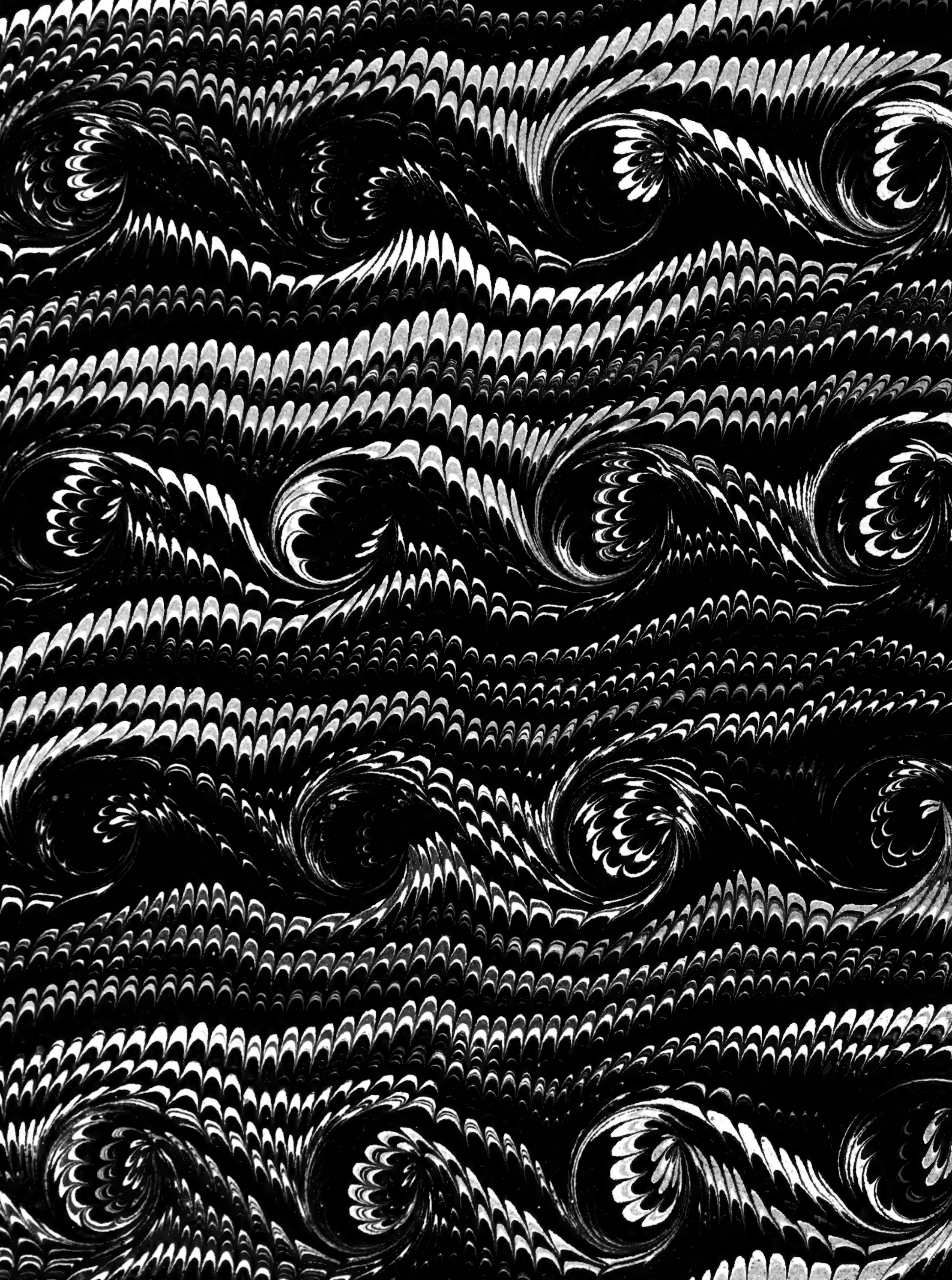

TABLES

D'INTÉGRALES DÉFINIES

PAR

D. BIERENS DE HAAN.

Publiées par l'Académie Royale des Sciences à Amsterdam.

IV

PREMIÈRE PARTIE.

AMSTERDAM,
C. G. VAN DER POST.
1856.

VERHANDELINGEN.

VERHANDELINGEN

DER

KONINKLIJKE AKADEMIE

VAN

WETENSCHAPPEN.

VIERDE DEEL.

AMSTERDAM,
C. G. VAN DER POST.
1858.

GEDRUKT BIJ W. J. KRÖBER.

TABLES

D'INTÉGRALES DÉFINIES

PAR

D. BIERENS DE HAAN.

SOMMAIRE.

PRÉFACE.

Dans la construction de ces *Tables d'Intégrales définies* j'avais en vue un objet quadruple.

En premier lieu je voulais réunir les uns auprès des autres les différents résultats, épars par-ci et par-là, que l'on avait obtenus au sujet de ces fonctions, par beaucoup de méthodes intrinsèquement différentes, et pour la plupart plus ou moins indirectes. Il résultait de cette dispersion des formules obtenues, que l'on ne pouvait en tirer tout le profit possible, tant pour la pratique, — c'est-à-dire pour les cas, où l'on pourrait avoir besoin des valeurs d'une certaine intégrale définie, — que pour la théorie elle-même, — c'est-à-dire pour l'emploi de ces formules dans la déduction d'autres intégrales définies, et pour la vérification de nouvelles formules de ce genre, à l'égard desquelles on pouvait entretenir des doutes, par rapport à la priorité ou à l'originalité. Une collection d'intégrales définies bien ordonnée peut certainement obvier à tous ces inconvenients.

De ce point de vue suivait naturellement une autre considération non moins importante. Après avoir réuni les diverses formules, il importait beaucoup de connaître leur méthode de déduction: et cela d'autant plus, que plusieurs méthodes employées en d'autres temps avec une confiance absolue, ne sont maintenant plus à l'abri d'objections, en quelques cas très-fondées. Dès-lors, pour être sûr d'un résultat quelconque, il fallait absolument que l'on fût à même de juger de la validité et de l'exactitude de la méthode employée. Or il ne pouvait entrer dans le plan de rédaction de ces Tables, déjà assez volumineuses, d'ajouter à côté de chaque intégrale la méthode à l'aide de laquelle on l'avait déduite, quand même il eût été possible de l'indiquer d'une manière courte, précise et certaine: de plus il y a beaucoup de formules, qui peuvent être trouvées de plus d'une manière. J'ai tâché de subvenir à cette difficulté d'une autre manière, qui, à ce que j'espère, ne manquera pas d'approbation. A côté de chaque formule in-

A

tégrale se trouve une notice bibliographique indiquant, où l'on peut en trouver la déduction, ou même plusieurs déductions diverses, s'il y en a. De cette façon chacun est mis en état de juger par lui-même de la validité des résultats indiqués, et de répéter lui-même les calculs nécessaires, s'il pourrait le juger convenable.

Mais par ces notices bibliographiques elles-mêmes il était en même temps possible de remplir un troisième but, celui de donner un tableau historique et bibliographique de cette branche de l'analyse. Pour que ce tableau fût complet, il aurait fallu que j'eusse pu parcourir tous les ouvrages, où pourraient se trouver des intégrales définies. C'était une entreprise à peu près impossible, puisque d'une part je n'aurais jamais pu m'assurer de n'avoir omis aucun livre, et que je ne me trouvais pas dans une condition de pouvoir les avoir tous sous les yeux: tandis que d'autre part le travail serait devenu d'une telle longueur que je n'aurais pas osé l'entreprendre. Néanmoins je dois confesser que de ce côté-lá mes désirs, peut-être trop ardents par l'intérêt personnel que je portais naturellement au succès de mon entreprise, n'ont pas été remplis comme je l'avais désiré, ni comme je l'avais espéré. J'avais demandé par la voie de quelques journaux scientifiques l'envoi des notes ou des mémoires monographiques, qui pourraient exister sur la théorie des intégrales définies: et j'avoue avoir assez compté sur l'intérêt que les formules, dont je me proposais la récolte, doivent inspirer aux Analystes, pour attendre quelques fruits de cette démarche: mais personne n'a repondu à l'appel. Tout dépendait donc de moi-même et c'est par le sommaire des livres, des journaux et des mémoires consultés (pages 20 et 21) que l'on pourra juger jusqu'à quel point les Tables peuvent être censées complètes. Si toutefois, comme je n'en doute guère, il y a des écrits, qui portent sur cette matière et qui pourtant ne se trouvent pas sur cette liste, qu'on veuille bien trouver l'excuse de leur omission dans ce que je viens de dire; l'espèce de reproche, qui s'y trouve, n'a son origine que dans mon désir de rendre mon excuse plus fondée. Quant à ce sommaire, il donne lieu à quelques observations. Les journaux mathématiques Anglais et Américains y manquent complétement, puisque je n'ai pas eu moyen de m'en procurer l'étude: la même observation se repête pour les livres et les monographies de ces pays, que l'on trouve peu chez nous. J'ai été bien fâché que tel ait été le cas, puisque bien certainement le fruit de leurs études m'eût fourni bien des données intéressantes: toutefois le Journal de Mathématiques pures et appliquées, rédigé par M. J. Liouville, nous donne quelques-unes de ces recherches, et j'ai dû me contenter de celles-là. Quant aux Mémoires des diverses Académies et Institutions scientifiques, il y avait de ce côté-là une occasion magnifique et unique pour notre patrie dans la Bibliothéque de l'Académie Royale des Sciences à Amsterdam; et comme l'entrée m'y était ouverte primitivement par les soins bienveillants de M. W. Vrolik, Sécrétaire de cette Académie, et plus tard par les droits acquis par mon titre de membre, j'ai tâché de ne rien laisser désirer à cet égard. Si l'on prend la peine de feuilleter les Tables, on voudra bien admettre, j'espère, que si tout le matériel ne s'y trouve pas, c'est bien au moins le plus grand nombre des formules trouvées, que l'on y rencontre. Je dois encore ajouter ici qu'en général je n'ai admis dans les Tables que toutes les intégrales définies, que j'ai pu trouver évaluées avant la fin de l'année 1853, lorsque j'ai commencé la rédaction en tables. Voici un sommaire — par ordre alphabétique des noms d'auteurs — des mémoires principaux, contenus dans les diverses Collections Académiques et dans les journaux scientifiques, que j'ai consultés: on y rencontre bien des noms éminents.

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Plana, Mém. Turin. 23. 1818. 7. — Mém. Turin. 25. 1820. — Mém. Brux. 10. 1837. — Cr. 17. 1. — Cr. 17. 163. — Cr. 17. 345.

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Enfin je m'étais encore proposé un quatrième but: savoir la critique des intégrales définies, que je trouvais. Mais de ce côté-là surgirent bien des obstacles. L'on ne pourrait exiger, que j'eusse fait la révision de tous les calculs, en général assez longs, dont les résultats seulement remplissent tant de volumes: mais la seule et la moindre difficulté ne se trouvait pas là; une autre était d'une importance bien plus grande pour la rédaction de ces Tables. Ce n'est pas seulement pourtant contre les anciennes méthodes, que se sont élevées des objections dont j'ai parlé précédemment, mais plusieurs des méthodes nouvelles ou récemment appliquées ont subi le même sort. En un mot telle, méthode employée, et par suite admise par tel Analyste, est rejetée comme fausse par un autre: donc les résultats obtenus par l'un ne sont pas admis par celui d'une opinion contraire. Fallait-il que je me fusse posé en juge? Je me suis souvent fait cette question: mais je n'osais le faire, je ne croyais pas les fondemens de cette partie de l'Analyse toujours basés sur des principes d'une telle stabilité, que l'on aurait le droit absolu de juger sans merci les pensées, les recherches d'un autre. C'est pourquoi j'ai aussi admis dans les Tables les résultats obtenus par des méthodes, que pour moi-même je ne saurais regarder comme valides. Je m'y suis résolu d'autant plus volontiers que l'annexion des noms des auteurs, qui ont déduit ces résultats au moins douteux, donne pour ainsi dire des poids, qui en indiquent et en mesurent la certitude et la validité. Dans cette catégorie tombent par exemple toutes les intégrales définies, qui se trouvent Partie I, Section IV, Tables 96, 97, 98, 102, en tant que les fonctions circulaires directes aient la première puissance de la variable pour argument. Ces intégrales de fonctions circulaires directes de x seulement, prises entre les limites zéro et l'infini, — dont les valeurs, entre autres d'après M. Raabe, ont obtenu leurs places aux Tables indiquées — sont évidemment indéterminées selon ma manière de voir: néanmoins, d'après les principes exposés j'ai cru devoir les admettre dans les Tables. Quiconque parcourt ces Tables peut s'assurer lui-même, par l'inspection des citations bibliographiques de l'exactitude et de la validité de la méthode de M. Raabe. Il y a encore une autre classe d'intégrales définies, que je m'occupe d'étudier, savoir celles qui contiennent un élément k, que l'on fait diverger vers l'infini: jusques à présent je ne suis pas toujours du même avis que plusieurs auteurs, qui en ont fait usage: mais puisque en premier lieu mes recherches n'ont pas encore atteint le but proposé, et que d'un autre côté les Tables ont déjà été rédigées avant la fin de l'année 1854, j'ai jugé à propos de ne pas admettre mes résultats dans les Tables: pourtant je dois avertir que les valeurs des intégrales mentionnées sont d'une grande influence sur plusieurs autres intégrales définies, dont on acquiert la valeur au moyen d'elles *). Néanmoins des observations critiques de ma part ne se font pas désirer: on les trouve là, où les résultats divergents de différents Mathématiciens demandaient un jugement: là aussi, où une faute de calcul, qui me frappait, rendait le résultat vicieux.

Voilà mes considérations, quant au but que je me suis proposé et à la manière dont j'ai cherché à l'atteindre — en recueillant les formules déduites par d'autres auteurs. Je ne manquais pas de reconnaître bientôt, que je me trouvais dans une position favorable pour en déduire des résultats nouveaux, et je vais m'expliquer dans quelle voie je me suis engagé à cet égard. En premier lieu il était aisé quelquefois de déduire d'autres intégrales par voie d'addition ou de soustraction des résultats déjà obtenus. D'une autre part plusieurs des sections me semblaient présenter des lacunes en quelques points, et n'être pas assez complètes; j'ai tâché d'y remédier en transformant les intégrales définies d'une autre catégorie, par la méthode connue de transformation de la variable, en d'autres formules telles, qu'elles pussent prendre place là, où ces lacunes se trouvaient à remplir. Il va sans dire que je n'ai pas eu l'intention d'épuiser toutes les substitutions possibles; j'ajouterai seulement que je ne les ai appliquées que là, où cette application était directement permise, c'est-à-dire où dans la détermination des limites elles n'offraient pas des maxima ou des minima, qui pouvaient donner lieu à des incertitudes ou au moins à des objections. De plus j'ai toujours donné la préférence à de telles intégrales résultantes, qui pouvaient se prêter à la méthode suivante, qui m'a fourni en grande partie les résultats nouveaux, que l'on trouvera dans les Tables.

Cette méthode, exposée et appliquée plus amplement dans une „Note sur une méthode pour la réduction d'intégrales définies et sur son application à quelques formules spéciales", que l'Académie Royale des Sciences m'a fait l'honneur de faire imprimer dans le deuxième Volume de ses Mémoires, revient à celle d'intégration partielle. Elle est contenue dans la formule

$$\int_a^b f(x)\,d.\mathrm{F}(x) - f(x).\mathrm{F}(x)\Big\}_a^b + \int_a^b \mathrm{F}(x)\,d.f(x) = 0.$$

On peut appliquer cette formule de transformation à une intégrale définie, aussitôt que la fonction intégrée se laisse diviser en deux facteurs dont l'un peut être considéré comme la différentielle de quelque fonction connue; car dès-lors cette intégrale définie rentre sous la forme du troisième

*) Depuis cette Note a été présentée et se trouvera dans le Volume VII de ces Mémoires.

terme de l'équation précédente, et la formule donnera lieu à la détermination du premier terme, — qui est une nouvelle intégrale définie, en général d'une forme tout-à-fait différente, — à moins que le deuxième terme $f(x).\ F(x)\Big\}_a^b$, c'est-à-dire pris de la limite a jusques à l'autre b, n'offre de difficultés ou d'obstacles, qu'il reste continu entre ces limites, et que sa valeur soit finie et assignable pour ces limites elles-mêmes. L'on observera sans peine dans les Tables les fruits, que cette méthode a portés.

Quant aux résultats — leur nombre est près de trois mille deux-cent — que j'obtenais ainsi par une des méthodes mentionnées, et qui n'étaient pas encore trouvés par d'autres, ils se distinguent par un manque de bibliographie; on y trouve seulement un renvoi vers une autre intégrale définie, dont elle a été déduite en général à l'aide d'une des trois méthodes précédentes. Je n'ai pas ajouté de quelle méthode j'ai fait usage dans chaque cas spécial, puisqu'en général on peut aisément s'en assurer par l'observation et par la comparaison de la formule employée et du résultat obtenu. De la sorte chacun peut lui-même reprendre les calculs nécessaires pour s'assurer de l'exactitude d'une telle intégrale définie, faculté qu'il m'importait beaucoup d'offrir, et qui était rigoureusement nécessaire afin que ces résultats nouveaux pussent être d'un même poids que les autres, que j'avais recueillis et munis de leurs passe-ports de bibliographie.

Ensuite j'ai encore à ajouter quelques observations explicatives, et justifiantes au besoin, sur la rédaction et la classification de ces Tables, qui n'ont pas laissé de me causer quelquefois maint embarras. Il me paraissait nécessaire en premier lieu que la division fût naturelle, d'une autre part que la recherche d'une intégrale définie put toujours se faire aisément. Mais quiconque veut se souvenir de la variété des formes, qui se fait observer parmi les intégrales définies, reconnaîtra qu'une division bonne, naturelle et simple n'est pas chose aussi facile, que cela peut paraître au premier abord. L'incertitude sur le nombre de formules à enrégistrer, dont dépendait naturellement le nombre des Tables, rendait cette division encore plus difficile au commencement, et j'ai été obligé de temps en temps à modifier les règles qui me servaient à la classification. C'est pourquoi l'exposition des principes que j'ai suivis pourra montrer de quelle manière j'ai cherché à atteindre ce but, aussi près qu'il m'était possible.

La première division (voir Page 3) en trois Parties est fondée sur le nombre de fonctions, qui se trouvent sous le signe d'intégration définie, suivant que ce nombre est d'une seule, de deux ou de plus de deux.

La deuxième division en trente-cinq Sections ne donnera guère lieu à plus de difficultés. J'ai pris en considération les cinq fonctions diverses: Algébriques, Exponentielles, Logarithmes, Circulaires Directes (autrement dites goniométriques), Circulaires Inverses: et chaque Section I à V contient les intégrales définies, dont l'argument ou la fonction intégrée appartient exclusivement à une seule de ces fonctions. La Section VI contient les autres fonctions, telles que fonctions Elliptiques, le Logarithme Intégral, l'Exponentielle Intégrale, la Sinus Intégrale, la Cosinus Intégrale, les fonctions $B'(x)$ et $B''(x)$ de M. RAABE. Les fonctions Hyperboliques, qui peuvent être représentées par des fonctions Exponentielles, ne sont pas admises comme distinctes, et l'on trouvera toujours leurs valeurs exprimées à l'aide de ces dernières. Dans la deuxième Partie, Sections VII à XX, qui contient les intégrales définies, dont les arguments sont composés de deux sortes de fonctions différentes, les six sortes de fonctions mentionnées précédemment se trouvent combinées deux à deux en respectant l'ordre donné à ces fonctions dans la Partie première. Enfin dans la Partie troisième, le même ordre est observé dans les combinaisons respectives. Elle contient Sections XXI à XXXIV les intégrales définies d'un argument, qui est composé de trois sortes différentes de fonctions, et dans la Section XXXV celles, qui en contiennent plus de trois. Les diverses combinaisons y sont à peu près toutes représentées; car, dans la Partie deuxième manque seulement la combinaison: Fonctions Circulaires Inverses et Autres; et dans la Partie troisième, — si l'on excepte la catégorie de „Autres Fonctions", qui ne s'y trouve que cinq fois — seulement la combinaison: Fonctions Exponentielles et Logarithmes et Circulaires Inverses. Il faut toutefois faire remarquer, que plusieurs de ces Sections ne sont représentées que par un petit nombre d'intégrales définies.

Il fallait subdiviser ces Sections en Tables. En premier lieu la considération des limites, entre lesquelles l'intégration définie doit avoir lieu, s'offrait comme argument principal: ces limites diffèrent généralement auprès des différentes fonctions et donc dans chaque Section. Ici ce sont les limites 0, ± 1 et $\pm \infty$, qui sont les plus naturelles, comme pour les fonctions Algébriques, Exponentielles, Logarithmiques, Circulaires Inverses: là ce sont au contraire les multiples et les parties aliquotes de π, comme pour les fonctions Circulaires Directes. Dans les Parties deuxième et troisième ce sont tantôt les limites de la première catégorie, tantôt celles de la seconde, qui s'offrent le plus, sans ordre apparent. Le choix des limites a donc dépendu en général du nombre des formules, qui venaient s'y soumettre; les limites, qui ne valaient que pour un petit nombre d'intégrales définies, se trouvant toutes réunies sous le nom de „Limites diverses." J'insère ici un extrait du sommaire des Tables pour offrir un coup d'oeil sur la division

B

à cet égard: cet aperçu servira bien mieux à donner une idée de cette classification que plusieurs réflexions ou observations ne pourraient le faire.

			Limites.
P. I.	S. I.	T. 1—16.	$0, 1$.
		T. 17.	$-1, 1$.
		T. 18—28.	$0, \infty$.
		T. 29, 30.	$-\infty, \infty$.
		T. 31, 32.	$1, \infty$.
		T. 33, 34.	$0, p$.
		T. 35.	diverses.
	S. II.	T. 36—39.	$0, \infty$.
		T. 40.	$-\infty, \infty$.
		T. 41.	diverses.
	S. III.	T. 42—44.	$0, 1$.
		T. 45.	diverses.
	S. IV.	T. 46—52.	$0, \frac{1}{4}\pi$.
		T. 53—77.	$0, \frac{1}{2}\pi$.
		T. 78—86.	$0, \pi$.
		T. 87—91.	$0, 2\pi$.
		T. 92.	$-\frac{1}{4}\pi, \frac{1}{4}\pi$.
		T. 93.	$-\frac{1}{2}\pi, \frac{1}{2}\pi$.
		T. 94.	$p\pi, q\pi$.
		T. 95.	$0, 1$.
		T. 96—99.	$0, \infty$.
		T. 100, 101.	$-\infty, \infty$.
		T. 102.	$\frac{\pi}{2}, \infty$.
		T. 103, 104.	$0, p$.

			Limites.
P. I.	S. IV.	T. 105—107.	λ, μ.
	S. V.	T. 108.	$0, 1$.
		T. 109.	$0, \infty$.
		T. 110.	diverses.
	S. VI.	T. 111.	diverses.
P. II.	S. VII.	T. 112.	$0, 1$.
		T. 113—141.	$0, \infty$.
		T. 142—148.	$-\infty, \infty$.
		T. 149.	$0, p$.
		T. 150.	$p, \pm\infty$.
	S. VIII.	T. 151—178.	$0, 1$.
		T. 179—185.	$0, \infty$.
		T. 187.	$1, \infty$.
		T. 188.	$0, p$.
		T. 189.	p, q.
		T. 190. (Log. de Log.)	$0, 1$.
		T. 191.	p, ∞.
	S. IX.	T. 192.	$0, 1$.
		T. 193—231.	$0, \infty$.
		T. 232—234.	$-\infty, \infty$.
		T. 235, 236.	$1, \infty$.
		T. 237.	$0, \frac{1}{4}\pi$.
		T. 238—243.	$0, \frac{1}{2}\pi$.
		T. 244—249.	$0, \pi$.
		T. 250.	$0, 2\pi$.
		T. 251, 252.	$0, p$.

			Limites.
P. II.	S. IX.	T. 253.	λ, μ.
		T. 254.	p, ∞.
		T. 255.	diverses.
	S. X.	T. 256—262.	$0, 1$.
		T. 263—269.	$0, \infty$.
		T. 270.	$1, \infty$.
		T. 271.	diverses.
	S. XI.	T. 272.	diverses.
	S. XII.	T. 273—275.	$0, \infty$.
		T. 276.	$-\infty, \infty$.
		T. 277.	diverses.
	S. XIII.	T. 278—285.	$0, \infty$.
		T. 286.	$-\infty, \infty$.
		T. 287—295.	$0, \frac{1}{2}\pi$.
		T. 296.	$0, \pi$.
		T. 297.	$-\frac{1}{2}\pi, \frac{1}{2}\pi$.
		T. 298.	diverses.
	S. XIV.	T. 299.	$0, \infty$.
	S. XV.	T. 300.	$0, \infty$.
	S. XVI.	T. 301.	$0, 1$.
		T. 302.	$0, \infty$.
		T. 303—329.	$0, \frac{1}{4}\pi$.
		T. 330—352.	$0, \frac{1}{2}\pi$.
		T. 353—355.	$0, \pi$.
		T. 356.	$0, 2\pi$.
		T. 357—360.	$\frac{1}{4}\pi, \frac{1}{2}\pi$.
		T. 361.	$0, p\pi$.
P. II.	S. XVI.	T. 362.	$0, \lambda$.
		T. 363.	$\lambda, \frac{1}{2}\pi$.
		T. 364.	λ, μ.
		T. 365.	diverses.
	S. XVII.	T. 366.	$0, 1$.
	S. XVIII.	T. 367.	diverses.
	S. XIX.	T. 368, 369.	$0, \frac{1}{2}\pi$.
		T. 370—372.	$0, \pi$.
		T. 373.	$0, 2\pi$.
		T. 374.	diverses.
	S. XX.	T. 375.	diverses.
P. III.	S. XXI.	T. 376.	$0, 1$.
		T. 377—381.	$0, \infty$.
		T. 382.	$-\infty, \infty$.
		T. 383.	diverses.
	S. XXII.	T. 384.	$0, \frac{1}{4}\pi$.
		T. 385—399.	$0, \infty$.
		T. 400.	diverses.
	S. XXIII.	T. 401.	$0, \infty$.
	S. XXIV.	T. 402.	$0, \infty$.
	S. XXV.	T. 403—409.	$0, 1$.
		T. 410.	$0, \frac{1}{4}\pi$.
		T. 411, 412.	$0, \frac{1}{2}\pi$.
		T. 413.	$0, \pi$.
		T. 414—419.	$0, \infty$.
		T. 420.	$-\infty, \infty$.
		T. 421.	diverses.

			Limites.
P. III.	S. XXVI.	T. 422—424.	0, 1.
		T. 425.	0, ∞.
		T. 426.	1, ∞.
		T. 427.	diverses.
	S. XXVII.	T. 428.	diverses.
	S. XXVIII.	T. 429.	$0, \frac{1}{2}\pi$.
		T. 430.	0, π.
		T. 431—433.	0, ∞.
		T. 434.	diverses.
P. III.	S. XXIX.	T. 435.	0, ∞.
	S. XXX.	T. 436—438.	$0, \frac{1}{2}\pi$.
		T. 439.	0, ∞.
		T. 440.	diverses.
	S. XXXI.	T. 441.	0, ∞.
	S. XXXII.	T. 442.	diverses.
	S. XXXIII.	T. 443.	diverses.
	S. XXXIV.	T. 444.	diverses.
	S. XXXV.	T. 445—447.	diverses.

Mais à présent les intégrales entre les mêmes limites, appartenant à une même Section, devaient entrer dans des cadres assez nuancés pour ainsi dire, pour pouvoir facilement faire saisir les distinctions établies entre elles. Il me semblait qu'il devait y avoir de l'inconvénient dans des tables trop étendues, puisqu'alors il serait nécessairement plus difficile de trouver une intégrale définie quelconque, que l'on chercherait. D'un autre côté il ne fallait pas rendre les tables trop petites et augmenter ainsi outre mesure le nombre des distinctions devenues par là nécessairement minutieuses. Là donc, où il était besoin d'une telle restriction, je me suis borné au nombre d'environ vingt-cinq formules pour chaque Table; j'ai dû régler la classification d'après cette limite arbitraire, et pour cela admettre des distinctions trop minutieuses pour être universellement admissibles. Toutefois ces divisions moins naturelles n'ont été nécessaires que dans un petit nombre de cas: quelquefois même je n'ai pas subdivisé des Tables d'une étendue plus grande (voir, p. a. T. 1, 40, 49, 68, 85, 127, 135, 195, 202, etc.).

En général je me suis demandé pour les fonctions Algébriques:

1°. si elles étaient rationnelles ou irrationnelles: — c'est-à-dire quant à la forme: p. ex. x^p, quoique p fût fractionnaire, est considéré comme rationnel, $x^{p-\frac{1}{2}}$ au contraire est considéré comme une fonction irrationnelle.

2°. si elles étaient entières ou fractionnaires: — de même quant à la forme; x^{p-1} est considéré entier, même dans le cas que p etait assujetti à la condition de ne pas surpasser l'unité, mais x^{-p} est regardé comme une fraction.

3°. si elles étaient monômes ou polynômes. Les formes $(a+x)^b$, quoique proprement des monômes, ont été rangés parmi les polynêmes, et bien comme des puissances de binômes.

Quelquefois la subdivision se règle d'après puissances, et alors aussi d'après puissances numériques (pour l'exposant *a* spécial) et puissances algébriques (pour cet exposant *a* général).

Auprès des fonctions Exponentielles et Logarithmes la même distinction de formes rationnelles ou irrationnelles, de formes entières ou fractionnaires, de formes monômes ou polynômes est retenue: cette distinction offrant là aussi beaucoup de facilité pour la classification.

Quant aux fonctions Circulaires Directes, j'ai toujours considéré la Sinus, la Cosinus et la Tangente comme des fonctions entières; pour la Cotangente, la Sécante et la Cosécante j'ai pris en général leurs valeurs fractionnaires exprimées en Sinus et en Cosinus; néanmoins j'ai pensé devoir quelquefois m'abstenir de cette distinction, quand pour la symétrie des résultats il importait de les réunir dans un même cadre.

Les fonctions Circulaires Inverses offraient peu de difficultés: quelquefois seulement j'ai été obligé de faire une distinction entre celles, qui avaient pour argument un simple x, et celles dont l'argument était une fonction quelconque de x.

C'est d'après les principes exposés que les intégrales définies sont rangées dans les Tables respectives: le sommaire (voir Pages 5 à 19) en fait voir le résultat: j'ose espérer que leur emploi prouvera que l'arrangement est convenable.

Quelques mots suffiront pour faire comprendre la construction des Tables elles-mêmes. En tête de chaque Table on trouve au milieu son numero, à gauche la description des fonctions intégrées, à droite les limites de l'intégration: ce sont les mêmes trois arguments principaux, qui figurent dans le sommaire des Tables. Alors viennent les intégrales définies elles-mêmes, numérotées, afin de pouvoir facilement les citer: les intégrales plus générales suivent celles qui sont spéciales ou les cas spéciaux des premières. Or ces cas spéciaux des formules générales ne sauraient toujours être omis comme sous-entendus dans celles-çi, puisque d'une part les valeurs deviennent pour la plupart beaucoup plus simples, et que d'un autre côté ces valeurs spéciales de quelque constante sont bien loin d'être toujours permises. Auprès de chaque formule sont notées, s'il le faut, les équations de limite auxquelles quelque constante peut être soumise: dans le cas contraire les premières lettres de l'alphabet $a, b, c, \ldots$ désignent en général des quantités entières, les lettres $p, q, r, \ldots$ au contraire des quantités quelconques, entières ou fractionnaires, rationnelles ou irrationnelles. Toutefois toutes ces quantités sont regardées comme positives, à moins que le contraire ne soit expressément énoncé; x est toujours réservé pour indiquer la variable de l'intégration.

Dans les valeurs des intégrales définies l'on observe diverses fonctions, outre celles dont il a été question déjà à l'occasion de la division des Tables: on les trouve Page 22, 23, avec les notations respectives, ainsi que je les ai employées. Ce sont: les quatre fonctions Hyperboliques, — les coefficiens du binôme, — les factorielles $c^{a/b}$, laquelle notation exprime le produit $c(c+b)(c+2b)\ldots\ldots(c+[a-6]b)$, — les coefficiens Bernouilliens B_{2a-1}, tandis que les fonctions correspondantes B_{2a} désignent les coefficiens de la série pour la sécante, — les trois séries hypergéométriques de M. KUMMER, — la fonction L (a) de M. LOBATSCHEWSKY. De plus la lettre α désigne souvent une quantité arbitraire ou indéterminée, et k une quantité qui devient infinie: i est la racine carrée de l'unité négative, la quantité ainsi dite imaginaire la plus simple, — A la constante du Logarithme Intégral, évaluée à 18 décimales (voir GRUNERT, Archiv der Mathematik und Physik, Th. XI. Seite 323), — e la base des Logarithmes naturels, évaluée à 105 décimales (voir GRUNERT, Archiv der Mathematik und Physik, Th. III. Seite 28), — π le rapport de la circonférence du cercle à son diamètre, évalué à 530 décimales.

Quelquefois on rencontre des sommations, c'est-à-dire des séries, soit finies, soit infinies; elles sont désignées par le signe $\sum\limits_{b}^{a}$, où a et b sont les limites entre lesquelles on doit donner à l'argument, qui est représenté par le lettre n, toutes les valeurs entières possibles. Lorsqu'il y a des sommations doubles, la première se fait ordinairement suivant l'argument n, la seconde suivant l'argument m: la forme des sommations elle-même en décide toujours aisément.

Encore une observation quant à la notation des fonctions Circulaires Directes. Il me semblait plus clair de prendre le signe $Sin.^2 x$ pour la seconde puissance de $Sin. x$, tandis que la Sinus d'une Sinus de x est désigné par $Sin.(Sin. x)$: on sait que dans les derniers temps on a proposé le premier signe pour la seconde fonction. Encore $Sin. x^2$ ou plutôt $Sin.(x^2)$ est ici la Sinus de x^2. De même j'ai donné la préférence aux signes $Arcsin. x$, $Arccos. x$ etc. sur les autres signes $\frac{1}{Sin.}. x$, $\frac{1}{Cos.}. x$, etc, et cela seulement pour l'exactitude de l'impression, car je craignais que dans les formules, où des fonctions Circulaires Directes se trouvaient mêlées à des fonctions Circulaires Inverses, l'on ne confondît entre les deux fonctions absolument diverses $\frac{1}{Sin.}. x$ et $\frac{1}{Sin. x}$. J'insiste sur ces raisons pour le choix de ces signes, puisque d'un point de vue purement théorique les autres notations pourraient bien être préférables.

Comme la publication de ces Tables est la première entreprise de ce genre, je ne doute pas qu'elles ne soient sujettes à des défauts: je n'ai qu'à prier ceux, qui en feront une étude particulière, de vouloir bien me faire part de leurs observations critiques, que je recevrai avec reconnaissance.

Il me reste enfin à faire observer, que je dois l'impression de ces Tables, dont la précision et l'élégance, faisant honneur à la typographie de M. Kröber, m'ont beaucoup facilité la correction, à la munificence de l'Académie Royale des Sciences, qui a bien voulu les insérer dans sa collection de Mémoires, et en a ainsi rendu la publication possible.

D. BIERENS DE HAAN.

Deventer, Décembre 1855.

Pendant que ces tables d'Intégrales Définies étaient livrées à l'impression je me suis occupé de la théorie de ces fonctions et de la critique des diverses méthodes d'évaluation. Lorsque ce travail était assez avancé, j'ai pu confronter mes résultats avec ceux, que j'avais accueillis dans mes Tables, sans toutefois en avoir alors revisé les calculs, comme je viens de dire plus haut. Le résultat de cette confrontation n'était pas toujours favorable à mes Tables; quelquefois une faute s'y était glissée par suite d'un signe ou d'une notation mal copiés, — et une transcription totale, quatre fois repétée durant la rédaction, n'en avait pas diminué le danger; tantôt j'avais admis un résultat lui-même fautif. Et, ce qui ne valait guère mieux, ces fautes s'étaient nécessairement repétées dans les nouvelles intégrales, que j'en avais déduites. Par la nature des fonctions en question, les formules juxtaposées ne donnent lieu en général à aucune comparaison mutuelle et sont tout-à-fait indépendantes les unes des autres, et même elles se déduisent souvent de formules qui se trouvent éparses dans toute l'étendue de cet ouvrage; de sorte que la correction des épreuves, — ouvrage naturellement assez épineux et dont je ne pouvais partager les difficultés avec un autre, — ne mettait pas ces fautes en évidence. Comme l'exactitude pourtant est de première nécessité pour ces Tables, j'ai pris le parti de faire moi-même le calcul de chaque formule, et ce travail, souvent assez pénible, m'a fourni la liste suivante de corrections et d'observations critiques: elle faillit plusieurs fois me décourager de mon ouvrage, qui contenait encore tant de fautes, notamment dans les quarante-trois feuilles déjà imprimées avant cette révision : toutefois je puis alléguer en ma faveur que près de six-cents formules fautives sont telles par suite d'une faute qui se trouvait dans un résultat acquis par un autre.

Mais en même temps j'étais contraint à présent de me déclarer à l'égard de la validité de certains résultats, déjà indiqués ci-devant: je l'ai fait en les pourvoyant d'un signe d'interrogation ou en les notant comme fautives dans la liste mentionnée. Parmi les résultats que je n'ai pas révisés, se trouvent ceux qui dépendent des facultés à un nombre fractionnaire ou négatif de termes de Mr. OETTINGER, savoir $a^{\frac{b}{c}/d}$, $a^{-b/d}$, ainsi que les résultats de M. LOBATSCHEWSKY, consignés dans un Mémoire en langue Russe.

Deventer, Mars 1858. B. D. H.

OBSERVATIONS ET CORRECTIONS,

EN PARTIE CRITIQUES.

Ad Page 22. Après les chiffres de la Constante du Logarithme Intégral ajoutez encore (après avoir ôté le dernier 1): 06065124, où les deux dernières figures ne sont pas certaines: voyez Grunerts Archiv, Th. 29, S. 240.

