\cos\theta$.

r = Resistance of conductor per mile. (From Tables VII–VIII.)

x = Reactance of conductor per mile. (From Tables IX–XII.)

l = Length of line in miles.

Then $P = \dfrac{1000 \text{ K.V.A.} \cos\theta}{E}$ = In-phase current at receiver end (in total amps.).

$Q = \dfrac{1000 \text{ K.V.A.} \sin\theta}{E}$ = Reactive current at receiver end (in total amps.) when current is lagging.

$= -\dfrac{1000 \text{ K.V.A.} \sin\theta}{E}$ when current is leading.

Find the following quantities:

Three phase or two phase.	*Single phase.*
$A = E + Prl + Qxl.$	$A = E + 2\,Prl + 2\,Qxl.$
$B = Pxl - Qrl.$	$B = 2\,Pxl - 2\,Qrl.$

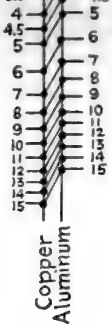
Formulas (capacity neglected):

(1) Voltage at supply end $= A + \dfrac{B^2}{2\,A}$.

(2) Regulation of line $= A + \dfrac{B^2}{2\,A} - E$. (Same as line drop.)

(3) Per cent regulation of line $= \dfrac{100\left(A + \dfrac{B^2}{2\,A} - E\right)}{E}$ per cent.

(Same as per cent line drop.)



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$$\text{Reg'n. Volts} = \frac{1000\,K.V.A. \times l\,V}{E}$$

% Line Drop (percent of E)

$$= \%\,\text{Reg'n.} - 100K\left(\frac{l}{1000}\right)^2$$

$$\text{Line Drop} = \text{Reg'n. Volts} - EK\left(\frac{l}{1000}\right)^2$$

where

$K = 2.16$ for 60 Cycles
$K = .375$ for 25 Cycles
E = Line Voltage at Load
K.V.A. = K.V.A. of Load
l = Length of Line in Miles

Equivalent Spacing of 3 Phase Line

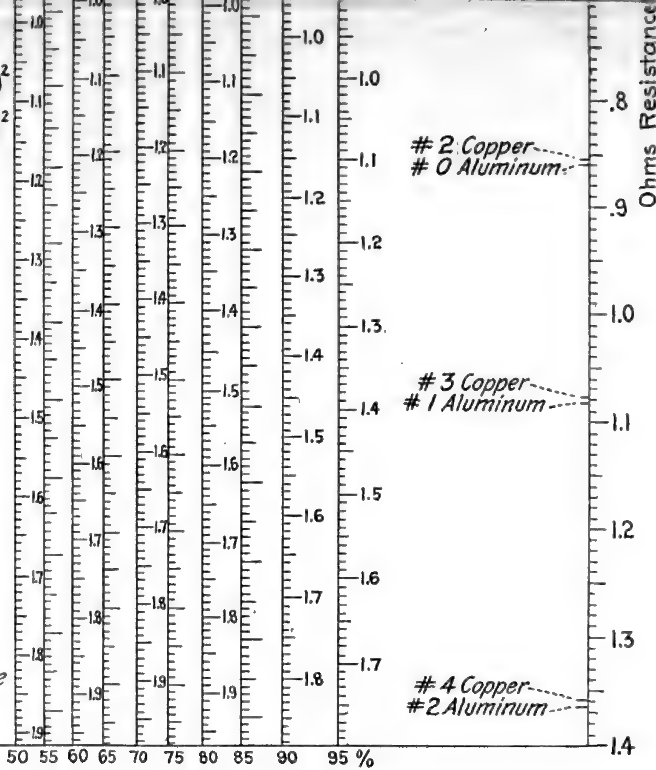
$$s = \sqrt[3]{abc}$$

$$s = 1.26\,a \quad \text{for Flat Spacing}$$

To determine the Size of Conductor required for a given % Regulation, use the following: –

$$V = \frac{\%\,\text{Reg'n.} \times E^2}{100{,}000\,K.V.A. \times l}$$

The above Equations are for 2 and 3 Phase Lines. For single Phase use 2 V in place of V.



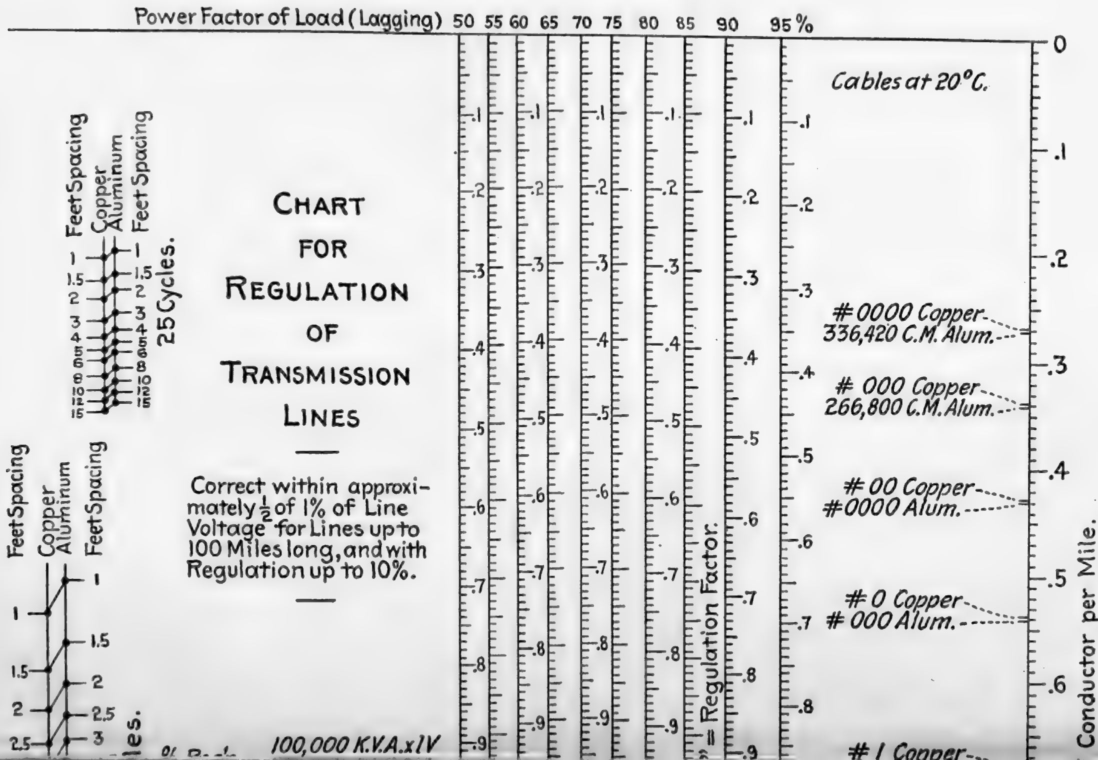


CHART FOR REGULATION OF TRANSMISSION LINES
Correct within approximately 1/2 of 1% of Line Voltage for Lines up to 100 Miles long, and with Regulation up to 10%.
Power Factor of Load (Lagging) 50 55 60 65 70 75 80 85 90 95 %
Cables at 20°C.
0000 Copper
336,420 C.M. Alum.
000 Copper
266,800 C.M. Alum.
00 Copper
0000 Alum.
0 Copper
000 Alum.
1 Copper
Feet Spacing
Copper
Aluminum
Feet Spacing
25 Cycles.
= Regulation Factor.
100,000 K.V.A. x 1V
Conductor per Mile.

TABLE I. (*Continued.*)

(4) K.V.A. at supply end $= \dfrac{A + \dfrac{B^2}{2A}}{E} \times$ K.V.A.

(5) K.W. at supply end $= \dfrac{1}{1000}(AP - BQ)$.

(6) Power factor at supply end $= \dfrac{1}{1000} \dfrac{(AP - BQ)\,E}{\left(A + \dfrac{B^2}{2A}\right) \times \text{K.V.A.}}$

(in decimals).

(7) In-phase current at supply end $= \dfrac{AP - BQ}{A + \dfrac{B^2}{2A}}$ in total amperes.*

(8) Reactive or quadrature current at supply end $= \dfrac{BP + AQ}{A + \dfrac{B^2}{2A}}$

in total amperes.*

When this quantity is positive, the current is lagging.
When this quantity is negative, the current is leading.

(9) K.W. loss in line $= \dfrac{1}{1000}(AP - BQ - EP)$.

(10) Per cent efficiency of line $= \dfrac{100\,EP}{AP - BQ}$ per cent.

* Amperes per wire, three phase, $= \dfrac{\text{Total amps.}}{\sqrt{3}}$.

Amperes per wire, two phase, $= \dfrac{\text{Total amps.}}{2}$.

TABLE II. — FORMULAS FOR SHORT LINES.

Conditions given at Supply End.

These formulas are exact when the line is short. When the line is 20 miles long, they are correct within approximately $\frac{1}{10}$ of 1% of line voltage.

Conditions given:

K.V.A. = K.V.A. at supply end.
E_s = Full load voltage at supply end.
$\cos\theta$ = Power factor at supply end.
K.W. = K.V.A. $\cos\theta$.
r = Resistance of conductor per mile. (From Tables VII–VIII.)
x = Reactance of conductor per mile. (From Tables IX–XII.)
l = Length of line in miles.

Then $P_s = \dfrac{1000 \text{ K.V.A. } \cos\theta}{E_s}$ = In-phase current at supply end (in total amps.).

$Q_s = \dfrac{1000 \text{ K.V.A. } \sin\theta}{E_s}$ = Reactive current at supply end (in total amperes) when current is lagging.

$= -\dfrac{1000 \text{ K.V.A. } \sin\theta}{E_s}$ when current is leading.

Find the following quantities:

Three Phase or Two Phase.

$F = E_s - P_s rl - Q_s xl$
$G = Q_s rl - P_s xl$

Single Phase.

$F = E_s - 2\,P_s rl - 2\,Q_s xl$
$G = 2\,Q_s rl - 2\,P_s xl.$

Formulas (capacity neglected):

(1) Voltage at receiver end $= F + \dfrac{G^2}{2F}$.

(2) Regulation of line $= E_s - F - \dfrac{G^2}{2F}$. (Same as line drop.)

(3) Per cent regulation of line $= \dfrac{100\left(E_s - F - \dfrac{G^2}{2F}\right)}{F + \dfrac{G^2}{2F}}$ per cent.

(Same as per cent line drop.)

(4) K.V.A. at receiver end $= \dfrac{F + \dfrac{G^2}{2F}}{E_s} \times$ K.V.A.

TABLE II. (*Continued.*)

(5) K.W. at receiver end $= \frac{1}{1000}(FP_8 - GQ_8)$.

(6) Power factor at receiver end $= \frac{1}{1000} \frac{(FP_8 - GQ_8)E_8}{\left(F + \frac{G^2}{2F}\right) \times \text{K.V.A.}}$

(in decimals).

(7) In-phase current at receiver end $= \frac{FP_8 - GQ_8}{F + \frac{G^2}{2F}}$ in total amperes.*

(8) Reactive or quadrature current at receiver end $= \frac{GP_8 + FQ_8}{F + \frac{G^2}{2F}}$

in total amperes.*

When this quantity is positive, the current is lagging.
When this quantity is negative, the current is leading.

(9) K.W. loss in line $= \frac{1}{1000}(E_8P_8 - FP_8 + GQ_8)$.

(10) Per cent efficiency of line $= \frac{100(FP_8 - GQ_8)}{E_8P_8}$ per cent.

* Amperes per wire, three phase, $= \frac{\text{Total amps.}}{\sqrt{3}}$.

Amperes per wire, two phase, $= \frac{\text{Total amps.}}{2}$.

TABLE III. — K FORMULAS FOR TRANSMISSION LINES.

Conditions given at Receiver End.

Accurate within approximately $\frac{1}{10}$ of 1% of line voltage up to 100 miles, and $\frac{1}{2}$ of 1% up to 200 miles, for lines with regulation up to 20%.

$K = 6 \dfrac{(\text{cycles})^2}{10{,}000}$. $K = 2.16$ for 60 cycles. $K = 0.375$ for 25 cycles.

Conditions given:

K.V.A. = K.V.A. at receiver end.
E = Full load voltage at receiver end.
$\cos\theta$ = Power factor at receiver end.
K.W. = K.V.A. $\cos\theta$.
r = Resistance of conductor per mile. (From Tables VII–VIII.)
x = Reactance of conductor per mile. (From Tables IX–XII.)
l = Length of transmission line in miles.

Then $P = \dfrac{1000 \text{ K.V.A.} \cos\theta}{E}$ = In-phase current at receiver end (in total amps.).

$Q = \dfrac{1000 \text{ K.V.A.} \sin\theta}{E}$ = Reactive current at receiver end (in total amps.), when current is lagging.

$= -\dfrac{1000 \text{ K.V.A.} \sin\theta}{E}$ when current is leading.

TABLE III. (*Continued.*)

Find the following quantities:

Full Load.

$$A = E\left\{1 - K\left(\frac{l}{1000}\right)^2\right\} + Prl\left\{1 - \frac{2}{3}K\left(\frac{l}{1000}\right)^2\right\}$$
$$+ Qxl\left\{1 - \frac{1}{6}K\left(\frac{l}{1000}\right)^2\right\}.$$

$$B = E\frac{rK}{x}\left(\frac{l}{1000}\right)^2 + Pxl\left\{1 - \frac{1}{6}K\left(\frac{l}{1000}\right)^2\right\} - Qrl\left\{1 - \frac{2}{3}K\left(\frac{l}{1000}\right)^2\right\}.$$

$$C = P\left\{1 - K\left(\frac{l}{1000}\right)^2\right\} + Q\frac{rK}{x}\left(\frac{l}{1000}\right)^2 - \frac{2}{3}E\frac{rK^2}{lx^2}\left(\frac{l}{1000}\right)^4.$$

$$D = P\frac{rK}{x}\left(\frac{l}{1000}\right)^2 - Q\left\{1 - K\left(\frac{l}{1000}\right)^2\right\}$$
$$+ 2E\frac{K}{lx}\left(\frac{l}{1000}\right)^2\left\{1 - \frac{1}{3}K\left(\frac{l}{1000}\right)^2\right\}.$$

No Load.

$$A_0 = E\left\{1 - K\left(\frac{l}{1000}\right)^2\right\}.$$

$$B_0 = E\frac{rK}{x}\left(\frac{l}{1000}\right)^2.$$

$$C_0 = -\frac{2}{3}E\frac{rK^2}{lx^2}\left(\frac{l}{1000}\right)^4.$$

$$D_0 = 2E\frac{K}{lx}\left(\frac{l}{1000}\right)^2\left\{1 - \frac{1}{3}K\left(\frac{l}{1000}\right)^2\right\}.$$

Note. — The above are for two- and three-phase lines. For single-phase lines use $2r$ and $2x$ in place of r and x.

TABLE III. (*Continued.*)

CONDITIONS GIVEN AT RECEIVER END.

Formulas:

Full Load. *No Load.*

Voltage at receiver end.

(1) $E.$

(2) $E_0 = \dfrac{A + \dfrac{B^2}{2A}}{A_0 + \dfrac{B_0^2}{2A_0}} E$

(for constant supply voltage).

Regulation at receiver end in volts, for constant supply voltage.

(3) $\dfrac{A + \dfrac{B^2}{2A}}{A_0 + \dfrac{B_0^2}{2A_0}} E - E.$

N.B. The regulation at receiver end may be expressed as a percentage of E.

Voltage at supply end.

(4) $E_s = A + \dfrac{B^2}{2A}.$

(5) $E_{0s} = A_0 + \dfrac{B_0^2}{2A_0}$

(for constant receiver voltage).

Regulation at supply end in volts, for constant receiver voltage.

(6) $A + \dfrac{B^2}{2A} - A_0 - \dfrac{B_0^2}{2A_0}.$

N.B. The regulation at supply end may be expressed as a percentage of E_s.

TABLE III. (*Continued.*)

*Current at supply end in total amperes.**

(7) $\sqrt{C^2 + D^2}$. (8) $\sqrt{C_0^2 + D_0^2}$.

K.V.A. at supply end.

(9) $\frac{1}{1000}\left(A + \frac{B^2}{2A}\right)\sqrt{C^2 + D^2}$. (10) $\frac{1}{1000}\left(A_0 + \frac{B_0^2}{2A_0}\right)\sqrt{C_0^2 + D_0^2}$.

K.W. at supply end.

(11) $\frac{1}{1000}(AC + BD)$. (12) $\frac{1}{1000}(A_0C_0 + B_0D_0)$.

Power factor at supply end, in decimals.

(13) $\frac{AC + BD}{\left(A + \frac{B^2}{2A}\right)\sqrt{C_2 + D^2}}$. (14) $\frac{A_0C_0 + B_0D_0}{\left(A_0 + \frac{B_0^2}{2A_0}\right)\sqrt{C_0^2 + D_0^2}}$.

*In-phase current at supply end in total amperes.**

(15) $\frac{AC + BD}{\left(A + \frac{B^2}{2A}\right)}$. (16) $\frac{A_0C_0 + B_0D_0}{A_0 + \frac{B_0^2}{2A_0}}$.

*Reactive current at supply end in total amperes.**

(17) $\frac{BC - AD}{A + \frac{B^2}{2A}}$. (18) $\frac{B_0C_0 - A_0D_0}{A_0 + \frac{B_0^2}{2A_0}}$.