Quant aux 530 décimales de π, je les donne ici, non puisque on en fera usage dans le calcul, mais comme un exemple intéressant de la perfection des méthodes dans l'Analyse, qui nous permettent une telle exactitude sans éxiger pour cela des travaux extraordinaires. J'insère un tableau des diverses recherches rélatives à cette constante, qui offre des données curieuses sur les progrès de ces méthodes dans le cours de vingt-et-un siècles: encore faut-il observer que les quelques décimales des siècles passés ont éxigé des calculs bien autrement longs et pénibles, que les derniers résultats.

Anneés.	*Calculateurs.*	*Déc. calc.*	*Déc. exactes.*	*Litérature.*
250 (a J.C.)	Archimède		2	*Archimedis,* de dimensione circuli.
1464	Regiomontanus		3	*Regiomontanus,* de quadratura circuli adversus Nic. de Cusa.
1580	Joh. Rheticus		8	*Rheticus,* Canon Doctrinae Triangulorum.
1585	Adr. Metius		8	*Metius,* Manuale Geometriae Practicae L. B.
1579	Fr. Vieta		11	*Vieta,* Canon Mathem. s. ad Triangula. Par 1579.
1597	Adr. Romanus		16	*Romanus,* In Archimedis Circ. Dimens. Exp. et Anal. Würzb.
1619	Lud. van Ceulen		32	*L. à Ceulen,* De circulo et adscriptis. Ed. Snellius L. B. 1619.
1621	Will. Snellius		34	*Snellius,* Cyclometricus de circ. dimens. L. B. 1621.
1717	Abr. Sharp	75	72	*A. S (harp) Philomath,* Geometry improv'd. Lond. 1717.
....	Machin		100	*Jones's,* Synopsis Palmariorum.
1719	de Lagny	128	114	Mém. de Paris 1719, p. 155.
1790	de Vega	141	136	N. Act. Petr. T. 9. Hist., p. 41.
1842	Rutherford	208	152	Phil. Trans. 1841. P. 2, p. 283.
....	(Anonyme)		154	Manuscrit de la Bibliothèque Radcliffe à Oxford.
1844	Dahse	205	200	Journ. v. Crelle, Bd. 27. S. 198.
	Clausen		256	Astron. Nachr., N. 184.
1853	Shanks		318	Proceed. Royal Society, Jan. 20. 1853, p. 273.
1853	Richter	353	330	Grunert's Archiv, Th. 21. S. 119.
1854	Richter		400	Grunert's Arch., Th. 22. S. 473 — corrigé. ib., Th. 23. S. 476.
1853	Rutherford		440	Proceed. Royal Society. Jan. 20. 1853, p. 273.
1855	Richter		500	Grunert's Arch., Th. 25. S. 472.
1853	Shanks		530	Proceed. Royal Society. Jan. 20. 1853, p. 273.

Ad. Page 23, 1ère Ligne, au lieu de: $q+2$ lisez: $q+1$

T.	N.	au lieu de:	lisez:
1.	15.	$b+c$	$b+c-1$
	16.	$b-c-1$	$b-c+1$
	19.	$=$	$=\frac{1}{2}$
2.	2, 5. fautives?		
3.	2.	0	-1
	11.	$\frac{1^{n/1}}{(p+2)}$	$\frac{1^{n-1/1}}{(p+1)}$
	20.	A	— A
	27.	$\frac{i}{2a}$	$\frac{i}{4a}$
4.	1.	$\left(\frac{1}{2}\right)^n$	$\left(-\frac{1}{2}\right)^n$
	10. fautive?		
	11.	$(1+p)^a$	$(1+p)^{1-a}$
	13.	$a^q(1+a)^p$	$a^p(1+a)^q$
	17.	$\sum_0^\infty \binom{a}{n}$	$\frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)} \sum_0^\infty \binom{-a}{n}$
	23.	$x^p + x^{-p}$	$x^{pi} + x^{-pi}$
5.	2.	$1+x$	$1-x$
	27.	$\frac{a}{b}$	$\frac{a+1}{b+1}$
6.	1.	$+\frac{1}{4}$	$-\frac{1}{4}$
	3.	$a-1$	$a+1$
	4.	$\frac{(1+a)^{q-1}}{a^q}$	$\frac{(1+a)^{-q}}{a^{1-q}}$
	5.	$p^q(1+p)^r$	$p^r(1+p)^q$
	8.	a (partout)	r
	18.	!	$(-1)^p$
7.	5.	$=$	$= -$
	6.	$\frac{1}{2}\lambda$	λ
	16.	dx	$x\,dx$
8.	3.	b	6
	7.	$q^{p+1}\,Sin.\,q\pi$	$q^p\,Sin.\,p\pi$
9.	1.	$1-x$	$1-x^a$
	2.	$a2$	$a+1/2$
	3.	$(2p-1)^{a-2}$	$(2p-1)^{a-1/2}\,1^{a-b+1/1}$

T.	N.	au lieu de:	lisez:
9.	8.	$a+b/2$	$a+b/1$
	10.	4	2^{a+b}
	13.	$2b+1$	$2b+3$
10.	11.	h/g	$\frac{h}{g^{/1}}$
	26.	$=$	$= (-1)^c$
11.	5 à 7. fautives?		
12.	4.	$(1-x)^p$	$(1-x)^{2p}$
	16.	$+\sqrt{3}$	$+3\sqrt{3}$
	19 à 22. fautives?		
13.	2.	$(1-x^3)$	$(1-x^3)^2$
	3.	$(1-x^3)^2$	$(1-x^3)$
	9'.	$\sqrt{3}$	1
14.	3.	$b-1$	$\frac{a+b}{2}-1$
	5.	$(1-x^q)$	$(1-x^q)^p$ V. Schubert.
	6.	$(1-x^a)$ (partout)	$(1-x^a)^{-}$
	7, 8, 10, 11, 18 à 21. fautives?		
15.	13. fautive?		
	18.	$\frac{\pi}{2b}$	$\frac{\pi}{b}$
16.	5.	$x^2 -$	$x^2 +$
	12.	$\sqrt{(\ )}$ (partout)	$\sqrt{(\ )^3}$
17.	1, 2. fautives: la valeur est 0.		
18.	9. ne vaut qu'entre a et $\pm\infty$.		
	10.	qr	q
	11.	$(1+x)^a$	$1+x^a$
	19.	$(1+x)^{2a+1}$, πp^2	$(1+x)^{2a}$, πp
	24.	$c-b$	$c+b+1$
	25.	$c-b+2$	$c-b+1$
	26.	$b-c-1$, $p\pi$	$b-c+1$, π
19.	3, 4', 4''. fautives; la valeur est 0.		
	11.	x^2	x
	12. fautive; elle est: $\frac{\pi}{3\sqrt{3}}$.		
	18.	$\pm q$	$\pm qi$
20.	9.	$\frac{\pi}{2a}$, $a>b$	$\frac{\pi}{2ac}$, $2a+1>2b$
	10.	$a<b$	$2a+1<2b$
	12.	$-x$	$+x$, Gr. 16. 94.

T.	N.	au lieu de:	lisez:
20.	16.	$a+1$	$a+2$
	17.	$-x^2$, $2b$	$+x^2$, $4b$. Gr. 16. 94.
21.	3.	$a+2$	$a+1$
	8.	$\frac{1}{2^a}$	$\frac{\pi}{2^{a+1}}$
22.	2, 11 à 13. fautives?		
	4.	$=$	$=2$
	10.	$b-1$, $-x$	-1, $+x$
25.	9.	ax	x
	10.	$\frac{\pi i}{b}$	$-\pi i$
	12.	q^p	q^{2-p}
26.	8.	$p\,\pi$. *Cos.*	$p\,\pi$. *Cosec.*
	12.	$\frac{2\pi}{a}$	$\frac{2\pi}{q}$
27.	1.	$=2$	$=$
	6.	$(2p-1)^{a+1/-2}$	$(1-2p)^{a-1/-2}1^{b-a+1/1}$
	12.	$b-\frac{1}{2}$	$b-1$
	15.	$=$	$=\frac{1}{3}$
	16.	$=$	$=\frac{2}{3}$
	19.	$\frac{b}{2}+1$	$\frac{1}{2}+b$
	24, 25. ne valent qu'entre 0 et 1.		
28.	5.	$p-$	$p+$
	6. fautive?		
	10.	$l\frac{}{q}$	$l\frac{}{p}$
	11.	p^2x	px^2
	22, 23.	x^b	a^6
29.	5. fautive, elle est 0.		
	7. fautive, elle est π.		
30.	4, 8, 9, 12. fautives?		
	18.	$\frac{1}{2}b^2$	$bc\,Cos.\,\lambda$
31.	6.	$(x-1)_{1+p}$	$(x-1)^{p-1}$
	7.	$^{b+c/1}$	$^{b+c+1\,1}$

T.	N.	au lieu de:	lisez:
31.	9.	$1^{c/}$, $_{b-c-1}$	$p^{c/}$, $_{b-c+1}$
	19.	$1+x^2+x^4$	$1-x^2+x^4$
	21.	$(x-1)^p$	$(x-1)^{-p}$
	22.	$(x-1)^{p-1}$	$(x-1)^p$
	8, 23. fautives?		
32.	5.	$\frac{-2p)^{b/2}}{(2p-1)^{a/-2}}$	$\frac{(1-2p)^{b/2}}{(2p-1)^{a-1/-2}1^{a-b+1/1}}$
	12.	$b-\frac{1}{2}$	$b-1$
34.	19.	$=$	$=-$
35.	2. elle est $\frac{1^{2q/1}}{(1^{q/1})^2}\cdot\frac{\pi}{p\,.\,2^{2q+1}}$.		
	6. $p\leqq 1$ est la condition de N°. 5.		
	7.	p^2	p
	19. fautive.		
	22.	$-x^2$	$-x$
	23.	$-2b)^{a-1}$	$-2b)^{a+1}$
36.	7. ajoutez: Schellbach, Cr. 48. 207.		
	8. 〃 Raabe, Cr. 48. 137.		
	11.	... 737 ...	... 777 ...
	14.	$\frac{1}{p}$	$\frac{1}{b}$
37.	5, 6, 8. fautives.		
	10.	$=$	$=(-1)^q$
	13.	$2q$	q
38.	4, 5.	$-px$, $-qx$	px, qx
	17. ajoutez: Poisson, P. 20. 222.		
	21, 22.	$=$	$dx=$
39.	2. ajoutez: Schellbach, Cr. 48. 207, où fautive.		
	10.	$e^{px}+1$, π	$e^{2px}+1$, $\pi+2$.
	11.	$=$	$=-$
	12.	$e^{-x}dx$	$e^x dx$
	16.	$+e^{(q-p)x}$	$+e^{(p-q)x}$
	19. ne vaut qu'entre $-\infty$ et $+\infty$.		
	20. seulement le $+$ des doubles signes.		
	21.	$q\,Sin.\,\lambda$	$q^2\,Sin.\,\lambda$
40.	8.	$\frac{\pi i}{4}$	$-\frac{\pi i}{4}$
	25, 26. fautives.		

T.	N.	au lieu de:	lisez:
41.	5.	pour $p < 1$: pour $p > 1$ elle est ∞.	
	17.	$(q e^{ri})^a$	$(q e^{ri})^{-a}$
43.	16.	$-\frac{1}{q}-$	$\frac{1}{q}+$
	17.	$=$	$=-$
44.	2.	$a-1$	a
	5.	$=-$	$=$
	6.	fautive?	
	8.	$-2l2$	$+2l2$
45.	6.	fautive: a est 1.	
	13.	ne vaut qu'entre 0 et 1.	
46.	16.	$=\frac{1}{2}$	$=$
	17.	*Tang.*	*Cot.*
	19.	$Tg.^p x + Cot.^p x, \pm\frac{1}{4}p\pi$	$Tg.^{pi} x + Cot.^{pi} x, \pm\frac{1}{2}p\pi$
47.	2, 3.	$2a+b+$	$2a+2b+$
	4.	dx	$\frac{dx}{Cos.^4 x}$
	21.	$Cos.^2 x$	$Cos.^2 x. Tang.^p x$
48.	6.	$\frac{Sin.^2 x}{1+3 Sin.^2 x. Cos.^2 x}$	$\frac{1}{1-3 Sin.^2 x. Cos.^2 x}$
	p. 92. (titre)	monôme.	binôme.
	8.	$-l$	$+l$
	19.	dx	$Tang. x\, dx$
49.	3.	b	6
	4.	$\pm\frac{2-a}{16}$	$\mp\frac{2-a}{16}$
	10.	$=$	$=-$
	12.	$\frac{1}{2} Sin.\lambda$	$Sin.\lambda$
	16.	$-Sin. x$	$+Sin. x$
	19, 21.	4	8
	24.	$Cos. 2x$	$Sin. 2x$
	25.	$1+p$	$1-p$ V.T.31.N°.25.
	28.	$q\Gamma(2q)$	$4\Gamma(2q)$
	29.	$\frac{p\pi}{2q}$	$\frac{p\pi}{q}$
50,	8, 9.	dx	$\frac{dx}{Cos. x}$
50.	14.	*Sec.*	*Cosec.*
51.	1.	$=\frac{1}{2}$	$=$
	9, 10.	fautives?	
	11.	$=$	$=\pi$
	18.	$2\sqrt{3}$	$3\sqrt{3}$
52.	18.	$\frac{Sin.^{\frac{1}{2}q} x. Cos.^q x}{\{(Cos. x - Sin. x)}$	$\frac{2 Sin.^{q+1} x. Cos.^{q-1} x}{\{(Cos.^2 x - Sin.^2 x)}$
	19.	$=$	$=\frac{1}{p^3}$
53.	18.	$Sin.^p x$	$Sin.^{p-1} x$
	22.	$\left(\frac{1}{2}p\right.$	$\frac{1}{2}(p$
54.	2.	$q-1$	$q-2$
	7.	$(2a-1)^2 p^2$	$(2a-1)^2-p^2$
	15.	$2p+1$	2^{p+1}
55.	12, 16 à 18.	Partout $a > b$.	
56.	4.	$1^{a/2}$	$2^{a/2}$
	8, 9.	$b+2$	$b+1$
	10 à 12.	$Cos. x$	$Cos.^{2(} x$
	16.	$\frac{a+b}{2}-1$	$\frac{a-b}{2}+1$
57.	1.	$2a-b$	$b-2a$
	5.	$2a-2b$	$2b-2a$
	8, 12.	$\frac{q\pi}{2}$	$\frac{1}{2}p\pi$
	14.	1^{b-1}	1^{-b}
58.	2.	ne vaut que pour $b=-1$.	
	3 à 7.	ne valent que pour $c=0$.	
59.	7, 12 à 15.	fautives?	
60.	2, 3.	a (partout)	$a\pi$
	4.	$2n+1$	$2n-1$
	9.	fautive.	
61.	14.	$=$	$=-$
62.	4, 5, 7, 8.	fautives?	
63.	1.	qx	$4qx$
	2.	$=\frac{1}{a}$	$=-a$
	4.	$p\pi$	$\frac{1}{2}p\pi$

T.	*N.*	*au lieu de:*	*lisez:*
63.	8.	$p-1$	$p-q$
	11.	$Cos.\ 2x$	$Cos.^2\ 2x$
64.	8.	$Cos.\ 2x$	$Cos.\ 2x.\ Cos.^2\ x$
	14.	$Cos^{p+q-1}\ x$	$Sin^{p+q-1}\ x$
65.	10.	$\overline{q}$	$\overline{p}$
	15.	$\overline{2}$	$\overline{3}$
66.	4, 5.	$=$	$=\frac{1}{2}$
	10.	$\frac{Cos.^2\ x}{1-}$	$\frac{1}{1-3}$
	21.	$1+a^2$	$-1+a^2$
	22.	$1+a^2$	$1-a^2$ V. T. 23. N°. 2.
67.	2, 3.	$1-p$	$1+p$ V. T. 31. N°. 22.
	4.	-2λ	$-2\lambda-Sin.\lambda$
	13.	$7p^4q^2+7$	$3p^4q^2+3$
	14, 15.	$6p^2q^2$	$2p^2q^2$
68.	2.	$=$	$=2$
	3.	$\overline{2}$	$\overline{3}$
	4.	$1-3$	$1-$
	26.	$Tang^p\ x$	$Tang^{p+2}\ x$
	27.	$1-a,\ )+a+1$	$1-q,\ )+a$
	30.	4	2
69.	9.	$\frac{1}{2}\lambda\ Sin.\lambda$	$\lambda\ Sin.\lambda-1$
70.	3, 4, 11 à 14. fautives?		
	20.	$-Cos.^2\ x$	$.Cos.^2\ x$
71.	3, 4, 7 à 10, 12 à 14, 17. fautives?		
	16.	$2n+1$	$2n-1$
	18, 19.	$=$	$=\frac{1}{2}$
72.	1, 11.	$n^{n/2}$	2^{n2}
	6.	4	3
	7. fautive.		
	13 à 15.	$Cos.\ x$	$Cos.^2(\ x$
	14.	$2b+1$	1
	21, 23.	$2\sqrt{3}$	$3\sqrt{3}$
74.	1.	F' (	$F\left(\frac{\pi}{4},\right.$
74.	3.	$\frac{2}{b\sqrt{(a+b)}}$	$\frac{2a}{b\sqrt{(a+b)}}$; c'est le coefficient de tout le reste.
		a F'	F'
		E'(, F(	$E\left(\frac{\pi}{4},\right.\ F\left(\frac{\pi}{4},\right.$
75.	20.	$3-p^2$	$1-p^2$
	24.	$+Sin.^2\lambda$	$-Sin.^2\lambda$
	26.	$a-b+\frac{1}{2}$	$a-b+1$
76.	4.	$\frac{1}{p^3},\frac{1}{(1-p^2)p}$	$\frac{1}{p(1-p^2)},\frac{1}{1-p^2}-\frac{1}{p}$
	5.	$\frac{1}{p^2}$	$\frac{1}{1-p^2}$
	11.	$p-\frac{1}{2}$	$p+\frac{1}{2}$
	13.	$(p+q-1),\frac{1}{2}(q+1)$	$(p-q-1),1-\frac{1}{2}(p+q)$
	18.	$Sin.^2\mu$	$Sin.\mu$
	20.	4	2
77.	1, 2.	pour $k=\infty$	pour $k=0$
78.	3.	2^{2a}	2^{2a+1}
	4.	-1	$+1$
	14. fautive: il y manque le facteur: x.		
	15.	$(-1)^a$	$(-1)^{a-1}$
79.	8.	$a^{b/1}$	$a^{b-1/1}$
82.	19.	$\}^3\ a+1$	$\}^2.2^a$
83.	1.	$=-$	$=$
	12, 13. ne valent qu'entre 0 et $\frac{1}{2}\pi$.		
	14.	$Sin.$	$Sin.^2$
84.	15, 20.	p^a	p^{a-1}
	25.	$Cos.\ 2cx$	$Sin.\ 2cx$
85.	10, 11.	$=$	$=\pi$
	24.	$)^{2a}$	$)^a$
	30.	2	$2pq$
	32. fautive, sa valeur est ∞.		
86.	5.	$=2$	$=1$

T.	*N.*	*au lieu de:*	*lisez:*
86.	13.	E	F
	14, 15.	p^2+q, p^2-q	$p^2+q^2,\ p^2-q^2$
	14.	$p^2 q$	p^2-q^2
	15.	$q\sqrt{}$	$q^2\sqrt{}$
87.	15.	$=$	$=\pi$
88.	8.	fautive, sa valeur est 0.	
89.	10, 12.	$a-1$	$a+1$
	15.	$2\pi a$	2π
	16.	$=$	$dx=$
90.	21.	$\frac{\pi}{2}$	π
	23.	$\frac{\pi}{2}$	$a\pi$
92.	12.	$=$	$=-$
93.	1.	$b-2$	$b-1$
	2.	$\left(\frac{a+b}{2}-1\right)\Gamma\left(\frac{b-a}{2}-1\right)$	$\left(\frac{a+b+1}{2}\right)\Gamma\left(\frac{b-a+1}{2}\right)$
94.	2.	ne vaut que pour $a=\infty$.	
	4.	fautive.	
	14.	$=$	$=a$
95.	5.	ax	πx
	6.	1	π
	9.	$=$	$=2$
96.	1 à 6, 12, 14.	fautives.	
97.	1 à 8.	fautives.	
98.	6.	$=$	$=-$
	7 à 9.	fautives.	
99.	1 à 4.	$\sqrt{}\,Sin.,\ \sqrt{}\,Cos.$	$Sin.\sqrt{},\ Cos.\sqrt{}$
	5, 6.	fautives.	
101.	1, 2.	q^2	q
102.	1, 3, 5 à 13.	fautives.	
104.	4.	$2\,Cos.^2\mu$	$2\,Cos.^3\mu$
	7 à 9.	$(Sin.\mu, q)$	$(Sin.\nu, q)$
	10 à 12.	$Sin.^2\lambda$	$Cos.^2\lambda$
	14.	$Sin.\mu$	$Sin.\nu$
105.	3.	$\frac{1}{Sin.^4\mu}\ \frac{1^{n/2}}{4^{n/2}}$	$\frac{3^{n/2}}{4^{n+1/2}}$
106.	5.	$Sin.^{2a+1}\lambda$	$Sin.^{2a-1}\lambda$
107.	17.	$\frac{1+Sin.^2\mu}{2\,Cos.\lambda}Sin.\mu$	$\frac{Sin.\mu}{Cos.\lambda}F'\left(\frac{Sin.\theta}{Sin.\mu}\right)-\frac{Sin.\mu.Sin.^2\lambda}{2\,Cos.\lambda}$
	21.	$\frac{1}{Sin.\mu}$	$Sin.\mu$
108.	5.	$-\frac{1}{2}p, -p$	$-p, -4p$
	6.	$Sin.\frac{1}{2}p$	$Sin.^{\frac{1}{2}}p$
	10.	$=-\frac{\pi}{16}$	$=\frac{\pi^2}{16}$
109.	2, 3, 7 à 10.	fautives: elles sont ∞.	
110.	2.	fautive: elle est ∞.	
	3.	π^2	$-\pi^2$
	4.	$=,\ \left(\frac{\pi}{2}\right),\ \frac{4}{p+2m}$	$=-,\left(\frac{\pi}{4}\right),\ \frac{2}{p+2m-1}$
112.	1.	3	2
	6.	ne vaut qu'entre 0 et ∞.	
	7, 12.	fautives.	
	11.	$\frac{d}{da}$	$\frac{d}{da}.l$
113.	8.	a^x	x^k
	9.	e^x	e^{-x}
	15.	$a+bi$	$a-bi$
	17.	ajoutez: Raabe, Cr. 48. 160.	
114.	4.	$=$	$=2^{-a}$
115.	10.	ôtez e^{c^b}	
116.	1.	fautive.	
	5.	$\frac{1}{3}$	$-\frac{1}{3}$
	8.	$\sum_0^\infty(-1)^n$	$\sum_0^a$
	9.	a	p
117.	3.	256	252
	4.	1680	240
	11, 12, 14.	0	1

T.	*N.*	*au lieu de:*	*lisez:*
117.	19.	x^2	x^3
	23.	$2a+1$	$2a-1$
119.	1.	x^2	$\frac{1}{2}x^2$
	2.	e^x-2	e^x V. T. 117. N°. 5.
	3, 4.	$\Gamma(q)$	$\Gamma(q+1)$ V. T. 117. N°. 16, 17.
	8.	e^{-x}	$(-e^x)$
120.	1.	$=-$	$=$
	8.	16	12
121.	5.	e^{qx}	$-e^{qx}$
	6.	$-e^{-qx}$	$+e^{-qx}$
	8.	$3\,Sec.^3$	$2\,Sec.^3$
	9.	$3\pi^3$	$2\pi^3$
	12.	e^{-x}	1
	19.	*Sin.*	*Cos.*
	19, 21.	$e^{bx}, e^{-bx}, \left(\frac{2\pi}{b}\right), b\pi$	$e^{qx}, e^{-qx}, (2b\pi), q\pi$, où $q^2=a^2b^2<1$, a arbitraire.
122.	1.	$-e^{-x}$	$+e^{-x}$
	6.	π	π^2
	7, 8.	changez les coefficients $p+q$ et $p-q$: dans les numérateurs liez les puissances de e par le signe $+$.	
	10.	$e^{\pi x}-, e^{\pi x}+$	$e^{\pi x}+, e^{\pi x}-$
123.	5.	$2a-1$	$\frac{2a-1}{4a}$
	8.	$2a+1$	$2a-1$
124.	9.	$-2\,Cos.\,\lambda, 3\,Sin.\,\lambda$	$+2\,Cos.\,\lambda, Sin.\,\lambda$
125.	8.	$\lambda+$	$\lambda-$
	12.	3	2
126.	3.	$=-$	$=$
	4 à 7, 17.	fautives?	
	8.	e^x	e^{-x}
	10, 11.	ne valent que pour a négatif.	
	12, 14.	ne valent que pour b négatif.	
127.	13.	$b-a$	$b-c$
	17.	$=$	$=-$
	27.	dx	$2dx$

T.	*N.*	*au lieu de:*	*lisez:*
127.	29.	$\sum_1$	$\sum_0$
	33.	$q^2\,lq$	$q^3\,lq$
128.	6.	$(-1)^c$	$(-1)^{q+1}$ V. T. 168. N°. 20.
129.	5.	x	xi
	10.	fautive.	
	11.	q	$-q$
	12.	$(-pq)^{n-1}$	$(pq)^{n-1}$
130.	11, 13.	fautives.	
131.	8.	fautive.	
132.	9, 10.	changez le dénominateur avec N°. 12, 13.	
	10, 13.	$=$	$=-$
	10.	$e^{-pq}\,Ei.\,(pq)$	$e^{pq}\,Ei.\,(-pq)$
	14.	$+e^{pq}$	$-e^{pq}$
134.	2.	fautive: sa valeur est $l\frac{\Gamma\left(\frac{q+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{q}{2}\right)}$	
	3.	$2p$	2
	6.	$(1-e^{-px})\,e^{-(q+1)x}$	$(e^{-qx}-1)\,e^{-(p+1)x}$
	14.	ôtez: $\left(\frac{c+1}{2}\right)^{\frac{b-1}{2}}$	
	16.	$e^{-px}+$	$e^{-px}-$
135.	10, 15.	ajoutez: Féaux, Funct. Transc.	
	12.	$+$	$-$
	18.	$dx\left\{\frac{e^{-x}}{x}-\right.$	$\frac{dx}{x}\left\{p-\frac{1}{1-e^{-x}}-\right.$
	19.	$-e^{-x}$	$+e^{-x}$
	24, 25.	$e^{(1-a)x}$	$e^{(1-p)x}$
	28.	$pq+$	$pq-$
	30.	$q-\frac{1}{2}$	$\left(q-\frac{1}{2}\right)$
	31.	$e^{\frac{1}{2}px}$	$e^{-\frac{1}{2}px}$
136.	3.	$=$	$=-$
	4.	$\frac{q\pi}{2p}$	$\frac{q\pi}{4p}$
	5.	$e^{qx}-e^{-qx}$	$(e^{qx}-e^{qx})^2$
	10.	fautive.	

T.	*N.*	*au lieu de:*	*lisez:*
136.	12.	$\overset{x}{-}$	$\frac{dx}{x}$
	18.	dx	$\frac{dx}{x^p}$; liez les fractions par $+$.
137.	1, 2. fautives.		
	3.	dx	$(1-e^{-x})^2\,dx$
	6.	$e^x +$, *Sec.*^{2}	$e^x -$, *Sec.*
	12.	$(p-n)$	$(p+n)$
138.	4, 7. ajoutez: Poisson, P. 20. 222, $p \leqq \pi$.		
	9.	$\frac{1}{4}q$	$\frac{1}{4q}$
	12.	dx	$2\,dx$
139.	8.	$c-n$	$c-n+1$
	10.	$(-q)$	q
140.	11.	q^n	$(-q)^n$
	13, 14.	$=$	$=-$
	15.	$xp\surd$	$p\surd x$
	17, 18.	$+x\}, -x\}$	$+p\}, -p\}$
141.	16, 17. fautives.		
	22.	π	2π
	23.	$-\pi$	2π
	24.	e^{-x} (partout)	e^x
		$a^{-p}, (a+b)^{-p}$	$a^{1-p}, (a+b)^{1-p}$
142.	3.	π	2π
143.	4, 5.	e^{2ax}	e^{-2ax}
	5.	e^x, *Tang.* $\frac{\pi}{2a}$	e^{-x}, *Tang.* $\frac{\pi}{a}$
	8, 9.	$=$	$=-$
	15.	$\frac{a\pi}{b}, \frac{a+2}{b}$	$\frac{a\pi}{2b}, \frac{a+2}{2b}$
144.	4.	$\frac{a}{2p}$	$\frac{a}{2b}$
	7.	$\frac{1^{a/1}}{2^{a/1}}$	$\frac{1^{a-1/1}}{a^{a/1}}$
	8.	$=$	$=-$
	9.	e^x	e^{-x} V. T. 182. N°. 7.
	10.	$q+1$	$q-1$
	12.	*Sin.* p $-$	*Sin.* $p\pi$ $+$
145.	3. Sa valeur est $\frac{1}{2}\pi^2$.		
145.	4, 8.	$=$	$=-$
	6.	$4\,lq$	lq
	7.	$\frac{1^{a/1}}{2^{a/2}}$	$\frac{1^{a-1/1}}{a^{a/1}}$
	9.	6	3
	12.	24	4
	13.	360	15
	14.	720	6
	15.	$q+1$	$q-1$
	19.	$=$	$=2$
	21, 22.	$e^{bx}, e^{-bx}, \left(\frac{2\pi}{b}\right), a\pi$	$e^{qx}, e^{-qx}, (2b\pi), q\pi$
		où $q^2 = a^2b^2 < 1$, α arbitraire.	
	27.	$(2\pi)^{a+1}$	$(2\pi)^{2a+1}$
146.	3 à 5.	$4pq$	$4\surd pq$
	4, 5.	$a^{2n/-1}$	$(a+n-1)^{2n/-1}$
147.	12, 13. changez les conditions.		
148.	1, 5. fautives?		
149.	5.	$(-1)^a\Sigma 1$	$\Sigma(-1)^n$
	8.	$e^{-x}, (e^{\frac{1}{2}x}-e^{-\frac{1}{2}x})^4, 2(l2)^2$	$2e^{-x}, (e^x+2e^{-x}-2)^2, (l2)^2$
	9.	e^{-x}	$2e^{-x}$
	11.	$(p^2-1)(e^x+e^{-x})-2(p^2+1)$	$(e^{-\frac{1}{2}x}-e^{\frac{1}{2}x})\surd\{(p^2-1)(e^x+e^{-x})+2(p^2+1)\}$
	12.	$\frac{e^x-1}{(p^2-1)(e^{2x}+1)+2(p^2+1)e^x}$	$\frac{1}{e^{2x}-1}$
	13, 14.	$=$	$=-$
150.	3. fautive.		
	11.	(limite) q	$2\,lq$
	13.	(〃) lp	$-lp$
151.	1, 14.	l	$-l$
	7.	$\Delta^a \frac{1}{qp^{b+1}}$	$\Delta^a_q \frac{1}{p^{b+1}}$
	11.	$(-1)^a\Sigma 1$	$\Sigma(-1)^n$
	13.	$p+1, p+3$	$p+2, p+4$
	15.	$\{$	$\{-$
152.	1. fautive: sa valeur est ∞.		
153.	5, 6, 20.	lx	$l\frac{1}{x}$

T.	N.	au lieu de:	lisez:
153.	12.	x^{p-q}	$-x^{p-q}$
	13.	$-x^{p+q}$	$+x^{p+q}$
	14.	$=\frac{1}{2}$	$=$
154.	8.	$=$	$=-$
155.	9.	1680	240
157.	4, 10.	Σ	$\sum\limits_{1}$
	8, 9.	$a/1$	$a-1/1$
	11, 12.	$b\;1$	$b-1/1$
158.	3, 9.	$x^b, x^{-b}, \left(\frac{2\pi}{b}\right), b\pi$	$x^q, x^{-q}, (2b\pi), q\pi$, où $q^2=\alpha^2b^2<1$, α arbitraire.
	5.	$2a-$	$2a-1$
	6.	$=(-1)^a$	$=-$
	12.	x^{-1}	x^{b-1}
160.	4.	$1+x^2$	$(1+x^2)^2$
	8.	$\frac{p}{1+p^2}, \frac{1}{4}p^2$	$\frac{-2}{1+p^2}, \frac{1}{2}\frac{p}{1+p^2}$
	9, 13, 14.	l	$-l$
	17.	$1+x^2$	$1-x^2$
	18.	$=2$	$=$
161.	8.	2560	1260
	12.	2^{a+2}	$2a+2$
	13.	$n+1$	n
	14, 15.	$\frac{1^{a+1/1}}{a}$	$1^{a-1/1}$
	16.	$q+1$	$q+2$
	17.	$\frac{1^{a+1/1}}{2^{a+1}}$	$-\frac{1^{a-1/1}}{2^a}$
162.	2.	$\frac{3}{4}$	$\frac{4}{3}$
163.	8, 9. fautives.		
165.	1, 2, 5, 11.	l	$-l$
	4.	$l(r$	$l(-r$; elle ne vaut qu'entre $-1+1$.
	6.	q^2	q
	8.	$-q^2$	$+q^2$
	13.	$F(p,\lambda)$	$\{F(p,\lambda)\}^2$
	14.	*Sin.*	*Sin.*^{2}
165.	19.	$-x$	$-x^2$
	21.	$F'(p)+$	$F'(p)-$
	25.	$1^{n/2}$	$3^{n/2}$
	27.	$F(p,\lambda)$	$F\{\sqrt{(1-p^2)},\lambda\}$
	28.	$2\sqrt{}$	$2\sqrt[4]{}$
166.	3.	*Cos.*$^2\mu$, *Cot.*	*Sin.*$^2\mu$, *Tang.*
	9.	$\sqrt{}$(*Cos.*^{2}	$\sqrt{}$(*Cos.*^{4}
	15, 16.	$=$	$=-$
	21.	$+\frac{2\,Sin.^2\lambda}{(1-x^2\,Sin.^2\lambda)^2}$	$-\frac{2\,Sin.\lambda}{x(1-x^2\,Sin.^2\lambda)}$
168.	9.	$\sum\limits_{0}$	$\sum\limits_{1}$
	11.	dx	$2dx$
	15.	$(pn+1)l$	$(pn+1)^2l$
	20.	r^{-b}	r^b
169.	1.	$=-$	$=$
	2.	$=$	$=-$
	3, 4.	$q^2+(lx)^2$	$\frac{1}{4}q^2+(lx)^2$
	3.	$\sum\limits_{1}$	$\frac{2}{q}\sum\limits_{0}$
	4.	$\sum\limits_{1}$	$\frac{1}{q^3}\sum\limits_{0}$
	6.	$2n-2$	$2n+2$
	14.	$e^{pq}\,Ei.(pq)$	$e^{pq}\,Ei.(-pq)$
	15.	$l, 1+pq$	$lx, 1-pq$
	16.	$1-pq$	$1+pq$
170.	3.	$1+\frac{1}{xlx}$	$-1-\frac{1}{xlx}$
171.	2.	Z' (partout)	$l\Gamma$
	3.	$(1-x^p)x^q$	$(1-x^q)x^p$
	5.	$l\left\{\left(\frac{a}{a+1}\right)^{\frac{a-1}{2}}\right.$	$l\{$
	13.	$\frac{1}{lx}+, dx, \Gamma(a+b+p)$	$\frac{p}{x}-, \frac{1}{(1-x)x}-, \frac{dx}{lx}, \Gamma(a-b+p)$
	19.	$l\frac{(1+p+s)(1+q+r)}{(1+p+r)(1+q+s)}$	$l\frac{\Gamma(1+p+s)\Gamma(1+q+r)}{\Gamma(1+p+r)\Gamma(1+q+s)}$

T.	N.	au lieu de:	lisez:
171.	26.	$\left.\frac{1}{1-x}\right)\frac{dx}{lx}$	$\left.\frac{x}{1-x}\right)\frac{dx}{xlx}$
172.	2, 7. fautives.		
	9.	$\frac{q\pi}{2p}$	$\frac{q\pi}{4p}$
173.	1.	dx	xdx
	2, 12.	dx	$-lxdx$
	2.	$\frac{q}{4\pi}$	$\frac{\pi}{q}$
	4.	π^{2n}	$(2\pi)^{2n}$
	8.	$\frac{1}{4}\pi$	$\frac{1}{4\pi}$
	9.	$2q+3, 2q+1$	$2q+3\pi, 2q+\pi$
	10.	$1+x^2$	$1-x^2$
	11.	$\frac{1}{2\pi}-l2$	$\frac{1}{2}\left(\frac{1}{2}-l2\right)$
	14.	$q^2\pi^2$	$-q^2\pi^2$
	17.	$\frac{dx}{1-x^2}=$	$\frac{dx}{1+x^2}=2$, V. T. 138. N°. 17.
	18.	$\sqrt{\pi}$	$\pi\sqrt{2}$ V.T. 138. N°. 18.
174.	2, 3. fautives.		
	5.	dx	$(1-x^2)dx$
	13.	$1+x^2$	$1-x^2$
175.	9.	$-x^{2p}+$	$-2x^{2p}+$
	12.	$1-x^2$	$1-x^q$ V.T. 135. N°. 27.
176.	1.	$\frac{1-x}{x}$	$\frac{1-x}{2}$
	4.	$1-x^2$	$1+x^2$
	5.	$\sum_{0}$	$\sum_{1}$
	4, 5.	π^{p-1}	1
	6.	$-\pi$	$-\frac{1}{\pi}$
	8.	(lx^2)	$(lx)^2$
	11.	$-\Sigma$	$-\pi\Sigma$
177.	6.	lx	$4lx$ V.T. 138. N°. 18.
	9.	$\frac{1}{2}\pi$	$\frac{1}{2\pi}$
	13, 14.	$2np\pi$	$np\pi$
177.	15.	$xq-\frac{1}{2}$	$x^{q-\frac{1}{2}}$
	19.	$\frac{1}{x}$	$\frac{1}{2}$
	20.	$\frac{2}{1-x^2}$	$\frac{x}{1-x^2}$
	21.	x^{a-1}	x^{p-1}
178.	4.	$\sum_{1}$	$\sum_{0}$
	6, 7.	lx	$-lx$
	8.	$q>\ >0$	$q>p>0$
179.	3.	$x^{1-p}, p+1$	$x^{2-p}, p-1$
180.	5.	B′	B″
181.	16.	*Cot.*	*Tang.*
	17.	n'est pas à sa place ici.	
182.	2.	$(lx)^2$	lx
	5.	$(lx)^2$	$l(x^2)$
	6.	$\frac{a}{2p}$	$\frac{a}{2b}$
	15.	$lq-$	$lq+$
	19.	$=-$	$=$
	22.	$2q$	q
183.	8.	$2q^{2a+1}$	$(2q)^{2a+1}$
	10.	$dx=p$	$\frac{dx}{x}=lq$
	14.	(lq^2)	$(lq)^2$
	17.	$p-q$	$p\pi$
184.	5, 6.	$x^b, x^{-b}, q\pi, a\pi$	$x^q, x^{-q}, b\pi, q\pi$, où $q^2=a^2b^2<1$.
	15.	lq	$\frac{1}{2}(lq)^2$
	16.	$\frac{1}{1-q}$	$\frac{-1}{1+q}$
	18.	$)^p$ (partout)	$)^{1-p}$
185.	1.	lx	$l(1+x)$
	2.	$\frac{1}{2}\pi$	2π
	10.	$\frac{1}{2}+a$	$\frac{1}{2}-a$
	14.	$2dx$	xdx