When this quantity is positive, the current is lagging.
When this quantity is negative, the current is leading.

K.W. loss in line.

(19) $\frac{1}{1000}(AC + BD - EP)$. (20) $\frac{1}{1000}(A_0C_0 + B_0D_0)$ [same as No. 12].

Per cent efficiency of line.

(21) $\frac{100\,EP}{AC + BD}$ per cent.

* Amperes per wire, three-phase, $= \frac{\text{Total amps.}}{\sqrt{3}}$.

Amperes per wire, two phase, $= \frac{\text{Total amps.}}{2}$.

TABLE IV.—K FORMULAS FOR TRANSMISSION LINES.

Conditions given at Supply End.

Accurate within approximately $\frac{1}{10}$ of 1% of line voltage up to 100 miles and $\frac{1}{2}$ of 1% up to 200 miles, for lines with regulation up to 20%.

$K = \dfrac{6\,(\text{cycles})^2}{10{,}000}$. $K = 2.16$ for 60 cycles. $K = 0.375$ for 25 cycles.

Conditions given:

K.V.A. = K.V.A. at supply end.

E_s = Full load voltage at supply end.

$\cos\theta$ = Power factor at supply end.

K.W. = K.V.A. $\cos\theta$.

r = Resistance of conductor per mile. (From Tables VII–VIII.)

x = Reactance of conductor per mile. (From Tables IX–XII.)

l = Length of transmission line in miles.

Then $P_s = \dfrac{1000 \text{ K.V.A. } \cos\theta}{E_s}$ = In-phase current at supply end (in total amps.).

$Q_s = \dfrac{1000 \text{ K.V.A. } \sin\theta}{E_s}$ = Reactive current at supply end (in total amps.), when current is lagging.

$= -\dfrac{1000 \text{ K.V.A. } \sin\theta}{E_s}$ when current is leading.

TABLE IV. (*Continued.*)

Find the following quantities:

Full Load.

$$F = E_s \left\{ 1 - K\left(\frac{l}{1000}\right)^2 \right\} - P_s rl \left\{ 1 - \frac{2}{3} K \left(\frac{l}{1000}\right)^2 \right\}$$
$$- Q_s xl \left\{ 1 - \frac{1}{6} K \left(\frac{l}{1000}\right)^2 \right\}.$$

$$G = E_s \frac{rK}{x}\left(\frac{l}{1000}\right)^2 - P_s xl \left\{ 1 - \frac{1}{6} K \left(\frac{l}{1000}\right)^2 \right\} + Q_s rl \left\{ 1 - \frac{2}{3} K \left(\frac{l}{1000}\right)^2 \right\}.$$

$$M = P_s \left\{ 1 - K\left(\frac{l}{1000}\right)^2 \right\} + Q_s \frac{rK}{x}\left(\frac{l}{1000}\right)^2 + \frac{2}{3} E_s \frac{rK^2}{lx^2}\left(\frac{l}{1000}\right)^4.$$

$$N = P_s \frac{rK}{x}\left(\frac{l}{1000}\right)^2 - Q_s \left\{ 1 - K\left(\frac{l}{1000}\right)^2 \right\}$$
$$- 2 E_s \frac{K}{lx}\left(\frac{l}{1000}\right)^2 \left\{ 1 - \frac{1}{3} K \left(\frac{l}{1000}\right)^2 \right\}.$$

No Load.

$$F_0 = E_s \left\{ 1 + K \left(\frac{l}{1000}\right)^2 \right\}.$$

$$G_0 = -E_s \frac{rK}{x}\left(\frac{l}{1000}\right)^2.$$

$$M_0 = \frac{4}{3} E_s \frac{rK^2}{lx^2}\left(\frac{l}{1000}\right)^4.$$

$$N_0 = 2 E_s \frac{K}{lx}\left(\frac{l}{1000}\right)^2 \left\{ 1 + \frac{2}{3} K \left(\frac{l}{1000}\right)^2 \right\}.$$

Note. — The above are for two- and three-phase lines. For single-phase lines use $2r$ and $2x$ in place of r and x.

TABLE IV. (*Continued.*)

CONDITIONS GIVEN AT SUPPLY END.

Formulas:

Full Load. | *No Load.*

Voltage at receiver end.

(1) $E = F + \frac{G^2}{2F}$.

(2) $E_0 = F_0 + \frac{G_0^2}{2F_0}$

(for constant supply voltage).

Regulation at receiver end in volts, for constant supply voltage.

(3) $F_0 + \frac{G_0^2}{2F_0} - F - \frac{G^2}{2F}$.

N.B. The regulation at receiver end may be expressed as a percentage of E.

Voltage at supply end.

(4) E_s.

(5) $E_{0s} = \dfrac{F + \frac{G^2}{2F}}{F_0 + \frac{G_0^2}{2F_0}} E_s$

(for constant receiver voltage).

Regulation at supply end, in volts, for constant receiver voltage.

(6) $E_s - \dfrac{F + \frac{G^2}{2F}}{F_0 + \frac{G_0^2}{2F_0}} E_s$.

N.B. The regulation at supply end may be expressed as a percentage of E_s.

TABLE IV. (*Continued.*)

*Current in total amperes.**

(7) $\sqrt{M^2+N^2}$ at receiver end. (8) $\sqrt{M_0^2+N_0^2}$ at supply end.

K.V.A.

(9) $\frac{1}{1000}\left(F+\frac{G_2}{2F}\right)\sqrt{M^2+N^2}$ at receiver end.

(10) $\frac{1}{1000}E_s\sqrt{M_0^2+N_0^2}$ at supply end.

K.W.

(11) $\frac{1}{1000}(FM+GN)$ at receiver end. (12) $\frac{1}{1000}E_sM_0$ at supply end.

Power factor, in decimals.

(13) $\dfrac{FM+GN}{\left(F+\frac{G^2}{2F}\right)\sqrt{M^2+N^2}}$ at receiver end.

(14) $\dfrac{M_0}{\sqrt{M_0^2+N_0^2}}$ at supply end.

*In-phase current in total amperes.**

(15) $\dfrac{FM+GN}{F+\frac{G^2}{2F}}$ at receiver end. (16) M_0 at supply end.

*Reactive current in total amperes.**

(17) $\dfrac{GM-FN}{F+\frac{G_2}{2F}}$ at receiver end. (18) N_0 at supply end.

When this quantity is positive, the current is lagging.
When this quantity is negative, the current is leading.

K.W. loss in line.

(19) $\frac{1}{1000}(E_sP_s-FM-GN)$. (20) $\frac{1}{1000}E_sM_0$ (same as No. 12).

Per cent efficiency of line.

(21) $\dfrac{100\,(FM+GN)}{E_sP_s}$ per cent.

* Amperes per wire, three phase, $=\dfrac{\text{Total amps.}}{\sqrt{3}}$.

Amperes per wire, two phase, $=\dfrac{\text{Total amps.}}{2}$.

TABLE V. — CONVERGENT SERIES FOR TRANSMISSION LINES.

Conditions given at Receiver End.

The convergent series give the results of the fundamental formulas as accurately as desired, if a sufficient number of terms is used.

When conditions are given at the receiver end, the same as with the K formulas, find the quantities:

Full Load.

$$A + jB = E\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2\cdot 3\cdot 4} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \text{etc.}\right)$$
$$+ (P - jQ)\, Z\left(1 + \frac{YZ}{2\cdot 3} + \frac{Y^2Z^2}{2\cdot 3\cdot 4\cdot 5} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \text{etc.}\right).$$

$$C + jD = (P - jQ)\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2\cdot 3\cdot 4} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \text{etc.}\right)$$
$$+ EY\left(1 + \frac{YZ}{2\cdot 3} + \frac{Y^2Z^2}{2\cdot 3\cdot 4\cdot 5} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \text{etc.}\right).$$

No Load.

$$A_0 + jB_0 = E\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2\cdot 3\cdot 4} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \text{etc.}\right),$$

$$C_0 + jD_0 = EY\left(1 + \frac{YZ}{2\cdot 3} + \frac{Y^2Z^2}{2\cdot 3\cdot 4\cdot 5} + \frac{Y^3Z^3}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \text{etc.}\right),$$

where $Z = (r + jx)\, l.$
r = resistance of conductor per mile.
x = reactance of conductor per mile.
l = length of transmission line in miles.
$Y = (g + jb)\, l.$
g = leakage conductance of conductor per mile.
b = capacity susceptance of conductor per mile.

Use A, B, C, D, etc., with the equations in the third and fourth pages of Table III to solve transmission line problems.

Note 1. — In the formulas, $A + \frac{B^2}{2A}$ is used instead of $\sqrt{A^2 + B^2}$. This approximation may be used for very accurate work, as it is correct within approximately $\frac{2}{100}$ of 1% when the regulation is not more than 20%.

Note 2. — The above are for two- and three-phase lines. For single-phase lines use $2r$ and $2x$ in place of r and x, and use $\frac{1}{2}g$ and $\frac{1}{2}b$ in place of g and b.

TABLE VI. — CONVERGENT SERIES FOR TRANSMISSION LINES.

Conditions given at Supply End.

The convergent series give the results of the fundamental formulas as accurately as desired, if a sufficient number of terms is used.

When conditions are given at the supply end, the same as with the K formulas, find the quantities:

Full Load.

$$F + jG = E_s\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \frac{Y^3Z^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \text{etc.}\right)$$
$$-(P_s - jQ_s)\, Z\left(1 + \frac{YZ}{2 \cdot 3} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{Y^3Z^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} + \text{etc.}\right).$$

$$M + jN = (P_s - jQ_s)\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4} + \frac{Y^3Z^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \text{etc.}\right)$$
$$- E_sY\left(1 + \frac{YZ}{2 \cdot 3} + \frac{Y^2Z^2}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{Y^3Z^3}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} + \text{etc.}\right).$$

No Load.

$$F_0 + jG_0 = E_s\left(1 - \tfrac{1}{2}YZ + \frac{5}{24}Y^2Z^2 - \frac{61}{720}Y^3Z^3 + \frac{277}{8064}Y^4Z^4 - \text{etc.}\right),$$

$$M_0 + jN_0 = E_sY\left(1 - \tfrac{1}{3}YZ + \frac{2}{15}Y^2Z^2 - \frac{17}{315}Y^3Z^3 + \frac{62}{2835}Y^4Z^4 - \text{etc.}\right),$$

where $Z = (r + jx)\, l.$
r = resistance of conductor per mile.
x = reactance of conductor per mile.
l = length of transmission line in miles.
$Y = (g + jb)\, l.$
g = leakage conductance of conductor per mile.
b = capacity susceptance of conductor per mile.

Use F, G, M, N, etc., with the equations in the third and fourth pages of Table IV to solve transmission line problems.

Note 1. — In the formulas, $F + \dfrac{G^2}{2F}$ is used instead of $\sqrt{F^2 + G^2}$. This approximation may be used for very accurate work, as it is correct within approximately $\frac{2}{100}$ of 1% when the regulation is not more than 20%.

Note 2. — The above are for two- and three-phase lines. For single-phase lines use $2\,r$ and $2\,x$ in place of r and x, and use $\frac{1}{2}\,g$ and $\frac{1}{2}\,b$ in place of g and b.

TABLE VII.—RESISTANCE OF COPPER WIRE AND CABLE.

Data assumed:

Temperature, 20° C. (68° F.).

Conductivity of hard drawn copper, 97.3% of the annealed copper standard.

Increase of resistance of cables due to spiralling, 1%.

Copper Wire.

B. & S. gauge.	Circular mils.	Diameter (2ρ) inches.		Resistance, ohms per mile.				
				Direct current.	25 cycles.	Increase.	60 cycles.	Increase.
						Per cent		Per cent
0000	211,600	.4600	...	.2655	.2657	.08	.2667	.44
000	167,800	.4096	...	.3348	.3350	.05	.3358	.27
00	133,100	.3648	...	.4221	.4223	.03	.4229	.17
0	105,500	.3249	...	.5326	.5327	.02	.5331	.11
1	83,690	.2893	...	.6714	.6714	.01	.6718	.07
2	66,370	.2576	...	.8466	.8466	.01	.8469	.04
3	52,630	.2294	...	1.068	1.068	...	1.068	.03
4	41,740	.2043	...	1.346	1.346	...	1.346	.02
5	33,100	.1819	...	1.697	1.697	...	1.698	.01
6	26,250	.1620	...	2.140	2.140	...	2.140	.01
7	20,820	.1443	...	2.699	2.699	...	2.699	...
8	16,510	.1285	...	3.403	3.403	...	3.403	...

TABLE VII. (*Continued.*)

Copper Cable.

B. & S. gauge.	Circular mils.	Diameter (2 ρ) inches.	No. of wires assumed.	Resistance, ohms per mile.				
				Direct current.	25 cycles.	Increase.	60 cycles.	Increase.
						Per cent		Per cent
....	500,000	.8111	19	.1135	.1140	.41	.1162	2.34
....	450,000	.7695	19	.1261	.1265	.33	.1285	1.90
....	400,000	.7255	19	.1419	.1422	.26	.1440	1.50
....	350,000	.6786	19	.1621	.1625	.20	.1640	1.16
....	300,000	.6211	7	.1892	.1894	.15	.1908	.85
....	250,000	.5669	7	.2270	.2272	.10	.2284	.60
0000	211,600	.5216	7	.2682	.2684	.07	.2693	.43
000	167,800	.4645	7	.3382	.3384	.05	.3391	.27
00	133,100	.4137	7	.4264	.4265	.03	.4271	.17
0	105,500	.3683	7	.5379	.5380	.02	.5385	.11
1	83,690	.3280	7	.6781	.6782	.01	.6785	.07
2	66,370	.2921	7	.8550	.8551	.01	.8554	.04
3	52,630	.2601	7	1.078	1.078	...	1.078	.03
4	41,740	.2317	7	1.360	1.360	...	1.360	.02

TABLE VII. (*Continued.*)—TEMPERATURE COEFFICIENTS OF COPPER.

For different initial temperatures and different conductivities.

Ohms per meter-gram at 20 deg. cent.	Per cent conductivity.	α_0	α_{15}	α_{20}	α_{25}	α_{30}	α_{50}
.16108	95	.00405	.00381	.00374	.00367	.00361	.00336
.15940	96	.00409	.00386	.00378	.00371	.00364	.00340
.15776	97	.00414	.00390	.00382	.00375	00368	.00343
.15727	97.3	.00415	.00391	.00383	.00376	.00369	.00344
.15614	98	.00418	.00394	.00386	.00379	.00372	.00346
.15457	99	.00423	.00398	.00390	.00383	.00375	.00349
.153022	100	.00428	.00402	.00394	.00386	.00379	.00352
.15151	101	.00432	.00406	.00398	.00390	.00383	.00355

$$R_t = R_{t1}(1 + \alpha_{t1}[t - t_1]),$$

where R_t is the resistance at any temperature t deg. cent.
and R_{t1} is the resistance at any "initial temperature" t_1 deg. cent.

From Appendix E, *Standardization Rules of the A. I. E. E.*, June 27, 1911.

TABLE VIII. — RESISTANCE OF ALUMINUM CABLE.

Data assumed:

Temperature, 20° C. (68° F.).
Conductivity of hard drawn aluminum, 60.86% of the annealed copper standard.
Increase of resistance of cables due to spiralling, 1%.

Aluminum Cable.

B. & S. gauge.	Circular mils.	Diameter (2ρ) inches.	No. of wires assumed.	Resistance, ohms per mile.				
				Direct current.	25 cycles.	Increase.	60 cycles.	Increase.
						Per cent.		Per cent.
....	556,500	.8557	19	.1630	.1634	.20	.1649	1.14
....	477,000	.7922	19	.1902	.1905	.15	.1918	.84
....	397,500	.7232	19	.2282	.2285	.10	.2296	.59
....	336,420	.6577	7	.2697	.2699	.07	.2708	.42
....	266,800	.5857	7	.3401	.3402	.05	.3410	.27
0000	211,600	.5216	7	.4288	.4289	.03	.4295	.17
000	167,800	.4645	7	.5407	.5407	.02	.5412	.10
00	133,100	.4137	7	.6816	.6817	.01	.6821	.07
0	105,500	.3683	7	.8600	.8600	.01	.8603	.04
1	83,690	.3280	7	1.084	1.084	...	1.084	.03
2	66,370	.2921	7	1.367	1.367	...	1.367	.02
3	52,630	.2601	7	1.724	1.724	...	1.724	.01
4	41,740	.2317	7	2.174	2.174	...	2.174	.01

Temperature coefficient of aluminum at 20° C., $\alpha_{20} = .0039$.

TABLE IX. — REACTANCE OF WIRE, 25 CYCLES.

$x = 2\pi f \times 741.1 \log_{10} \frac{s}{0.779\rho} \times 10^{-6}$ ohms per mile.

ρ = radius of wire.
s = spacing.