T.	N.	au lieu de:	lisez:
187.	2.	Σ	$\Sigma(-1)^{n-1}$
	4, 13.	lx	$-lx$
	8.	$\frac{1}{6}\pi^2 -$	$\frac{1}{12}\pi^2 +$ V. T. 160. N°. 9, et T. 184. N°. 14
	12.	dx	$q\,dx$
	15.	$-\frac{1}{2}q$	$+\frac{1}{2}$
	16.	$\frac{(lx)^b}{x^2} = \frac{b}{a}$	$\frac{(lx)^{b-1}}{x} = \frac{\pi}{a}$ V. T. 20. N°. 1.
	17.	$\sqrt{(2n+1)}$	$\sqrt{(2n+1)^3}$
188.	1, 14.	$=$	$= -$
	2.	$= -$	$=$
	4.	$\int_0$	$\int_1$
	8.	$\left(l\frac{-1+\sqrt{5}}{2}\right)^2$	$\left(l\frac{-1+\sqrt{5}}{2}\right)$
	12.	$l\frac{3-\sqrt{5}}{2} +$	$l\frac{3-\sqrt{5}}{2} -$
	15.	$=$	$= \frac{1}{2}$
189.	2.	$\frac{dx}{x}$	$\frac{dx}{x^2}$
	10.	$-l$	$+l$
	11, 12.	fautives.	
	13.	$q - px^2$	$q^2 - px^2$
	18.	$(lx)^2$	$l(x^2)$
190.	4.	$\sqrt{(l\,2n+1)}$	$\sqrt{(2n+1)}$ V. T. 381. N°. 14.
	5.	$\Sigma\{-\}$	$\Sigma(-1)^n\{+\}$
	6.	$+A$	$-A$
	8.	$\sqrt{(2n+1)}$	$\sqrt{n}$
192.	4, 13.	n'appartiennent pas ici: dans 4) il doit être q^{x-1}.	
	9.	Σ	$\Sigma(-1)^n$
	10.	$\frac{1}{2}\pi$	$\frac{1}{4}\pi$
193.	1 à 11, 21 à 23.	fautives?	
195.	15 à 17.	Dans les conditions changez p en q et q en p.	
197.	20.	125	115
199.	5.	où $s = -p$.	
200.	6.	$a-1$	$a+1$
201.	5, 13, 14.	fautives.	
202.	14.	$r-1$	$r+1$
	15.	$=$	$= 2^r$
	25.	$-2n)^a -$	$-2n)^a +$
	32.	$a-1$	$a+1$
	36.	$\frac{b}{2}$	$\frac{b}{2}+1$
	40, 41.	$2p+q$	$2p+a$
203.	10.	1_{2n1}	$1^{2n/1}$
	11, 12.	a y doit être l'unité.	
204.	9, 10.	fautives: elles sont infinies.	
	15.	ajoutez: Liouville, P. 21. 71.	
205.	3, 4, 12 à 18, 26, 27.	fautives: elles sont infinies.	
206.	3 à 8, 11, 12, 15 à 19.	fautives: elles sont infinies.	
	13, 14.	$a^2 - x^2$	$x^2 - a^2$
	20.	$=$	$= -$
207.	1 à 3.	fautives: elles sont infinies.	
	17.	$Cos.\left(\frac{n\pi}{b} - a\right.$	$Cos.\left(\frac{2n\pi}{b} - a\right.$
	18.	$Cos.\left(\frac{2n+1}{2b}\pi - a\right.$	$Cos.\left(\frac{2n+1}{b}\pi - a\right.$
	26.	$\frac{\pi}{2}$	$\frac{\pi}{2\sqrt{2}}$
	27.	$q^{2p-1)}$	$q^{2(p-1)}$
209.	10.	$3aq$	$2aq$
210.	1 à 4.	ne valent qu'entre les limites $-\infty$ et ∞.	
	1, 2.	$g + 2hx$	$g^2 + 2hx$
	5.	fautive.	
	11 à 13.	fautives: elles sont infinies.	
212.	5, 8 à 11.	fautives: elles sont infinies.	
	14, 15.	e^{-pq}	$(e^{-pq} - 1)$
	15.	$Cos.px$	$Cos.px - 1$
	16.	4	2
213.	13, 14.	$=$	$= \frac{1}{2}$
	19.	$(b \pm xi)^{-a}$ (partout)	$(b \pm xi)^{-a-1}$

T.	N.	*au lieu de:*	*lisez:*
214 à 218.	ne valent pas.		
219.	6.	$=$	$=-$
220.	5, 6.	$e^{2ac}+e^{-2ac}$	$e^{2aq}+e^{-2ac}$
	6.	$q+x^2$	q^2+x^2
	7.	$Cos.\,2\,a\,x\,d\,x$	$(p+Cos.\,2\,a\,x)\,d\,x$
	8, 9. fautives.		
	10.	$d\,x$	$x\,d\,x$
	11.	$+p\,e^{-a}$	$+p\,e^{-2a}$
	13.	p^2 (numérat.)	p
221.	1 à 4, 18 à 20. fautives.		
	7.	$1-p\,e^{q}$	$1+p\,e^{q}$
	10.	$2\,q$	$2\,p$
	15.	$Cos.\,r\,x$	$Cos.\,2\,r\,x$
222.	2, 13, 14, fautives,		
	3.	$\frac{\pi}{2\,q}$	$\frac{\pi}{2\,q^2}$
	7, 11.	$d\,x$	$2\,d\,x$
	8, 10.	$-e^{-2ac}$	$+e^{-2ac}$
223.	fautive.		
224.	16 à 23. fautives.		
225.	3 à 10, 22, 24, 26 à 33. fautives.		
	16.	$\bar{2}$	$\bar{3}$
	23. n'est pas fautive.		
226.	5, 6. fautives,		
227.	4.	$+\sqrt{\frac{\pi}{2\,b}}$	$-\sqrt{\frac{\pi}{2\,b}}$
	9, 10. sans restrictions pour a.		
228.	2 à 6, 8 à 12. fautives, quoique Cauchy ordonne la différentiation.		
229.	1.	$=-$	$=$
	5.	$\sum_1^\infty$	$\sqrt{2}\sum_1^\infty Sin.\left(\frac{2n-1}{4}\pi\right)$
	6.	$\frac{1}{b}\sum_1^\infty$	$\frac{1}{a}\sqrt{2}\sum_1^\infty Cos.\left(\frac{2n-1}{4}\pi\right)$
	8.	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
	9.	$\frac{1}{4\,q}\left(1+\frac{1}{q}\right)$	$\frac{1}{2\,q\sqrt{2\,q}}\left(b+\frac{1}{2\,q}\right)$
229.	10.	$\frac{1}{4\sqrt{q}}$	$\frac{1}{2\sqrt{2\,q}}$
230.	1, 7.	$\sqrt{2\,\pi}$	$\sqrt{2\,a\,\pi}$
	2 à 6, 8 à 12. fautives, quoique Cauchy ordonne la différentiation.		
231.	7.	$Cos.^2$	$Sin.^2$
232.	4, 7.	$r\pm q\,i$	$q\pm r\,i$
	14.	$=($	$=(-$
	19, 20.	3	2
233.	3, 7.	$r\pm q\,i$	$q\pm r\,i$
	20.	$Sin.\,r\,t\,-$	$Sin.\,r\,t\,+$
	22.	$b-c$	$b+1$
	23.	$-2\,p\,x$	$+2\,p\,x$
	24.	$a+1$	$2\,a+1$
234.	2 à 4. fautives.		
235.	2 à 7, 10 à 13. } fautives, quoique Cauchy ordonne		
236.	3 à 8, 11 à 14. } la différentiation.		
	21.	pour a	pour $\frac{1}{a}$
237.	8.	$\{1$	$\{2$ V. T. 238. N°. 19.
	9.	$d\,x$	$a\,d\,x$
	10.	$Cos.^2\,x$	$Tang.\,x$
	11.	$=$	$=-$
	16.	$+l$	$-l$
238.	2.	$\frac{1}{2}\pi\,l\,2$	$\frac{1}{4}\pi$ V. T. 267. N°. 2.
	19.	$\{1-$	$\{2-$
239.	2.	$+\pi$	$-\pi$
	3.	$=$	$=-$
	4.	$(-1)^{2n-1}$	$(-1)^{m-1}$
	5.	2^{m-2}	2^{2m-2}
	7, 17. fautives; elles sont infinies.		
	15.	$+(-1)^n 2a,2n-1\ \}$	$+\sum_1^\infty(-1)^n 2a,2n$
	20.	$2^{p-2}\left\{\Gamma\left(\frac{1}{2}p\right)\right\}^2$	$\frac{2^{p-1}}{p}\left\{\Gamma\left(\frac{p+1}{2}\right)\right\}^2$
	22.	$\frac{\pi}{p}$	$\frac{\pi}{4p}$

T.	N.	au lieu de:	lisez:
240.	4.	$\{1$	$\{2$
	5.	$Cos.\, n\lambda$	$(-1)^{n-1}\, Cos.\, n\lambda$
	7, 8.	$(2n)^{2m+2}$	$(2n+1)^{2m+2}$
		, où $c = q - \sqrt{(q^2-1)}$.	
	14.	$\sqrt{(1+q)}$	$\sqrt{(1+g^2)}$
241.	1.	ne vaut qu'entre les limites 0 et 2π.	
	12.	$= -$	$=$
	14.	dx	$q\,dx$
	16.	$=$	$= \pi -$
242.	7.	9	3
	8.	$3\sqrt{3}$	$\sqrt{3}$
243.	6.	$\frac{\pi}{2} - \frac{1}{2}$	$-\frac{\pi}{\sqrt{2}} +$
	18.	$Sin.^2$	$Cos.^2$
	19.	$Sin.\, 2x, = \frac{2}{p^3}$	$Cot.x + p^2 Sin.x.Cos.x, =$
244.	12.	$\frac{1}{q^2}$	$\frac{1}{q^3}$
	13.	a^3	a^2
	15.	9Σ	$12\,\Sigma\,(-1)^n$
245.	13.	πq	$\pi q \lambda$
246.	4.	$2n-1$	$2n+1$
	8.	fautive. elle ne vaut que pour $Cos\, hp.\lambda$ et $Sin\, hp.\lambda$ au lieu de $Cos.\,\lambda$ et de $Sin.\,\lambda$.	
	9.	$= 2$	$=$
	21.	$p^2 - 1 +$	$1 - p^2 +$
247.	5.	où $a < 1$.	
	24.	$p^2 + pq$	$2p^2 + pq$
249.	4, 6.	gh	qh
	6.	$= -2$	$= -$
	22.	ne vaut qu'entre 0 et ∞.	
	23.	fautive par suite de N°. 22.	
250.	3, 4.	a^n	p^n
251.	5 à 8.	ne valent qu'entre 0 et ∞.	
	9, 10.	ne valent que pour $r = 1$.	
253.	2.	$Sin.^2\lambda.\, Sin.^2\mu$	$Sin.^2\lambda.\, Sin.\mu$
255.	3.	$2\, Sin.^2\mu\,(Cos.\lambda$	$2\, Sin.^3\mu\,(Cos.\lambda$

T.	N.	au lieu de:	lisez:
257.	6.	$\frac{\pi}{2}$	$\frac{\pi}{2p}$
	14.	$\frac{1}{2^{2p+2}}$ [1	$\frac{1}{2^{2p+1}}$ [2
258.	28.	$x - \frac{1}{x}$	$x^2 - 1$
259.	1.	$-\pi$	$+\pi$
	2.	$(-1)^n$	$(-1)^{n-1}$
	5.	$(2n)^{2m+2}$	$(2n+1)^{2m+2}$
260.	15.	$4p$	4
265.	28.	$x\,dx$	dx
267.	24.	$2\, Sin.^2\lambda$	$2\, Cos.^2\lambda$
269.	1.	$1+x$	$1+x^2$
271.	9.	$= -$	$=$
272.	2.	$=$	$= -$
277.	8.	$2p$	p
279.	1.	$2a+2$	$2a-2$
	8.	$= 2$	$= r$
281.	11.	$\sum_0^\infty \frac{n}{n^2+(a-1)^2}$	$\sum_0^{a-2} \frac{p}{p^2+(n+1)^2}$
282.	4.	4	2
283.	2.	$(-1)^n$	$(-1)^{n-1}$
286.	13, 14.	$-s^2, \frac{\pi}{p^2+s^3}$	$+s^2, \frac{\pi}{\sqrt{(p^2+s^2)}}$
291.	4.	$\frac{1}{2}$	$\frac{1}{2^a}$
	6.	ôtez $\sqrt{\pi}$.	
293.	17.	$=$	$= -$
296.	22.	$=$	$= \frac{1}{2}\pi$
	23.	$= \frac{1}{2}$	$=$
297.	6.	-1	1
	7.	$e^{(q+1)xi+\frac{1}{2}}$	$e^{-(q+1)xi-\frac{1}{2}}$
301.	15.	$=$	$= -$
	23.	$q\sqrt{}, = 4$	$2q\sqrt{}, = 2$

T.	*N.*	*au lieu de:*	*lisez:*
303.	13.	$-x$	$+x$
306.	7.	$= -$	$= -\frac{1}{4}$
	8.	$\sum_{0}^{\infty} \frac{n^{2a+1}}{1}$	$\sum_{1}^{\infty} \frac{1}{n^{2a+1}}$
	9.	$2x)^{a}$	$2x)^{a-1}$
307.	1.	0	1
309.	7.	$4a$	$4a^{2}$
	10.	$\frac{1}{2}$	$\frac{1}{3}$
	12.	$)^{p}$	$)^{p-1}$
310.	12.	$x+$	$x-$
311.	5.	*l Tang.*	*l Cot.*
312.	10.	$2a+2$	$2a+1$
	11.	$2a-1$	$2a+1$
313.	5.	π	π^{2}
	17.	$-(p+q)$	$+(p+q)$
314.	2.	16	8
316.	9.	π	1
	11.	l	$-l$
318.	10.	$Cot.^{p} x, \Sigma$	$-Cot.^{p} x, \Sigma_{0}$
319.	9.	*Sin.*	$Sin.^{3}$
320.	3, 10.	$2\pi^{2a}$	$(2\pi)^{2a}$
	18.	dx	$Sec. 2x\,dx$
	20.	4	2
321.	16.	$l\,Tang. \left(\frac{\pi}{4} \pm, = \pm\right.$	$Tang. \left(\frac{\pi}{4} -\right), = -$
322.	22.	fautive.	
325.	1.	8	4
	13.	$=$	$= -$
327.	2.	dx	$dx\,l\,Tang.\,x$
329.	15.	$\frac{l\,Tang.\,x}{Cos.\,2x}$	$l\,Cot.\,x$
	22.	*Sin.*	*Cos.*
331.	10.	$0, \frac{1}{2}$	$1, \frac{1}{n}$
	12.	$Sin.\,p$	$Sin.\,x$
333.	8.	$(q-1)x$	$(q+1)x$
336.	5.	$2p-1, 4p$	$p-1, 2p$
338.	6.	$\frac{1}{2}$	$\frac{1}{4}$
339.	14, 16.	$\frac{1}{2}$	$\frac{1}{2}\pi$
340.	15.	4	2
344.	8.	$Sin.\,x$	$Sin.^{2}\,x$
	14.	$\frac{2}{q}$	$\frac{2}{q^{2}}$
345.	14.	π	π^{2}
350.	5.	$Sin.^{q}\,x +$	$Sin.^{q}\,x -$
352.	2.	$l\,Cos.\,x.dx$	$l\,Sin.\,x.dx$
	7.	$)^{n}$	$)^{2n}$
353.	16.	*Cot.*, =	*Cos.*, = 2
362.	9.	$-Sin.\,x)$	$-Sin.^{2}\,x)$
363.	8.	$\pi +$	$\pi + \pi$
369.	7.	$\frac{1}{2}\pi$	$\frac{1}{8}\pi$
372.	2.	$\sqrt{}(1+a^{2}-2a, F', -\frac{2}{b}$	$\sqrt{}(1+b^{2}-2b, \Pi', + \frac{(1+b)(b-a^{2})}{ab} F'(b) - \frac{2}{b}$
	7.	c	p
374.	2.	fautive.	
	3, 5.	$Cos.^{a+1}$	$Cos.^{a-1}$
375.	14.	où $p = Sin.\,\lambda$.	
379.	2.	$pq+a-$	$pq+a-1$
388.	20.	$(-1)^{n-1}$	$(-1)^{n}$ V. T. 439. N°. 13.
	22.	ax	$2ax$ V. T. 439. N°. 11.
390.	15, 16.	ne valent pas, puisque T. 116. N°. 4 ne vaut plus pour $q > 2$.	
396.	22, 29.	$\frac{e^{q}-e^{-q}}{4}$	$\frac{e^{q}-e^{-q}}{2}$
	24, 25.	$\frac{e^{q}-e^{-q}}{2}$	$(e^{q}-e^{-q})$
399.	2 à 8, 10 à 16.	fautives?	

T.	N.	au lieu de:	lisez:
408.	5, 9.	$p^2 <$	$q^2 <$
409.	5, 6, 12, 13.	$p^2 <$	$q^2 <$
	9.	$+ (e^{p\pi}$	$- (e^{p\pi}$
	11.	$+ (e^{p\pi}$	$+ (- e^{p\pi}$
	13.	$=$	$= 1$
410.	6.	$=$	$= -$
411.	10.	$\frac{\pi}{1-p^2}$	$\frac{\pi}{\sqrt{(1-p^2)}}$
418.	9.	$1 -$	$1 +$
422.	8.	$\frac{dx}{x}$	$\frac{dx}{x^2}$
	11.	$+ \frac{2}{1-x^2}$	$- \frac{2}{1-x^2}$
	34.	$=$	$= \frac{1}{2}$
423.	24.	changez $\sqrt{(1-x^2)}$ et $(1-x^2)$.	
	30.	$x^2\, Cos.^2\, \lambda$	$x^2\, Sin.^2\, \lambda$
439.	15.	$p\pi$ (partout)	px

TABLES

D'INTÉGRALES DÉFINIES

PAR

D. BIERENS DE HAAN.

DIVISION DES TABLES.

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PARTIE DEUXIÈME:

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VII. FONCTIONS ALGÉBRIQUES ET EXPONENTIELLES. T. 112 à 150.

116.	F. Alg. rat. ent.	et Expon.	monôme d'autre forme	Lim. 0 et ∞
117.	〃 〃 〃 〃 monôme	〃 〃	bin. $e^{ax} \pm 1$ en dén. Num. alg.	〃 〃 〃 〃
118.	〃 〃 〃 〃 〃	〃 〃	〃 〃 〃 〃. 〃 . 〃 et exp.	〃 〃 〃 〃
119.	〃 〃 〃 〃 〃	〃 〃	〃 $(e^{ax} \pm 1)^2$ en dén.	〃 〃 〃 〃
120.	〃 〃 〃 〃 〃	〃 〃	〃 $e^{ax} \pm e^{-ax}$ 〃 〃. Num. alg.	〃 〃 〃 〃
121.	〃 〃 〃 〃 〃	〃 〃	〃 〃 〃 〃. 〃 〃 et exp.	〃 〃 〃 〃
122.	〃 〃 〃 〃 〃	〃 〃	〃 $(e^{ax} \pm e^{-ax})^2$	〃 〃 〃 〃
123.	〃 〃 〃 〃 binôme	〃 〃	〃 en dén.	〃 〃 〃 〃
124.	〃 〃 〃 〃	〃 〃	polyn. en dén. Num. alg.	〃 〃 〃 〃
125.	〃 〃 〃 〃	〃 〃	〃 〃 〃. 〃 〃 et exp.	〃 〃 〃 〃
126.	〃 〃 〃 fract. à dén monôme	〃 〃	monôme en num.	〃 〃 〃 〃
127.	〃 〃 〃 〃 〃 〃 x^a pour a spéc.	〃 〃	polyn. 〃 〃	〃 〃 〃 〃
128.	〃 〃 〃 〃 〃 〃 〃 〃 〃 génér.	〃 〃	〃 〃 〃	〃 〃 〃 〃
129.	〃 〃 〃 〃 〃 〃 $x \pm q$	〃 〃	monôme	〃 〃 〃 〃
130.	〃 〃 〃 〃 〃 〃 $x^2 \pm q^2$	〃 〃	〃	〃 〃 〃 〃
131.	〃 〃 〃 〃 〃 〃 $(x^a \pm q^a)^b$	〃 〃	〃	〃 〃 〃 〃
132.	〃 〃 〃 〃 〃 autre dén.	〃 〃	〃	〃 〃 〃 〃
133.	〃 〃 〃 〃 〃 dén. prod. de polyn.	〃 〃	〃	〃 〃 〃 〃
134.	〃 〃 〃 〃 〃 〃 x	〃 〃	bin. $e^{ax} \pm 1$ en dén. à un terme	〃 〃 〃 〃
135.	〃 〃 〃 〃 〃 〃 monôme	〃 〃	〃 〃 〃 〃 〃 plus. termes	〃 〃 〃 〃
136.	〃 〃 〃 〃 〃 〃 〃	〃 〃	〃 $e^{ax} \pm e^{-ax}$ en dén.	〃 〃 〃 〃
137.	〃 〃 〃 〃 〃 〃 〃	〃 〃	trinôme en dén.	〃 〃 〃 〃
138.	〃 〃 〃 〃 〃 〃 binôme	〃 〃	binôme 〃 〃	〃 〃 〃 〃
139.	〃 〃 irrat. ent.	〃 〃		〃 〃 〃 〃
140.	〃 〃 〃 fract.	〃 〃		〃 〃 〃 〃
141.	〃 〃 rat. ent.	〃 〃	sous forme irrat.	〃 〃 〃 〃
142.	〃 〃 〃 〃	〃 〃	monôme	Lim. — ∞ et ∞
143.	〃 〃 〃 〃 x	〃 〃	binôme en dén.	〃 〃 〃 〃
144.	〃 〃 〃 〃 〃	〃 〃	polynôme en dén.	〃 〃 〃 〃
145.	〃 〃 〃 〃 x^a	〃 〃	〃 〃 〃	〃 〃 〃 〃
146.	〃 〃 〃 fract.	〃 〃	dén. à fact. x^a	〃 〃 〃 〃
147.	〃 〃 〃 〃	〃 〃	〃 sans fact. x^a	〃 〃 〃 〃
148.	〃 〃 irrat.	〃 〃		〃 〃 〃 〃
149.	〃 〃	〃 〃		Lim. div. 0 et p
150.	〃 〃	〃 〃		Lim. div. p et $\pm \infty$

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IX. FONCTIONS ALGÉBRIQUES ET CIRCULAIRES DIRECTES. T. 192 à 255.

221.	F. Alg. rat. fract. à dén. binôme	et Circ. Dir. en num. trinôme $a - b\ Cos.\ x + c$.	Lim. 0 et ∞
222.	″ ″ ″ ″ ″ ″ polynôme	″ ″ ″ ″ ″ ″	″ ″ ″ ″
223.	″ ″ irrat ent.	″ ″ ″	″ ″ ″ ″
224.	″ ″ ″ fract. à dén. $\sqrt{x}$	″ ″ ″ en num. monôme à un fact. circ. de x	″ ″ ″ ″
225.	″ ″ ″ ″ ″ ″ $x^a\sqrt{x}$	″ ″ ″ ″ ″ ″ ″ ″ ″ ″ ″	″ ″ ″ ″
226.	″ ″ ″ ″ ″ ″ monôme	″ ″ ″ ″ ″ ″ ″ deux ″ ″ ″ ″	″ ″ ″ ″
227.	″ ″ ″ ″ ″ ″ ″	″ ″ ″ ″ ″ binôme	″ ″ ″ ″
228.	″ ″ ″ ″ ″ ″ ″	″ ″ ″ ″ ″ circ. de $x - \frac{1}{x}$	″ ″ ″ ″
229.	″ ″ ″ ″ ″ autre dén.	″ ″ ″ ″ ″	″ ″ ″ ″
230.	″ ″ ″ ″ ″ dén. binôme	″ ″ ″ ″ ″ circ. de $x - \frac{1}{x}$	″ ″ ″ ″
231.	″ ″ ″ ″	″ ″ ″ ″ dén.	″ ″ ″ ″
232.	″ ″ rat. ″	″ ″ ″ ″ num. *Sin.* x	Lim. $-\infty$ et ∞
233.	″ ″ ″ ″	″ ″ ″ ″ ″ *Cos.* x	″ ″ ″ ″
234.	″ ″ ″ ″	″ ″ ″ ″ ″ d'autre forme	″ ″ ″ ″
235.	″ ″ fract.	″ ″ ″ *Sin.* x	Lim. 1 et ∞
236.	″ ″ ″	″ ″ ″ *Cos.* x	″ ″ ″ ″
237.	″ ″ rat. ent.	″ ″ ″ en dén.	Lim. 0 et $\frac{\pi}{4}$
238.	″ ″ ″ ″	″ ″ ″ ent.	Lim. 0 et $\frac{\pi}{2}$
239.	″ ″ ″ ″	″ ″ ″ en dén. monôme	″ ″ ″ ″
240.	″ ″ ″ ″	″ ″ ″ ″ ″ binôme	″ ″ ″ ″
241.	″ ″ ″ ″	″ ″ ″ ″ ″ d'autre forme	″ ″ ″ ″
242.	″ ″ ″ ″	″ ″ ″ sous forme irrat. à dén. monôme	″ ″ ″ ″
243.	″ ″ ″ ″	″ ″ ″ ″ ″ ″ ″ ″ polynôme	″ ″ ″ ″
244.	″ ″ ″ ″	″ ″ ″ ent.	Lim. 0 et π
245.	″ ″ ″ ″	″ ″ ″ en dén. binôme $a + b$	″ ″ ″ ″
246.	″ ″ ″ ″	″ ″ ″ ″ ″ ″ $a - b$	″ ″ ″ ″
247.	″ ″ ″ ″	″ ″ ″ ″ ″ puiss. de binôme	″ ″ ″ ″
248.	″ ″ ″ ″	″ ″ ″ ″ ″ trinôme $1 - a\,Cos.\,x + b$	″ ″ ″ ″
249.	″ ″ ″	″ ″ ″ ″ ″ d'autre forme	″ ″ ″ ″
250.	″ ″ ″	″ ″ ″	Lim. 0 et 2π
251.	″ ″ ″	″ ″ ″	Lim. 0 et γ
252.	″ ″ ″	″ ″ ″ sous forme irrat	Lim. 0 et λ
253.	″ ″ ″	″ ″ ″ ″ ″ ″	Lim. λ et μ
254.	″ ″ ″ fract.	″ ″ ″	Lim. div. p et $+\infty$
255.	″ ″ ″	″ ″ ″	Lim. diverses

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X. FONCTIONS ALGÉBRIQUES ET CIRCULAIRES INVERSES. T. 256 à 271.

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XIX. FONCTIONS CIRCULAIRES DIRECTES ET CIRCULAIRES INVERSES. T. 368 à 374.

XX. FONCTIONS CIRCULAIRES DIRECTES ET AUTRES FONCTIONS. T. 375.

PARTIE TROISIÈME:

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376.	F. Alg. ent.	et Expon. monôme	et Log.	Lim. 0 et 1
377.	〃 〃 〃	〃 〃 〃	〃 〃	Lim. 0 et ∞
378.	〃 〃 fract. à dén. monôme et binôme	〃 〃 〃	〃 〃	〃 〃 〃 〃
379.	〃 〃 〃 〃 〃 puiss. de binôme	〃 〃 〃	〃 〃	〃 〃 〃 〃
380.	〃 〃 rat.	〃 〃 en dén. polynôme	〃 〃	〃 〃 〃 〃
381.	〃 〃 irrat.	〃 〃	〃 〃	〃 〃 〃 〃
382.	〃 〃	〃 〃	〃 〃	Lim. — ∞ et ∞
383.	〃 〃	〃 〃	〃 〃	Lim. diverses.

XXII. FONCTIONS ALGÉBRIQUES ET EXPONENTIELLES ET CIRCULAIRES DIRECTES. T. 384 à 400.

384.	F. Alg. rat. ent.	et Expon.	et Circ. Dir.	Lim. 0 et $\frac{\pi}{4}$
385.	〃 〃 〃 〃 x^a pour a spécial	〃 〃 $e^{\pm px}$	〃 〃 〃 monôme . . .	Lim. 0 et ∞
386.	〃 〃 〃 〃 〃 〃 〃 général	〃 〃 〃	〃 〃 〃 〃 . . .	〃 〃 〃 〃
387.	〃 〃 〃 〃	〃 〃 〃	〃 〃 〃 polynôme. . .	〃 〃 〃 〃
388.	〃 〃 〃 〃	〃 〃 e^{-x^2}	〃 〃 〃	〃 〃 〃 〃
389.	〃 〃 〃 〃	〃 〃 e^{-ax^2}	〃 〃 〃	〃 〃 〃 〃
390.	〃 〃 〃 〃	〃 〃 d'autre forme	〃 〃 〃	〃 〃 〃 〃
391.	〃 〃 〃 〃	〃 〃 en dén. binôme	〃 〃 〃	〃 〃 〃 〃
392.	〃 〃 〃 fract. à dén. x	〃 〃 monôme	〃 〃 〃 monôme . . .	〃 〃 〃 〃
393.	〃 〃 〃 〃 〃 〃 〃	〃 〃	〃 〃 〃 (F. polyn. en num.)	〃 〃 〃 〃
394.	〃 〃 〃 〃 〃 〃 x^a	〃 〃 en num.	〃 〃 〃	〃 〃 〃 〃
395.	〃 〃 〃 〃 〃 〃 $x^2 + a^2$	〃 〃 〃 〃	〃 〃 〃	〃 〃 〃 〃
396.	〃 〃 〃 〃	〃 〃 en dén. polyn.	〃 〃 〃	〃 〃 〃 〃
397.	〃 〃 irrat. ent.	〃 〃	〃 〃 〃	〃 〃 〃 〃
398.	〃 〃 〃 fract. à dén. $\sqrt{x}$	〃 〃	〃 〃 〃	〃 〃 〃 〃
399.	〃 〃 〃 〃 à autre dén.	〃 〃	〃 〃 〃	〃 〃 〃 〃
400.	〃 〃	〃 〃	〃 〃 〃	Lim. diverses.

XXIII. FONCTIONS ALGÉBRIQUES ET EXPONENTIELLES ET CIRCULAIRES INVERSES. T. 401.

401.	F. Alg.	et Expon.	et Circ. Inv.	Lim. 0 et ∞

XXIV. FONCTIONS ALGÉBRIQUES ET EXPONENTIELLES ET AUTRES. T. 402.

402.	F. Alg.	et Expon.	et autres Fonct.	Lim. 0 et ∞

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XXV. FONCTIONS ALGÉBRIQUES ET LOGARITHMES ET CIRCULAIRES DIRECTES. T. 403—421.

XXVI. FONCTIONS ALGÉBRIQUES ET LOGARITHMES ET CIRCULAIRES INVERSES. T. 422—427.

XXVII. FONCTIONS ALGÉBRIQUES ET LOGARITHMES ET AUTRES. T. 428.

XXVIII. FONCTIONS ALGÉBRIQUES ET CIRCULAIRES DIRECTES ET CIRCULAIRES INVERSES. T. 429—434.

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ABBRÉVIATIONS ET NOTATIONS.

Mém. Inst.	Mémoires de l'Institut. — Classe des Sciences physiques et mathém. Paris.
Mém. Acad.	Mémoires de l'Académie Royale des Sciences. Paris.
Sav. Étr.	Mémoires présentés à l'Acad. Royale des Sc. par divers Savans. Paris.
C. R.	Comptes Rendus des Séances hebdomadaires de l'Acad. des Sc. Paris.
Comm. Petr.	Commentaria Petropolitana.
N. C. Petr.	Nova Commentaria Petropolitana.
A. Petr.	Acta Petropolitana.
N. A. Petr.	Nova Acta Petropolitana.
Mém. Petr.	Mémoires de l'Académie de St. Pétersbourg.
Mém. Turin.	Mémoires de l'Académie de Turin.
Mém. Brux.	Nouv. Mém. de l'Acad. Roy. des Sc. et Belles Lettres de Bruxelles.
Mém. Kasan.	Mémoires de l'Académie de Kasan.
Abh. Berlin.	Abhandlungen der K. Akademie der Wissenschaften zu Berlin.
Phil. Trans.	Philosophical Transactions.
Verh. K. Ak. Wet.	Verhandelingen der K. Akademie van Wetenschappen. Amsterdam.
Handl. Stockh.	Kongl. Vetenskaps Academiens Handlingar. Stockholm.
Danske Handl.	Danske Videnskap Akademiens Handlingar.
Overs. Handl.	Overs. over det Kongl. Danske Videnskap. Selskabs Forhandl.
Gött. Stud.	Göttinger Studien.
P.	Journal de l'École Polytechnique.
Bull. Phil.	Bulletin de la Société Philomatique.
C.	Crelle, Journal für reine und angewandte Mathematik.
Gr.	Grunert, Archiv der Mathematik und Physik.
L.	Liouville, Journal de Mathématiques pures et appliquées.
Lim. Imag.	A. L. Cauchy, Mém. sur les intégrales définies prises entre des limites imaginaires. Paris. Debure. 1825. 4°. 69 Pages.
Rés. Leç.	A. L. Cauchy, Résumé des Leçons données à l'Éc. Polyt. sur le Calcul Infinitésimal. T. I (et seul). Paris. Debure. 1823. 4°. XII et 172 Pages.
Exerc.	Cauchy, Exercices Mathématiques. Paris. 4°.
Eul. Int.	R. Dedekind, Ueber die Elemente der Theorie der Euler'schen Integrale. Göttingen. Huth. 1852. 4°. 23 S.
Calc. Int.	Euler, Institutiones Calculi Integralis. IV Vol. Petrop. 1792—1794. 4°.
Funct. Transc.	B. J. Féaux, De functione transcendente, quae littera Γ ( ) obsignatur: sive de integrali Euleriano secundae speciei. Monast. Coppenrath 1844. 4°. 43 Pages.
Chal.	Fourier, Théorie Analytique de la Chaleur. Paris. Firmin Didot. 1822. 4°. XXII et 639 Pages.
Transf.	Ph. P. Helmling, Transformation und Ausmittelung bestimmter Integrale. Dorpat. Laakman. 1851. 4°. 35 S.
Transf. II.	Ph. P. Helmling, Transformation und Ausmittelung bestimmter Integrale mit besonderer Rücksicht auf grössere Werthe der Gränzen und implicirten Coustanten. Mitau und Leipzig. Reyher. 1854. 4°. IV und 146 S.

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ABBRÉVIATIONS ET NOTATIONS.

Réfr.	Kramp, Analyse des réfractions astronomiques et terrestres. Leipzic. Schwickert. 1799. 4°. XX et 210 S.
Probab.	Laplace, Théorie analytique des Probabilités. Paris. Courcier. 1812. 4°. 465 Pages et quatre Suppléments.
Exerc.	A. M. Legendre, Exercices de Calcul Intégral sur divers ordres de transcendantes et sur les Quadratures. 3 Vol. Paris. Courcier. 1811—1818. 4°.
Int.	R. Lobatto, Lessen over de Integraal-Rekening. I. 's Gravenh. Van Cleef. VI en 466 Bladz.
Adn.	L. Mascheroni, Adnotationes ad Calculem Integralem Euleri. Ticini. Galeatis. 1790. 4°. 72 Pag.
Int. Déf.	A. Meyer, Exposé élément. de la Théorie des Intégrales définies. Bruxelles. Muquardt. 1851. 8°. 513 Pages.
Int.	Moigno, Leçons de Calcul Intégral. I. Paris. Bachelier. 1844. 8°. XLVIII et 783 Pages.
Def. Int.	H. Mosely, Definite Integrals (Encycl. Metropol. Re-issue). London. Griffin. 1849. 4°. 54 Pages.
Ausw.	M. Ohm, Die Auswerthungsmethoden bestimmter Integrale, so wie die Theorie der Reihen und der Integrale des Fourier. Nürnberg. Korn. 1852. 8°. XII und 437 S.
Chal.	S. D. Poisson, Théorie mathématique de la Chaleur. Paris. Bachelier. 1835. 4°. 532 Pag. et Supplement. Paris. Bachelier. 1837. 4°. 72 Pag.
Int.	J. L. Raabe, Die Integralrechnung. III Th. Zürich. Orell. 1839, 1843, 1847. 8°.
J. B. Funct.	J. L. Raabe, Die Jacob-Bernoullische Function. Zürich. Orell. 1848. 4°. 51 S.
Mat.	Rogner, Materialien aus der höheren Analysis. Gratz. Hesse. 1853. 8°. XIV und 463 S.
Beitr.	O. Schlömilch, Beiträge zur Theorie bestimmter Integrale. Jena. Frommann. 1843. 4°. 103 S.
An. Stud.	O. Schlömilch, Analytische Studien. II Th. Leipzig. Engelmann. 1848. 8°. 209 und 197 S.
Int.	O. Schlömilch, Handbuch der Integralrechnung. Greifswald. Otte. 1847. 8°. 214 S.
Höh. An.	O. Schlömilch, Compendium der höhern Analysis. Braunschweig. Vieweg. 1853. 8°. XVI und 550 S.
Samml.	J. A. Schubert, Sammlung von Differential- und Integral-Formeln. Dresden. Arnold. 1845. 8°. XIV und 173 S.
Samml.	L. A. Sohnke, Sammlung von Aufgaben aus der Differential- und Integral-Rechnung. Halle. Schmidt. 1850. 8°. VI und 338 S.
Transf.	A. F. Svanberg, Observations sur la transformation des Intégrales multiples. Ups. Leffler 1845. 4°. 13 Pag.
Anal.	J. Vieille, Cours complémentaire d'Analyse et de Mécanique rationnelle. Paris. Bachelier. 1851. 8°. VII en 400 Pages.