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

B. & S. gauge.	No. 0000	No. 000	No. 00	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	
Circular mils.	211,600	167,800	133,100	105,500	83,690	66,370	52,630	41,740	33,100	26,250	20,820	16,510	Spacing.
Diameter (2ρ) inches.	.4600	.4096	.3648	.3249	.2893	.2576	.2294	.2043	.1819	.1620	.1443	.1285	
Spacing (s) feet.													
1	.213	.219	.224	.230	.236	.242	.248	.254	.259	.265	.271	.277	1
1.5	.233	.239	.245	.251	.256	.262	.268	.274	.280	.286	.292	.298	1.5
2	.248	.254	.259	.265	.271	.277	.283	.289	.295	.300	.306	.312	2
2.5	.259	.265	.271	.276	.282	.288	.294	.300	.306	.312	.318	.323	2.5
3	.268	.274	.280	.286	.292	.297	.303	.309	.315	.321	.327	.333	3
3.5	.276	.282	.288	.293	.299	.305	.311	.317	.323	.329	.335	.340	3.5
4	.283	.289	.294	.300	.306	.312	.318	.324	.330	.335	.341	.347	4
4.5	.289	.295	.300	.306	.312	.318	.324	.330	.335	.341	.347	.353	4.5
5	.294	.300	.306	.311	.317	.323	.329	.335	.341	.347	.353	.358	5
6	.303	.309	.315	.321	.327	.333	.338	.344	.350	.356	.362	.368	6
7	.311	.317	.323	.329	.334	.340	.346	.352	.358	.364	.370	.375	7
8	.318	.324	.330	.336	.341	.347	.353	.359	.365	.370	.376	.382	8
9	.324	.330	.335	.341	.347	.353	.359	.365	.370	.376	.382	.388	9
10	.329	.335	.341	.347	.352	.358	.364	.370	.376	.382	.388	.394	10
11	.334	.340	.346	.351	.357	.363	.369	.375	.381	.387	.392	.398	11
12	.338	.344	.350	.356	.362	.368	.374	.379	.385	.391	.397	.403	12
13	.342	.348	.354	.360	.366	.372	.378	.383	.389	.395	.401	.407	13
14	.346	.352	.358	.364	.369	.375	.381	.387	.393	.399	.405	.411	14
15	.350	.356	.361	.367	.373	.379	.385	.391	.396	.402	.408	.414	15
16	.353	.359	.365	.370	.376	.382	.388	.394	.400	.406	.411	.417	16
18	.359	.365	.371	.376	.382	.388	.394	.400	.406	.411	.417	.423	18
20	.364	.370	.376	.382	.388	.394	.399	.405	.411	.417	.422	.428	20

TABLE X. — REACTANCE OF CABLE, 25 CYCLES.

$x = 2\pi f \times 741.1 \log_{10} \frac{s}{0.728\rho} \times 10^{-6}$ ohms per mile for a 7-wire stranded cable. $\left[\text{Use } \log_{10} \frac{s}{0.758\rho} \text{ for 19 wires.}\right]$

ρ = maximum radius of cable.
s = spacing.

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

	Sizes for copper cables.						Sizes for aluminum cables.					
Circular mils...	500,000	450,000	400,000	350,000	300,000	250,000	556,500	477,000	397,500	336,420	266,800	Spacing.
No. of wires assumed......	19	19	19	19	7	7	19	19	19	7	7	
Diameter (2ρ) inches...	.8111	.7695	.7255	.6786	.6211	.5669	.8557	.7922	.7232	.6577	.5857	
Spacing (s) feet.												
1	.185	.188	.191	.194	.201	.205	.183	.186	.191	.198	.204	1
1.5	.206	.208	.211	.215	.221	.226	.203	.206	.211	.218	.224	1.5
2	.220	.223	.226	.229	.236	.240	.218	.221	.226	.233	.239	2
2.5	.232	.234	.237	.241	.247	.252	.229	.233	.237	.244	.250	2.5
3	.241	.244	.246	.250	.256	.261	.238	.242	.246	.253	.259	3
3.5	.249	.251	.254	.258	.264	.269	.246	.249	.254	.261	.267	3.5
4	.255	.258	.261	.264	.271	.275	.253	.256	.261	.268	.274	4
4.5	.261	.264	.267	.270	.277	.281	.259	.262	.267	.274	.280	4.5
5	.267	.269	.272	.276	.282	.287	.264	.268	.272	.279	.285	5
6	.276	.279	.281	.285	.291	.296	.273	.277	.282	.288	.294	6
7	.284	.286	.289	.293	.299	.304	.281	.285	.289	.296	.302	7
8	.290	.293	.296	.299	.306	.311	.288	.292	.296	.303	.309	8
9	.296	.299	.302	.305	.312	.316	.294	.297	.302	.309	.315	9
10	.301	.304	.307	.311	.317	.322	.299	.303	.308	.314	.320	10
11	.306	.309	.312	.316	.322	.327	.304	.308	.312	.319	.325	11
12	.311	.314	.317	.320	.327	.331	.308	.312	.317	.324	.330	12
13	.315	.318	.321	.324	.331	.335	.312	.316	.321	.328	.334	13
14	.319	.321	.325	.328	.334	.339	.316	.320	.325	.331	.337	14
15	.322	.325	.328	.331	.338	.342	.320	.323	.328	.335	.341	15
16	.326	.328	.331	.335	.341	.346	.323	.327	.331	.338	.344	16
18	.331	.334	.337	.341	.347	.352	.329	.332	.337	.344	.350	18
20	.337	.340	.342	.346	.352	.357	.334	.338	.343	.349	.356	20

TABLE X.—REACTANCE OF CABLE, 25 CYCLES. (*Continued.*)

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

Sizes for both Copper and Aluminum Cables.

B. & S. gauge.	No. 0000	No. 000	No. 00	No. 0	No. 1	No. 2	No. 3	No. 4	Spacing.
Circular mils.	211,600	167,800	133,100	105,500	83,690	66,370	52,630	41,740	
No. of wires assumed....	7	7	7	7	7	7	7	7	
Diameter (2ρ) inches.	.5216	.4645	.4137	.3683	.3280	.2921	.2601	.2317	
Spacing (s) feet.									
1	.210	.215	.221	.227	.233	.239	.245	.251	1
1.5	.230	.236	.242	.248	.254	.259	.265	.271	1.5
2	.245	.251	.256	.262	.268	.274	.280	.286	2
2.5	.256	.262	.268	.274	.279	.285	.291	.297	2.5
3	.265	.271	.277	.283	.289	.294	.300	.306	3
3.5	.273	.279	.285	.290	.206	.302	.308	.314	3.5
4	.280	.286	.291	.297	.303	.309	.315	.321	4
4.5	.286	.292	.297	.303	.309	.315	.321	.327	4.5
5	.291	.297	.303	.309	.315	.320	.326	.332	5
6	.300	.306	.312	.318	.324	.330	.335	.341	6
7	.308	.314	.320	.326	.331	.337	.343	.349	7
8	.315	.321	.326	.332	.338	.344	.350	.356	8
9	.321	.326	.332	.338	.344	.350	.356	.362	9
10	.326	.332	.338	.344	.350	.355	.361	.367	10
11	.331	.337	.343	.348	.354	.360	.366	.372	11
12	.335	.341	.347	.353	.359	.365	.371	.376	12
13	.339	.345	.351	.357	.363	.369	.375	.380	13
14	.343	.349	.355	.361	.367	.372	.378	.384	14
15	.347	.352	.358	.364	.370	.376	.382	.388	15
16	.350	.356	.361	.367	.373	.379	.385	.391	16
18	.356	.362	.367	.373	.379	.385	.391	.397	18
20	.361	.367	.373	.379	.385	.390	.396	.402	20

TABLE XI. — REACTANCE OF WIRE, 60 CYCLES.

$$x = 2\pi f \times 741.1 \log_{10} \frac{s}{0.779\rho} \times 10^{-6} \text{ ohms per mile.}$$

ρ = radius of wire.
s = spacing.
For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

B. & S. gauge.	No. 0000	No. 000	No. 00	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	Spacing.
Circular mils.	211,600	167,800	133,100	105,500	83,690	66,370	52,630	41,740	33,100	26,250	20,820	16,510	
Diameter (2ρ) inches.	.4600	.4096	.3648	.3249	.2893	.2576	.2294	.2043	.1819	.1620	.1443	.1285	
Spacing (s) feet.													
1	.510	.524	.539	.553	.567	.581	.595	.609	.623	.637	.651	.665	1
1.5	.560	.574	.588	.602	.616	.630	.644	.658	.672	.686	.700	.714	1.5
2	.594	.608	.623	.637	.651	.665	.679	.693	.707	.721	.735	.749	2
2.5	.621	.636	.650	.664	.678	.692	.706	.720	.734	.748	.762	.776	2.5
3	.644	.658	.672	.686	.700	.714	.728	.742	.756	.770	.784	.798	3
3.5	.662	.676	.690	.705	.719	.733	.747	.761	.775	.789	.803	.817	3.5
4	.678	.693	.707	.721	.735	.749	.763	.777	.791	.805	.819	.833	4
4.5	.693	.707	.721	.735	.749	.763	.777	.791	.805	.819	.833	.847	4.5
5	.706	.720	.734	.748	.762	.776	.790	.804	.818	.832	.846	.860	5
6	.728	.742	.756	.770	.784	.798	.812	.826	.840	.854	.868	.882	6
7	.747	.761	.775	.789	.803	.817	.831	.845	.859	.873	.887	.901	7
8	.763	.777	.791	.805	.819	.833	.847	.861	.875	.889	.903	.917	8
9	.777	.791	.805	.819	.833	.847	.861	.875	.889	.903	.918	.932	9
10	.790	.804	.818	.832	.846	.860	.874	.888	.902	.916	.930	.944	10
11	.801	.816	.830	.844	.858	.872	.886	.900	.914	.928	.942	.956	11
12	.812	.826	.840	.854	.868	.882	.896	.910	.924	.938	.952	.967	12
13	.822	.836	.850	.864	.878	.892	.906	.920	.934	.948	.962	.976	13
14	.831	.845	.859	.873	.887	.901	.915	.929	.943	.957	.971	.985	14
15	.839	.853	.867	.881	.895	.909	.923	.937	.951	.965	.980	.994	15
16	.847	.861	.875	.889	.903	.917	.931	.945	.959	.973	.987	1.001	16
18	.861	.875	.889	.903	.917	.931	.946	.960	.973	.987	1.002	1.016	18
20	.874	.888	.902	.916	.930	.944	.958	.972	.986	1.000	1.014	1.029	20

TABLE XII. — REACTANCE OF CABLE, 60 CYCLES.