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ABBRÉVIATIONS ET NOTATIONS.

A = 0, 577215 664901 532861.... Constante du Logarithme intégral.

e = 2, 718281 828459 045235 360287 471352 662497 757247 093699 959574 966967 627724 076630 353547 594571 382178 525166 427427 466.... Base des Logarithmes naturels.

π = 3, 141592 653589 793238 462643 383279 502884 197169 399375 105820 974944 592307 816406 286208 998628 034825 342117 067982 148086 513282 306647 093844 609550 582231 725359 408128 481117 450284 102701 938521 105559 644622 948954 930381 964428 810975 665933 446128 475648 233786 783165 271201 909145 648566 923460 348610 454326 648213 393607 260249 141273 724587 006606 315588 174881 520920 962829 254091 715364 367892 590360 011330 530548 820466 521384 146951 941511 609433 057270 365759 591953 092186 117381 932611 793105 118548 074462 379962 749567 351885 752724 891227 938183 011949 129833 673362 440656 643086 021394 88.... Circonférence du cercle dont le diamètre est l'unité.

$i = \sqrt{-1}$

$$Sinhp.\ a = \frac{e^a - e^{-a}}{2},\ \text{Sinus hyperbolique}$$

$$Coshp.\ a = \frac{e^a + e^{-a}}{2},\ \text{Cosinus} \quad "$$

$$Tghp.\ a = \frac{e^a - e^{-a}}{e^a + e^{-a}},\ \text{Tangente} \quad "$$

$$Cothp.\ a = \frac{e^a + e^{-a}}{e^a - e^{-a}},\ \text{Cotangente} \quad "$$

Ces notations ne sont employées, qu'autant qu'elles portent sur des constantes; elles ne sont donc pas admises comme argument dans les tables, mais dans les formules, où elles se trouvent, on y a substitué les valeurs équivalentes en exponentielles.

$$la = \int_1^a \frac{dx}{x},\ \text{le Logarithme naturel}$$

$$li\ (a) = \int_0^a \frac{dx}{lx},\ \text{le Logarithme intégral}$$

$$Ei\ (a) = \int_{-a}^{\infty} \frac{e^{-x}\,dx}{x},\ \text{l'Exponentielle intégrale}$$

$$Si.\ (a) = \int_0^a \frac{Sin.\,x\,dx}{x},\ \text{le Sinus intégral}$$

$$Ci.\ (a) = \int_{\infty}^a \frac{Cos.\,x\,dx}{x},\ \text{le Cosinus intégral}$$

$$Z'\ (a) = \frac{d\,.\,l\Gamma(a)}{da}$$

Ces fonctions sont comprises sous la dénomination d' *"autres fonctions."*

$\binom{a}{b}$ le coefficient $b^{ième}$ de la puissance $a^{ième}$ du binôme.

$c^{a/b}$ factorielle (Notation de Kramp).

B_{2p-1} coefficient ou nombre Bernoullien.
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$$\varphi(p, q, r) = 1 + \frac{p}{q}\frac{r}{1} + \frac{p \,.\, p+1}{q \,.\, q+2}\,\frac{r^2}{1 \,.\, 2} + \ldots$$

$$\psi(p, q) = 1 + \frac{q}{1 . p} + \frac{q^2}{1 \,.\, 2 \,.\, p \,.\, p+1} + \ldots$$

$$\chi(p, q, r) = 1 - \frac{p}{1}\frac{q}{r} + \frac{p \,.\, p+1}{1 \,.\, 2}\,\frac{q \,.\, q+1}{r^2} - \ldots$$

Notations employées par Kummer, Cr. 17. 228.

$$B'(x) = \frac{x^{2a+2}}{2a+2} - \tfrac{1}{2}x^{2a+1} + \tfrac{1}{2}\binom{2a+1}{1}B_1 x^{2a} - \tfrac{1}{4}\binom{2a+1}{3}B_3 x^{2a-2} + \ldots + \frac{(-1)^{a-1}}{2a}\binom{2a+1}{2a-1}B_{2a-1}x$$

$$B''(x) = \frac{x^{2a+1}}{2a+1} - \tfrac{1}{2}x^{2a} + \tfrac{1}{2}\binom{2a}{1}B_1 x^{2a-1} - \tfrac{1}{4}\binom{2a}{3}B_3 x^{2a-3} + \ldots + \frac{(-1)^{a-1}}{2a}\binom{2a}{2a-1}B_{2a-1}x$$

Notations employées par Raabe Cr. 42. 348 et comprises parmi les „*autres fonctions.*"

$$L(a) = \int_a^0 dx\, l\, Cos.\, x = a l 2 - \tfrac{1}{2}\sum_1^{\infty}(-1)^n \frac{Sin.\, 2na}{n^2}$$

. Lobatschewsky, Mém. Kasan. 1836. 1.

$$Y(p, \varphi) = \int_0^{\varphi} \frac{E(p, \varphi)}{\sqrt{(1 - p^2 Sin.^2 \varphi)}}\, d\varphi.$$

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ABBRÉVIATIONS DANS LE SOMMAIRE DES TABLES.

F.	Fonction.
Alg.	Algébrique.
Exp.	Exponentielle.
Log.	Logarithme.
Circ. Dir.	Circulaire Directe.
Circ. Inv,	Circulaire Inverse.
rat.	rationnelle.
irrat.	irrationnelle.
ent.	entière.
fract.	fractionnaire.
mon.	monôme.
bin.	binôme.
trin.	trinôme.
polyn.	polynôme.
num.	numérateur.
dén.	dénominateur.
fact.	facteur.
prod.	produit.
puiss.	puissance.
comp.	composée.
arg.	argument.

PARTIE PREMIÈRE.

TABLES
D'INTÉGRALES DÉFINIES

PARTIE PREMIÈRE.

F. Alg. rat. ent.	TABLE 1.	Lim. 0 et 1.

1) $\int x^{p-1}\,dx = \frac{1}{p}$ Cauchy, Cours Leç. 32. — Plana, Cr. 17. 1. Il observe que Cavalleri a trouvé 2) et Wallis 3).

2) $\int x^{a}\,dx = \frac{1}{a+1}$

3) $\int x^{\frac{a}{b}}\,dx = \frac{b}{a+b}$

4) $\int (1-x^2)^a\,dx = \frac{2^{2a}}{2a+1}\,\frac{(1^{a/1})^2}{1^{2a/1}}$ Plana, Cr. 17. 1.

5) $\int (1-x)\,x^{p-1}\,da = \frac{1}{p(1+p)}$ Cisa de Grésy, Mém. Turin 1821. 209. I. N°. 5.

6) $\int (1-x)^{a-1}\,x^{a-1}\,dx = 2\,\frac{1^{a-1/1}}{(a+1)^{a/1}}$ Euler, Calc. Int. 4. S. 3. 13.

7) $\int (1-x)^p\,x^{1-p}\,dx = p\,\frac{1-p}{2}\,\frac{\pi}{Sin.\,p\,\pi} = \int (1-x)^{1-p}\,x^p\,dx$ Oettinger, Cr. 38. 162.

8) $\int (1-x)^{p-1}\,x^{q-1}\,dx = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}$ Poisson, P. 19. 404. N°. 72. — Jacobi, Cr. 11. 307. — Plana, Cr. 17. 1. — Grunert, Gr. 2. 266.

9) $= \frac{1^{p/1}}{q^{p/1}}\cdot\frac{1}{p}$ pour p et q entiers; Legendre, Exerc. 3. 34. — Schlömilch, Gr 4. 23. — Cisa de Grésy, Mém. Turin 1821. 209. I. N°. 2.

10) $= \frac{1^{p-1/1}\,1^{q-1/1}}{1^{p+q-1/1}}$ pour tout p et q; Oettinger, Cr. 35. 13. — Lobatschewsky, Mém. Kasan. 1835. 211.

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11) $\int (1-x)^{p-1} x^{q-1}\, dx = \frac{p+q}{pq} \cdot \frac{1.p+q+1}{p+1.q+1} \cdot \frac{2.p+q+2}{p+2.q+2} \cdots\cdots$ Cisa de Grésy, Mém. Turin 1821. 209. I. N°. 4.

C'est l'intégrale Eulérienne de première espèce $B(p,q)$ ou $\left[\begin{matrix} p \\ q \end{matrix}\right]$. Binet en traite P. 27. 123. — Lejeune Dirichlet, Cr. 15. 258. — Schaar, Mém. Cour. Brux. T. 22.

12) $\int (1-x)^{q-p-1} x^{p+a-1}\, dx = \frac{p^{a/1}}{q^{a/1}} \frac{\Gamma(p)\,\Gamma(q-p)}{\Gamma(q)}$ Schlömilch, Stud. I. 24.

13) $\int (1-x)^{c\pm b-1} x^{a\pm b-1}\, dx = \frac{1^{c\pm b-1/1}\, 1^{a\pm b-1/1}}{1^{a+c\pm 2b-1/1}} = \int (1-x)^{a\pm b-1} x^{c\pm b-1}\, dx$

14) $\int (1-x)^{b-a-1} x^{a-1}\, dx = \frac{1^{a-1/1}\, 1^{b-a-1/1}}{1^{b-1/1}} = \int (1-x)^{a-1} x^{b-a-1}\, dx$

(13–14) Oettinger, Cr. 35. 13.

15) $\int (1-x)^{b-p} x^{p+c}\, dx = \frac{(1+p)^{c/1}\,(1-p)^{b/1}}{1^{b+c/1}} \frac{p\pi}{Sin.\, p\pi} = \int (1-x)^{p+c} x^{b-p}\, dx$

16) $\int (1-x)^{b-p} x^{p-c}\, dx = \frac{(1-p)^{b/1}}{p^{c/-1}\, 1^{b-c-1/1}} \frac{p\pi}{Sin.\, p\pi} = \int (1-x)^{p-c} x^{b-p}\, dx$

17) $\int (1-x^2)^q x^{2a-1}\, dx = \frac{1^{a/1}\, 1^{q/1}}{2a \,.\, 1^{a+q/1}}$

18) $\int (1-x^2)^q x^{2a}\, dx = \frac{2^{q/2}}{(2a+1)^{q+1/2}}$

(15–18) Oettinger, Cr. 38 162.

19) $\int (1-x^2)^b x^{2a+1}\, dx = \frac{2^{a/2}\, 2^{b/2}}{4^{a+b/2}}$ Ohm, Ausw 49.

20) $\int (1-x^b)^p x^{q-1}\, dx = b^p \frac{1^{p/1}}{q^{p+1/b}}$ pour p et q entiers; Euler, Calc. Int. 4. S. 3. 1. — Id., N. C. Petr. 16. 91. — Kramp, Réfr. 3. 76. — Plana, Cr. 17. 1. — Oettinger, Cr. 35. 13.

21) $= \frac{\Gamma(\frac{b}{q})\,\Gamma(p+1)}{q\,\Gamma(\frac{b}{q}+p+1)} = \frac{p}{q+bp} \frac{\Gamma(\frac{b}{q})\,\Gamma(p)}{\Gamma(\frac{b}{q}+p)}$ pour q aussi des fractions; Plana, Cr. 17. 1.

22) $= \frac{1^{p/1}}{b(\frac{q}{b})^{p+1/1}} = \frac{1^{\frac{q}{b}/1}\, 1^{p/1}}{q\,.\,1^{\frac{q}{b}+p/1}} = \frac{1^{\frac{q}{b}/1}}{q(p+1)^{\frac{q}{b}/1}}$ pour p et q entiers; Oettinger, Cr. 35. 13.

23) $\int (1-x^b)^{a-1} x^{ab-1}\, dx = \frac{2}{b} \cdot \frac{1^{a-1/1}}{(a+1)^{a/1}}$ Euler, Calc. Int. 4. S. 3. 14.

24) $\int (1-x^a)^{b-1} x^{a-1}\, dx = \frac{1}{ab}$ Euler, Calc. Int. 4. S. 3. 17.

25) $\int (x^a-1)(1-x)^b\, dx = 1^{b/1} \left\{ \frac{1^{a/1}}{2^{a+b/1}} - \frac{1}{2^{b/1}} \right\}$ Lobatschewsky, Mém. Kasan. 1835. 211.

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26) $\int (x^b - x^{b+c})^g\, x^{a-1}\, dx = \dfrac{1^{\frac{a+bg}{c}/1} . 1^{g/1}}{(a+bg) . 1^{\frac{a+bg+cg}{g}/1}} = c^g . \dfrac{1 . 2 \ldots g-1}{a+bg . a+bg+c . a+bg+2c \ldots a+(b+c)g}$ Oettinger, Cr. 35. 13.

27) $\int (1-x)^{a-1}(1+qx^b)^c x^{p-1}\, dx = 1^{a-1/1} \sum_0^\infty \binom{c}{n} \dfrac{q^n}{(p+nb)^{a/1}},\ q^2 < 1$

28) $\int (1-x)^{a-1}(1+x^b)^c x^{p-1}\, dx = 1^{a-1/1} \sum_0^\infty \binom{c}{n} \dfrac{1}{(p+nb)^{a/1}}$

29) $\int (1-x)^{a-1}(1-x^b)^c x^{p-1}\, dx = 1^{a-1/1} \sum_0^\infty \binom{c}{n} (-1)^n \dfrac{1}{(p+nb)^{a/1}},\ a+c > 0$

(27–29) Schlömilch, Stud. I. 13.

30) $\int \left\{(1+x)^{p-1}(1-x)^{q-1} + (1+x)^{q-1}(1-x)^{p-1}\right\} dx = 2^{p+q-1}\,\mathrm{B}(p, q)$ Binet, P. 27. 123. N°. 3.

F. Alg. rat. fract. à dén. monôme. TABLE 2. Lim. 0 et 1.

1) $\int \dfrac{dx}{x^p} = \infty$ Cauchy, Cours Leç. 32.

2) $\int (1 - \dfrac{1}{x^b})^c\, x^{a-1}\, dx = \dfrac{1}{a} b^c \dfrac{1^{c/1}}{(b-a)^{c/b}} = \dfrac{1^{c/1}}{a(1-\frac{a}{b})^{c/1}}$

3) $\int (\dfrac{1}{x} - 1)^p\, dx = \dfrac{p\pi}{Sin.\, p\pi}$

4) $\int \dfrac{(1-x)^p}{x^{p+1}} dx = \dfrac{-\pi}{Sin.\, p\pi}$

(2–4) Oettinger, Cr. 35. 13.

5) $\int \dfrac{(1-x)^{p+b}}{x^{p+a}} dx = \dfrac{(1+p)^{a/1}}{2^{a-b+1/1} p^{b/1}} . \dfrac{p\pi}{Sin.\, p\pi}$

6) $\int \dfrac{(1-x)^{p+1}}{x^p} dx = \dfrac{1+p}{2} \dfrac{p\pi}{Sin.\, p\pi}$

(5–6) Oettinger, Cr. 38. 162.

F. Alg. fract. à dén. $a \pm bx^c$. TABLE 3. Lim. 0 et 1.

1) $\int \dfrac{x^{p-1}\, dx}{1+x} = \frac{1}{2} Z'\left(\dfrac{p+1}{2}\right) - \frac{1}{2} Z'\left(\dfrac{p}{2}\right)$ Legendre, Exerc. 5. 4.

2) $= \sum_0^\infty \dfrac{(-1)^{n-1}}{p+n+1}$ pour p entier; Arndt, Gr. 6. 434.

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3) $\int \frac{dx}{p-x} = l\,\frac{p}{1-p},\ p<1$; Cauchy, Exerc. 1827. p. 125.

4) $\int \frac{1-x^{p-1}}{1-x}\,dx = A + Z'(p)$ Legendre, Exerc. 4. 50. — Cauchy, P. 28. 147. I. § 6. — Cisa de Grésy, Mém. Turin 1821. 209. I. N°. 28. — Lobatschewsky, Mém. Kasan 1835, 211. — Schlömilch, Gr. 4. 167. — Id., Gr. 9. 5. — Id., Stud. I. 6.

5) $= \sum_{0}^{p-2} \frac{1}{n+1}$ où p entier; Cauchy, Cours Leç. 32. — Dienger, Gr. 8. 451.

6) $\int \frac{1-x^k}{1-x}\,dx = A + lk$, pour $k = \infty$ Legendre, Exerc. 5. 12.

7) $\int \frac{1-x^p}{1-x}\,x^{q-1}\,dx = Z'(p+q) - Z'(q)$ Legendre, Exerc. 4. 50.

8) $\int \frac{x^q - x^p}{1-x}\,dx = Z'(1+p) - Z'(1+q)$ Legendre, Exerc. 4. 50. — Schlömilch, Gr. 4. 167.

9) $\int \frac{dx}{p+qx} = \frac{1}{q}\,l\,\frac{p+q}{p}$ Meyer, Int. Déf. 95.

10) $\int \frac{(1-x)^{b-a-1}}{1-px}\,x^{a-1}\,dx = \frac{\Gamma(a)\,\Gamma(b-a)}{\Gamma(b)}\,\xi(a,b,p),\ b > a > 0$ Schaeffer, Cr. 37. 127.

11) $\int \frac{1-a^c x^c}{1-ax}(1-x)^p\,dx = \sum_{1}^{c} \frac{a^{n-1}\,.\,1^{n/1}}{(p+2)^{n-1/1}}$ Lindmann, Stockh. Handl. 1850.

12) $\int \frac{dx}{1+x^2} = \frac{1}{4}\pi$ Ohm, Ausw. 2. — Raabe, Int. 136.

13) $\int \frac{x^p\,dx}{1+x^2} = \frac{1}{4} Z'\left(\frac{p+3}{4}\right) - \frac{1}{4} Z'\left(\frac{p+1}{4}\right)$ Legendre, Exerc. 5. 16. — Lindmann, Stockh. Handl. 1850.

14) $\int \frac{x^p - x^q}{1-x^2}\,dx = \frac{1}{2} Z'\left(\frac{q+1}{2}\right) - \frac{1}{2} Z'\left(\frac{p+1}{2}\right)$ Malmsten, Cr. 38. 1.

15) $\int \frac{1-x^3}{1-x^4}\,dx = \frac{\pi}{8} + \frac{3}{4}\,l2$

16) $\int \frac{1-x}{1-x^4}\,x^2\,dx = -\frac{\pi}{8} + \frac{3}{4}\,l2$

(15, 16) Poisson, Mém. Inst. 1811, 163.

17) $\int \frac{x^{b-1} + x^{a-b-1}}{1+x^a}\,dx = \frac{\pi}{a}\,Cosec.\,\frac{b\pi}{a}$ Euler, Calc. Int. T. 4, S. 3. 70. — Id., N. C. P. 19. 3.

18) $\int \frac{x^{a-1}\,dx}{1+x^b} = \frac{1}{2b} Z'\left(\frac{a+b}{2b}\right) - \frac{1}{2b} Z'\left(\frac{a}{2b}\right)$ Legendre, Exerc. 5. 4.

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19) $\int \frac{x^{b-1}-x^{a-b-1}}{1-x^a}dx = \frac{\pi}{a}\,Cot.\frac{b\pi}{a}$ Euler, Calc. Int. T. 4, S. 3. 70. — Id., N. C. P. 19. 3. — Legendre, Mém. Inst. 1809. 416. N°. 45. — Id., Exerc. 2. 44. — Id., ib. 5. 13.

20) $\int \frac{x^{a-1}-x^{b-1}}{1-x^b}dx = \frac{1}{b}\left\{A - Z'\left(\frac{a}{b}\right)\right\}$ Raabe, Cr. 25. 160.

21) $\int \frac{x^{a-1}-x^{ap+b-1}}{1-x^a}dx = \frac{1}{a}\left\{A + Z'\left(p+\frac{b}{a}\right)\right\}$ Schlömilch, Stud. I. 7.

22) $\int \frac{x^{q-1}dx}{1-x^b} = -\frac{1}{b}\sum_1^b Cos.\frac{2qn\pi}{b}\,l\,2\,Sin.\frac{n\pi}{b} + \frac{1}{b}\sum_1^b Sin.\frac{2qn\pi}{b}\left(\frac{\pi}{2}-\frac{n\pi}{b}\right)$

23) $\quad = -\frac{1}{b}\sum_1^b Cos.\frac{2qn\pi}{b}\,l\,Sin.\frac{n\pi}{b} - \frac{\pi}{b^2}\sum_1^b n\,Sin.\frac{2qn\pi}{b}$

(22, 23) Lebesgue, L. 15. 215.

24) $\int \frac{x^{q-1}+x^{p-1}}{x^{p+q}+1}dx = \frac{\pi}{p+q}\,Sec.\frac{q-p}{q+p}\frac{\pi}{2}$

25) $\int \frac{x^{q-1}-x^{p-1}}{x^{p+q}-1}dx = \frac{\pi}{p+q}\,Tang.\frac{q-p}{q+p}\frac{\pi}{2}$

(24, 25) Raabe, Int. 146. — Ohm, Ausw. 14.

26) $\int \frac{e^{pi}dx}{1+e^{api}x^a} = -\frac{1}{a}\sum_0^{\frac{1}{2}(a-1)}\left\{Cos.\frac{2n+1}{a}\pi\,l\left(Cos.p - Cos.\frac{2n+1}{a}\pi\right)\right\} + \frac{\pi}{2a\,Sin.\frac{\pi}{a}} +$

$+\frac{i}{2a}\sum_0^{\frac{1}{2}(a-1)}\left\{Sin.\frac{2n+1}{a}\pi\,l\frac{1-Cos.\left(\frac{2n+1}{a}\pi+p\right)}{1-Cos.\left(\frac{2n+1}{a}\pi-p\right)}\right\}$ Dienger, Cr. 38. 331.

27) $\int \frac{e^{bpi}x^{b-1}dx}{1+e^{2api}x^{2a}} = -\frac{1}{2a}\sum_0^{a-1}\left\{Cos.\frac{2n+1}{2a}b\pi l\left(Cos.p - Cos.\frac{2n+1}{2a}\pi\right)\right\} + \frac{\pi}{4a}\,Cosec.\frac{b\pi}{2a} +$

$+\frac{i}{2a}\sum_0^{a-1}\left\{Sin.\frac{2n+1}{2a}b\pi\,l\frac{1-Cos.\left(\frac{2n+1}{2a}\pi+p\right)}{1-Cos.\left(\frac{2n+1}{2a}\pi-p\right)}\right\}$ Dienger, Cr. 38. 331.

F. Alg. rat. fract. à dén. $(a \pm bx^c)^d$. TABLE 4. Lim. 0 et 1.

1) $\int \frac{x^{a-1}dx}{(1+x)^b} = (\tfrac{1}{2})^a\sum_0^\infty\binom{b-a-1}{n}\frac{(\frac{1}{2})^n}{a+n}$ Legendre, Exerc. 5. N°. 6.

2) $\int \frac{x^{a-1}dx}{(1+x)^{2a+b}} = \frac{1^{a-1/1}\Gamma(\frac{1}{2})}{2^{2a+b}\Gamma(a+\frac{1}{2})}\sum_0^\infty\binom{b}{2n}\frac{1^{n/1}}{(2a+1)^{n/2}} + \frac{1}{a.2^{2a+b}}\sum_0^\infty\binom{b}{2n+1}\frac{1^{n/1}}{(a+1)^{n/2}}$ Legendre, Exerc. 5. N°. 8.

3) $\int \frac{x^{p-1}\,dx}{(1+x)^{2p}} = \frac{1}{2^{2p}} \frac{\Gamma(p)\Gamma(\frac{1}{2})}{\Gamma(p+\frac{1}{2})}$ Legendre, Exerc. 5. N°. 7.

4) $\int \frac{x^{q-1} + x^{p-q-1}}{(1+x)^p}\,dx = \frac{\Gamma(q)\Gamma(p-q)}{\Gamma(p)}$ Legendre, Exerc. 4. N°. 101.

5) $\int \frac{x^{q-1} + x^{p-1}}{(1+x)^{p+q}}\,dx = B(q,p)$ Binet, P. 27. 123.

6) $\int \frac{x^{p-1}\,dx}{(1-x)^p} = \frac{\pi}{Sin.\,p\pi}$, $p^2 < 1$; Cisa de Grésy, Mém. Turin 1821. 209. I. §7.— Oettinger, Cr. 35. 13.

7) $\int \frac{x^p\,dx}{(1-x)^p} = \frac{p\pi}{Sin.\,p\pi}$

8) $\int \frac{x^p\,dx}{(1-x)^{p+1}} = -\frac{\pi}{Sin.\,p\pi}$

(7, 8) $p^2 < 1$ Oettinger, Cr. 35. 13.

9) $\int \frac{x^{p+1}\,dx}{(1-x)^p} = \frac{1+p}{2} \frac{p\pi}{Sin.\,p\pi}$

10) $\int \frac{x^{p+a}\,dx}{(1-x)^{p+b}} = \frac{(1+p)^{a/1}}{1^{a-b+1/1}\,p^{b/1}} \cdot \frac{p\pi}{Sin.\,p\pi}$

(9, 10) $p^2 < 1$ Oettinger, Cr. 38. 162.

11) $\int \frac{x^{a-2}\,dx}{(1+px)^a} = \frac{(1+p)^a}{a-1}$ Legendre, Exerc. 4. 118.

12) $\int \frac{x^{a-1}\,dx}{(1+px)^b} = \frac{1}{(a-1)(1+p)^b}\,\xi\left(b, a, \frac{p}{1+p}\right)$, $a > 1$, $p \geqq -\frac{1}{2}$; Schaeffer, Cr. 37. 127.

13) $\int \frac{x^{q-1}(1-x)^{p-1}}{(x+a)^{p+q}}\,dx = \frac{\Gamma(q)\Gamma(p)}{\Gamma(p+q)} \frac{1}{a^q(1+a)^p}$ Abel, Cr. 2. 22.

14) $\int \frac{x^{p-1}(1-x)^{q-1}\,dx}{(r+sx)^{p+q}} = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \frac{1}{(r+s)^p r^q}$ Schlömilch, Höh. Anal. 85.

15) $\int \frac{x^{p-1}(1-x)^{q-1}\,dx}{(1+ax)^{p+q}} = \frac{1}{(1+a)^p} B(q,p)$

16) $\int \frac{x^{p-1}(1-x)^{q-1}\,dx}{(1+ax)^r} = B(p,q)\left\{1 - \binom{r}{1}\frac{p}{p+q}a + \binom{r}{2}\frac{p \cdot p+1}{p+q \cdot p+q+1}a^2 + \dots\right\}$

(15, 16) Boncompagni, Cr. 25. 74.

17) $\int \frac{x^{b-1}(1-x)^{c-b-1}}{(1-qx)^a}\,dx = \sum_0^\infty \binom{a}{n} \frac{b^{n/1}}{c^{n/1}} q^n$ Schlömilch, Stud. I. 24.

18) $\int \frac{x^q\,dx}{(1+x^2)^2} = \frac{1-q}{8}\left\{Z'\left(\frac{q+3}{4}\right) - Z'\left(\frac{q+1}{4}\right)\right\} + \frac{1}{4}$ Legendre, Exerc. 5. 17.

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19) $\int \frac{x^{2p-2}\,dx}{(1-x^2)^p} = \frac{\Gamma(2p-1)\,\Gamma(1-p)}{2^{2p-1}\,\Gamma(p)}$ Legendre, Exerc. 4. 117.

20) $\int \frac{x^{a-1}\,dx}{(1-x^b)^c} = \infty$ Oettinger, Cr. 35. 13.

21) $\int \left(\frac{d^{2a}}{dp^{2a}} \cdot \frac{1}{x-p}\right) dx = \frac{d^{2a}}{dp^{2a}} . l \frac{1-p}{p}$

22) $\int \left(\frac{d^{2a-1}}{dp^{2a-1}} \cdot \frac{1}{x-p}\right) dx = \infty$

(21, 22) Cauchy, P. 19. 511.

23) $\int \frac{x^p + x^{-p}}{(1+x^q)^2}\, x^{q-1}\,dx = \frac{\pi}{q^2} \frac{p}{e^{\frac{p\pi}{q}} - e^{-\frac{p\pi}{q}}}$ Euler, N. A. Petr. 3. 3.

F. Alg. rat. fract. à dén. $(a \pm b x^c)^d x^e$. TABLE 5. Lim. 0 et 1.

1) $\int \frac{x^{p-1} + x^{-p}}{1+x}\,dx = \pi\, Cosec.\, p\pi$ Legendre, Exerc. 4. 96.

2) $\int \frac{x^p - x^{1-p}}{1+x} \frac{dx}{x} = \pi\, Cot.\, p\pi$ Legendre, Exerc. 4. 54.

3) $\int \frac{x^{-p} - 1}{1-x}\,dx = -A - Z'(1-p)$

4) $\int \frac{x^{-p} - x^{-q}}{1-x}\,dx = Z'(1-q) - Z'(1-p)$

5) $\int \frac{x^{-p} - x^p}{1-x}\,dx = \frac{1}{p} - \pi\, Cot.\, p\pi$

(3, 4, 5) $p < 1$; Legendre, Exerc. 5. 3.

6) $\int \frac{x^{p-1} - x^{-p}}{1-x}\,dx = \pi\, Cot.\, p\pi$ Legendre, Exerc. 4. 98. — Serret, L. 8. 1.

7) $\int \left(\frac{1-x}{x}\right)^p \frac{dx}{1-x} = \pi\, Cosec.\, p\pi$, $p < 1$; Oettinger, Cr. 35. 13.

8) $\int \frac{x^q - x^p}{1-x} \frac{dx}{x} = Z'(p) - Z'(q)$ Legendre, Exerc. 4. 50. — Id., ib. 5. 3. — Stern, Cr. 21. 377. — Schlömilch, Beitr. III. 9.

9) $\int \frac{(x^p - x^{-p})(x^q - x^{-q})}{1+x^2}\,dx = 2\pi \frac{Sin. \frac{1}{2} p\pi . Sin. \frac{1}{2} q\pi}{Cos.\, p\pi + Cos.\, q\pi}$, $p < 1$, $q < 1$; V. T. 38. N°. 15.

10) $\int \frac{(x^p + x^{-p})(x^q + x^{-q})}{1+x^2}\,dx = 2\pi \frac{Cos. \frac{1}{2} p\pi . Cos. \frac{1}{2} q\pi}{Cos.\, p\pi + Cos.\, q\pi}$, $p < 1$, $q < 1$; V. T. 38. N°. 14.

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11) $\int \frac{x^p + x^{-p}}{1 + x^2} dx = \frac{\pi}{2} Sec. \frac{p\pi}{2}$ V. T. 38. N°. 16.

12) $\int \frac{x^{p-1} - x^{1-p}}{1 - x^2} dx = \frac{\pi}{2} Cot. \frac{p\pi}{2}$ Legendre, Exerc. 4. 98. — Cauchy, P. 19. 511.

13) $\int \frac{x^p - x^{-p}}{1 - x^2} dx = -\frac{\pi}{2} tg. \frac{p\pi}{2}$ Legendre, Exerc. 4. 98.

14) $\int \frac{x^p - x^{-p}}{1 - x^2} x dx = -\frac{1}{p} + \frac{\pi}{2} Cot. \tfrac{1}{2} p\pi$ V. T. 38. N°. 18.

15) $\int \frac{(x^p - x^{-p})(x^q + x^{-q})}{1 - x^2} dx = \frac{-\pi\, Sin. p\pi}{Cos. p\pi + Cos. q\pi}, p < 1$; V. T. 38. N°. 18.

16) $\int \frac{x^p + x^{q-p}}{1 + x^q} \frac{dx}{x} = \frac{\pi}{q} Cosec. \frac{p\pi}{2q}$ Euler, Calc. Int. 4. S. 5. N°. 155.

17) $\int \frac{x^{p-q} + x^{p+q}}{1 + x^{2p}} \frac{dx}{x} = \frac{\pi}{2p} Sec. \frac{q\pi}{2p}$

18) $\int \frac{x^{p+q} - x^{p-q}}{1 - x^{2p}} \frac{dx}{x} = -\frac{\pi}{2p} Tang. \frac{q\pi}{2p}$

(17, 18) Euler, Calc. Int. 4. S. 3. N°. 71. — Id., N. C. P. 19. 30.

19) $\int \frac{x^{p(q+r)} + x^{p(q-r)}}{1 + x^{2pq}} \frac{dx}{x} = \frac{\pi}{2pq} Sec. \frac{r\pi}{2q}$

20) $\int \frac{x^{p(q+r)} - x^{p(q-r)}}{1 - x^{2pq}} \frac{dx}{x} = -\frac{\pi}{2pq} Tang. \frac{r\pi}{2q}$

(19, 20) Euler, Calc. Int. 4. S. 3. N°. 72.

21) $\int \frac{x^q - x^{-q}}{x^p - x^{-p}} \frac{dx}{x} = \frac{\pi}{2p} Tang. \frac{q\pi}{2p}$

22) $\int \frac{x^q + x^{-q}}{x^p + x^{-p}} \frac{dx}{x} = \frac{\pi}{2p} Sec. \frac{q\pi}{2p}$

(21, 22) Euler, N. A. Petr. 3. 3. — Poisson, P. 18. 295. N°. 22. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

23) $\int \frac{dx}{x^{1-p} + x^{1+p}} = \frac{\pi}{4p}$ V. T. 38. N°. 8.

24) $\int \frac{1}{(x + x^{-1})^{2p}} \frac{dx}{x} = \frac{\{\Gamma(p)\}^2}{4\Gamma(2p)}$ Schlömilch, Gr. 6. 213.

25) $\int \frac{x^{q-p} + x^{p-q}}{(x + x^{-1})^{p+q}} \frac{dx}{x} = \tfrac{1}{2} B(p,q)$ V. T. 39. N°. 16.

26) $\int \frac{x^{2p} + x^{-2p}}{(x + x^{-1})^{2q}} \frac{dx}{x} = \frac{\Gamma(q+p)\Gamma(q-p)}{2\Gamma(2q)}$ V. T. 39. N°. 18.

27) $\int \frac{x^{ak} - x^{bk}}{x - 1} \frac{dx}{x} = l \frac{a}{b}$, pour $k = \infty$; Euler, N. C. Petr. 20. 59.

1) $\int \frac{x}{1+x^2} \frac{dx}{1+px} = -\frac{1}{1+p^2}\left\{l\frac{1+p}{\sqrt{2}} + \frac{1}{4}p\pi\right\}$ Bertrand, L. 8. 110.

2) $\int \frac{x^{q-1}}{(1-x)^q} \frac{dx}{1+px} = \frac{\pi}{(1+p)^q}$ $Cosec.\ q\pi$ Legendre, Exerc. 4. 118. — Boncompagni, Cr. 25. 74.

3) $\int \frac{1-x^a}{(1+x)^{a+1}} \frac{dx}{1-x} = \frac{1}{2^{a-1}} \sum_1^a \frac{2^n}{n}$ Serret, L. 8. 1.

4) $\int \frac{x^{q-1}}{(1-x)^q} \frac{dx}{x+a} = \frac{(1+a)^{q-1}}{a^q} \pi\ Cosec.\ q\pi$

5) $\int \frac{x^{q-1}}{(1-x)^{1-r}} \frac{dx}{(x+p)^{q+r}} = \frac{\Gamma(q)\Gamma(r)}{\Gamma(q+r)} \frac{1}{p^q(1+p)^r}$

(4, 5) Abel, Cr. 2. 22.

6) $\int \frac{x^{r-1}\,dx}{(1-x)^r(1+px)^a} = \frac{\pi}{Sin.\ r\pi} \frac{1}{(1+p)^r} \sum_0^\infty (-1)^n \binom{a-1}{n}\binom{r}{n}\left(\frac{p}{1+p}\right)^n$ Legendre, Exerc. 4. 119.

7) $\int \frac{x^{r+p-2}}{(1-x)^p} \frac{dx}{(1+qx)^r} = (1+q)^{1-r-p} \frac{\Gamma(r+p-1)\ \Gamma(1-p)}{\Gamma(r)}$ Legendre, Exerc. 4. 115, où $r+p>1, p<1, q+1>0$.

8) $\int \frac{x^{r-1}}{(1-x)^r} \frac{dx}{(1+px)(1+qx)} = \frac{\pi}{(p-q)\ Sin.\ r\pi}\left\{\frac{p}{(1+p)^a} - \frac{q}{(1+q)^a}\right\}$ Legendre, Exerc. 4. 120.

9) $\int \frac{1}{(1-x)^{1-p}x^p} \frac{dx}{a-bx} = \frac{\pi}{(a-b)^{1-p}a^p\ Sin.\ p\pi}$

10) $\int \frac{1}{(1-x)^{1-p}x^p} \frac{dx}{(a-bx)^{c+1}} = \frac{p^{c/1}}{1^{c/1}} \frac{\pi\ Cosec.\ p\pi}{a^p(a-b)^{c-1-p}} \sum_0^c \frac{1-p.2-p....c-p-n}{c+p-1.c+p-2....p+n}\binom{c}{n}\left(\frac{a-b}{a}\right)^n$

(9, 10) $1>p>0$; a ni $=0$, ni $=b$; Dienger, Cr. 42. 283.

11) $\int \left(\frac{x^{q-1}}{1+px} + \frac{x^{-q}}{p+x}\right) dx = \frac{\pi}{p^q} Cosec.\ q\pi$ Legendre, Éxerc. 4. 137.

12) $\int \left(\frac{x^{rp-1}}{1-x} - \frac{r\,x^{r^2p-1}}{1-x^r}\right) dx = lr$ Stern, Cr. 21. 377.

13) $\int \left(\frac{x^{p-1}}{1-x} - \frac{q\,x^{pq-1}}{1-x^q}\right) dx = lq$ Legendre, Exerc. 4. 56. — Stern, Cr. 21. 377. — Arndt, Gr. 10. 253.

14) $\int \left(\frac{b\,x^{b-1}}{1-x^b} - \frac{x^{ab-1}}{1-x}\right) dx = \mathrm{A} + \frac{1}{b}\sum_1^b \mathrm{Z}'\left(a + \frac{b-n}{b}\right)$

15) $\int \left(\frac{b\,x^{b-1}}{1-x^b} - \frac{1}{1-x}\right) dx = -\frac{1}{b}\mathrm{Z}'(ab) + \frac{1}{b}\sum_1^b \mathrm{Z}'\left(a + \frac{b-n}{b}\right)$

(14, 15) Arndt, Gr. 10. 253. — Schlömilch, Stud. I. 7.

16) $\int \left(\frac{1}{1-x} - \frac{p\,x^{p-1}}{1-x^p}\right) dx = lp$ Legendre, Exerc. 5. 12. — Schlömilch, Stud. I. 7. — Arndt, Gr. 10. 253.

17) $\int \left(\frac{k x^{k-1}}{1-x^k} - \frac{x^k}{1-x}\right) dx = A$, pour $k = \infty$; Legendre, Exerc. 5. 12.

18) $\int \left(\frac{e^{pi}}{1+e^{api}x^a} + \frac{e^{-pi}}{1+e^{-api}x^a}\right) dx = 2 \sum_0^\infty \frac{1}{na+1} \operatorname{Cos.}(na+1)p$

19) $\int \left(\frac{e^{pi}}{1+e^{api}x^a} - \frac{e^{-pi}}{1+e^{-api}x^a}\right) dx = 2 \sum_0^\infty \frac{(-1)^n}{na+1} \operatorname{Sin.}(na+1)p$

(18, 19) $a^2 p^2 < \pi^2$; Dienger, Cr. 38. 331.

F. Alg. rat. fract. à dén. trinôme. TABLE 7. Lim. 0 et 1.

1) $\int \frac{dx}{1-x+x^2} = \frac{2\pi}{3\sqrt{3}}$

2) $\int \frac{dx}{1+x+x^2} = \frac{\pi}{3\sqrt{3}}$

(1, 2) Euler, Calc. Int. 4. S. 3. § 105.

3) $\int \frac{dx}{1-2x\operatorname{Cos.}\lambda+x^2} = \frac{1}{\operatorname{Sin.}\lambda} \operatorname{Arctg.} \frac{\operatorname{Sin.}\lambda}{1-\operatorname{Cos.}\lambda}$ Euler, Calc. Int. 4. S. 5. 46.

4) $\int \frac{dx}{1+2x\operatorname{Cos.}\lambda+x^2} = \frac{\lambda}{2\operatorname{Sin.}\lambda}$ Legendre, Exerc. 4. 105.

5) $\int \frac{\operatorname{Cos.}\lambda - x}{1-2x\operatorname{Cos.}\lambda+x^2} dx = l(2\operatorname{Sin.}\tfrac{1}{2}\lambda)$ Euler, Calc. Int. 4. S. 5. N. 55.

6) $\int \frac{1-x^2}{1+2x\operatorname{Cos.}\lambda+x^2} dx = \operatorname{Cos.}\lambda\, l\{2(1+\operatorname{Cos.}\lambda)\} + \tfrac{1}{2}\lambda \operatorname{Sin.}\lambda - 1$ Poisson, Mém. Inst. 1811, 163. N°. 27.

7) $\int \frac{x^p + x^{-p}}{1+2x\operatorname{Cos.}\lambda+x^2} dx = \frac{\pi}{\operatorname{Sin.} p\pi} \cdot \frac{\operatorname{Sin.} p\lambda}{\operatorname{Sin.}\lambda}$, $p < 1$; Legendre, Exerc. 4. 103.

8) $\int \frac{x^a dx}{1+x+x^2} = \frac{1}{3}\left\{Z'\left(\frac{a+2}{3}\right) - Z'\left(\frac{a+1}{3}\right)\right\}$

9) $\int \frac{x^a dx}{1-x+x^2} = \frac{1}{6}\left\{Z'\left(\frac{a+5}{6}\right) - Z'\left(\frac{a+2}{6}\right) + Z'\left(\frac{a+4}{6}\right) - Z'\left(\frac{a+1}{6}\right)\right\}$

10) $\int \frac{x^a dx}{1 \pm x\sqrt{2}+x^2} = \frac{1}{8}\left\{Z'\left(\frac{a+7}{8}\right) - Z'\left(\frac{a+1}{8}\right) + Z'\left(\frac{a+5}{8}\right) - Z'\left(\frac{a+3}{8}\right)\right\}$
$\mp \frac{1}{8}\sqrt{2}\left\{Z'\left(\frac{a+6}{8}\right) - Z'\left(\frac{a+2}{8}\right)\right\}$

(8–10) Legendre, Exerc. 5. 16.

11) $\int \frac{x\,dx}{1-2px+x^2} = \tfrac{1}{2} l\{2(1-p)\} + \frac{p}{2} \frac{\operatorname{Arccos}(-p)}{\operatorname{Sin.}\{\operatorname{Arccos}(-p)\}}$, $p < 1$; Cauchy, Sav. Etr. 1827. 599. Suppl. 1.

12) $\int \frac{x\,dx}{1-2px+x^2} = \frac{1}{2} l\{2(p-1)\} - \frac{p}{2\sqrt{(p^2-1)}} l\{p+\sqrt{(p^2-1)}\},\ p>1;$ Cauchy, Sav. Etr. 1827. 599. Suppl. 1.

13) $\int \frac{x^c\,dx}{1+2x\,Cos.\frac{a\pi}{b}+x^2} = \frac{1}{Sin.\frac{a\pi}{b}} \sum_0^{c-1} (-1)^{n-1} Sin.\frac{na\pi}{b}\left\{Z'\left(\frac{b+c+n}{2b}\right) - Z'\left(\frac{c+n}{2b}\right)\right\}$ pour $a+b$ impair.

14) $= \frac{1}{Sin.\frac{a\pi}{b}} \sum_0^{\frac{1}{2}(c-1)} (-1)^{n-1} Sin.\frac{na\pi}{b}\left\{Z'\left(\frac{b+c-n}{b}\right) - Z'\left(\frac{c+n}{b}\right)\right\}$ pour $a+b$ pair.

Ces deux formules chez Malmsten, Cr. 38. 1.

15) $\int \frac{1-x\,Cos.\lambda - x^{a+1}\,Cos.(a+1)\lambda + x^{a+2}\,Cos.a\lambda}{1-2x\,Cos.\lambda+x^2}dx = \sum_0^a \frac{Cos.n\lambda}{n+1}$

16) $\int \frac{Sin.\lambda - x^a\,Sin.(a+1)\lambda + x^{a+1}\,Sin.a\lambda}{1-2x\,Cos.\lambda+x^2}dx = \sum_0^a \frac{Sin.n\lambda}{n+1}$

Dienger, Gr. 8. 450.

17) $\int \frac{Sin.\lambda - q^a x^a\,Sin.(a+1)\lambda + q^{a+1}x^{a+1}\,Sin.a\lambda}{1-2qx\,Cos.\lambda+q^2x^2}(1-x)^p\,dx = \frac{\Gamma(p+1)}{q}\sum_1^a \frac{q^n\,\Gamma(n)\,Sin.n\lambda}{\Gamma(n+p+1)}$

18) $\int \frac{Cos.\lambda - qx - q^a x^a\,Cos.(a+1)\lambda + q^{a+1}x^{a+1}\,Cos.a\lambda}{1-2qx\,Cos.\lambda+q^2x^2}(1-x)^p\,dx = \frac{\Gamma(p+1)}{q}\sum_1^a \frac{q^n\,\Gamma(n)\,Cos.n\lambda}{\Gamma(n+p+1)}$

Lindmann, Stockh. Handl. 1850. I.

19) $\int \frac{1+x^2}{1-x^2+x^4}dx = \frac{1}{2}\pi$ Raabe, Cr. 37. 356.

20) $\int \frac{x^{b-1}+x^{2a-b-1}}{1+2x^a\,Cos.\lambda+x^{2a}}dx = \frac{\pi\,Sin.\frac{a-b}{a}\lambda}{a\,Sin.\lambda.\,Sin.\frac{b\pi}{a}}$ Euler, Calc. Int. 4. S. 5. N°. 185.

21) $\int \frac{x^{a-b-1}+x^{a+b-1}}{1+2x^a\,Cos.\lambda+x^{2a}}dx = \frac{\pi\,Sin.\frac{b\lambda}{a}}{a\,Sin.\lambda.Sin.\frac{b\pi}{a}}$ Euler, Calc. Int. 4. S 5. N°. 191.

22) $\int \frac{1-x}{1-2x\,Cos.\lambda+x^2}dx = \frac{1}{Sin.\lambda}\sum_1^\infty \frac{Sin.n\lambda}{n(n+1)}$ Schlömilch, Gr. 4. 23.

1) $\displaystyle\int \frac{x^{1+p} + x^{1-p}}{(1 + 2\,x\,Cos.\,\lambda + x^2)^2}\,dx = \frac{\pi}{2\,Sin.\,p\pi}\,\frac{p\,Sin.\,\lambda.\,Cos.\,p\,\lambda - Cos.\,\lambda.\,Sin.\,p\,\lambda}{Sin^3.\,\lambda}$ Legendre, Exerc. 4. 108.

2) $\displaystyle\int \frac{x^a\,dx}{(1 + x + x^2)^2} = 2\,\frac{1-a}{9}\,Z'\left(\frac{a+2}{3}\right) - \frac{2-a}{9}\,Z'\left(\frac{a+1}{3}\right) + \frac{a}{9}\,Z'\,\frac{a}{3} + \frac{1}{3}$

3) $\displaystyle\int \frac{x^a\,dx}{(1 - x + x^2)^2} = \frac{1-a}{9}\left\{Z'\left(\frac{a+5}{6}\right) - Z'\left(\frac{a+2}{6}\right)\right\} - \frac{2-a}{18}\left\{Z'\left(\frac{a+4}{6}\right) - Z'\left(\frac{a+1}{6}\right)\right\}$

$\displaystyle\qquad + \frac{a}{18}\left\{Z'\left(\frac{a+3}{6}\right) - Z'\left(\frac{a}{b}\right)\right\} + \frac{1}{3}$

4) $\displaystyle\int \frac{x^a\,dx}{(1 \pm x\sqrt{2} + x^2)^2} = \frac{1-a}{8}\left\{Z'\left(\frac{a+7}{8}\right) - Z'\left(\frac{a+3}{8}\right)\right\} - \frac{1}{8}\left\{Z'\left(\frac{a+5}{8}\right) - Z'\left(\frac{a+1}{8}\right)\right\}$

$\displaystyle\qquad \mp \frac{2-a}{16}\sqrt{2}\left\{Z'\left(\frac{a+6}{8}\right) - Z'\left(\frac{a+2}{8}\right)\right\} \mp \frac{a\sqrt{2}}{16}\left\{Z'\left(\frac{a+4}{8}\right) - Z'\left(\frac{a}{8}\right)\right\} + \frac{1}{2}$

(2–4) Legendre, Exerc. 5. 17.

5) $\displaystyle\int \frac{x^{q-1}}{1 + p\,x\,Cos.\,\lambda + p^2\,x^2}\,\frac{dx}{(1-x)^q} = \frac{\pi}{p^q\,Sin.\,q\,\pi.\,Sin.\,\lambda}\,Sin.\left\{\lambda - q\,Arctg.\,\frac{p\,Sin,\,\lambda}{1 + p\,Cos.\,\lambda}\right\}$ Legendre, Exerc. 4. 120.

6) $\displaystyle\int \left(\frac{x^p}{1 + 2\,q\,x\,Cos.\,\lambda + q^2\,x^2} + \frac{x^{-p}}{x^2 + 2\,q\,x\,Cos.\,\lambda + q^2}\right)dx = \frac{\pi\,Sin.\,p\,\lambda}{q^{p+1}\,Sin.\,p\pi.\,Sin.\,\lambda}$

7) $\displaystyle\int \left\{\frac{1 + q\,x\,Cos.\,\lambda}{1 + 2\,q\,x\,Cos.\,\lambda + q^2\,x^2}\,x^{p-1} + \frac{x + q\,Cos.\,\lambda}{x^2 + 2\,q\,x\,Cos.\,\lambda + q^2}\,\frac{1}{x^p}\right\}dx = \frac{\pi}{q^{p+1}\,Sin.\,q\,\pi}\,Cos.\,p\lambda$

(6–7) Legendre, Exerc. 4. 138.

8) $\displaystyle\int \left(\frac{Sin.\,\lambda}{1 + 2\,x\,Cos.\,\lambda + x^2} - \frac{\lambda}{(1 + x)^2}\right)dx = 0$ Malmsten, Cr. 38. 1.

9) $\displaystyle\int \frac{x^p - 2\,Cos.\,\lambda + x^{-p}}{x^q - 2\,Cos.\,\mu + x^{-q}}\,\frac{dx}{x} = \frac{\pi\,Sin.\left(\frac{\pi-\mu}{q}\,p\right)}{q\,Sin.\,\mu.\,Sin.\,\frac{p\,\pi}{q}} - \frac{\pi - \mu}{q\,Sin,\,\mu}\,Cos.\,\lambda$

10) $\displaystyle\int \frac{x^p + x^{-p}}{x^q - 2\,Cos.\,\lambda + x^{-q}}\,\frac{dx}{x} = \frac{\pi\,Sin.\left(p\,\frac{\pi-\lambda}{q}\right)}{q\,Sin.\,\lambda.\,Sin.\,\frac{p\,\pi}{q}}$

11) $\displaystyle\int \frac{x^p + x^{-p}}{x^q + \left(a + \frac{1}{a}\right) + x^{-q}}\,\frac{dx}{x} = \frac{a^{\frac{p}{q}} - a^{-\frac{p}{q}}}{a - \frac{1}{a}}\,\frac{\pi}{q\,Sin.\,\frac{p\,\pi}{q}}$

(9–11) Euler, N. A. Petr. 3, 3.

12) $\displaystyle\int \frac{x^q + x^{-q}}{x^p + 2\,Cos.\,\lambda + x^{-p}}\,\frac{dx}{x} = \frac{\pi\,Sin.\,\frac{q\,\lambda}{p}}{p\,Sin.\,\lambda.\,Sin.\,\frac{q\pi}{p}}$ Poisson, P. 18. 295. N°. 28.

1) $\int x^{a-b-1}(1-x)^{\frac{b-a}{a}}dx = \frac{\pi}{a} Cosec. \frac{b\pi}{a}$ Euler, N. A. P. I. 2. p. 3.

2) $\int x^{a+p+\frac{1}{2}}(1-x)^{b-p-\frac{1}{2}}dx = \frac{(1+2p)^{a/2}.(1-2p)^{b/2}}{1^{a+b+1/1}} \frac{\pi Sec. p\pi}{2^{a+b+1}} = \int x^{b-p-\frac{1}{2}}(1-x)^{a+p+\frac{1}{2}}dx$

3) $\int x^{-a+p+\frac{1}{2}}(1-x)^{b-p-\frac{1}{2}}dx = \frac{(1-2p)^{b/2}}{(2p-1)^{a/-2}} \frac{2^{a-b-1}\pi}{Cos. p\pi} = \int x^{b-p-\frac{1}{2}}(1-x)^{-a+p+\frac{1}{2}}dx$

(2, 3) Oettinger, Cr. 38. 162.

4) $\int dx \sqrt{(1-x^2)} = \frac{1}{4}\pi$ Euler, Calc. Int. I. P. 1. S. 1. 8. 340. — Plana, Cr. 17. 1.

5) $\int (1-x^2)^{a-\frac{1}{2}}dx = \frac{(a+1)^{a/1}}{1^{a/1}} \frac{\pi}{2^{2a+1}}$ Laplace, Prob. I. 34.

6) $\int x^{2a-1}dx \sqrt{(1-x^2)} = \frac{2^{a-1/2}}{3^{a/2}}$

7) $\int x^{2a}dx \sqrt{(1-x^2)} = \frac{3^{a-1/2}}{4^{a/2}} \frac{\pi}{4}$

(6, 7) Oettinger, Cr. 35. 13.

8) $\int x^{2a}(1-x^2)^{b-\frac{1}{2}}dx = \frac{1^{a/2}1^{b/2}}{1^{a+b/2}} \frac{\pi}{2^{a+b+1}}$ Euler, Calc. Int. I. P. I. S. 1. 8. 340. — Oettinger, Cr. 35. 13.

9) $= \frac{1^{a/2}1^{b/2}}{2^{a/2}(2+2a)^{b/2}} \frac{\pi}{2}$ Kramp, Réfr. 3. 79.

10) $\int x^{2a}(1-x^2)^{b+\frac{1}{2}}dx = \frac{3^{b/2}3^{a-1/2}}{3^{a+b/2}} \frac{\pi}{4}$ Oettinger, Cr. 35. 13.

11) $\int x^{2a-1}(1-x^2)^{b-\frac{1}{2}}dx = \frac{2^{a-1/2}1^{b/2}}{1^{a+b/2}}$ Euler, Calc. Int. I. P. 1. S. 1. 8. 340. — Oettinger, Cr. 35. 13.

12) $= \frac{2^{a-1/2}1^{b/2}}{1^{a/2}(1+2a)^{b/2}}$ Kramp, Réfr. 3. 79.

13) $\int x^{2a-1}(1-x^2)^{b+\frac{1}{2}}dx = \frac{2^{a-1/2}}{(2b+1)^{a/2}}$ Oettinger, Cr. 35. 13.

14) $\int (1-x^2)^{1-\frac{1}{2}a}(1-p^2x^2)^{1-\frac{1}{2}a}x^a dx = \frac{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(2-\frac{a}{2}\right)}{\sqrt{\left\{\pi . a-1 . a-3 . a-5\right\}}} \frac{1}{p^3}$

$\left\{\frac{1+(a-3)p+p^2}{(1+p)^{a-3}} - \frac{1-(a-3)p+p^2}{(1-p)^{a-3}}\right\}$ Ramus, Overs. Danske Forh. 1844.

1) $\int x^{a-1}(1-x^b)^{\frac{c-b}{b}}\,dx = \frac{b}{ac}\cdot\frac{2b\,.\,a+c}{a+b\,.\,c+b}\cdot\frac{3b\,.\,a+c+b}{a+2b\,.\,c+2b}\cdots\cdots$ Euler, Calc. Int. I. P. 1. S. 1. 8. 364.

2) $= \frac{1^{\frac{a}{b}/1}\;1^{\frac{c-b}{b}/1}}{a\,.\,1^{\frac{a+c}{b}-1/1}}$ Oettinger, Cr. 35. 13.

3) $= \frac{\Gamma\left(\frac{a}{b}\right)\,\Gamma\left(\frac{c}{b}\right)}{b\,\Gamma\left(\frac{a+c}{b}\right)}$, où b peut être fractionnaire; Legendre, Mém. Inst. 1809. 416. N°. 55. — Id., Exerc. 2. 56. — Plana, Cr. 17. 1.

4) $\int x^{a-1}(1-x^a)^{\frac{b-a}{a}}\,dx = \frac{1}{b}$ Euler, N. A. P. I. 2. p. 3. — Id., Calc. Int. 4. S. 3. 129.

5) $\int (1-x^a)^{\frac{b-a}{a}}\,x^{a-b-1}\,dx = \frac{\pi}{a}\,Cosec.\frac{b\pi}{a}$ Euler, Calc. Int. 4. S. 3. 131.

6) $\int x^{a+b-1}(1-x^b)^{\frac{c}{d}}\,dx = \frac{1^{\frac{a}{b}+1/1}\;1^{\frac{c}{d}/1}}{(a+b)\,1^{\frac{a}{b}+\frac{c}{d}+1/1}}$

7) $\int x^{a+bd-1}(1-x^b)^{\pm\frac{h}{g}+c}\,dx = \frac{1}{a+bd}\,\frac{(\pm h+g)^{c/g}(a+b)^{d/b}}{(ag\pm bh+bg)^{c+d/bg}}\,\frac{1^{\pm\frac{h}{g}/1}\;1^{\frac{a}{b}/1}}{1^{\frac{a}{b}\pm\frac{h}{g}/1}}\,b^c\,g^d$

8) $\int x^{a-bd-1}(1-x^b)^{\frac{h}{g}-c}\,dx = \frac{\left(\frac{a}{b}+\frac{h}{g}\right)^{c+d/-1}}{\left(\frac{h}{g}\right)^{c/-1}\left(\frac{a}{b}\right)^{d/-1}}\,\frac{1}{a-bd}\,\frac{1^{\frac{a}{b}/1}\;1^{\frac{h}{g}/1}}{1^{\frac{a}{b}+\frac{h}{g}/1}}$

9) $\int x^{\pm a+bd-1}(1-x^b)^{\frac{h}{g}-c}\,dx = \frac{1}{a+bd}\,\frac{\left(1\pm\frac{a}{b}\right)^{d/1}}{\left(\frac{h}{g}\right)^{c/-1}\left(1\pm\frac{a}{b}+\frac{h}{g}\right)^{d-c/1}}\,\frac{1^{\pm\frac{a}{b}/1}\;1^{\frac{h}{g}/1}}{1^{\pm\frac{a}{b}+\frac{h}{g}/1}}$

10) $\int x^{a-bd-1}(1-x^b)^{\pm\frac{h}{g}+c}\,dx = \frac{1}{a-bd}\,\frac{\left(1\pm\frac{h}{g}\right)^{c/1}}{\left(\frac{a}{b}\right)^{d/-1}\left(1+\frac{a}{b}\pm\frac{h}{g}\right)^{c-d/1}}\,\frac{1^{\frac{a}{b}/1}\;1^{\pm\frac{h}{g}/1}}{1^{\frac{a}{b}\pm\frac{h}{g}/1}}$

11) $\int x^{-a+bd-1}(1-x^b)^{\frac{h}{g}+c}\,dx = \frac{1}{bd-a}\,\frac{(h+g)^{c/g}(b-a)^{d/b}}{(bh+bg-ag)^{c+d/bg}}\cdot\frac{1^{h/g}\,1^{-\frac{a}{b}/1}}{1^{\frac{h}{g}-\frac{a}{b}/1}}\,b^c\,g^d$

12) $\int x^{a-1}(1-x^b)^{\pm\frac{h}{g}+c}\,dx = \frac{(g\pm h)^{c/g}}{(ag+bg+bh)^{c/bg}}\,\frac{1^{\frac{a}{b}/1}\;1^{\pm\frac{h}{g}/1}}{1^{\frac{a}{b}\mp\frac{h}{g}/1}}\,\frac{b^c}{a}$

13) $\int x^{a-1}(1-x^b)^{+\frac{h}{g}-c}\,dx = \frac{(ag+bh)^{c/-bg}}{h^{c/-g}}\,\frac{1^{\frac{a}{b}/1}\,1^{\frac{h}{g}/1}}{1^{\frac{a}{b}+\frac{h}{g}/1}}\,\frac{1}{ab^c}$

14) $\int x^{a-1}(1-x^b)^{-\frac{a}{b}+c}\,dx = \frac{(b-a)^{c/b}}{b^{c/b}}\,\frac{1^{\frac{a}{b}/1}\,1^{-\frac{a}{b}/1}}{a}$

15) $\int x^{a+bc-1}(1-x^b)^{\frac{h}{g}}\,dx = \frac{(a+b)^{c/b}}{(ag+bh+bg)^{c/bg}}\,\frac{1}{a+bc}\,\frac{1^{\frac{a}{b}/1}\,1^{\frac{h}{g}/1}}{1^{\frac{a}{b}+\frac{h}{g}/1}}\,g^c$

16) $\int x^{a-bc-1}(1-x^b)^{\frac{h}{g}}\,dx = \frac{(ag+bh)^{c/-bg}}{a^{c/-b}}\,\frac{1}{a-bc}\,\frac{1^{\frac{a}{b}/1}\,1^{\frac{h}{g}/1}}{1^{\frac{a}{b}+\frac{h}{g}/1}}\,\frac{1}{g^c}$

17) $\int x^{ab-1}(1-x^b)^{\frac{h}{g}}\,dx = \frac{g^{a/g}}{ab\,(h+g)^{a/g}}$

18) $\int x^{ab}(1-x^b)^{\frac{h}{g}}\,dx = \frac{(b+1)^{a/b}}{(g+bh+bg)^{a/bg}}\,\frac{1^{\frac{h}{g}/1}\,1^{\frac{1}{a}/1}}{1^{\frac{h}{g}+\frac{1}{a}/1}}\,\frac{g^a}{1+ab}$

19) $\int x^{ab-1}(1-x^b)^{\pm\frac{h}{g}+c}\,dx = \frac{(g+h)^{c/g}\,g^{a/g}}{ab\,(h+g)^{c+a/g}}$

20) $\int x^{ab}(1-x^b)^{\pm\frac{h}{g}+c}\,dx = \frac{(g+h)^{c/g}\,(1+b)^{a/b}}{(g+bh+bg)^{a+c/bg}}\,\frac{1^{\frac{h}{g}/1}\,1^{\frac{1}{a}/1}}{1^{\frac{h}{g}+\frac{1}{a}/1}}\,\frac{b^c\,g^a}{1+ab}$

21) $\int x^{ab-1}(1-x^b)^{\frac{h}{g}-c}\,dx = \frac{g^{a/g}}{ab\,.\,h^{c/-g}\,(h+g)^{a-c/g}}$

22) $\int x^{ab}(1-x^b)^{\frac{h}{g}-c}\,dx = \frac{(1+b)^{a/b}}{h^{c/-g}\,(g+bh+bg)^{a-c/bg}}\,\frac{1^{\frac{h}{g}/1}\,1^{\frac{1}{a}/1}}{1^{\frac{h}{g}+\frac{1}{a}/1}}\,\frac{g^a}{1+ab}$

Les Intégrales 6 à 22 se trouvent toutes: Oettinger, Cr. 35. 13.

23) $\int x^{b-a-1}(1-x^b)^{\frac{a}{b}-1}\,dx = \frac{\pi}{b}\,Cosec.\frac{a\pi}{b}$ Euler, Calc. Int. T. 1. S. 1. 8. 352. — Oettinger, Cr. 35. 13.

24) $\int x^{a+bc-1}(1-x^b)^{-\frac{a}{b}+g}\,dx = \frac{(b-a)^{g/b}\,(b+a)^{c/b}}{b^{c+g/b}}\,\frac{a}{a+bc}\,\frac{\pi}{b\,Sin.\,\frac{a\pi}{b}}$

25) $\int x^{a+bc-1}(1-x^b)^{-\frac{a}{b}+c}\,dx = \left(1-\frac{a^2}{b^2}\right)\left(4-\frac{a^2}{b^2}\right)\,\cdots\cdots\left(c^2-\frac{a^2}{b^2}\right)\,\frac{a}{1^{2c/1}}\,\frac{1}{a+bc}\,\frac{\pi}{b\,Sin.\,\frac{a\pi}{b}}$

26) $\int x^{a-bc-1}(1-x^b)^{-\frac{a}{b}+g}\,dx = \frac{(b-a)^{g/b}}{(b-a)^{c/b}}\,\frac{b^{c-g}}{1^{g-c/1}}\,\frac{\pi}{b\,Sin.\,\frac{a\pi}{b}}$

27) $\int x^{a-bc-1}\,(1-x^b)^{-\frac{a}{b}+c}\,dx = (-1)^c\,\frac{\pi}{b}\,Cosec.\,\frac{a\pi}{b}$

Des formules 24—27 voyez Oettinger, Cr. 38. 162.

F. Alg. irrat. fract. à dén. monôme. TABLE 11. Lim. 0 et 1.

1) $\int \frac{(1-x)^{p+\frac{1}{2}}}{x^{p+\frac{1}{2}}}\,dx = \frac{2p+1}{2}\,\pi\,Sec.\,p\pi$

2) $\int \frac{(1-x)^{p-\frac{1}{2}}}{x^{p+\frac{1}{2}}}\,dx = \pi\,Sec.\,p\pi$

3) $\int \frac{(1-x)^{p-\frac{1}{2}}}{x^{p-\frac{1}{2}}}\,dx = \frac{1-2p}{2}\,\pi\,Sec.\,p\pi$

4) $\int \frac{(1-x)^{p+a+\frac{1}{2}}}{x^{p+b+\frac{1}{2}}}\,dx = \frac{(1+2p)^{a+1/2}}{(1+2p)^{b/2}\,1^{a-b+1/1}}\,(-1)^b\,2^{b-a}\,\pi\,Sec.\,p\pi$

Oettinger, Cr. 38. 162.

5) $\int \frac{(1-x^b)^{\frac{h}{g}-c}}{x^{a+bd+1}}\,dx = \frac{-1}{a+bd}\,\frac{\left(-\frac{a}{b}+\frac{h}{g}\right)^{c+d/-1}}{\left(\frac{h}{g}\right)^{c/-1}\left(\frac{a}{-b}\right)^{d/-1}}\cdot\frac{1^{-\frac{a}{b}/1}\,1^{\frac{h}{g}/1}}{1^{\frac{h}{g}-\frac{a}{b}/1}}$

6) $\int \frac{(1-x^b)^{\mp\frac{h}{g}+c}}{x^{a+bd+1}}\,dx = \frac{-1}{a+bd}\,\frac{\left(1\pm\frac{h}{g}\right)^{c/1}}{\left(-\frac{a}{b}\right)^{d/-1}\left(1-\frac{a}{b}\pm\frac{h}{g}\right)^{c-d/1}}\,\frac{1^{-\frac{a}{b}/1}\,1^{\pm\frac{h}{g}/1}}{1^{\pm\frac{h}{g}-\frac{a}{b}/1}}$

7) $\int \frac{(1-x^b)^{\frac{a}{b}-c}}{x^{a+bg+1}}\,dx = 0$

Oettinger, Cr. 35. 13.

F. Alg. irrat. fract. à dén. $(1\pm x)^a$ et $(1\pm x^2)^a$. TABLE 12. Lim. 0 et 1.

1) $\int \frac{x^{a-\frac{1}{2}}\,dx}{\sqrt{(1-x)}} = \frac{1^{a/2}}{2^{a/2}}\,\pi$ Schlömilch, Stud. I. § 2.

2) $\int \frac{x^a\,dx}{\sqrt{(1-x)}} = \frac{2^{a/2}}{3^{a/2}}\,2$ Schlömilch, Gr. 5. 90.

3) $\int \frac{x^{\frac{a}{b}}\,dx}{(1-x)^{\frac{a}{b}}} = \frac{a}{b}\,\pi\,Cosec.\,\frac{a\pi}{b}$ Euler, N. C. P. 6. 115.

4) $\int \frac{x^{p-\frac{1}{2}}\,dx}{(1-x)^p} = \frac{2^{1-2p}}{1-2p}\,\frac{\Gamma(p+\frac{1}{2})\,\Gamma(1-p)}{\sqrt{\pi}}$, $p < \frac{1}{2}$; Legendre, Exerc. 4. 126.

5) $\int \frac{x^{a+p+\frac{1}{2}}\,dx}{(1-x)^{p+b+\frac{1}{2}}} = \frac{(1+2p)^{a+1/2}}{(1+2p)^{b/2}\,1^{a-b+1/1}}\,(-1)^b\,2^{b-a}\,\pi\,Sec.\,p\,\pi$

6) $\int \frac{x^{p+\frac{1}{2}}\,dx}{(1-x)^{p+\frac{1}{2}}} = \frac{2p+1}{2}\,\pi\,Sec.\,p\,\pi$

7) $\int \frac{x^{p-\frac{1}{2}}\,dx}{(1-x)^{p+\frac{1}{2}}} = \pi\,Sec.\,p\,\pi$

8) $\int \frac{x^{p-\frac{1}{2}}\,dx}{(1-x)^{p-\frac{1}{2}}} = \frac{1-2p}{2}\,\pi\,Sec.\,p\,\pi$

(5–8) Oettinger, Cr. 38. 162.

9) $\int dx\sqrt{\frac{1-x^2}{1+x^2}} = \frac{\{\Gamma(\frac{1}{4})\}^2}{4\sqrt{2\pi}} - \frac{\pi\sqrt{2\pi}}{\{\Gamma(\frac{1}{4})\}^2}$ Catalan, L. 4. 323.

10) $\int \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2}\pi$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. § 330, 356. — Oettinger, Cr. 35. 13.

11) $\int \frac{x\,dx}{\sqrt{(1-x^2)}} = 1$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 330.

12) $\int \frac{x^{2a}\,dx}{\sqrt{(1-x^2)}} = \frac{3^{a-1/2}}{2^{a/2}}\,\frac{\pi}{2}$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 330. — Oettinger, Cr. 35. 13. — Schlömilch, Stud. I. 2.

13) $\int \frac{x^{2a-1}\,dx}{\sqrt{(1-x^2)}} = \frac{2^{a-1/2}}{1^{a/2}}$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 330. — Plana, Cr. 17. 1. — Oettinger, Cr. 35. 13. — Dienger, Cr. 38. 266. — Schlömilch, Stud. I. 2.

14) $\int dx\sqrt{\frac{1-p^2x^2}{1-x^2}} = 1 - \sum_1^\infty \left\{\frac{1^{n-1/2}}{2^{n/2}}\right\}^2 (2n-1)\,p^{2n}$ Ohm, Ausw. 26.

15) $\int \frac{dx\sqrt[3]{x}}{\sqrt{(1-x^2)}} = \frac{1-\sqrt{3}}{\sqrt[4]{3}}\,F'\left(Cos.\frac{\pi}{12}\right) + \frac{2\sqrt{3}}{\sqrt[4]{3}}\,E'\left(Cos.\frac{\pi}{12}\right)$ Legendre, Exerc. 1. 39.

16) $\int \frac{dx\sqrt[3]{x^2}}{\sqrt{(1-x^2)}} = \frac{3\sqrt{3}}{\sqrt[4]{3}}\,E'\left(Sin.\frac{\pi}{12}\right) - \frac{3+\sqrt{3}}{2\sqrt[4]{3}}\,F'\left(Sin.\frac{\pi}{12}\right)$ Legendre, Exerc. 1. 40.

17) $\int \frac{x^{2p}\,dx}{(1-x^2)^{p+\frac{1}{2}}} = \frac{\pi}{2}\,Sec.\,p\,\pi$ Euler, N. C. P. 6. 115.

18) $\int \frac{x^{p-\frac{1}{2}}\,dx}{(1-x^2)^p} = \frac{2^{\frac{1}{2}-p}}{2p-1}\,\frac{\Gamma(p+\frac{1}{2})\,\Gamma(1-p)}{\sqrt{\pi}}\,Sin.\left(\frac{2p-1}{4}\pi\right)$, $p < 1$; Legendre, Exerc. 4. 124.

19) $\int \frac{x^{2a-1}\,dx}{(1-x^2)^{b-\frac{1}{2}}} = (-1)^{b-1}\,\frac{2^{a/2}}{2a\,.\,1^{b-1/2}\,3^{a-b/2}}$ Oettinger, Cr. 35. 13.

20) $\int \frac{x^{2a-1}\,dx}{(1-x^2)^{b-\frac{1}{2}}} = (-1)^{b-1} \frac{2^{a-1/2}(2a-2b+1)^{b/2}}{1^{a/2}\,1^{b/2}}$ Kramp, Refr. 3. 81.

21) $\int \frac{x^{2a}\,dx}{(1-x^2)^{b-\frac{1}{2}}} = (-1)^{b-1} \frac{3^{a-1/2}}{1^{b-1/2}\,4^{a-b/2}} \frac{\pi}{4}$ Oettinger, Cr. 35. 13.

22) $= (-1)^{b-1} \frac{1^{a/2}(2a-2b+2)^{b/2}}{2^{a/2}\,1^{b/2}} \frac{\pi}{2}$ Kramp, Refr. 3. 81.

23) $\int \frac{x^{p-1}+x^{q-1}}{(1-x^2)^{\frac{p+q}{2}}}\,dx = \frac{Cos.\left(\frac{q-p}{4}\pi\right)}{2\,Cos.\left(\frac{q+p}{4}\pi\right)} B\left(\frac{p}{2}, \frac{q}{2}\right)$

24) $\int \frac{x^{p-1}-x^{q-1}}{(1-x^2)^{\frac{p+q}{2}}}\,dx = \frac{Sin.\left(\frac{q-p}{4}\pi\right)}{2\,Sin.\left(\frac{q+p}{4}\pi\right)} B\left(\frac{p}{2}, \frac{q}{2}\right)$

(23, 24) Serret, L. 8. 1.

25) $\int \frac{dx}{\sqrt{(1+px^2)}} = \frac{1}{\sqrt{p}}\, l\{\sqrt{p}+\sqrt{(1+p)}\}$ Schlömilch, Beitr. III. 7.

F. Alg. irrat. fract. à dén. $(1-x^a)^b$, pour a spécial. TABLE 13. Lim. 0 et 1.

1) $\int \frac{dx}{\sqrt{(1-x^3)}} = \frac{2}{\sqrt[4]{27}} F'\left(Sin.\frac{\pi}{12}\right)$ Legendre, Exerc. 1. 145.

2) $\int \frac{dx}{\sqrt[3]{(1-x^3)}} = \frac{4}{3\sqrt[3]{4}\sqrt[4]{3}} F'\left(Cos.\frac{\pi}{12}\right)$ Legendre, Exerc. 1. 41.