$x = 2\pi f \times 741.1 \log_{10} \frac{s}{0.728\rho} \times 10^{-6}$ ohms per mile for a 7-wire stranded cable. $\left[\text{Use } \log_{10} \frac{s}{0.758\rho} \text{ for 19 wires.}\right]$

ρ = maximum radius of cable.
s = spacing.

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

	Sizes for copper cables.						Sizes for aluminum cables.					
Circular mils.	500,000	450,000	400,000	350,000	300,000	250,000	556,500	477,000	397,500	336,420	266,800	
No. of wires assumed....	19	19	19	19	7	7	19	19	19	7	7	Spacing.
Diameter (2ρ) inches.	.8111	.7695	.7255	.6786	.6211	.5669	.8557	.7922	.7232	.6577	.5857	
Spacing (s) feet.												
1	.445	.451	.458	.466	.482	.493	.438	.447	.459	.475	.489	1
1.5	.494	.500	.507	.515	.531	.542	.487	.496	.508	.524	.538	1.5
2	.529	.535	.542	.550	.566	.577	.522	.532	.543	.559	.573	2
2.5	.556	.562	.569	.577	.593	.604	.549	.559	.570	.586	.600	2.5
3	.578	.584	.591	.599	.615	.626	.571	.581	.592	.608	.622	3
3.5	.597	.603	.610	.618	.634	.645	.590	.599	.611	.627	.641	3.5
4	.613	.619	.626	.634	.650	.661	.606	.616	.627	.643	.657	4
4.5	.627	.634	.641	.649	.664	.675	.621	.630	.641	.657	.672	4.5
5	.640	.646	.653	.662	.677	.688	.633	.643	.654	.670	.684	5
6	.662	.668	.676	.684	.699	.710	.655	.665	.676	.692	.706	6
7	.681	.687	.694	.702	.718	.729	.674	.684	.695	.711	.725	7
8	.697	.703	.710	.719	.734	.745	.690	.700	.711	.727	.741	8
9	.711	.718	.725	.733	.749	.760	.705	.714	.725	.742	.756	9
10	.724	.730	.738	.746	.761	.772	.717	.727	.738	.754	.768	10
11	.736	.742	.749	.757	.773	.784	.729	.738	.749	.766	.780	11
12	.746	.752	.760	.768	.784	.794	.740	.749	.760	.776	.791	12
13	.755	.762	.769	.778	.793	.804	.749	.759	.770	.786	.800	13
14	.765	.771	.778	.786	.802	.813	.758	.768	.779	.795	.809	14
15	.773	.780	.787	.795	.811	.822	.767	.776	.787	.804	.818	15
16	.781	.787	.795	.803	.818	.829	.774	.784	.795	.811	.825	16
18	.795	.802	.809	.817	.833	.844	.789	.796	.809	.826	.840	18
20	.808	.815	.822	.830	.845	.856	.802	.811	.822	.838	.852	20

TABLE XII. — REACTANCE OF CABLE, 60 CYCLES. (*Continued.*)

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

Sizes for both Copper and Aluminum Cables.

B. & S. gauge	No. 0000	No. 000	No. 00	No. 0	No. 1	No. 2	No. 3	No. 4	Spacing.
Circular mils	211,600	167,800	133,100	105,500	83,690	66,370	52,630	41,740	
No. of wires assumed	7	7	7	7	7	7	7	7	
Diameter (2ρ) inches	.5216	.4645	.4137	.3683	.3280	.2921	.2601	.2317	
Spacing (s) feet.									
1	.503	.517	.531	.545	.560	.574	.588	.602	1
1.5	.552	.566	.580	.595	.609	.623	.637	.651	1.5
2	.587	.601	.615	.629	.644	.658	.672	.686	2
2.5	.614	.628	.642	.656	.671	.685	.699	.713	2.5
3	.636	.651	.664	.678	.693	.707	.721	.735	3
3.5	.655	.669	.683	.697	.711	.725	.739	.754	3.5
4	.671	.685	.699	.714	.728	.742	.756	.770	4
4.5	.686	.700	.714	.728	.742	.756	.770	.784	4.5
5	.698	.712	.727	.741	.755	.769	.783	.797	5
6	.721	.734	.749	.763	.777	.791	.805	.819	6
7	.739	.753	.767	.781	.796	.810	.824	.838	7
8	.755	.770	.784	.798	.812	.826	.840	.854	8
9	.770	.784	.798	.812	.826	.840	.854	.868	9
10	.782	.797	.811	.825	.839	.853	.867	.881	10
11	.794	.808	.822	.836	.850	.865	.878	.893	11
12	.805	.819	.833	.847	.861	.875	.889	.903	12
13	.814	.828	.842	.857	.871	.885	.899	.913	13
14	.823	.837	.851	.866	.880	.894	.908	.922	14
15	.832	.846	.860	.874	.888	.902	.916	.930	15
16	.839	.854	.868	.882	.896	.910	.924	.938	16
18	.854	.868	.882	.896	.910	.924	.938	.952	18
20	.867	.881	.895	.909	.923	.937	.951	.965	20

TABLE XIII. — CAPACITY SUSCEPTANCE OF WIRE, 25 CYCLES.

$$b \text{ (approx.)} = \frac{2\pi f \times 38.83 \times 10^{-9}}{\log_{10}\left(\frac{s}{\rho} - \frac{\rho}{s}\right)} \text{ mhos per mile.}$$

ρ = radius of wire.
s = spacing.

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

B. & S. gauge.	No. 0000	No. 000	No. 00	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	Spacing.
Circular mils.	211,600	167,800	133,100	105,500	83,690	66,370	52,630	41,740	33,100	26,250	20,820	16,510	
Diameter (2ρ) inches.	.4600	.4096	.3648	.3249	.2893	.2576	.2294	.2043	.1819	.1620	.1443	.1285	
Spacing (s) feet.	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	
1	3.55	3.45	3.35	3.26	3.18	3.10	3.02	2.95	2.88	2.81	2.75	2.69	1
1.5	3.22	3.14	3.06	2.98	2.91	2.84	2.78	2.72	2.66	2.60	2.54	2.49	1.5
2	3.02	2.95	2.88	2.81	2.75	2.69	2.63	2.57	2.52	2.47	2.42	2.37	2
2.5	2.88	2.82	2.75	2.69	2.63	2.58	2.52	2.47	2.42	2.38	2.33	2.29	2.5
3	2.78	2.72	2.66	2.60	2.55	2.49	2.44	2.40	2.35	2.30	2.26	2.22	3
3.5	2.70	2.64	2.58	2.53	2.48	2.43	2.38	2.33	2.29	2.25	2.21	2.17	3.5
4	2.63	2.57	2.52	2.47	2.42	2.37	2.33	2.28	2.24	2.20	2.16	2.12	4
4.5	2.57	2.52	2.47	2.42	2.37	2.33	2.28	2.24	2.20	2.16	2.12	2.09	4.5
5	2.52	2.47	2.42	2.38	2.33	2.29	2.24	2.20	2.16	2.13	2.09	2.05	5
6	2.44	2.40	2.35	2.30	2.26	2.22	2.18	2.15	2.10	2.07	2.03	2.00	6
7	2.38	2.34	2.29	2.25	2.21	2.17	2.13	2.09	2.06	2.02	1.99	1.96	7
8	2.33	2.28	2.24	2.20	2.16	2.12	2.09	2.05	2.02	1.98	1.95	1.92	8
9	2.28	2.24	2.20	2.16	2.12	2.09	2.05	2.02	1.98	1.95	1.92	1.89	9
10	2.24	2.20	2.16	2.13	2.09	2.05	2.02	1.99	1.96	1.92	1.89	1.86	10
11	2.21	2.17	2.13	2.10	2.06	2.03	1.99	1.96	1.93	1.90	1.87	1.84	11
12	2.18	2.14	2.11	2.07	2.03	2.00	1.97	1.94	1.91	1.88	1.85	1.82	12
13	2.15	2.12	2.08	2.04	2.01	1.98	1.95	1.92	1.89	1.86	1.83	1.80	13
14	2.13	2.09	2.06	2.02	1.99	1.96	1.93	1.90	1.87	1.84	1.81	1.79	14
15	2.11	2.07	2.04	2.00	1.97	1.94	1.91	1.88	1.85	1.82	1.80	1.77	15
16	2.09	2.05	2.02	1.99	1.95	1.92	1.89	1.86	1.84	1.81	1.78	1.75	16
18	2.05	2.02	1.99	1.95	1.92	1.89	1.86	1.83	1.81	1.78	1.75	1.73	18
20	2.02	1.99	1.96	1.92	1.90	1.87	1.84	1.81	1.78	1.76	1.73	1.71	20