3) $\int \frac{x\,dx}{\sqrt[3]{(1-x^3)^2}} = \frac{\sqrt[3]{4}}{2\sqrt[4]{3}} \frac{\pi}{F'\left(Cos.\frac{\pi}{12}\right)}$ Legendre, Exerc. 1. 118.

4) $\int \frac{x^{3a}\,dx}{\sqrt[3]{(1-x^3)}} = \frac{1^{a/3}}{3^{a/3}} \frac{2\pi}{3\sqrt{3}}$

5) $\int \frac{x^{3a-1}\,dx}{\sqrt[3]{(1-x^3)}} = \frac{3^{a-1/3}}{2^{a/3}}$

(4, 5) Kausler, Mém. Pétersb. 1811. T. 3.

6) $\int \frac{dx}{\sqrt{(1-x^4)}} = \frac{1}{2}\sqrt{2}.F'\left(Sin.\frac{\pi}{4}\right)$ Legendre, Exerc. 1. 146.

7) $\int \frac{x^3\,dx}{\sqrt{(1-x^4)}} = \frac{1}{2}$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 337.

8) $\int dx \sqrt{\frac{1-p^2x^4}{1-x^4}} = \frac{c\mathrm{F}'(c)+b\mathrm{F}'(b)}{(b+c)^2} + \frac{b-c}{(b+c)^2}\{\mathrm{E}'(b)-\mathrm{E}'(c)\}$, où $b^2 = \frac{(1+\sqrt{p})^2}{2(1+p)}$, $c^2 = \frac{(1-\sqrt{p})^2}{2(1+p)}$; Legendre, Exerc. 6. 308.

9) $\int \frac{dx}{\sqrt{(1-x^6)}} = \frac{1}{\sqrt[4]{27}} \mathrm{F}'\left(Sin.\frac{5\pi}{12}\right)$

9') $= \frac{\sqrt{3}}{\sqrt[4]{3}} \mathrm{E}'\left(Sin.\frac{\pi}{12}\right)$

Legendre, Exerc. 1. 147.

10) $\int \frac{dx}{\sqrt{(1-x^8)}} = \frac{1}{\sqrt{2}} \mathrm{F}'\left(Tg.\frac{\pi}{8}\right)$ Legendre, Exerc. 1. 148.

11) $\int \frac{dx}{\sqrt{(1-x^{12})}} = \frac{1}{2\sqrt[4]{3}} \mathrm{F}'\left(Sin.\frac{\pi}{4}\right) + Sin.\frac{\pi}{12}\,\mathrm{F}'\left(\frac{\sqrt{2}-\sqrt[4]{3}}{1+\sqrt{3}}\right)$ Legendre, Exerc. 1. 150.

F. Alg. irrat. fract. à dén. $(1-x^a)^b$ pour a général. TABLE 14. Lim. 0 et 1.

1) $\int \frac{x^{a-b-1}\,dx}{(1-x^a)^{\frac{a-b}{a}}} = \frac{\pi}{a} Cosec.\frac{b\pi}{a}$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 352.

2) $\int \frac{x^{b-1}\,dx}{(1-x^a)^{\frac{b}{a}}} = \frac{\pi}{a} Cosec.\frac{b\pi}{a}$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 352, 366. — Id., N. C. P. 6. 115. — Oettinger, Cr. 35. 13.

3) $\int \frac{x^{b-1}\,dx}{(1-x^a)^{\frac{a+b}{2a}}} = \frac{\pi}{a} Sec.\frac{b\pi}{2a}$ Euler, Calc. Int. T. 1. P. 1. S. 1 8. 366.

4) $\int \frac{dx}{\sqrt[q]{(1-x^q)}} = \frac{\pi}{q} Cosec.\frac{\pi}{q}$ Ohm, Ausw. 14. — Raabe, J. 147.

5) $\int \frac{x^{p-1}\,dx}{\sqrt[q]{(1-x^q)}} = \frac{\pi}{q} Cosec.\frac{p\pi}{q}$, $p > q-1$; Minding, Samml. 115.

6) $\int \frac{x^{p-1}\,dx}{(1-x^a)^{\frac{q-a}{a}}} = \frac{\Gamma\left(\frac{q}{a}\right)\Gamma\left(\frac{p}{a}\right)}{a\,\Gamma\left(\frac{p+q}{a}\right)} = \int \frac{x^{q-1}\,dx}{(1-x^a)^{\frac{p-a}{a}}}$ Raabe, J. 217.

7) $\int \frac{x^{a-bd-1}\,dx}{(1-x^b)^{\frac{h}{g}+c}} = \frac{\left(\frac{a}{b}-\frac{h}{g}\right)^{c+d/-1}}{\left(-\frac{h}{g}\right)^{c/-1}\left(\frac{a}{b}\right)^{d/-1}} \frac{1}{a-bd} \frac{1^{\frac{a}{b}/1}\,1^{-\frac{h}{g}/1}}{1^{\frac{a}{b}-\frac{h}{g}/1}}$

8) $\int \frac{x^{a-bd-1}\,dx}{(1-x^b)^{\frac{h}{g}+c}} = \frac{1}{a+bd} \frac{\left(1+\frac{a}{b}\right)^{d/1}}{\left(-\frac{h}{g}\right)^{c/-1}\left(1+\frac{a}{b}-\frac{h}{g}\right)^{d-c/1}} \cdot \frac{1^{\frac{a}{b}/1}\,1^{-\frac{h}{g}/1}}{1^{\frac{a}{b}-\frac{h}{g}/1}}$

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9) $\displaystyle\int \frac{x^{a-1}\,dx}{(1-x^b)^{\frac{a}{b}}} = \frac{1}{a}\,\frac{b^2}{b^2-a^2}\cdot\frac{4b^2}{4b^2-a^2}\cdots$

10) $\displaystyle\int \frac{x^{a-1}\,dx}{(1-x^b)^{\frac{h}{g}+c}} = \frac{(bh-ag)^{c/bg}}{ab^c\,h^{c/g}}\,\frac{1^{\frac{a}{b}/1}\;1^{-\frac{h}{g}/1}}{1^{\frac{a}{b}-\frac{h}{g}/1}}$

11) $\displaystyle\int \frac{x^{a-1}\,dx}{(1-x^b)^{\frac{a}{b}+c}} = 0$

12) $\displaystyle\int \frac{x^{a-bc-1}\,dx}{(1-x^b)^{\frac{a}{b}}} = 0$

13) $\displaystyle\int \frac{x^{a+bc-1}}{(1-x^b)^{\frac{h}{g}}}\,dx = \frac{1}{a+bc}\,\frac{(a+b)^{c/b}}{(ag-bh+bg)^{c/bg}}\,\frac{1^{\frac{a}{b}/1}\;1^{-\frac{h}{g}/1}}{1^{\frac{a}{b}-\frac{h}{g}/1}}\,g^c$

14) $\displaystyle\int \frac{x^{a-bc-1}}{(1-x^b)^{\frac{h}{g}}}\,dx = \frac{1}{a-bc}\,\frac{(bh-ag)^{c/bg}}{a^{c/-b}}\,\frac{1^{\frac{a}{b}/1}\;1^{-\frac{h}{g}/1}}{1^{\frac{a}{b}-\frac{h}{g}/1}}\,\frac{1}{g^c}$

15) $\displaystyle\int \frac{x^{h+ab-1}}{(1-x^b)^{\frac{h}{b}}}\,dx = \frac{1}{ab+h}\,\frac{(b+h)^{a/b}\;1^{\frac{h}{b}/1}\;1^{-\frac{h}{b}/1}}{b^{a/b}}$

16) $\displaystyle\int \frac{x^{ab-1}\,dx}{(1-x^b)^{\frac{h}{g}}} = \frac{g^{a/g}}{ab(g-h)^{a/g}}$

17) $\displaystyle\int \frac{x^{ab}\,dx}{(1-a^b)^{\frac{h}{g}}} = \frac{1}{1+ab}\,\frac{(b+1)^{a/b}}{(g-bh+bg)^{a/bg}}\,\frac{1^{\frac{1}{a}/1}\;1^{-\frac{h}{g}/1}}{1^{\frac{1}{a}-\frac{h}{g}/1}}\,g^a$

18) $\displaystyle\int \frac{x^{ab-1}\,dx}{(1-x^b)^{\frac{h}{g}+c}} = \frac{1}{ab}\,\frac{g^{a/g}}{h^{c/g}\,(g-h)^{a-c/g}}$

19) $\displaystyle\int \frac{x^{ab}\,dx}{(1-x^b)^{\frac{h}{g}+c}} = \frac{1}{1+ab}\,\frac{(1+b)^{a/b}}{h^{c/g}\,(bh-g-bg)^{a-c/-bg}}\,\frac{1^{\frac{1}{a}/1}\;1^{-\frac{h}{g}/1}}{1^{\frac{1}{a}-\frac{h}{g}/1}}\,g^a$

20) $\displaystyle\int \frac{x^{h-cg-1}\,dx}{(1-x^g)^{\frac{h}{g}+c}} = 0$

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(9–20) Oettinger, Cr. 35. 13.

21) $\int \frac{x^{a+bc-1}\,dx}{(1-x^b)^{\frac{a}{b}+g}} = (-1)^g \frac{(a+b)^{c/b}}{a^{g/b}\,1^{c-g/1}} \frac{a\,b^{g-c}}{a+bc} \frac{\pi}{b\,Sin.\,\frac{a\pi}{b}}$

22) $\int \frac{x^{a+bc-1}\,dx}{(1-x^b)^{\frac{a}{b}+c}} = (-1)^c \frac{\pi}{b}\,Cosec.\,\frac{a\pi}{b}$

Oettinger, Cr. 38. 162.

23) $\int \frac{dx}{(1-\sqrt[b]{x})^b} = b\pi\,Cosec.\,b\pi$

24) $\int \frac{x^{q-1}\,dx}{(1-\sqrt[b]{x})^{bq}} = b\pi\,Cosec.\,bq\pi$

Kramp, Réfr. 3. 83.

F. Algébr. irrat. fract. à dén. comp. avec fact. monôme. TABLE 15. Lim. 0 et 1.

1) $\int \frac{(1-x)^{-\frac{1}{2}}-1}{x}\,dx = 2\,l\,2$ Arndt, Gr. 6. 187.

2) $\int \frac{\sqrt{x}+\sqrt{\frac{1}{x}}}{1+x^2}\,dx = \frac{1}{2}\pi\sqrt{2}$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 2. — Plana, Mém. Brux. T. 10. — Mascheroni, Adn. p. 53 la trouvait fautivement $=\pi\sqrt{2}$.

3) $\int \frac{x^a+x^{a+\frac{1}{2}}-2x^{2a}}{1-x}\frac{dx}{x} = 2\,l\,2$

4) $\int \frac{x^a+x^{a+\frac{1}{3}}+x^{a+\frac{2}{3}}-3x^{3a}}{1-x}\frac{dx}{x} = 3\,l\,3$

Legendre, Exerc. 4. 55.

5) $\int \frac{\sqrt{\frac{1}{x}-1}}{1-x}\,dx = l\,4$ Poisson, Mém. Inst. 1811. 163. N°. 54.

6) $\int \frac{dx}{\sqrt{(x-x^2)}} = \pi$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 335. — Dienger, Cr. 42. 283.

7) $\int \frac{(1-x)^a}{\sqrt{x(1-x)}}\,dx = \frac{1^{a/2}}{2^{a/2}}\pi$

8) $\int \frac{(1-x)^a x^b\,dx}{\sqrt{x(1-x)}} = \frac{1^{a/2}\,1^{b/2}}{2^{a+b/2}}\pi$

Ohm, Ausw. N°. 46.

9) $\int \frac{x^a\,dx}{\sqrt{x(1-x)}} = \frac{1^{a/2}}{2^{a/2}}\pi$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 335. — Oettinger, Cr. 35. 13. — Ohm, Ausw. 14 trouve $\frac{\pi}{2}$ au lieu de π faut.

10) $\int \frac{dx}{\sqrt{x(1+px)}} = \frac{2}{\sqrt{p}}\,l\left\{\sqrt{p}+\sqrt{(1+p)}\right\}$ Schlömilch, Beitr. III. 7.

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11) $\int \frac{dx}{x^{\frac{2}{3}} \sqrt{(1-x^2)}} = \frac{3}{\sqrt[4]{3}} F'\left(Sin. \frac{\pi}{12}\right)$ Legendre Exerc. 1. 39.

12) $\int \frac{dx}{x^{\frac{1}{3}} \sqrt{(1-x^2)}} = \frac{1}{\sqrt[4]{3}} F'\left(Cos. \frac{\pi}{12}\right)$ Legendre, Exerc. 1. 40.

13) $\int \frac{dx}{x^{a+bd+1} (1-x^b)^{\frac{h}{g}+c}} = \frac{\left(-\frac{a}{b}-\frac{h}{g}\right)^{c+d/-1}}{\left(-\frac{h}{g}\right)^{c/-1} \left(-\frac{a}{b}\right)^{d/-1}} \frac{1^{-\frac{a}{b}/1}\, 1^{-\frac{h}{g}/1}}{1^{-\left(\frac{a}{b}+\frac{h}{g}\right)/1}} \frac{1}{a+bd}$

14) $\int \frac{x^{ab-1}\, dx}{\sqrt{(x^{2a}-x^{3a})}} = \frac{2^{b-1/2}}{(b-1)a . 1^{b-1/2}}$

(13, 14) Oettinger, Cr. 35. 13.

15) $\int \frac{1}{a-bx} \frac{dx}{\sqrt{x(1-x)}} = \frac{\pi}{\sqrt{a(a-b)}}$

16) $\int \frac{1}{(a-bx)^{c+1}} \frac{dx}{\sqrt{x(1-x)}} = \frac{1^{c/2}}{2^{c/2}} \frac{\pi}{(a-b)^c \sqrt{(a^2-ab)}} \sum_0^\infty \binom{c}{n} \frac{1^{n/2}}{(2c-1)^{n/-2}} \left(\frac{a-b}{a}\right)^n$

(15, 16) pour b ni $= a$, ni $= 0$; Dienger, Cr. 42. 483.

17) $\int \left(\frac{x}{1-x}\right)^p \frac{dx}{\sqrt{x(1-x)}} = \pi\, Sec.\, p\pi, \quad p^2 < \frac{1}{4};$ Ohm, Ausw. N°. 46.

18) $\int \frac{x^{-\frac{a}{2b}} - x^{\frac{a}{2b}}}{1-x} \frac{dx}{\sqrt{x}} = \frac{\pi}{2b} Tang. \frac{a\pi}{2b}$ Malmsten, Cr. 38. 1.

F. Alg. irrat. fract. à dén. comp. sans fact. monôme. TABLE 16. Lim. 0 et 1.

1) $\int \frac{x^{p-\frac{1}{2}}\, dx}{(1-x)^p (1+qx)^p} = \frac{2\Gamma(p+\frac{1}{2})\Gamma(1-p)}{\sqrt{\pi}} Cos.^{2p}(Arctg. \sqrt{q}) . \frac{Sin. \{(2p-1) Arctg. \sqrt{q}\}}{(2p-1) Sin. (Arctg. \sqrt{q})}$

2) $\int \frac{x^{p-\frac{1}{2}}\, dx}{(1-x)^p (1-qx)^p} = \frac{2\Gamma(p+\frac{1}{2})\Gamma(1-p)}{\sqrt{\pi}} \frac{(1-\sqrt{q})^{1-2p} - (1+\sqrt{q})^{1-2p}}{(2p-1) 2\sqrt{q}}$

(1, 2) $p < 1$, $1 > q > 0$; Legendre, Exerc. 4. 124, 145.

3) $\int \frac{x^2}{1+x^4} \frac{dx}{\sqrt{(1-x^4)}} = \frac{\pi}{8}$

4) $\int \frac{1+px^2}{1+x^4} \frac{dx}{\sqrt{(1-x^4)}} = \frac{1+p}{8}\pi + \frac{1}{4}\sqrt{2} . F'\left(Sin. \frac{\pi}{4}\right)$

5) $\int \frac{1+p+2x^2-(1-p)x^4}{1+x^4} \frac{dx}{\sqrt{(1-x^4)}} = \frac{1+p}{4}\pi + \frac{1}{2}\sqrt{2} . F'\left(Sin. \frac{\pi}{4}\right)$

(3, 4, 5) Legendre, Exerc. 6. 308.

6) $\int \frac{x\, dx}{\sqrt{(p^2+x^2)(1-x^2)}} = \frac{\pi}{2} - Arctg.\, p$ Poisson, P. 16. 215. N°. 11.

7) $\int \frac{x^{\frac{q}{2}}\,dx}{\{(1-x)(1-p^2 x)\}^{\frac{q+1}{2}}} = \frac{(1-p)^{-q}-(1+p)^{-q}}{2pq}\; B\left(\frac{q+2}{2}, \frac{1-q}{2}\right)$ Boncompagni, Cr. 25. 74.

8) $\int \frac{x^{\frac{q}{2}}\,dx}{\left\{(1-x)(1-x\, Tang.^2\lambda)\right\}^{\frac{q+1}{2}}} = \frac{Sin.\, q\,\lambda\; Cos.^q\,\lambda}{q\; Tang.\,\lambda}\; B\left(\frac{q+2}{2}, \frac{1-q}{2}\right)$ Boncompagni, Cr. 25. 74.

9) $\int \left(\frac{x^{a-1}}{1-\sqrt[p]{x}} - \frac{p x^{pa-1}}{1-x}\right) dx = p\, l p$ Arndt, Gr. 10. 253.

10) $\int \left(\frac{a}{1-x} - \frac{x^{p-1}}{1-\sqrt[a]{x}}\right) dx = a l p + \sum_{1}^{a} Z'\left(p + \frac{b-n}{a}\right)$ Arndt, Gr. 10. 253.

11) $\int \left(\frac{k}{1-x} - \frac{\sqrt[k]{x}}{1-\sqrt[k]{x}}\right) dx = k\,A$, pour $k = \infty$; Legendre, Exerc. 5. 12.

12) $\int \frac{1-x^2}{\sqrt{(1+a^2 x^2)}} \; \frac{1-a^2 b^2 x^2}{\sqrt{(1+b^2 x^2)}} \, x^2\, dx = 0$ Ramus, Overs. Danske Forh. 1844.

13) $\int \left(\frac{p x^{pq-1}}{1-x} - \frac{x^{q-1}}{1-\sqrt[p]{x}}\right) dx = \frac{d}{dq} \cdot l \frac{\Gamma(q)\,\Gamma\left(q+\frac{1}{p}\right)\Gamma\left(q+\frac{2}{p}\right)\ldots\Gamma\left(q+\frac{p-1}{p}\right)}{\Gamma(qp)}$ Schaar, Mém. Cour. Brux. T. 22.

F. Algébr. TABLE 17. Lim. — 1 et + 1.

1) $\int \frac{dx}{x} = l\,\alpha$, où α arbitraire; Cauchy, Cours. Leç. 24.

2) $= -(2k+1)\pi i$, où k arbitraire; Poisson, P. 18. 295. N°. 33.

3) $\int \frac{dx}{x^2} = -2$ Poisson, P. 18. 295. N°. 33.

4) $\int \frac{dx}{x^a} = \frac{(-1)^a}{a-1}\left[Cos.\left\{(a-1)(2k+1)\pi\right\} - 1\right]$, où k arbitraire; Poisson, P. 18. 295. N°. 34.

5) $= 0$ pour a impair Poisson, P. 18. 295. N°. 34.

6) $= \frac{2}{1-a}$ pour a pair Poisson, P. 18. 295. N°. 34.

7) $\int \frac{dx}{\sqrt{(1-2px+p^2)}} = 2$, pour $p < 1$; Poisson, Chal. 113.

8) $\int \frac{dx}{\sqrt{(1-2px+p^2)}} = \frac{2}{p}$, pour $p > 1$;

9) $\int \frac{p-x}{\sqrt{(1-2px+p^2)^3}}\, dx = 0$, pour $p < 1$;

10) $= \frac{2}{p^2}$, pour $p > 1$;

Poisson, Chal. 113.

11) $\int \frac{px-q}{\sqrt{(p^2-pqx+q^2)^3}}\, dx = 0$, pour $p > q$;

12) $= -\frac{2}{q^2}$, pour $p < q$;

Plana, Mém. Turin. 1820. 389. N°. 9.

13) $\int \frac{dx}{\sqrt{\left\{(1-2pqx+p^2q^2)\left(1-\frac{2p}{q}x+\frac{p^2}{q^2}\right)\right\}}} = \frac{1}{p} l \frac{1+p}{1-p}$ Legendre, Exerc. 5. 124. — Liouville, L. 2. 135.

14) $\int \frac{dx}{\sqrt{\{(1-2px+p^2)(1-2qx+q^2)\}}} = \frac{1}{\sqrt{pq}} l \frac{1+\sqrt{pq}}{1-\sqrt{pq}}$, pour $p^2 < 1$, $q^2 < 1$;

15) $= \frac{1}{\sqrt{pq}} l \frac{\sqrt{p}+\sqrt{q}}{\sqrt{p}-\sqrt{q}}$, pour $q^2 < 1 < p^2$;

16) $= \frac{1}{\sqrt{pq}} l \frac{\sqrt{q}+\sqrt{p}}{\sqrt{q}-\sqrt{p}}$, pour $p^2 < 1 < q^2$;

17) $= \frac{1}{\sqrt{pq}} l \frac{\sqrt{pq}+1}{\sqrt{pq}-1}$, pour $p^2 > 1$, $q^2 > 1$;

Poisson, P. 19. 404. N°. 82.

F. Alg. rat. fract. à dén. x^a et $(1 \pm x)^a$. TABLE 18. Lim. 0 et ∞.

1) $\int \frac{dx}{x} = \infty$ Cauchy, Cours. Leç. 24. — Meijer, Int. déf. 98.

2) $\int \frac{x^{p-1}\, dx}{1+x} = \frac{\pi}{Sin.\, p\pi}$

3) $= \Gamma(p)\,\Gamma(1-p)$

où $0 < p < 1$; Legendre, Exerc. 4. 95. — Poisson, P. 19. 404. N°. 72. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Id., Lim. Imag. Add. 12. — Id., P. 28. 147. I. N°. 2. — Bonnet, L. 6. 238. — Lejeune-Dirichlet, Cr. 15. 258. — Jürgensen, Cr. 23. 142. — Oettinger, Cr. 35. 13. — Winckler, Cr. 45. 102. — Grunert, Gr. 2. 266. — Schlömilch, Gr. 3. 278. — Serret, L. 8. 1.

4) $\int \frac{x^{1-p}\, dx}{1+x} = -\pi\, Cosec.\, p\pi$ Oettinger, Cr. 35. 13.

5) $\int \frac{x^{p-1}\, dx}{x+a} = \pi a^{p-1}\, Cosec.\, p\pi$, $0 < p < 1$; Schlömilch, Gr. 12. 198. — Dedekind, Cr. 45. 370.

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6) $\int \frac{x^{p-1}\,dx}{x+e^{qi}} = \frac{\pi}{Sin.\,p\pi} e^{(p-1)qi}$, $0 < p < 1$, $q^2 < \pi^2$; Minding, Taf. II.

7) $\int \frac{x^{p-1}\,dx}{qx+1} = \frac{\pi}{q^p} Cosec.\,p\pi$, $p < 1$; Dedekind, Cr. 45. 370.

8) $\int \frac{x^{p-1}\,dx}{1-x} = \pi\,Cot.\,p\pi$, $0 < p < 1$; Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Id., Lim. Imag. Add. 13. — Grunert, Gr. 2. 266. — Schlömilch, Gr. 3. 278. — Meijer, Int. Déf. 155 a faut. $\frac{1}{2}\pi\,Cot.\,p\pi$.

9) $\int \frac{(x-a)^{p-1}dx}{b-x} = \pm \frac{(-1)^p \pi}{(b-a)^{1-p} Sin.\,p\pi}$, $\pm$ selon que $\begin{matrix} a > b \\ a < b \end{matrix}$; Jürgensen, Cr. 23. 142.

10) $\int \frac{x^{p-r-1}(1+x)^{1-p}}{1-q\,r+x}\,dx = \frac{\Gamma(r)\,\Gamma(p-r)}{\Gamma(p)} \xi(r,p,q)$, $p > r$; Schaeffer, Cr. 37. 127.

11) $\int \frac{x^{b-1}\,dx}{(1+x)^a} = \frac{\pi}{a} Cosec.\,\frac{b\pi}{a}$ Euler, Calc. Int. T. 4. S. 5. 124.

12) $\int \frac{x^{q-1}\,dx}{(1+x)^p} = \frac{\Gamma(q)\,\Gamma(p-q)}{\Gamma(p)}$, $q < p$; Legendre, Exerc. 4. 99. — Poisson, P. 19. 404. N°. 72. — Cauchy, P. 28. 147. I. N°. 2.

13) $\int \frac{x^{p-1}\,dx}{(1+x)^{p+q}} = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}$ Lobatschewsky, Mém. Kasan. 1835. 1. — Grunert, Gr. 2. 266.

14) $= \mathrm{B}(p,q) = \int \frac{x^{q-1}\,dx}{(1+x)^{p+q}}$ Binet, P. 27. 123. — Schaar, Mém. Cour. Brux. T. 22.

15) $= \left(\frac{p}{q}\right)$ Lejeune-Dirichlet, Cr. 15. 258.

C'est l'Intégrale Eulérienne de première espèce.

16) $\int \frac{x^{q-1}+x^{p-1}}{(1+x)^{p+q}}\,dx = 2\,\mathrm{B}(q,p)$ Binet, P. 27. 123.

17) $\int \frac{x^{p-1}\,dx}{(b+x)^{a+1}} = \frac{(-1)^a}{1^{a/1}} \frac{(p-1)^{a/-1}}{Sin.\,p\pi} \pi\, b^{p-a-1}$

18) $\int \frac{x^{p-1}\,dx}{(1+x)^{a+1}} = \frac{(p-1)^{a/-1}}{1^{a/1}} \frac{(-1)^a \pi}{Sin.\,p\pi}$

19) $\int \frac{x^{a+p}\,dx}{(1+x)^{2a+1}} = \frac{(-1)^a \pi}{Sin.\,p\pi} \cdot \frac{p^2 \cdot p^2-1^2 \cdot p^2-2^2 \ldots\ldots p^2-a^2}{1^{2a+1/1}}$

(17–19) Schlömilch, Gr. 12. 198.

20) $\int \frac{x^a\,dx}{(1+x)^{a+p+1}} = \Delta^a \left(\frac{1}{p}\right)$ Cauchy, P. 28. 147. I. § 2.

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21) $\int \frac{x^{1-p}\,dx}{(1+x)^3} = \frac{1-p}{2}\, p\pi\, Cosec.\, p\pi$

22) $\int \frac{x^{p}\,dx}{(1+x)^3} = \frac{1-p}{2}\, p\pi\, Cosec.\, p\pi$

23) $\int \frac{x^{p+1}\,dx}{(1+x)^3} = \frac{1+p}{2}\, p\pi\, Cosec.\, p\pi$

24) $\int \frac{x^{p+c}\,dx}{(1+x)^{b+c+2}} = \frac{(1+p)^{c/1}\,(1-p)^{b/1}}{1^{c-b/1}}\, p\pi\, Cosec.\, p\pi = \int \frac{x^{b-p}\,dx}{(1+x)^{b+c+2}}$

25) $\int \frac{x^{p+c}\,dx}{(1+x)^{b-c+2}} = \frac{(1+p)^{c/1}}{p^{b/1}\,1^{c-b+2/1}}\, p\pi\, Cosec.\, p\pi$

26) $\int \frac{x^{p-c}\,dx}{(1+x)^{b-c+2}} = \frac{(1-p)^{b/1}}{(1-p)^{c-1/1}\,1^{b-c-1/1}}\, p\pi\, Cosec.\, p\pi = \int \frac{x^{b-p}\,dx}{(1+x)^{b-c+2}}$

(21–26) Oettinger, Cr. 38. 162

27) $\int \frac{x^{b-a-1}\,dx}{(1+x)^{b}} = \frac{1^{a-1/1}\,1^{b-a-1/1}}{1^{b-1/1}} = \int \frac{x^{a-1}\,dx}{(1+x)^{b}}$

28) $\int \frac{x^{a\pm b-1}\,dx}{(1+x)^{a+c\pm 2b}} = \frac{1^{a\pm b-1/1}\,1^{c\pm b-1/1}}{1^{a+c\pm 2b-1/1}} = \int \frac{x^{c\pm b-1}\,dx}{(1+x)^{a+c\pm 2b}}$

(27–28) Oettinger, Cr. 35. 13.

29) $\int \left(x^{q-p} - \frac{x^{q}}{(1+x)^{p}}\right) dx = \frac{q}{q-p+1}\,\frac{\Gamma(q)\,\Gamma(p-q)}{\Gamma(p)}$ Legendre, Exerc. 4. 109.

F. Alg. rat. fract. à dén. $1 \pm x^a$ pour a spécial. TABLE 19. Lim. 0 et ∞.

1) $\int \frac{dx}{1+x^2} = \frac{1}{2}\pi$ Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 353. — Poisson, P. 18. 295. N°. 26.

2) $\int \frac{dx}{p^2+x^2} = \frac{\pi}{2p},\ p > 0$; Liouville, Cr. 13. 219. — Schlömilch, Gr. 4. 71. — Id., Gr. 10. 440. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 58.

3) $\int \frac{dx}{1-x^2} = \frac{1}{2}\, l(-1)$ Poisson, P. 18. 295. N°. 26. — Plana, Cr. 17. 1.

3') $= 0$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Schlömilch, Gr. 3. 278.

4) $\int \frac{dx}{b^2-x^2} = 0$ Legendre, Exerc. 5. 13. — Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 31. — Plana, Mém. Turin. 1818. 7. Art. 1. 3, 4. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 56. — Schlömilch, Gr. 7. 270.

4') $= \frac{1}{2b}\, l(-1)$ Poisson, P. 18. 295. N°. 38. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 58.

4'') $= \alpha$, où α arbitraire; Arndt, Gr. 10. 240. C'est la vraie valeur.

5) $\int \frac{x^{2p-1}}{1+x^2}\, dx = \frac{\pi}{2}\, Cosec.\, p\pi,\ 1 > p > 0$; Schlömilch, Beitr. III. 4.

6) $\int \frac{x^{p-1}}{1+x^2}\, dx = \frac{\pi}{2}\, Cosec. \frac{p\pi}{2},\ 2 \geqq p \geqq 0$; Cauchy, Cours. Leç. 34. — Id., P. 19. 511. — Id., Exerc. 1826. p. 95.

7) $= \frac{\pi}{i^{p-1} + (-i)^{p-1}},\ 2 > p > 0$; Meijer, Int. Déf. 154.

8) $\int \frac{x^p\, dx}{1+x^2} = \frac{\pi}{2}\, Sec. \frac{p\pi}{2},\ 1 \geqq p \geqq 0$; Schlömilch, Gr. 3. 278.

9) $\int \frac{x^{p-1} dx}{1-x^2} = \frac{\pi}{2}\, Cot. \frac{p\pi}{2},\ 1 > p$; Cauchy, Cours. Leç. 34. — Meijer, Int. Déf. 155.

10) $\int \frac{dx}{1+x^3} = \frac{2\pi}{9}\sqrt{3}$

11) $\int \frac{x^2\, dx}{1+x^3} = \frac{2\pi}{9}\sqrt{3}$

(10, 11) Euler, N. C. P. 6. 115. — Id., Calc. Int. T. 1. P. 1. S. 8. 353.

12) $\int \frac{dx}{1-x^3} = \frac{1}{9}\pi - \frac{1}{3} l(-1)$ Plana, Mém. Turin. 1820.

13) $\int \frac{dx}{1+x^4} = \frac{1}{4}\pi\sqrt{2}$ Euler, N. C. P. 6. 115. — Id., Calc. Int. T. 1. P. 1. S. 1. 8. 353. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

14) $\int \frac{x^2\, dx}{1+x^4} = \frac{1}{4}\pi\sqrt{2}$ Euler, N. C. P. 6. 115. — Id., Calc. Int. T. 1. P. 1. S. 1. 8. 353.

15) $\int \frac{dx}{1+x^6} = \frac{1}{3}\pi$

16) $\int \frac{x^2\, dx}{1+x^6} = \frac{1}{6}\pi$

17) $\int \frac{x^4\, dx}{1+x^6} = \frac{1}{3}\pi$

(15–17) Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 353. — Poisson, L. 2. 224.

18) $\int \frac{dx}{(\pm p + qi)^2 + x^2} = \frac{\pi}{2(p \pm q)}$ Ohm, Ausw. 2.

F. Alg. rat. fract. à dén. $(1 \pm x^a)$ pour a général. TABLE 20. Lim. 0 et ∞.

1) $\int \frac{x^{q-1}\, dx}{1+x^p} = \frac{\pi}{p}\, Cosec. \frac{q\pi}{p},\ p > q > 0$; Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 351. — Poisson, P. 16. 215. N°. 6. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Mascheroni, Adnot. p. 63. — Oettinger, Cr. 35. 13. — Schlömilch, Gr. 3. 278.

2) $= \infty,\ q > p$; Poisson, P. 16. 215. N°. 6.

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3) $\int \frac{x^{p-q-1}\,dx}{1+x^p} = \frac{\pi}{p} Cosec. \frac{q\pi}{p}, p>q>0$; Euler, Calc. Int. T. 1. P. 1. S. 1. 8. 351. — Oettinger, Cr. 35. 13.

4) $\int \frac{x^q\,dx}{p^2+x^{2q}} = \frac{\pi \sqrt[q]{p}}{2pq} Sec. \frac{\pi}{2q}$

5) $\int \frac{dx}{p^2+x^{2q}} = \frac{\pi \sqrt[q]{p}}{2p^2 q} Cosec. \frac{\pi}{2q}$

(4, 5) $q \geqq 1$; Raabe, Int. 416.

6) $\int \frac{x^{2b}\,dx}{1+x^{2a}} = \frac{\pi}{2a} Cosec. \left(\frac{2b+1}{2a}\pi\right), 2a+1>2b$; Poisson, P. 16. 215. N°. 6. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Serret, L. 8. 1. — Grunert, Gr. 2. 266.

7) $= \infty,\ a<b$; Poisson, P. 16. 215. N°. 6.

8) $\int \frac{x^{q-1}+x^{p-1}}{x^{p+q}+1}\,dx = \frac{2\pi}{p+q} Sec. \frac{q-p}{q+p}\frac{\pi}{2}$ Ohm, Ausw. 14. — Raabe, Int. 147.

9) $\int \frac{x^{2bc+c-1}\,dx}{1+x^{2ac}} = \frac{\pi}{2a} Cosec. \left(\frac{2b+1}{2a}\pi\right), a>b$

10) $= \infty \qquad , a<b$

(9, 10) Poisson, P. 16. 215. N°. 6.

11) $\int \frac{x^{a-1}\,dx}{e^{qi}+x^b} = \frac{\pi}{b} e^{\left(\frac{a}{b}-1\right)qi} Cosec. \frac{a\pi}{b}, a<b, q^2<\pi^2$; Minding, Taf. II.

12) $\int \frac{1+x^2}{1-x^{2(2b+1)}} x^{a-1}\,dx = \frac{\pi}{4b+2}\left\{Cosec. \frac{a\pi}{4b+2} + Cosec. \left(\frac{a+2}{4b+2}\pi\right)\right\}, 4b>a>0$; Lindmann, Gr. 14. 94.

13) $\int \frac{x^{q-1}\,dx}{1-x^p} = \frac{\pi}{p} Cot. \frac{q\pi}{p}, p>q$; Mascheroni. Adn. p. 64. — Legendre, Exerc. 5. 13. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Schlömilch, Gr. 3. 278.

14) $\int \frac{x^{2b}\,dx}{1-x^{2a}} = \frac{\pi}{2a} Cot. \left(\frac{2b+1}{2a}\pi\right), 2a+1>2b$; Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Grunert, Gr. 2. 266.

15) $\int \frac{x^{q-1}-x^{p-1}}{x^{p+q}-1}\,dx = \frac{2\pi}{p+q} Tang. \left(\frac{q-p}{q+p}\frac{\pi}{2}\right)$ Ohm, Ausw. 14. — Raabe, Int. 147.