TABLE XIV. — CAPACITY SUSCEPTANCE OF CABLE, 25 CYCLES.

$$b \text{ (approx.)} = \frac{2\pi f \times 38.83 \times 10^{-9}}{\log_{10}\left(\frac{s}{\rho} - \frac{\rho}{s}\right)} \text{ mhos per mile.}$$

ρ = maximum radius of cable.
s = spacing.

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

	Sizes for copper cables.						Sizes for aluminum cables.					
Circular mils.	500,000	450,000	400,000	350,000	300,000	250,000	556,500	477,000	397,500	336,420	266,800	
No. of wires assumed....	19	19	19	19	7	7	19	19	19	7	7	Spacing.
Diameter (2ρ) inches.	.8111	.7695	.7255	.6786	.6211	.5669	.8557	.7922	.7232	.6577	.5857	
Spacing (s) feet.	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	
1	4.15	4.08	4.01	3.94	3.84	3.75	4.21	4.12	4.01	3.91	3.78	1
1.5	3.70	3.65	3.60	3.54	3.46	3.38	3.76	3.68	3.60	3.51	3.41	1.5
2	3.44	3.40	3.35	3.30	3.23	3.16	3.49	3.42	3.35	3.27	3.19	2
2.5	3.26	3.23	3.18	3.13	3.07	3.01	3.31	3.25	3.18	3.11	3.03	2.5
3	3.13	3.10	3.06	3.01	2.95	2.90	3.17	3.11	3.05	2.99	2.92	3
3.5	3.03	2.99	2.95	2.91	2.86	2.81	3.06	3.01	2.95	2.90	2.83	3.5
4	2.94	2.91	2.87	2.84	2.79	2.74	2.98	2.93	2.87	2.82	2.75	4
4.5	2.87	2.84	2.81	2.77	2.72	2.68	2.90	2.86	2.81	2.75	2.69	4.5
5	2.81	2.78	2.75	2.71	2.67	2.62	2.84	2.80	2.75	2.70	2.64	5
6	2.71	2.68	2.65	2.62	2.58	2.54	2.74	2.70	2.65	2.61	2.55	6
7	2.63	2.61	2.58	2.55	2.51	2.47	2.66	2.62	2.58	2.53	2.48	7
8	2.57	2.54	2.52	2.49	2.45	2.41	2.59	2.56	2.52	2.47	2.42	8
9	2.52	2.49	2.47	2.44	2.40	2.36	2.54	2.50	2.46	2.42	2.38	9
10	2.47	2.45	2.42	2.39	2.36	2.32	2.49	2.46	2.42	2.38	2.34	10
11	2.43	2.41	2.38	2.35	2.32	2.29	2.45	2.42	2.38	2.34	2.30	11
12	2.39	2.37	2.35	2.32	2.29	2.25	2.41	2.38	2.35	2.31	2.27	12
13	2.36	2.34	2.32	2.29	2.26	2.23	2.38	2.35	2.32	2.28	2.24	13
14	2.33	2.31	2.29	2.26	2.23	2.20	2.35	2.32	2.29	2.25	2.21	14
15	2.31	2.29	2.26	2.24	2.21	2.18	2.32	2.30	2.26	2.23	2.19	15
16	2.28	2.26	2.24	2.21	2.19	2.15	2.30	2.27	2.24	2.21	2.17	16
18	2.24	2.22	2.20	2.17	2.15	2.11	2.26	2.23	2.20	2.16	2.13	18
20	2.20	2.18	2.16	2.14	2.11	2.08	2.22	2.19	2.16	2.13	2.09	20

TABLE XIV. — CAPACITY SUSCEPTANCE OF CABLE, 25 CYCLES. (*Continued.*)

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

Sizes for both Copper and Aluminum Cables.

B. & S. gauge.	No. 0000	No. 000	No. 00	No. 0	No. 1	No. 2	No. 3	No. 4	Spacing.
Circular mils.	211,600	167,800	133,100	105,500	83,690	66,370	52,630	41,740	
No. of wires assumed....	7	7	7	7	7	7	7	7	
Diameter (2ρ) inches..	.5216	.4645	.4137	.3683	.3280	.2921	.2601	.2317	
Spacing (s) feet.	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	
1	3.67	3.56	3.46	3.36	3.27	3.19	3.10	3.03	1
1.5	3.32	3.23	3.14	3.06	2.99	2.92	2.85	2.78	1.5
2	3.10	3.03	2.95	2.88	2.82	2.75	2.69	2.63	2
2.5	2.96	2.89	2.82	2.76	2.70	2.64	2.58	2.53	2.5
3	2.85	2.79	2.72	2.66	2.61	2.55	2.50	2.45	3
3.5	2.76	2.70	2.64	2.59	2.53	2.48	2.43	2.38	3.5
4	2.68	2.63	2.58	2.52	2.47	2.42	2.38	2.33	4
4.5	2.63	2.58	2.52	2.47	2.42	2.38	2.33	2.29	4.5
5	2.58	2.53	2.48	2.43	2.38	2.33	2.29	2.25	5
6	2.50	2.45	2.40	2.35	2.31	2.26	2.22	2.18	6
7	2.43	2.38	2.34	2.29	2.25	2.21	2.17	2.13	7
8	2.38	2.33	2.29	2.25	2.20	2.16	2.13	2.09	8
9	2.33	2.29	2.24	2.20	2.16	2.13	2.09	2.05	9
10	2.29	2.25	2.21	2.17	2.13	2.09	2.06	2.02	10
11	2.25	2.21	2.17	2.14	2.10	2.06	2.03	2.00	11
12	2.22	2.18	2.14	2.11	2.07	2.04	2.00	1.97	12
13	2.20	2.16	2.12	2.08	2.05	2.01	1.98	1.95	13
14	2.17	2.13	2.10	2.06	2.03	1.99	1.96	1.93	14
15	2.15	2.11	2.07	2.04	2.01	1.97	1.94	1.91	15
16	2.13	2.09	2.05	2.02	1.99	1.96	1.92	1.89	16
18	2.09	2.05	2.02	1.99	1.96	1.92	1.89	1.86	18
20	2.06	2.02	1.99	1.96	1.93	1.90	1.87	1.84	20

TABLE XV. — CAPACITY SUSCEPTANCE OF WIRE, 60 CYCLES.

$$b \text{ (approx.)} = \frac{2\pi f \times 38.83 \times 10^{-9}}{\log_{10}\left(\frac{s}{\rho} - \frac{\rho}{s}\right)} \text{ mhos per mile.}$$

ρ = radius of wire.
s = spacing.