16) $\int \frac{1-x^2}{1-x^{2b}} x^{a-1}\,dx = \frac{\pi}{2b} Sin. \frac{\pi}{b}. Cosec. \frac{a\pi}{2b}. Cosec. \left(\frac{a+1}{2b}\pi\right), b-1>\frac{1}{2}a>0$

17) $\int \frac{1-x^2}{1-x^{4b}} x^{a-1}\,dx = \frac{\pi}{2b}\left\{Cot. \frac{a\pi}{4b} + Cot. \left(\frac{a+1}{4b}\pi\right)\right\}, 2b-1>\frac{1}{2}a>0$

(16, 17) Lindmann, Gr. 14. 94.

$$1)\ \int \frac{dx}{(1+x^2)^a} = \frac{1^{a-1/2}}{2^{a-1/2}} \frac{\pi}{2}$$

$$2)\ \int \frac{dx}{(p+x^2)^{a+1}} = \pm \frac{\pi}{2} \frac{d^a}{dp^a} \cdot \frac{1}{\sqrt{p}}$$

$$3)\ \qquad = \frac{1^{a/2}}{1^{a/1}} \frac{\pi}{2^{a+2} p^a \sqrt{p}}$$

Cauchy, Cours. Leç. 33.

$$4)\ \int \frac{dx}{(1+x^2)^3} = \frac{3}{16} \pi$$

$$5)\ \int \frac{x^2 dx}{(1+x^2)^3} = \frac{1}{16} \pi$$

$$6)\ \int \frac{x^4 dx}{(1+x^2)^3} = \frac{3}{16} \pi$$

Poisson, L. 2. 224.

$$7)\ \int \frac{dx}{(q^2-x^2)^2} = 0$$

Plana, Mém. Turin. 1818. 7. Art. 1. N°. 3.

$$8)\ \int \frac{dx}{(q^2+x^2)^{a+1}} = \frac{1}{2^a q^{2a+1}} \frac{1^{a/2}}{1^{a/1}}$$

$$9)\ \int \frac{x^{p-1}\, dx}{(1+x^2)^a} = \frac{\Gamma(\frac{1}{2}p)\, \Gamma(a-\frac{1}{2}p)}{2\, \Gamma(a)}, \quad 1 > p > 0;$$

Schlömilch, Beitr. III. 12, 13.

$$10)\ \int \frac{x^{b-1} dx}{(1+x^a)^c} = \left(1-\frac{b}{a}\right)\left(1-\frac{b}{2a}\right) \ldots\ldots \left(1-\frac{b}{(c-1)a}\right) \frac{\pi}{a} \operatorname{Cosec.} \frac{b\pi}{a}$$

Euler, Calc. Int. T. 4. S. 5. 159.

$$11)\ \int \frac{x^{-a+c(b+1)-1}\, dx}{(1+x^b)^{g+c-1}} = \frac{(b-a)^{c/b}\, (b+a)^{g-1/b}}{b^{g+c/b}} \frac{a\pi}{b} \operatorname{Cosec.} \frac{a\pi}{b}$$

$$12)\ \int \frac{x^{-a+c(b+1)-1}\, dx}{(1+x^b)^{2c-1}} = \left(1-\frac{a^2}{b^2}\right)\left(4-\frac{a^2}{b^2}\right) \ldots \left(c^2-\frac{a^2}{b^2}\right) \frac{a}{1^{2c/1}} \frac{1}{a+bc} \frac{\pi}{b} \operatorname{Cosec.} \frac{a\pi}{b}$$

$$13)\ \int \frac{x^{-a+b(g+1)-1}}{(1+x^b)^{g-c+1}}\, dx = (-1)^c \frac{(b-a)^{g/b}}{(b-a)^{c/b}\, 1^{g-c/1}} \frac{\pi}{b^{g-c+1}} \operatorname{Cosec.} \frac{a\pi}{b}$$

Oettinger, Cr. 38. 162.

$$14)\ \int \frac{x^{p+q-1}\, dx}{(1+x^q)^2} = \frac{p\pi}{q^2} \operatorname{Cosec.} \frac{p\pi}{q}, \quad p < q;$$

$$15)\ \int \frac{x^{p-1}\, dx}{(1+x^q)^2} = \frac{p-q}{q^2} \pi \operatorname{Cosec.} \left(\frac{p-q}{q}\pi\right), \quad 2q > p > q;$$

Ohm, Ausw. N° 20.

1) $\int \frac{dx}{(1+x)\,x^p} = \pi\ Cosec.\, p\pi,\ 1 > p > 0$; Ramus, Cr. 24. 257. — Oettinger, Cr. 35. 13.

2) $\int \frac{dx}{(1+x)^{c-b+2}\,x^{b+p}} = \frac{(1+p)^{c/1}}{1^{c-b+2/1}\,p^{b/1}}\,p\,\pi\ Cosec.\,p\,\pi$ Oettinger, Cr. 38. 162.

3) $\int \frac{(1+x)^q-1}{(1+x)^{p+q}}\frac{dx}{x} = Z'(p+q) - Z'(q)$ Lindmann, Stockh. Handl. 1850. II.

4) $\int \frac{x^{p-1}+x^{-p}}{1+x}\,dx = \pi\ Cosec.\,p\,\pi,\ p < 1$; Dedekind, Cr. 45. 370.

5) $\int \frac{x^{q-1}-x^{-p}}{1-x}\,dx = \pi\ Sin.\left\{(q+p)\,\pi\right\}.\,Cosec.\,q\pi.\,Cosec.\,p\pi$ Svanberg, Transf. § 5.

6) $\int \frac{x^p - a^{p-q}\,x^q}{x}\frac{dx}{x-a} = \pi\,a^{p-1}\,(Cot.\,q\,\pi - Cot.\,p\,\pi),\ \begin{matrix}1 > p > 0\\ 1 > q > 0\end{matrix}$; Minding, Taf. II.

7) $\int \frac{x^p - x^{-p}}{1-x^2}\,dx = -\frac{\pi}{2}\,Tang.\frac{p\,\pi}{2},\ 1 > p > 0$; Schlömilch, Gr. 3. 278.

8) $\int \frac{dx}{(1+x^3)\,x^p} = \frac{1+p}{2}\,p\,\pi\ Cosec.\,p\,\pi$ Oettinger, Cr. 38. 162.

9) $\int \frac{x^b}{1+x^a}\frac{dx}{x} = \frac{\pi}{a}\,Cosec.\frac{b\,\pi}{a}$ Euler, Calc. Int. 4. S. 5. 155.

10) $\int \frac{x^{bc+b-1}}{1-x^b}\frac{dx}{x^a} = (-1)^c\,\frac{\pi}{b}\,Cosec.\frac{a\,\pi}{b}$

11) $\int \frac{dx}{x^{a+(b+1)g+1}\,(1+x^b)^{c-g+1}} = \frac{(-1)^g}{a+bc}\,\frac{(b-a)^{c/b}}{(b-a)^{g/b}\,1^{c-g/1}}\,\frac{\pi}{b^{c-g+1}}\,Cosec.\,\frac{a\,\pi}{b}$

12) $\int \frac{dx}{x^{a+(b+1)c+1}\,(1+x^b)} = (-1)^c\,\frac{\pi}{b}\,Cosec.\frac{a\,\pi}{b}$

(10–12) Oetinger, Cr. 38. 162.

13) $\int \frac{x^{-p}}{x^q - x^{-q}}\frac{dx}{x} = -\frac{\pi}{2q}\,Tang.\frac{p\,\pi}{2q}$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

14) $\int \frac{x^p + x^{-p}}{x^q + x^{-q}}\frac{dx}{x} = \frac{\pi}{q}\,Sec.\frac{p\,\pi}{2q}$ V. T. 40. N°. 29.

15) $\int \frac{x^p - x^{-p}}{x^q - x^{-q}}\frac{dx}{x} = \frac{\pi}{q}\,Tang.\frac{p\,\pi}{2q}$ Malmsten, Cr. 38. 1, où faut. $\frac{\pi}{2q}$.

16) $\int \left\{\frac{1}{x^p} - \frac{1}{(1+x)^p}\right\} x^q\,dx = \frac{q}{q-p+1}\,\frac{\Gamma(q)\,\Gamma(p-q)}{\Gamma(p)}$ Legendre, Exerc. 4. 109.

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17) $\int\left\{\frac{1}{1+x}-\frac{1}{(1+x)^p}\right\}\frac{dx}{x}=\mathrm{A}+\mathrm{Z}'(p)$ Schlömilch, Stud. I. § 6. — Id., Gr. 9. 5.

18) $\int\left\{\frac{1}{(1+x)^p}-\frac{1}{(1+x)^q}\right\}\frac{dx}{x}=\mathrm{Z}'(q)-\mathrm{Z}'(p)$ Schlömilch, Beitr. III. 9.

19) $\int\frac{x^p-a^p}{x-1}\,\frac{x^{-p}-1}{x-a}\,dx=\frac{1}{a-1}\left\{2\pi(a^p-1)\,Cot.\,p\pi-(a^p+1)\,l\,a\right\},\ p^2<1;$ Minding, Taf. II.

20) $\int\left\{\frac{q^p\,x^{p-1}}{(1+q\,x)^p}-\frac{(1+q\,x)^{p-1}}{q^{p-1}\,x^p}\right\}dx=\pi\,Cot.\,p\pi$ Legendre, Exerc. 4. 143.

F. Alg. rat. fract. à dén. à fact. binômes $(1\pm x)^a$. TABLE 23. Lim. 0 et ∞ .

1) $\int\frac{dx}{(1+x)(2+x)}=l\,2$ Schlömilch, Gr. 9. 5.

2) $\int\frac{dx}{(ax+b)(cx+d)}=\frac{1}{ad-bc}\,l\,\frac{ad}{bc}$, où $a<b,\ c<d;$ Dedekind, Cr. 45. 370.

3) $\int\frac{dx}{(x+1)(x+a)}=\frac{1}{a-1}\,l\,a$ Dedekind, Cr. 45. 370.

4) $\int\frac{dx}{(ax+1)(a+x)}=\frac{1}{a^2-1}\,l\,a^2$ Dedekind, Cr. 45. 370.

5) $\int\frac{x^{p-1}}{1+x}\,\frac{dx}{1+ax}=\frac{1-a^{1-p}}{1-a}\,\pi\,Cosec.\,p\pi,\ p<1;$ Svanberg, Transf. § 5.

6) $\int\frac{x^p}{1+x}\,\frac{dx}{x+a}=\frac{a^p-1}{a-1}\,\pi\,Cosec.\,p\pi,\ p^2<1;$ Minding, Taf. II.

7) $\int\frac{x-1}{qx+1}\,\frac{x^{p-1}\,dx}{x+q}=\frac{q^{a-1}-q^{-a}}{q-1}\,\pi\,Cosec.\,p\pi,\ p<1;$ Dedekind, Cr. 45. 370.

8) $\int\left(\frac{x^q}{(1+x)^{1+q}}-\frac{x^p}{(1+x)^{1+p}}\right)dx=\mathrm{Z}'(1+p)-\mathrm{Z}'(1+q)$ Legendre, Exerc. 4. 110.

9) $\int\frac{(a-xi)^{-p}+(a+xi)^{-p}}{2}\,\frac{(b-xi)^{-q}+(b+xi)^{-q}}{2}\,dx=\frac{\pi}{2}(a+b)^{1-p-q}\,\frac{\Gamma(p+q-1)}{\Gamma(p)\,\Gamma(q)}$ Cauchy, Lim. Imag. 108.

10) $\int\frac{(a-xi)^{-p}-(a+xi)^{-p}}{2}\,\frac{(b-xi)^{-q}-(b+xi)^{-q}}{2}\,dx=-\frac{\pi}{2}(a+b)^{1-p-q}\,\frac{\Gamma(p+q-1)}{\Gamma(p)\,\Gamma(q)}$ Cauchy, Lim. Imag. 108.

11) $\int\frac{(a-xi)^{-b}+(a+xi)^{-b}}{2}\,x^{2c}\,dx=0\ ,\ b>2c+1;$ Cauchy, P. 28. 147. I. N°. 3.

12) $\int \frac{(a-xi)^{-b}-(a+xi)^{-b}}{2} x^{2c-1}\, dx = 0$, $b > 2c$; Cauchy, P. 28. 147. I. N°. 3.

13) $\int \frac{(1+px)^{-b}+(1+qx)^{-b}}{2} x^{a-1}\, dx = (pq)^{\frac{1}{2}a} \frac{\Gamma(a)\Gamma(b-a)}{\Gamma(b)} Cos.\left(a\, Arccos.\, \frac{p+q}{2\sqrt{pq}}\right)$

14) $\int \frac{(1+px)^{-b}-(1+qx)^{-b}}{2} x^{a-1}\, dx = -(pq)^{\frac{1}{2}a} \frac{\Gamma(a)\Gamma(b-a)}{i\,\Gamma(b)} Sin.\left(a\, Arcsin.\, \frac{p-q}{2i\sqrt{pq}}\right)$

(13, 14) $, a < b$; Legendre, Exerc. 4. 102.

15) $\int \frac{x^p-x^q}{x-1} \frac{dx}{x+a} = \frac{\pi}{1+a}\left\{\frac{a^p - Cos.\, p\pi}{Sin.\, p\pi} - \frac{a^q - Cos.\, q\pi}{Sin.\, q\pi}\right\}$, $p^2 < 1$, $q^2 < 1$;

16) $\int \frac{x^p-1}{x-1} \frac{dx}{x+a} = \frac{\pi}{1+a}\left\{\frac{a^p - Cos.\, p\pi}{Sin.\, p\pi} - \frac{1}{\pi}\, l\, a\right\}$, $p^2 < 1$;

17) $\int \frac{x^p-x^{p-q}}{x-1} \frac{x^q-a^q}{x-a} dx = \frac{\pi}{a-1} \frac{Sin.\, q\pi}{Sin.\, p\pi}\left\{\frac{a^{p+q}-1}{Sin.\{(p+q)\pi\}} + \frac{a^q-a^p}{Sin.\{(p-q)\pi\}}\right\}$, $(p+q)^2 < 1$, $(p-q)^2 < 1$;

18) $\int \frac{x^p-a^p}{x-a} \frac{x^p-1}{x-1} dx = \frac{\pi}{a-1}\left\{\frac{a^{2p}-1}{Sin.\, 2p\pi} - \frac{a^p}{\pi}\, l\, a\right\}$, $4p^2 < 1$;

(15–18) Minding, Taf. II.

F. Alg. rat. fract. à dén. à fact. binômes $(1 \pm x^a)^b$. TABLE 24. Lim. 0 et ∞ .

1) $\int \frac{x}{a^2+x^2} \frac{dx}{1+x} = \frac{1}{1+a^2}\left(\frac{a\pi}{2} - l\, a\right)$

2) $\int \frac{x}{a^2+x^2} \frac{dx}{1-x} = \frac{-1}{1+a^2}\left(\frac{a\pi}{2} + l\, a\right)$

3) $\int \frac{1}{a^2+x^2} \frac{dx}{1+x} = \frac{1}{1+a^2}\left(\frac{\pi}{2a} + l\, a\right)$

4) $\int \frac{1}{a^2+x^2} \frac{dx}{1-x} = \frac{1}{1+a^2}\left(\frac{\pi}{2a} - l\, a\right)$

(1–4) Schlömilch, Gr. 5. 204.

5) $\int \frac{1-x}{1+x} \frac{dx}{1+x^2} = 0$ Arndt, Gr. 10. 225.

6) $\int \frac{ax-1}{x+a} \frac{dx}{1+x^2} = \frac{1}{2}\, l\, a^2$ Arndt, Gr. 11. 70.

7) $\int \frac{dx}{(1+x^2)(p^2+x^2)} = \frac{\pi}{2p(p+1)}$ Schlömilch, Gr. 4. 71. — Id., Gr. 10. 440.

8) $\int \frac{x^2}{p^2+x^2} \frac{dx}{q^2+x^2} = \frac{\pi}{2(p+q)}$ Cisa de Grésy, Mém. Turin. 1821. 209. II. 62.

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9) $\int \frac{g + hx^2}{b^2 + x^2} \frac{dx}{c^2 - x^2} = \frac{g - hb^2}{b^2 + c^2} \frac{\pi}{2b}$

10) $\int \frac{x^{p-1}}{b^2 + x^2} \frac{dx}{c^2 - x^2} = \frac{b^{p-2} + c^{p-2} Cos. \frac{p\pi}{2}}{b^2 + c^2} \frac{\pi}{2} Cosec. \frac{p\pi}{2}$ } Legendre, Exerc. 5. 13.

11) $\int \frac{x^2}{q^2 + x^2} \frac{dx}{x^2 - p^2} = \frac{q}{2} \frac{\pi}{p^2 + q^2}$ Cisa de Grésy, Mém. Turin. 1821. 209 II. 61.

12) $\int \frac{x}{q^2 + x^2} \frac{dx}{1 - x^2} = -\frac{1}{1 + q^2} lq$ V. T. 24. N°. 1, 2.

13) $\int \frac{1}{q^2 + x^2} \frac{dx}{1 - x^2} = \frac{1}{1 + q^2} \frac{\pi}{2q}$ V. T. 24. N°. 3, 4.

14) $\int \frac{x^2}{q^2 + x^2} \frac{dx}{1 - x^2} = -\frac{1}{1 + q^2} \frac{q\pi}{2}$ V. T. 24. N°. 1, 2.

15) $\int \frac{dx}{(1 + x^4)(1 + x^6)} = \frac{\pi}{4} \left(\sqrt{2} - \frac{1}{3}\right)$

16) $\int \frac{x^{2b}}{1+x^{2a}} \frac{dx}{1+x^{3a}} = \frac{\pi}{4a} \left\{ Cosec. \left(\frac{2b+1}{2a}\pi\right) + Sec. \left(\frac{2b+1}{2a}\pi\right) \right\} + \frac{\pi}{6a} \frac{1 + 4\, Cos. \left(\frac{2b+1}{3a} 2\pi\right) + 4\, Cos. \left(\frac{2b+1}{3a} 2\pi - \frac{4\pi}{3}\right)}{Sin. \left(\frac{2b+1}{a}\pi\right)}$

17) $\int \frac{x^{p-1}}{1+x^a} \frac{dx}{1+x^b} = \frac{\pi}{2a Sin. p\pi} \left\{ \frac{Cos. \left(\frac{1-a}{a} p\pi\right) + Cos. \left(\frac{(1-a)(p-b)}{a}\pi\right)}{1 + Cos. \left(\frac{b}{a}\pi\right)} + \frac{Cos. \left(\frac{3-a}{a} p\pi\right) + Cos. \left(\frac{(3-a)(p-b)}{a}\pi\right)}{1 + Cos. \left(\frac{b}{a} 3\pi\right)} + \dots \right\}$

$+ \frac{\pi}{2b Sin p\pi} \left\{ \frac{Cos. \left(\frac{1-b}{b} p\pi\right) + Cos. \left(\frac{(1-b)(p-a)}{b}\pi\right)}{1 + Cos. \left(\frac{a}{b}\pi\right)} + \frac{Cos. \left(\frac{3-b}{b} p\pi\right) + Cos. \left(\frac{(3-b)(p-a)}{b}\pi\right)}{1 + Cos. \left(\frac{a}{b} 3\pi\right)} + \dots \right\}$

Dans 16), 17) a et b sont des entiers quelconques; $1 + x^a$ et $1 + x^b$ n'ont pas de racine commune; p est > 0 et quelconque, rationnel ou irrationnel, mais $< (a + b)$; les dénominateurs des derniers termes sont pour les deux séries respectivement: $1 + Cos. \left(\frac{2a-1}{a} b\pi\right)$ et $1 + Cos. \left(\frac{2b-1}{b} a\pi\right)$. — Des formules (15) à (17) voyez Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

18) $\int \left(\frac{x}{p^2 r^2 + x^2} - \frac{x}{q^2 r^2 + x^2}\right) dx = \frac{1}{2} l \frac{q}{p}$ Schlömilch, Gr. 5. 152.

1) $\int \frac{dx}{1+x+x^2} = \frac{2\pi}{3\sqrt{3}}$ Dienger, Gr. 10. 107.

2) $\int \frac{dx}{1-2x\,Cos.\lambda+x^2} = \frac{\pi-\lambda}{Sin.\lambda}, \frac{\pi}{2} > \lambda > 0;$ Schlömilch, Gr. 10. 424. — Ohm, Ausw. 3, la trouve fautivement égale à $-\lambda : Sin.\lambda$.

3) $\int \frac{dx}{1+2x\,Cos.\lambda+x^2} = \frac{\lambda}{Sin.\lambda}, \frac{\pi}{2} > \lambda > 0;$ Schlömilch, Gr. 10. 424.

4) $\int \frac{x^{p-1}\,dx}{1+x+x^2} = \frac{2\pi}{\sqrt{3}} Sin.\left(\frac{1-p}{3}\pi\right).Cosec.\,p\pi,\ 0 < p < 2;$ Dienger, Gr. 10. 107.

5) $\int \frac{x^p\,dx}{1+2x\,Cos.\lambda+x^2} = \frac{\pi}{Sin.\,p\pi}\,\frac{Sin.\,p\lambda}{Sin.\lambda},\ p^2 < 1,\ \lambda^2 < \pi^2;$ Legendre, Exerc. 4. 102. — Schlömilch, Gr. 12. 198.

6) $\int \frac{x^p\,dx}{1+2qx\,Cos.\lambda+q^2x^2} = \frac{\pi}{q^{p+1}Sin.p\pi}\,\frac{Sin.\,p\lambda}{Sin.\lambda},\ p^2 < 1,\ \lambda^2 < \pi^2;$ Plana, Mém. Brux. T. 10. — Legendre, Exerc. 4. 102, trouve fautivement q^p au lieu de q^{p+1}.

7) $\int \frac{1+ax}{1+2ax+(a^2+b^2)x^2}\,x^{p-1}\,dx = \frac{\pi}{\sqrt{(a^2+b^2)^p}} Cosec.\,p\pi.Cos.\left(p\,Arctg.\frac{b}{a}\right)$

8) $\int \frac{bx}{1+2ax+(a^2+b^2)x^2}\,x^{p-1}\,dx = \frac{\pi}{\sqrt{(a^2+b^2)^p}} Cosec.\,p\pi.Sin.\left(p\,Arctg.\frac{b}{a}\right)$

9) $\int \frac{x^p\,dx}{1+x^2+2ax\,Cos.\left(Arctg.\frac{b}{a}\right)} = \pi\,Cosec.\,p\pi.Sin.\left(p\,Arctg.\frac{b}{a}\right).Cosec.\left(Arctg.\frac{b}{a}\right)$

(7–9) Cauchy, Sav. Etr. 1827. 599 P.1.§3.

10) $\int \frac{x^{1-p}\,dx}{b^2+(x+a)^2} = \frac{\pi i}{b\,Sin.\,p\pi}\,\frac{(a-bi)^{1-p}-(a+bi)^{1-p}}{2}$

11) $= \frac{\pi}{\sqrt{(a^2+b^2)^p}}\,\frac{Sin.\left\{(1-p)\,Arctg.\frac{b}{a}\right\}}{Sin.\,p\pi.Sin.\left(Arctg.\frac{b}{a}\right)}$

12) $\int \frac{x^{1-p}\,dx}{1+2qx\,Cos.\lambda+q^2x^2} = \frac{\pi}{q^p}\,\frac{Sin.\{(1-p)\lambda\}}{Sin.\lambda.Sin.\{(1-p)\pi\}},\ p<1, q<1;$

(10–12) Plana, Mém. Brux. 1837.

13) $\int \frac{x^p\,dx}{q^2+2qx\,Cos.\lambda+x^2} = \frac{\pi q^{p-1}}{Sin.\,p\pi}\,\frac{Sin.\,p\lambda}{Sin.\lambda},\ p^2<1, \lambda^2<\pi^2;$ Schlömilch, Gr. 12. 198.

14) $\int \frac{dx}{1-2x\,Cos.\frac{2a\pi}{b}+x^2} = \frac{1}{2b\,Sin.\frac{2a\pi}{b}}\sum_1^{b-1} Sin.\frac{2na\pi}{b}.Cot.\frac{n\pi}{b}$ Schlömilch, Gr. 10. 424.

15) $\int \frac{x^{p-1}\,dx}{1+x^2+x^4} = \frac{\pi}{\sqrt{3}}\,Cosec.\frac{p\pi}{2}.\,Sin.\left(\frac{\pi}{3}-\frac{p\pi}{6}\right),\ 0<p<4;$ Dienger, Gr. 10. 341.

16) $\int \frac{1+x^2}{1-x^2+x^4}\,dx = \pi$ Raabe, Cr. 37. 356.

17) $\int \frac{dx}{a+bx^2+cx^4} = \frac{\pi}{2\sqrt{\{ab+2a\sqrt{ac}\}}}$

18) $\int \frac{x^2\,dx}{a+bx^2+cx^4} = \frac{\pi}{2\sqrt{\{bc+2c\sqrt{ac}\}}}$

19) $\int \frac{dx}{x^2+(a+bx^2)^2} = \frac{\pi}{2a\sqrt{(1+4ab)}}$

20) $\int \frac{x^2\,dx}{x^2+(a+bx^2)^2} = \frac{\pi}{2b\sqrt{(1+4ab)}}$

(17–20) Plana, Mém. Turin. 1820.

21) $\int \frac{p^2-q^2+x^2}{(p^2+q^2)^2+2(p^2-q^2)x^2+x^4}\,dx = \frac{1}{2}\,\frac{p\pi}{p^2+q^2}$

22) $\int \frac{dx}{(p^2+q^2)^2+2(p^2-q^2)x^2+x^4} = \frac{1}{4p}\,\frac{\pi}{p^2+q^2}$

(21–22) Ohm, Ausw. N°. 2.

23) $\int \frac{x^{b-1}\,dx}{1-2x^a\,Cos.\lambda+x^{2a}} = \frac{\pi}{a}\,Cosec.\lambda.\,Cosec.\frac{b\pi}{a}.\,Sin.\left\{\frac{b(\pi-\lambda)+a\lambda}{a}\right\}$ Euler, Calc. Int. T. 4. S. 5. 179.

24) $\int \frac{x^{b-1}\,dx}{1+2x^a\,Cos.\lambda+x^{2a}} = \frac{\pi}{a}\,Cosec.\lambda.\,Cosec.\frac{b\pi}{a}.\,Sin.\left(\frac{a-b}{a}\lambda\right)$ Euler, Calc. Int. T. 4. S. 5. 183.

25) $\int \frac{x^{a\pm b-1}\,dx}{1+2x^a\,Cos.\lambda+x^{2a}} = \frac{\pi}{a}\,Cosec.\lambda.\,Cosec.\frac{b\pi}{a}.\,Sin.\frac{b\lambda}{a}$ Euler, Calc. Int. T. 4. S. 5. 191.

F. Alg. rat. fract. à autre dén. polynôme. TABLE 26. Lim. 0 et ∞.

1) $\int \frac{x^{p+1}\,dx}{(1+2x\,Cos.\lambda+x^2)^2} = \frac{\pi}{2\,Sin.p\pi}\,\frac{p\,Sin.\lambda.\,Cos.p\lambda - Cos.\lambda.\,Sin.p\lambda}{Sin^3\lambda}$ Legendre, Exerc. 4. 108.

2) $\int \frac{1}{1+2x\,Cos.\lambda+x^2}\,\frac{dx}{x^p} = \frac{\pi}{Sin.p\pi}\,\frac{Sin.p\lambda}{Sin.\lambda},\ p^2<1;$ Legendre, Exerc. 4. 102.

3) $\int \frac{x+a}{b^2+(x+a)^2}\,\frac{dx}{x^p} = \frac{\pi}{\sqrt{(a^2+b^2)^p}}\,.\,\frac{Cos.\,(p\,Arctg.\frac{b}{a})}{Sin.\,p\pi}$ Cauchy, P. 28. 147. I. § 2.

4) $= \frac{\pi}{Sin.p\pi}\,\frac{(a-bi)^{-p}+(a+bi)^{-p}}{2}$ Plana, Mém. Brux. 1837.

5) $\int \frac{1}{b^2+(x+a)^2} \frac{dx}{x^p} = \frac{\pi}{\sqrt{(a^2+b^2)^p}} \frac{Sin.\left(p\, Arctg. \frac{b}{a}\right)}{Sin.\, p\pi}$ Cauchy, P. 28. 147. I. § 2.

6) $= \frac{\pi i}{Sin.\, p\pi} \frac{(a-bi)^{-p}-(a+bi)^{-p}}{2}$

7) $= \frac{\pi}{\sqrt{(a^2+b^2)^{p+1}}} \frac{Sin.\left(p\, Arctg. \frac{b}{a}\right)}{Sin.\, p\pi .\, Sin.\left(Arctg. \frac{b}{a}\right)}$

8) $\int \frac{1}{1+2qx\, Cos.\, \lambda + q^2x^2} \frac{dx}{x^p} = \frac{\pi}{q^{p-1}} Sin.\, p\lambda .\, Cosec.\, p\pi .\, Cos.\, \lambda ,\ p<1,\ q<1;$

Plana, Mém. Brux. 1837.

9) $\int \frac{x^p - 2\, Cos.\, \mu + x^{-p}}{x^q - 2\, Cos.\, \lambda + x^{-q}} \frac{dx}{x} = \frac{2\pi\, Sin.\left(p\frac{\pi-\lambda}{q}\right)}{q\, Sin.\, \lambda .\, Sin.\frac{p\pi}{q}} - \frac{2(\pi-\lambda)\, Cos.\, \mu}{q\, Sin.\, \lambda}$

10) $\int \frac{x^p + x^{-p}}{x^q - 2\, Cos.\, \lambda + x^{-q}} \frac{dx}{x} = \frac{2\pi\, Sin.\left(p\frac{\pi-\lambda}{q}\right)}{q\, Sin.\, \lambda\ Sin.\frac{p\pi}{q}}$

11) $\int \frac{x^{\pm p}}{x^q - 2\, Cos.\, \lambda + x^{-q}} \frac{dx}{x} = \frac{\pi\, Sin.\left(p\frac{\pi-\lambda}{q}\right)}{q\, Sin.\, \lambda .\, Sin.\frac{p\pi}{q}}$

12) $\int \frac{x^p + x^{-p}}{x^q + \left(a+\frac{1}{a}\right) + x^{-q}} \frac{dx}{x} = \frac{a^{\frac{p}{q}} - a^{-\frac{p}{q}}}{a-\frac{1}{a}} \frac{2\pi}{a}\, Cosec.\frac{p\pi}{q}$

Euler, N. A. Petr. III. 3.

13) $\int \frac{dx}{x^6+ax^4+bx^2+c} = \frac{p\pi}{p(p^2-a)\sqrt{c}-2c}$

14) $\int \frac{x^2\, dx}{x^6+ax^4+bx^2+c} = \frac{\pi\sqrt{c}}{p(p^2-a)\sqrt{c}-2c}$

15) $\int \frac{x^4\, dx}{x^6+ax^4+bx^2+c} = \frac{1}{2}\pi\sqrt{c}\, \frac{p^2-a}{p(p^2-a)\sqrt{c}-2c}$

, où p la plus grande racine de $(Z^2-a)^2 - 8Z\sqrt{c} - 4b = 0$;

Poisson. L. 2. 224.

1) $\int \frac{x^{\frac{1}{2}p-1}\, dx}{(1+x)^a} = 2\, \frac{\Gamma\left(\frac{1}{2}p\right)\Gamma\left(a-\frac{1}{2}p\right)}{\Gamma(a)}$, $1 > p > 0$; Schlömilch, Beitr. III. 13.

2) $\int \frac{x^{p-\frac{1}{2}}\, dx}{1+x} = \pi\, Sec.\, p\pi$

3) $\int \frac{x^{p-\frac{1}{2}}\, dx}{(1+x)^2} = \frac{1-2p}{2}\, \pi\, Sec.\, p\pi$

4) $\int \frac{x^{p+\frac{1}{2}}\, dx}{(1+x)^2} = \frac{2p+1}{2}\, \pi\, Sec.\, p\pi$

5) $\int \frac{x^{p+a+\frac{1}{2}}\, dx}{(1+x)^{a+b+2}} = \frac{(1+2p)^{a+1/2}(1-2p)^{b/2}}{1^{a+b+1/1}\; 2^{a+b+1}}\, \pi\, Sec.\, p\pi = \int \frac{x^{b-p-\frac{1}{2}}\, dx}{(1+x)^{a+b+2}}$

6) $\int \frac{x^{p-a+\frac{1}{2}}\, dx}{(1+x)^{-a+b+2}} = \frac{(1-2p)^{b/2}}{(2p-1)^{a+1/-2}}\, 2^{a-b-1}\, \pi\, Sec.\, p\pi = \int \frac{x^{b-p-\frac{1}{2}}\, dx}{(1+x)^{-a+b+2}}$

7) $\int \frac{x^{p+a+\frac{1}{2}}\, dx}{(1+x)^{a-b+2}} = (-1)^b\, \frac{(1+2p)^{a+1/2}}{(1+2p)^{b/2}\; 1^{a-b+1/1}}\, 2^{b-a}\, \pi\, Sec.\, p\pi$

Oettinger, Cr. 38. 162. (2–7)

8) $\int \frac{x^{\frac{1}{2}(p-q-1)}\, dx}{(1+x)^{p+\frac{1}{2}}} = \frac{\Gamma\left(\frac{p-q+1}{2}\right)\Gamma\left(\frac{p+q}{2}\right)}{\Gamma\left(p+\frac{1}{2}\right)}$

9) $\int \frac{x^{\frac{1}{2}(p-q-1)}\, dx}{(1+x)^{p-\frac{1}{2}}} = \frac{\Gamma\left(\frac{p-q+1}{2}\right)\Gamma\left(\frac{p+q}{2}-1\right)}{\Gamma\left(p-\frac{1}{2}\right)}$

10) $\int \frac{x^{p-q}\, dx}{(1+x^2)^{p+\frac{1}{2}}} = \frac{\Gamma\left(\frac{p-q+1}{2}\right)\Gamma\left(\frac{p+q}{2}\right)}{2\,\Gamma\left(p+\frac{1}{2}\right)}$

11) $\int \frac{x^{p-q}\, dx}{(1+x^2)^{p-\frac{1}{2}}} = \frac{\Gamma\left(\frac{p-q+1}{2}\right)\Gamma\left(\frac{p+q}{2}-1\right)}{2\,\Gamma\left(p-\frac{1}{2}\right)}$

Meijer, Int. Déf. 329. (8–11)

12) $\int \frac{x^{2a}\, dx}{(2+x^2)^{b+\frac{1}{2}}} = 2^{a-b-\frac{1}{2}}\, \frac{\Gamma\left(a+\frac{1}{2}\right)\Gamma(b-a)}{\Gamma\left(b+\frac{1}{2}\right)}$ Schlömilch, Gr. 6. 213.

13) $\int \frac{dx}{\sqrt{(1+x^4)}} = F'\left(Sin.\frac{\pi}{4}\right)$ Legendre, Exerc. 1. 146.

14) $\int \frac{1-x}{\sqrt[4]{(1-x^4)^3}}\, dx = 0$ Euler, N. C. Petr. 6. 115.

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15) $\int \frac{dx}{\sqrt{(1+x^6)}} = \frac{2}{\sqrt[4]{3}} \mathrm{F}'\left(Sin.\frac{5\pi}{12}\right)$

16) $= \sqrt[4]{3} \,.\, \mathrm{F}'\left(Sin.\frac{\pi}{12}\right)$

Legendre, Exerc. 1. 147. (15, 16)

17) $\int \frac{dx}{\sqrt{(1+x^8)}} = Sec.\frac{\pi}{8}\sqrt{\frac{1}{2}}.\ \mathrm{F}'\left(Tg.\frac{\pi}{8}\right)$ Legendre, Exerc. 1. 148.

18) $\int \frac{dx}{\sqrt{(1+x^{12})}} = \frac{1}{2\sqrt[4]{3}} Sec.\frac{\pi}{12} \mathrm{F}'\left(Sin.\frac{\pi}{4}\right) + Tg.\frac{\pi}{12} \mathrm{F}'\left(\frac{\sqrt{2}-\sqrt[4]{3}}{1+\sqrt{3}}\right)$ Legendre, Exerc. 1. 149.

19) $\int \frac{x^{2a}\,dx}{\left(\frac{1}{x^2}+x^2\right)^{b+\frac{1}{2}}} = \frac{\Gamma\left(\frac{b+a+1}{2}\right)\Gamma\left(\frac{b-a}{2}\right)}{4\Gamma\left(\frac{b}{2}+1\right)}$ Schlömilch, Gr. 6. 213.

20) $\int \frac{x^{\frac{1}{2}b\pm a-1}\,dx}{1+x^b} = \frac{\pi}{b} Sec.\frac{a\pi}{b}$ Euler, N. C. P. 6. 115.

21) $\int \frac{x^{p-1}\,dx}{\sqrt{(1+x^q)}} = 2^{\frac{2p}{q}} \mathrm{B}(q-2p\,,\,p) \quad ,q>2p;$

22) $\int \frac{x^{q-p-1}\,dx}{\sqrt{(1+x^q)}} = 2^{2-\frac{2p}{q}} \mathrm{B}(2p-q\,,\,q-p),\ q<2p;$

Plana, Cr. 17. 163. (21, 22)

23) $\int \frac{x^{b(a+1)-1}\,dx}{(1+x^b)^{a+\frac{c}{b}+1}} = \frac{1}{c}\,\frac{1^{\frac{c}{b}|1}\,1^{a|1}}{1^{\frac{c}{b}+a|1}}$ Oettinger, Cr. 35. 13.

24) $\int \frac{x^{a-1}\,dx}{(1-x^b)^{\frac{a}{b}}} = \frac{\pi}{b} Cosec.\frac{a\pi}{b}$

25) $\int \frac{x^{a+b-1}\,dx}{(1-x^b)^{\frac{a}{b}}} = \frac{a\pi}{b^2} Cosec.\frac{a\pi}{b}$

Euler, N. C Petr. 6. 115. (24, 25)

F. Alg. irrat. fract. à autre dén. TABLE 28. Lim. 0 et ∞.

1) $\int \frac{1}{1+x}\frac{dx}{\sqrt{x}} = \pi$ Dedekind, Cr. 45. 370.

2) $\int \frac{1}{1-x}\frac{dx}{\sqrt{x}} = 0$ Schlömilch, Beitr. III. § 6.

3) $\int \frac{dx}{(1+x)^2\,x^{p+\frac{1}{2}}} = \frac{2p+1}{2}\pi\,Sec.\,p\pi$ Oettinger, Cr. 38. 162.

4) $\int \frac{dx}{(1+x)^2 x^{p-\frac{1}{2}}} = \frac{1-2p}{2} \pi \, Sec.\, p\pi$

5) $\int \frac{dx}{(1+x) x^{p-\frac{1}{2}}} = \pi \, Sec.\, p\pi$

6) $\int \frac{dx}{x^{p+b+\frac{1}{2}}(1+x)^{a-b+2}} = (-1)^b \frac{(1+2p)^{a+1/2}}{(1+2p)^{b/2} \, 1^{a-b+1/1}} 2^{b-a} \pi \, Sec.\, p\pi$

Oettinger, Cr. 38. 162.

7) $\int \left(\frac{x^{\frac{1}{2}p} - x^{-\frac{1}{2}p}}{x-1}\right)^2 dx = 2(1 - p\pi \, Cot.\, p\pi), \; p^2 < 1;$ Minding, Taf. II.

8) $\int \frac{1}{\left(x+\frac{1}{x}\right)^{2(p-a)}} \frac{dx}{x} = \frac{\{\Gamma(p-a)\}^2}{2\Gamma(2p-2a)}$ Schlömilch, Gr. 6. 213.