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

B. & S. gauge.	No. 0000	No. 000	No. 00	No. 0	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	Spacing.
Circular mils.	211,600	167,800	133,100	105,500	83,690	66,370	52,630	41,740	33,100	26,250	20,820	16,510	
Diameter (2ρ) inches	.4600	.4096	.3648	.3249	.2893	.2576	.2294	.2043	.1819	.1620	.1443	.1285	
Spacing (s) feet.	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	
1	8.52	8.28	8.05	7.83	7.63	7.43	7.25	7.07	6.90	6.74	6.59	6.45	1
1.5	7.73	7.53	7.34	7.16	6.99	6.82	6.67	6.52	6.38	6.24	6.11	5.98	1.5
2	7.25	7.08	6.91	6.75	6.60	6.45	6.31	6.18	6.05	5.92	5.80	5.69	2
2.5	6.92	6.76	6.61	6.46	6.32	6.18	6.06	5.93	5.81	5.70	5.59	5.48	2.5
3	6.67	6.52	6.38	6.24	6.11	5.98	5.86	5.75	5.64	5.53	5.44	5.33	3
3.5	6.47	6.33	6.20	6.07	5.94	5.83	5.71	5.60	5.49	5.39	5.29	5.20	3.5
4	6.31	6.18	6.05	5.93	5.81	5.69	5.58	5.48	5.38	5.28	5.19	5.10	4
4.5	6.18	6.05	5.92	5.81	5.69	5.58	5.48	5.38	5.28	5.18	5.09	5.01	4.5
5	6.06	5.93	5.82	5.70	5.59	5.49	5.39	5.29	5.19	5.10	5.01	4.93	5
6	5.87	5.75	5.64	5.53	5.43	5.33	5.23	5.15	5.05	4.97	4.88	4.80	6
7	5.71	5.60	5.50	5.40	5.30	5.20	5.11	5.02	4.94	4.85	4.77	4.70	7
8	5.59	5.48	5.38	5.28	5.19	5.10	5.01	4.92	4.84	4.76	4.69	4.61	8
9	5.48	5.38	5.28	5.19	5.09	5.01	4.92	4.84	4.76	4.69	4.61	4.54	9
10	5.39	5.29	5.19	5.10	5.02	4.93	4.85	4.77	4.69	4.62	4.55	4.48	10
11	5.31	5.21	5.12	5.03	4.95	4.86	4.78	4.70	4.63	4.56	4.49	4.42	11
12	5.23	5.14	5.05	4.97	4.88	4.80	4.72	4.65	4.58	4.50	4.44	4.37	12
13	5.17	5.08	4.99	4.91	4.83	4.75	4.67	4.60	4.53	4.46	4.39	4.32	13
14	5.11	5.02	4.94	4.86	4.78	4.70	4.62	4.55	4.48	4.41	4.35	4.29	14
15	5.06	4.97	4.89	4.81	4.73	4.65	4.58	4.51	4.44	4.37	4.31	4.25	15
16	5.01	4.93	4.84	4.76	4.69	4.61	4.54	4.47	4.40	4.34	4.27	4.21	16
18	4.92	4.84	4.76	4.69	4.61	4.54	4.47	4.40	4.34	4.27	4.21	4.15	18
20	4.85	4.77	4.69	4.62	4.55	4.48	4.41	4.34	4.28	4.22	4.16	4.10	20

TABLE XVI. — CAPACITY SUSCEPTANCE OF CABLE, 60 CYCLES.

$$b \text{ (approx.)} = \frac{2\pi f \times 38.83 \times 10^{-9}}{\log_{10}\left(\frac{s}{\rho} - \frac{\rho}{s}\right)} \text{ mhos per mile.}$$

ρ = maximum radius of cable.
s = spacing.

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

	Sizes for copper cables.						Sizes for aluminum cables.					
Circular mils.	500,000	450,000	400,000	350,000	300,000	250,000	556,500	477,000	397,500	336,420	266,800	Spacing.
No. of wires assumed....	19	19	19	19	7	7	19	19	19	7	7	
Diameter (2ρ) inches.	.8111	.7695	.7255	.6786	.6211	.5669	.8557	.7922	.7232	.6577	5857	
Spacing (s) feet.	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	
1	9.95	9.80	9.63	9.45	9.22	9.00	10.11	9.88	9.62	9.37	9.08	1
1.5	8.89	8.77	8.63	8.49	8.30	8.12	9.01	8.83	8.63	8.42	8.19	1.5
2	8.26	8.16	8.04	7.91	7.75	7.59	8.37	8.21	8.04	7.86	7.65	2
2.5	7.83	7.74	7.64	7.52	7.37	7.23	7.95	7.79	7.63	7.47	7.28	2.5
3	7.51	7.43	7.33	7.23	7.09	6.96	7.61	7.48	7.33	7.18	7.01	3
3.5	7.26	7.18	7.09	7.00	6.87	6.74	7.35	7.23	7.09	6.95	6.79	3.5
4	7.06	6.98	6.90	6.81	6.69	6.57	7.14	7.03	6.90	6.76	6.61	4
4.5	6.89	6.82	6.74	6.65	6.53	6.42	6.97	6.86	6.73	6.61	6.46	4.5
5	6.75	6.68	6.60	6.51	6.40	6.29	6.82	6.71	6.59	6.47	6.33	5
6	6.51	6.44	6.37	6.29	6.19	6.09	6.58	6.48	6.37	6.26	6.12	6
7	6.32	6.26	6.19	6.12	6.02	5.92	6.38	6.29	6.19	6.08	5.96	7
8	6.17	6.11	6.04	5.97	5.88	5.79	6.23	6.14	6.04	5.94	5.82	8
9	6.04	5.98	5.92	5.85	5.76	5.67	6.09	6.01	5.91	5.82	5.70	9
10	5.92	5.87	5.81	5.74	5.66	5.57	5.98	5.90	5.81	5.71	5.60	10
11	5.83	5.77	5.72	5.65	5.57	5.48	5.88	5.80	5.71	5.62	5.52	11
12	5.74	5.69	5.63	5.57	5.49	5.41	5.79	5.72	5.63	5.54	5.44	12
13	5.66	5.61	5.56	5.50	5.42	5.34	5.72	5.64	5.56	5.47	5.37	13
14	5.59	5.54	5.49	5.43	5.36	5.28	5.64	5.57	5.49	5.41	5.31	14
15	5.53	5.48	5.43	5.37	5.30	5.22	5.58	5.51	5.43	5.35	5.25	15
16	5.47	5.43	5.37	5.32	5.24	5.17	5.52	5.45	5.37	5.29	5.20	16
18	5.37	5.33	5.28	5.22	5.15	5.08	5.42	5.35	5.27	5.20	5.10	18
20	5.28	5.24	5.19	5.14	5.07	5.00	5.33	5.26	5.19	5.11	5.03	20

TABLE XVI. — CAPACITY SUSCEPTANCE OF CABLE, 60 CYCLES. (*Continued.*)

For three-phase irregular spacing use $s = \sqrt[3]{abc}$.
For three-phase regular flat spacing use $s = 1.26\,a$.
For a two-phase line the spacing is the average distance between centers of conductors of the same phase.

Sizes for both Copper and Aluminum Cables.

B. & S. gauge.	No. 0000	No. 000	No. 00	No. 0	No. 1	No. 2	No. 3	No. 4	
Circular mils.	211,600	167,800	133,100	105,500	83,690	66,370	52,630	41,740	
No. of wires assumed....	7	7	7	7	7	7	7	7	Spacing.
Diameter (2ρ) inches.	.5216	.4645	.4137	.3683	.3280	.2921	.2601	.2317	
Spacing (s) feet.	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	
1	8.80	8.54	8.30	8.07	7.85	7.64	7.45	7.26	1
1.5	7.96	7.75	7.55	7.36	7.17	7.00	6.84	6.68	1.5
2	7.45	7.27	7.09	6.92	6.76	6.61	6.46	6.32	2
2.5	7.10	6.93	6.77	6.62	6.47	6.33	6.20	6.07	2.5
3	6.84	6.68	6.53	6.39	6.25	6.12	5.99	5.87	3
3.5	6.63	6.48	6.34	6.21	6.08	5.95	5.83	5.72	3.5
4	6.46	6.32	6.19	6.06	5.94	5.82	5.70	5.59	4
4.5	6.32	6.19	6.06	5.93	5.81	5.70	5.59	5.49	4.5
5	6.20	6.07	5.94	5.82	5.71	5.60	5.49	5.39	5
6	6.00	5.88	5.76	5.65	5.54	5.44	5.34	5.24	6
7	5.84	5.72	5.61	5.50	5.40	5.30	5.21	5.12	7
8	5.70	5.59	5.49	5.39	5.29	5.19	5.10	5.02	8
9	5.59	5.49	5.39	5.29	5.19	5.10	5.01	4.93	9
10	5.50	5.39	5.30	5.20	5.11	5.02	4.94	4.85	10
11	5.41	5.31	5.22	5.13	5.04	4.95	4.87	4.79	11
12	5.34	5.24	5.15	5.06	4.97	4.89	4.81	4.73	12
13	5.27	5.18	5.09	5.00	4.92	4.83	4.75	4.68	13
14	5.21	5.12	5.03	4.95	4.86	4.78	4.70	4.63	14
15	5.16	5.07	4.98	4.90	4.82	4.74	4.67	4.59	15
16	5.11	5.02	4.93	4.85	4.77	4.69	4.62	4.55	16
18	5.02	4.93	4.85	4.77	4.69	4.62	4.55	4.48	18
20	4.94	4.86	4.78	4.70	4.63	4.55	4.48	4.41	20

TABLE XVII.—POWER FACTOR TABLE.

Cos θ.	Sin θ (approx.).	Sin θ.
.95	.312	.312250
.90	.436	.435890
.85	.527	.526783
.80	.600	.600000
.75	.661	.661438
.70	.714	.714143
.65	.760	.759934
.60	.800	.800000
.55	.835	.835165
.50	.866	.866025

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