9) $\int \frac{dx}{(q^2+x^2)\sqrt{(p^2+x^2)}} = \frac{1}{q\sqrt{(p^2-q^2)}} Arctg. \frac{\sqrt{(p^2-q^2)}}{q}, \; q < p;$

10) $= \frac{1}{q\sqrt{(q^2-p^2)}} \, l \frac{q+\sqrt{(q^2-p^2)}}{q}, \; q > p;$

Lobatto, Int. § 53.

11) $\int \frac{dx}{(1+px^2)\sqrt[3]{(1+9p^2x)}} = \frac{\pi}{4\sqrt{p}}$ Legendre, Exerc. 1. 140.

12) $\int \left\{1 - \frac{x^2+1}{\sqrt{(x^4+1)}}\right\} \frac{dx}{x} = -l2$

13) $\int \left\{1 - \frac{ax^2+c}{\sqrt{\{a^2x^4+2(ac-2b^2)x^2+c^2\}}}\right\} \frac{dx}{x} = l \frac{ac-b^2}{ac}$

Winckler, Cr. 45. 102.

14) $\int \frac{dx}{\sqrt{(1+p^2x)(1+q^2x)(1+r^2x)}} = \frac{2}{\sqrt{(p^2-r^2)}} F\left(\varphi, \sqrt{\frac{p^2-q^2}{p^2-r^2}}\right)$

15) $\int \frac{dx}{\sqrt{x(x+p^2)(x+q^2)(x+r^2)}} = \frac{2}{\sqrt{(p^2-r^2)}} F\left(\varphi, \sqrt{\frac{p-q^2}{p^2-r^2}}\right)$

16) $\int \frac{dx}{\sqrt{(p^2+l^2x)(q^2+m^2x)(r^2+n^2x)}} = \frac{\pi}{2pmn\,Sin.\,\psi} F\left(\psi, \frac{n}{m}\sqrt{\frac{p^2m^2-q^2l^2}{p^2n^2-r^2l^2}}\right)$

, où $Cos.\varphi = \frac{r}{p}$, et $Cos.^2\psi = \frac{rl}{pn}$; Jacobi, Cr. 10. 101.

17) $\int \frac{x^{p-q}\,dx}{(a+bx+cx^2)^{p+\frac{1}{2}}} = \frac{1}{\Gamma(p+\frac{1}{2})}\left(\frac{c}{a}\right)^{\frac{q}{2}} \sqrt{\frac{\pi}{c}} \sum_0^\infty \frac{(q-n)^{2n/1}}{2^{n/2}(2\sqrt{ac})^n} \frac{\Gamma(p-n)}{(b+2\sqrt{ac})^{p-n}}$

18) $\int \frac{x^{p+q}\,dx}{(a+bx+cx^2)^{p+\frac{1}{2}}} = \frac{1}{\Gamma(p+\frac{1}{2})}\left(\frac{a}{c}\right)^{\frac{q}{2}} \sqrt{\frac{\pi}{c}} \sum_0^\infty \frac{(q-n+1)^{2n/1}}{2^{n/2}(2\sqrt{ac})^n} \frac{\Gamma(p-n)}{(b+2\sqrt{ac})^{p-n}}$

Schlömilch, Cr. 33. 268. — Id., Stud. I. 17.

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19) $\int \frac{p+\sqrt{\frac{1}{2}x}}{x+p\sqrt{2x}+p^2} \frac{dx}{q^2-x^2} = \frac{\pi}{2\sqrt{2q}\,.\,(q+p\sqrt{2q}+p^2)}$ Poisson, Chaleur. N°. 159.

20) $\int \frac{q+\frac{1}{p}\sqrt{2x}}{q^2+\frac{q}{p}\sqrt{2x}+\frac{x}{p^2}} \frac{dx\sqrt{\frac{1}{2}x}}{1+r^2x^2} = \frac{\pi}{2\sqrt{r}} \frac{p}{1+pq\sqrt{r}}$

21) $\int \frac{q+\frac{1}{p}\sqrt{2x}}{q^2+\frac{q}{p}\sqrt{2x}+\frac{x}{p^2}} \frac{dx}{1+r^2x^2} = \frac{\pi}{2r} \frac{p}{1+pq\sqrt{r}}$

(20–21) Poisson, Chaleur. Suppl. Note B.

22) $\int \frac{x^b}{\sqrt{\{1+(2-4p^2)x^2+x^4\}}} \frac{dx}{(1+x^2)^3} = \frac{3}{8p^2}\{E'(p)-F'(p)\} + \frac{1}{2} F'(p)$, où $p < 1$;

23) $\int \frac{x^b}{\sqrt{\{1+(2-4p^2)x^2+x^4\}}} \frac{dx}{(1+x^2)^5} = \frac{2p^2+1}{8p^2} E'(p) - \frac{1-p^2}{8p^2} F'(p)$

(22–23) Schlömilch, Beitr. III. 16.

24) $\int \frac{x^b-\sqrt{x}}{1-x} \frac{dx}{x} = \pi \, Cot.\, b\pi$ Dedekind, Eul. Int. S. 22.

F. Alg. rat. fract. à dén. $1 \pm x^a$. TABLE 29. Lim. $-\infty$ et ∞.

1) $\int \frac{dx}{x \pm q} = 0$ Grunert, Gr. 2. 266.

2) $\int \frac{dx}{1+x^2} = \pi$ Ohm, Ausw. N° 2.

3) $\int \frac{dx}{x^2+p^2} = \frac{1}{p}\pi$

4) $\int \frac{x\,dx}{x^2+p^2} = 0$

5) $= l\,\alpha$, pour $-\infty = -\alpha(\infty)$;

(3–5) Cauchy, Cours. Leç. 32.

6) $\int \frac{dx}{x^2-p^2} = 0$ Poisson, P. 18. 295. N°. 41.

7) $\int \frac{(-xi)^{p-1}}{1+x^2} dx = \pi \left\{(-i)^{p-1} + (i)^{p-1}\right\}$, $p < 2$; Cauchy, Cours. Leç. 34.

8) $= \pi$

9) $\int \frac{(-xi)^{p-1}}{1-x^2} dx = \pi \, Cos.\, \frac{p\pi}{2}$

(8–9) Meijer, Int. Déf. 154.

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10) $\int \frac{(-xi)^{p-1}}{1-x^2}\,dx = \frac{\pi}{2}\{(-i)^p + (i)^p\}$, $p < 2$; Cauchy, Cours. Leç. 34.

11) $\int \frac{x^{2b}\,dx}{1+x^{2a}} = \frac{\pi}{a}\,Cosec.\left\{\frac{2b+1}{2a}\pi\right\}$, $2b < 2a-1$; Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. (pour a et b quelconques). — Serret, L. 8. 1. — Grunert, Gr. 2. 266.

12) $\int \frac{x^{2b-1}\,dx}{1+x^{2a}} = 0$

13) $\int \frac{x^{2b}\,dx}{1+x^{2a-1}} = \frac{\pi}{2a-1}\,Cot.\left\{\frac{2b+1}{2a-1}\,\frac{\pi}{2}\right\}$

14) $\int \frac{x^{2b-1}\,dx}{1+x^{2a-1}} = \frac{\pi}{2a-1}\,Tang.\,\frac{b\pi}{2a-1}$

(12–14): $2b < 2a-1$; Grunert, Gr. 2. 266.

15) $\int \frac{x^{2b}\,dx}{1-x^{2a}} = \frac{\pi}{a}\,Cot.\left\{\frac{2b+1}{2a}\pi\right\}$, $2b < 2a-1$; Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. (pour a et b quelconques). — Grunert, Gr. 2. 266.

16) $\int \frac{x^{2b-1}\,dx}{1-x^{2a}} = 0$

17) $\int \frac{x^{2b}\,dx}{1-x^{2a-1}} = \frac{\pi}{2a-1}\,Cot.\left\{\frac{2b+1}{2a-1}\,\frac{\pi}{2}\right\}$

18) $\int \frac{x^{2b-1}\,dx}{1-x^{2a-1}} = -\frac{\pi}{2a-1}\,Tang.\,\frac{b\pi}{2a-1}$

(16–18): $2b < 2a-1$; Grunert, Gr. 2. 266.

F. Alg. rat. fract. à autre dén. TABLE 30. Lim. — ∞ et ∞ .

1) $\int \frac{dx}{(r+xi)^p(s-xi)^q} = 2\pi(r+s)^{1-p-q}\,\frac{\Gamma(p+q-1)}{\Gamma(p)\,\Gamma(q)}$

2) $\int \frac{dx}{(r+xi)^p(s+xi)^q} = 0$

3) $\int \frac{dx}{(r-xi)^p(s-xi)^q} = 0$

(1–3): $r > 0,\ s > 0,\ 1 > p > 0,\ 1 > q > 0$; Cauchy, Lim. Imag. N°. 102, 106.

4) $\int \left(\frac{p-qi}{x-r-si} + \frac{p+qi}{x-r+si}\right)dx = 2\pi q + 2p\alpha$, pour $-\infty = -\alpha(\infty)$; Cauchy, Cours. Leç. 32.

5) $= 2\pi q$ Cauchy, Cours. Leç. 32. — Grunert, Gr. 2. 266.

6) $\int \left(\frac{1}{x-r-si} + \frac{1}{x-r+si}\right)dx = 0$ Grunert, Gr. 2. 266.

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7) $\int \frac{1}{(1-qxi)^p} \frac{dx}{1+x^2} = \frac{\pi}{(1+q)^p}$ V. T. 113. N°. 17. et T. 147. N°. 8.

8) $\int \frac{(p+xi)^{a-1}}{(1-p-xi)^{a+b}} dx = 2\, Sin.\, a\pi \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, -\infty < a < 1;$

9) $\int \frac{(p+xi)^{q-1}}{(1-p-xi)^{1-r}} dx = \frac{2\, Sin.\, r\pi.\, Sin.\, q\pi}{Sin.\, \{(q+r)\pi\}} \cdot \frac{\Gamma(q)\Gamma(r)}{\Gamma(q+r)}, -\infty < (q+r) < 1;$

} Cayley, L. 12. 231.

10) $\int \frac{dx}{1+x+x^2} = \frac{2\pi}{\sqrt{3}}$ Ohm, Ausw. 8.

11) $\int \frac{dx}{1-x+x^2} = \frac{2\pi}{\sqrt{3}}$ Ohm, Ausw. 9.

12) $\int \frac{x-a}{(x-a)^2+b^2} dx = \alpha$, pour $-\infty = -\alpha(\infty)$; Cauchy, Cours. Leç. 32.

13) $\qquad = 0$

14) $\int \frac{dx}{(x-a)^2+b^2} = \frac{1}{b}\pi$

} Cauchy, Cours. Leç. 32. — Grunert, Gr. 2. 266.

15) $\int \frac{dx}{1 \pm x\sqrt{3}+x^2} = 2\pi$

16) $\int \frac{1+x^2}{1-x^2+x^4} dx = 2\pi$

} Raabe, Cr. 37. 356.

17) $\int \frac{dx}{1-2x\, Cos.\, \lambda + x^2} = \pi\, Cosec.\, \lambda$ Schlömilch, Int. 117.

18) $\int \frac{a+bx}{x^2+2cx\, Cos.\, \lambda + c^2} dx = \frac{\pi}{c\, Sin.\, \lambda}(a - \frac{1}{2}b^2)$ Plana, Mém. Turin. 1818. 7. Art. 1. N°. 12.

F. Algébr. rat. fract. TABLE 31. Lim. 1 et ∞.

1) $\int \frac{(x-1)^p}{x} dx = -\pi\, Cosec.\, p\pi$

2) $\int \frac{(x-1)^{1-p}}{x} dx = -\pi\, Cosec.\, p\pi$

} Oettinger, Cr. 35. 13.

3) $\int \frac{dx}{x^p} = \frac{1}{p-1}, p > 1;$

4) $\qquad = \infty \quad , p \leqq 1;$

} Raabe, Int. 120.

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5) $\int \frac{(x-1)^{1-p}\,dx}{x^3} = \frac{1-p}{2}\, p\,\pi\, Cosec.\, p\,\pi$

6) $\int \frac{(x-1)^{1+p}\,dx}{x^3} = \frac{1+p}{2}\, p\,\pi\, Cosec.\, p\,\pi$

7) $\int \frac{(x-1)^{p+c}\,dx}{x^{b+c+2}} = \frac{(1+p)^{c/1}\,(1-p)^{b/1}}{1^{b+c/1}}\, p\,\pi\, Cosec.\, p\,\pi = \int \frac{(x-1)^{b-p}\,dx}{x^{b+c+2}}$

8) $\int \frac{(x-1)^{p+c}\,dx}{x^{c-b+2}} = \frac{(1+p)^{c/1}}{(1+p)^{b-1/1}\, 1^{c-b+2/1}}\, \pi\, Cosec.\, p\,\pi$

9) $\int \frac{(x-1)^{p-c}\,dx}{x^{b-c+2}} = \frac{(1-p)^{b|1}}{1^{c/-1}\,1^{b-c-1|1}}\, p\,\pi\, Cosec.\, p\,\pi = \int \frac{(x-1)^{b-p}\,dx}{x^{b-c+2}}$

(5–9: Oettinger, Cr. 38. 162.)

10($\int \frac{(x-1)^{a-1}\,dx}{x^b} = \frac{1^{a-1/1}\; 1^{b-a-1/1}}{1^{b-1/1}} = \int \frac{(x-1)^{b-a-1}}{x^b}\,dx$

11) $\int \frac{(x-1)^{a\pm b-1}\,dx}{x^{a\pm 2b+c}} = \frac{1^{a\pm b-1/1}\; 1^{c\pm b-1/1}}{1^{a\pm 2b+c-1/1}} = \int \frac{(x-1)^{c\pm b-1}}{x^{a\pm 2b+c}}\,dx$

12) $\int \frac{(x^b-1)^a\,dx}{x^{ab+c-1}} = \frac{1}{c}\,\frac{1^{\frac{c}{b}/1}\,1^{a/1}}{1^{\frac{c}{b}+a/1}}$

(10–12: Oettinger, Cr. 35. 13.)

13) $\int \frac{dx}{1+x} = \infty$

14) $\int \frac{x^p\,dx}{1+x} = \infty$

(13–14: Schlömilch, Stud. I. 11.)

15) $\int \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}\,dx = \mathrm{B}(p,q)$ Binet, P. 27. 123.

16) $\int \frac{dx}{1+x^2} = \frac{\pi}{4}$ Raabe, Int. 136. — Ohm, Ausw. 3.

17) $\int \frac{x^{q-1}+x^{p-1}}{x^{p+q}+1}\,dx = \frac{\pi}{p+q}\, Sec. \left\{\frac{q-p}{q+p}\,\frac{\pi}{2}\right\}$

18) $\int \frac{x^{q-1}-x^{p-1}}{x^{p+q}-1}\,dx = \frac{\pi}{p+q}\, Tang. \left\{\frac{q-p}{q+p}\,\frac{\pi}{2}\right\}$

(17–18: Raabe, Int. 147. — Ohm, Ausw. 14.)

19) $\int \frac{1+x^2}{1+x^2+x^4}\,dx = \frac{1}{2}\pi$ Raabe, Cr. 37. 356.

20) $\int \frac{dx}{x\,(x-1)^p} = \pi\, Cosec.\, p\,\pi$ Oettinger, Cr. 35. 13.

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$$21)\int\frac{dx}{x^3(x-1)^p}=\frac{1-p}{2}\,p\,\pi\,\mathrm{Cosec.}\,p\,\pi$$

$$22)\int\frac{dx}{x^3(x-1)^{p-1}}=\frac{1+p}{2}\,p\,\pi\,\mathrm{Cosec.}\,p\,\pi$$

$$23)\int\frac{dx}{x^{b+c+2}(x-1)^{b+p}}=\frac{(1+p)^{c/1}}{(1+p)^{b-1/1}\,1^{c-b+1/1}}\,\pi\,\mathrm{Cosec.}\,p\,\pi$$

Oettinger, Cr. 38. 162.

$$24)\int\frac{x^{p-1}-x^{-p-1}}{x^q-x^{-q}}\,dx=\frac{\pi}{2q}\,\mathrm{Sec.}\,\frac{p\pi}{2q}$$ Malmsten, Cr. 38. 1.

F. Algébr. irrat. fract. TABLE 32. Lim. 1 et ∞.

$$1)\int\frac{(x-1)^{p-\frac{1}{2}}}{x}\,dx=\pi\,\mathrm{Sec.}\,p\,\pi$$

$$2)\int\frac{(x-1)^{p-\frac{1}{2}}}{x^2}\,dx=\frac{1-2p}{2}\,\pi\,\mathrm{Sec.}\,p\,\pi$$

$$3)\int\frac{(x-1)^{p+\frac{1}{2}}}{x^2}\,dx=\frac{2p+1}{2}\,\pi\,\mathrm{Sec.}\,p\,\pi$$

$$4)\int\frac{(x-1)^{p+a+\frac{1}{2}}}{x^{a+b+2}}\,dx=\frac{(1+2p)^{a+1/2}(1-2p)^{b/2}}{1^{a+b+1/1}\,2^{a+b+1}}\,\pi\,\mathrm{Sec.}\,p\,\pi=\int\frac{(x-1)^{p+b-\frac{1}{2}}}{x^{a+b+2}}\,dx$$

$$5)\int\frac{(x-1)^{p-a+\frac{1}{2}}}{x^{-a+b+2}}\,dx=\frac{-2p)^{b/2}}{(2p-1)^{a/-2}}\,2^{a-b-1}\,\pi\,\mathrm{Sec.}\,p\,\pi=\int\frac{(x-1)^{p+b-\frac{1}{2}}}{x^{b-a+2}}\,dx$$

$$6)\int\frac{(x-1)^{p+a+\frac{1}{2}}}{x^{a-b+2}}\,dx=(-1)^b\,\frac{(1+2p)^{a+1/2}}{(1+2p)^{b/2}\,1^{a-b+1/1}}\,2^{b-a}\,\pi\,\mathrm{Sec.}\,p\,\pi=\int\frac{(x-1)^{p-b-\frac{1}{2}}}{x^{a-b+2}}\,dx$$

Oettinger, Cr. 38. 162.

$$7)\int\frac{(x^b-1)^{-1+\frac{a}{b}}}{x}\,dx=\frac{\pi}{b}\,\mathrm{Cosec.}\,\frac{a\pi}{b}$$ Oettinger, Cr. 35. 13.

$$8)\int\frac{(x^b-1)^{-\frac{a}{b}+c}}{x^{(c+g)b+1}}\,dx=\frac{(b-a)^{c/b}(b+a)^{g/b}}{b^{c+g/b}}\,\frac{a}{a+bc}\,\frac{\pi}{b}\,\mathrm{Cosec.}\,\frac{a\pi}{b}$$

$$9)\int\frac{(x^b-1)^{-\frac{a}{b}+c}}{x^{2bc+1}}\,dx=\left(1-\frac{a^2}{b^2}\right)\left(4-\frac{a^2}{b^2}\right)\ldots\left(c^2-\frac{a^2}{b^2}\right)\frac{a}{1^{2c/1}}\,\frac{1}{a+bc}\,\frac{\pi}{b}\,\mathrm{Cosec.}\,\frac{a\pi}{b}$$

$$10)\int\frac{(x^b-1)^{-\frac{a}{b}+g}}{x^{b(g-c)+1}}\,dx=(-1)^c\,\frac{(b-a)^{c/b}}{(b-a)^{g/b}}\,\frac{b^{g-c-1}}{1^{c-g/1}}\,\pi\,\mathrm{Cosec.}\,\frac{a\pi}{b}$$

Oettinger, Cr. 38. 162.

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11) $\int \frac{(x^b - 1)^{-\frac{a}{b}+c}}{x} dx = (-1)^c \frac{\pi}{b} \, Cosec. \frac{a\pi}{b}$ Oettinger, Cr. 38. 162.

12) $\int \frac{\left(1+\frac{1}{x^2}\right)\left(x-\frac{1}{x}\right)^{2a}}{\left(x^2+\frac{1}{x^2}\right)^{b+\frac{1}{2}}} dx = 2^{a-b-\frac{1}{2}} \frac{\Gamma(a+\frac{1}{2})\,\Gamma(b-a)}{\Gamma(b+\frac{1}{2})}$ Schlömilch, Gr. 6. 213.

13) $\int \frac{dx}{x\sqrt[q]{(x^2-1)}} = \frac{1}{2}\pi \, Cosec. \frac{\pi}{q}$

14) $\int \frac{dx}{x\sqrt[q]{(x^p-1)}} = \frac{\pi}{p} \, Cosec. \frac{\pi}{q}$

(13–14) Raabe, Int. 147. — Ohm, Ausw. 14.

15) $\int \frac{dx}{x(x-1)^{p-\frac{1}{2}}} = \pi \, Sec. \, p\pi$

16) $\int \frac{dx}{x^2(x-1)^{p-\frac{1}{2}}} = \frac{1-2p}{2} \pi \, Sec. \, p\pi$

17) $\int \frac{dx}{x^2(x-1)^{p+\frac{1}{2}}} = \frac{2p+1}{2} \pi \, Sec. \, p\pi$

18) $\int \frac{dx}{x(x^b-1)^{\frac{a}{b}}} = \frac{\pi}{b} \, Cosec. \frac{a\pi}{b}$

19) $\int \frac{dx}{x(x^b-1)^{\frac{a}{b}+c}} = (-1)^c \frac{\pi}{b} \, Cosec. \frac{a\pi}{b}$

20) $\int \frac{dx}{x^{(c-g)b+1}(x^b-1)^{\frac{a}{b}+g}} = (-1)^g \frac{(b+a)^{c/b}}{a^{g/b}} \frac{ab^{g-c-1}}{1^{c-g/1}} \frac{}{a+bc} \pi \, Cosec. \frac{a\pi}{b}$

(15–20) Oettinger, Cr. 38. 162.

1) $\int dx \sqrt{(p^2-x^2)} = \frac{1}{4} p^2 \pi$

2) $\int x dx \sqrt{(p^2-x^2)} = \frac{1}{3} p^3$

3) $\int x^2 dx \sqrt{(p^2-x^2)} = \frac{1}{16} p^4 \pi$

(1–3) Sohnke, Samml.

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$$4)\int x^{2b}\,dx\,\sqrt{(p^2-x^2)} = \frac{1^{b/2}}{2^{b+1/2}}\,p^{2b+2}\,\frac{\pi}{2}$$

$$5)\int x^{2b+1}\,dx\,\sqrt{(p^2-x^2)} = \frac{2^{b/2}}{3^{b+1/2}}\,p^{2b+3}$$

Sohnke, Samml. (4, 5)

$$6)\int dx\,\sqrt{(p^2-x^2)^{2b}} = \frac{2^{b/2}}{3^{b/2}}\,p^{2b+1}$$

$$7)\int dx\,\sqrt{(p^2-x^2)^{2b-1}} = \frac{1^{b/2}}{2^{b/2}}\,p^{2b}\,\frac{\pi}{2}$$

$$8)\int x\,dx\,\sqrt{(p^2-x^2)^{b}} = \frac{1}{b+2}\,p^{b+2}$$

$$9)\int x^{2b+1}\,dx\,\sqrt{(p^2-x^2)^{c}} = \frac{2^{b/2}}{(c+2)^{b+1/2}}\,a^{2b+c+2}$$

$$10)\int x^{2b}\,dx\,\sqrt{(p^2-x^2)^{2c-1}} = \frac{1^{b/2}\,1^{c/2}}{2^{b+c/2}}\,a^{2(b+c)}\,\frac{\pi}{2}$$

$$11)\int x^{2b}\,dx\,\sqrt{(p^2-x^2)^{2c}} = \frac{1^{b/2}\,2^{c/2}}{3^{b+c/2}}\,a^{2(b+c)+1}$$

Dienger, Cr. 38. 266. (6–11)

1) $\int \frac{dx}{1+x^2} = Arctg.\,p$ Raabe, Int. 136.

2) $\int \frac{k\,dx}{k^2+x^2} = \frac{1}{2}\pi$, pour $k=0$; Schlömilch, Gr. 11. 63.

3) $\int \frac{dx}{x^2-p^2} = -\infty$

4) $\qquad = \frac{1}{2p}\,l\,\frac{0}{2p}$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 31. — Plana, Mém. Turin. 1818. 7. Art. 1. N. 4. (3, 4)

5) $\int \frac{dx}{\sqrt{(1-x^2)}} = Arcsin.\,p$, $p^2<1$; Raabe, Int. 135.

6) $\int \frac{dx}{\sqrt{(p^2-x^2)}} = \frac{1}{2}\pi$ Cauchy, Cours. Leç. 32.

7) $\int \frac{x\,dx}{\sqrt{(p^2-x^2)}} = p$ Sohnke, Samml.

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8) $\int \frac{x^2\, dx}{\sqrt{(p^2 - x^2)}} = \frac{1}{4} p^2 \pi$

9) $\int \frac{x^3\, dx}{\sqrt{(p^2 - x^2)}} = \frac{2}{3} p^3$

10) $\int \frac{x^{2b}\, dx}{\sqrt{(p^2 - x^2)}} = \frac{1^{b/2}}{2^{b/2}} p^{2b} \frac{\pi}{2}$

11) $\int \frac{x^{2b+1}\, dx}{\sqrt{(p^2 - x^2)}} = \frac{2^{b/2}}{3^{b/2}} p^{2b+1}$

(8–11) Sohnke, Samml.

12) $\int \frac{dx}{\sqrt{(p^2 - x^2)(b^2 - x^2)}} = \frac{1}{b} F'\left(\frac{p}{b}\right)$

13) $\int \frac{x^2\, dx}{\sqrt{(p^2 - x^2)(b^2 - x^2)}} = b \left\{ F'\left(\frac{p}{b}\right) - E'\left(\frac{p}{b}\right) \right\}$

14) $\int \frac{x^4\, dx}{\sqrt{(p^2 - x^2)(b^2 - x^2)}} = \frac{1}{3} b^3 \left\{ \left(2 + \frac{p^2}{b^2}\right) F'\left(\frac{p}{b}\right) - 2\left(1 + \frac{p^2}{b^2}\right) E'\left(\frac{p}{b}\right) \right\}$

(12–14) Poisson, L. 2. 184.

15) $\int \frac{x^b\, dx}{\sqrt{(p x - x^2)}} = \frac{1^{b/2}}{2^{b/2}} p^b \pi$

16) $\int \frac{dx}{x} \sqrt{\frac{p^4 - c^2 x^2}{p^2 - x^2}} = \frac{p\pi}{2} \left[1 - \sum_0^\infty \frac{1}{2n-1} \left\{ \frac{1^{n/2}}{2^{n/2}} \left(\frac{c}{p}\right)^n \right\}^{n+1} \right]$

17) $\int \frac{dx}{\sqrt{(p - x)(2bx - x^2)}} = \frac{\pi}{\sqrt{2b}} \sum_0^\infty \left(\frac{1^{n/2}}{2^{n/2}}\right)^{n-1} \left(\frac{p}{2b}\right)^n$

(15–17) Rogner, Mater.

18) $\int \frac{dx}{(q^2 + x)\sqrt{(1 - x)}} = \frac{1}{\sqrt{(1 + q^2)}} \left\{ l \frac{\sqrt{(1+q^2)} - (\sqrt{1-p})}{\sqrt{(1+q^2)} + \sqrt{(1-p)}} + l \frac{\sqrt{(1+q^2)} + 1}{\sqrt{(1+q^2)} - 1} \right\}, p \leqq 1$; Raabe, Int. 421.

19) $\int \frac{2x^2 - b^2 - p^2}{\sqrt{\{(b^2 + p^2 - x^2)(b^2 p^2 - (b^2 + p^2) x^2 + x^4)\}}} dx = \frac{1}{2} \pi, b \geqq p$; Dienger, Gr. 10. 341.

20) $\int \frac{x^{b-1} (p - x)^{c-1}\, dx}{\{(g - h) x + hp + k\}^{b+1}} = \frac{\Gamma(b)\Gamma(c)}{\Gamma(b + c)} \frac{p^{b+c-1}}{(gp + k)^b (hp + k)^c}$ Winckler, Cr. 45. 102.

F. Algébrique. TABLE 35. Limites diverses.

1) $\int_{-1}^{0} \frac{dx}{x} = -\infty$ Cauchy, Cours. Leç. 24.

2) $\int_0^{\frac{1}{p}} (1 - p^2 x^2)^{q - \frac{1}{2}}\, dx = \frac{3^{q/1}}{2^q \Gamma(q + 1)} \frac{\pi}{2p}$ Lindmann, Stockh. Handl. 1850. III.

3) $\displaystyle\int_0^{\frac{1}{p}} \frac{x^q\,dx}{1+p^2x^2} = \frac{1}{4\,p^{q+1}}\left\{ Z'\left(\frac{q+3}{4}\right) - Z'\left(\frac{q+1}{4}\right)\right\}$ Lindmann, Stockh. Handl. 1850. III.

4) $\displaystyle\int_0^{\sqrt{(1-p^2)}} (x^2+p^2)^{\frac{2b+1}{2}}\,dx = \frac{1}{2b+2}\left\{\frac{1-p^{2b+2}}{\sqrt{(1-p^2)}} + \frac{(-1)^b\,\eta^{b+\frac{1}{2}}\,p^{2b+2}}{1^{b/1}}\,\frac{d^b}{d\eta^b}\cdot\left\{\frac{1}{2\sqrt{\eta^3}}\,l\,\frac{\sqrt{\eta}+\sqrt{(1-p^2)q}}{\sqrt{\eta}-\sqrt{(1-p^2)q}}\right\}\right.$,

après la différentiation on doit mettre q au lieu de η;

5) $\displaystyle\int_0^{\sqrt{(1-p^2)}} (x^2+p^2)^{2a}\,dx = \sum_0^\infty p^{2(2a-n)}\,(1-p^2)^{n+\frac{1}{2}}\binom{2a}{n}\frac{1}{2n+1}$

Sur les intégrales 4), 5) voyez Cauchy, Sav. Etr. 1827. 124. Note 16.

6) $\displaystyle\int_1^p \frac{dx}{\sqrt{(x^2-1)}} = l\,\{p+\sqrt{(p^2-1)}\}\ ,\ p \leqq 1$; Ohm, Ausw. 10.

7) $\displaystyle\int_1^p \frac{dx}{(q^2+x)\sqrt{(1-x)}} = \frac{-2i}{\sqrt{(1+q^2)}}\ Arctg.\ \frac{\sqrt{(p^2-1)}}{\sqrt{(1+q^2)}}$ Raabe, Int. 421.

8) $\displaystyle\int_a^b \frac{dx}{\sqrt{(x^2-a^2)(b^2-x^2)}} = \frac{1}{b}\,F'\left\{\frac{1}{b}\sqrt{(b^2-a^2)}\right\}$

9) $\displaystyle\int_a^b \frac{x^2\,dx}{\sqrt{(x^2-a^2)(b^2-x^2)}} = \frac{1}{b}\,E'\left\{\frac{1}{b}\sqrt{(b^2-a^2)}\right\}$

10) $\displaystyle\int_a^b \frac{x^4\,dx}{\sqrt{(x^2-a^2)(b^2-x^2)}} = \frac{b^3}{3}\left[2\left(1+\frac{a^2}{b^2}\right)E'\left\{\frac{1}{b}\sqrt{(b^2-a^2)}\right\} - \frac{a^2}{b^2}F'\left\{\frac{1}{b}\sqrt{(b^2-a^2)}\right\}\right]$

(8–10) Poisson, L. 2. 184.

11) $\displaystyle\int_a^\infty \frac{dx}{x^2-a^2} = \infty$

12) $\displaystyle = -\frac{1}{2a}\,l\,\frac{0}{2a}$

(11–12) Bidone, Mém. Turin. 1812. 231. Art. 2. N. 31.— Plana, Mém. Turin. 1818. 7. Art. 1. N. 4.

13) $\displaystyle\int_p^\infty \frac{dx}{x(x+1)} = l\,\frac{1+p}{p}$ Schlömilch, Gr. 4. 71. — Arndt, Gr. 10. 225.

14) $\displaystyle\int_{Sec.\,\lambda}^\infty \frac{dx}{x^2-1} = \frac{1}{2}\,l\,\frac{1+Cos.\,\lambda}{1-Cos.\,\lambda}$ Legendre, Exerc. 5. 76.

15) $\displaystyle\int_{-p}^p \frac{dx}{x} = 0$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 3. — Matzka, Gr. 20. 1.

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16) $\int_a^\infty \frac{dx}{(b-x)(x-a)^p} = \frac{\pi}{(-1)^{1-p}(b-a)^p} \operatorname{Cosec.} p\pi,\ b < a;$

17) $\int_{-\infty}^a \frac{dx}{(b-x)(x-a)^p} = \frac{\pi}{(-1)^p(b-a)^p} \operatorname{Cosec.} p\pi,\ b > a;$

Ramus, Cr. 24. 257.

18) $\int_{-a}^a \frac{(a+x)^{b-1}(a-x)^{c-1}\,dx}{\{g(a+x)+h(a-x)+2k\}^{b+c}} = \frac{\Gamma(b)\,\Gamma(c)}{2\,\Gamma(b+c)} \frac{a^{b+c-1}}{(ga+k)^b\,(ha+k)^c}$ Winckler, Cr. 45. 102.

19) $\int_p^q \frac{dx}{x} = \frac{1}{2}\, l \left(\frac{q}{p}\right)^2 - l\,\alpha$ Arndt, Gr. 10. 240.

20) $= l\,\frac{q}{p}$ Ohm, Ausw. 1, 2.

21) $= \frac{1}{2}\, l \left(\frac{p}{q}\right)^2$ Matzka, Gr. 20. 1.

22) $\int_p^q \frac{dx}{(r^2+x)\sqrt{(1-x^2)}} = \frac{1}{\sqrt{(1-r^2)}}\, l \left\{\frac{\sqrt{(1+r^2)}+\sqrt{(1-p)}}{\sqrt{(1+r^2)}+\sqrt{(1-q)}} \frac{\sqrt{(1+r^2)}-\sqrt{(1-q)}}{\sqrt{(1+r^2)}-\sqrt{(1-p)}}\right\}$ Raabe, Int. 421.

23) $\int_0^{a-2b} x(a-2b-x)^{a-1}\,dx = \frac{(a-2b)^{a-1}}{a(a+1)}$ Hoppe, Cr. 40. 142.

F. Exponent.. Forme e^{x^a}. TABLE 36. Lim. 0 et ∞ .

1) $\int e^{-x}\,dx = 1$ Cauchy, Cours. Leç. 32. — Grunert, Gr. 2. 266.

2) $\int e^{-px}\,dx = \frac{1}{p}$ Cauchy, Cours. Leç. 32. — Cisa de Grésy, Mém. Turin. 1821. 209. II. N°. 45. — Liouville, L. 4. 317. — Oettinger, Cr. 35. 13. — Grunert, Gr. 2. 266.

3) $\int p^x\,dx = -\frac{1}{lp},\ p < 1;$ Poisson, P. 19. 404. N°. 76.

4) $\int e^{px}\,dx = \infty$ Cisa de Grésy, Mém. Turin. 1821. 209. II. N. 45. — Cauchy, Cours. Leç. 24.

5) $\int e^{-pxi}\,dx = \frac{1}{pi}$

6) $\int e^{-(p+qi)x}\,dx = \frac{p-qi}{p^2+q^2}$

Meyer, Int. Déf. 99.

7) $\int e^{-x^2}\,dx = \frac{1}{2}\sqrt{\pi}$

Sur cette intégrale on peut voir: Laplace, Mém. de l'Ac. 1778. 227. § 23. — Id., ib. 1782. I. § 4. — Id., Mém. Inst. 1809. 353. § 3. — Id., Probabilités. L. 1. N. 24. — Legendre, Exerc. 2. 81. — Fourier, Chaleur. 360. — Kramp, Réfr. 3. N. 66. — Bidone, Mém. Turin. 1812. 231. Art. 1. N. 20. — Binet, P. 27. 123. — Oettinger, Cr. 35. 13. — Roberts, L. 16. 1. — Grunert, Gr. 2. 266.

8) $\int e^{-p^2x^2}\,dx = \frac{1}{2p}\sqrt{\pi}$ Bidone, Mém. Turin. 1812. 231. Tableau. — Cisa de Grésy, Mém. Turin. 1821. 209. II. N°. 35. — Boncompagni, Cr. 25. 74. — Winckler, Cr. 45. 102. — Schlömilch, Gr. 5. 90. — Id., Stud. I. 12.

9) $\int e^{px^2i}\,dx = \frac{1}{2}e^{\frac{\pi i}{4}}\sqrt{\frac{\pi}{p}}$ Schlömilch, Stud. I. 13. — Schaar, Mém. Brux. T. 24.

10) $\int e^{\frac{2\pi x^2 i}{p}}\,dx = \frac{1+i}{4}\sqrt{p}$ Schaar, Mém. Brux. T. 24.

11) $\int e^{-x^4}\,dx = \frac{1}{2}\sqrt{(\pi_1\sqrt{2\pi})}$, où $\pi_1 = 1,31102\ 87371\ 46059\ 87$; Laplace, Mém. Ac. 1782. § 5.

12) $\int e^{-x^p}\,dx = \frac{1}{p}\Gamma\left(\frac{1}{p}\right)$ Legendre, Mém. Inst. 1809. 416. N°. 81. — Id., Exerc. 2. 81.

13) $= 1^{\frac{1}{p}/1}$ Oettinger, Cr. 35. 13.

14) $\int e^{-pe^{bx}}\,dx = -\frac{1}{p}\,li\,(e^{-p})$ Winckler, Cr. 45. 102.

15) $\int e^{-x^{\frac{2}{1+2a}}}\,dx = \frac{1^{a+1/2}}{2^{a+1}}\sqrt{\pi}$ Kramp, Réfr. 3. 67.

16) $\int e^{-\frac{1}{x^2}}\,dx = \sqrt{\pi}$ Kramp, Réfr. 3. 74.

17) $\int e^{-\frac{1}{x^a}}\,dx = \frac{\sqrt[a]{a}}{(a-1)^{\frac{1}{a}/a}}$ Kramp, Réfr. 3. 73.

1) $\int e^{-x^2+p-\frac{p^2}{4x^2}}\,dx = \frac{1}{2}\sqrt{\pi}$ Cauchy, Sav. Etr. 1827. 124. Notes. N°. 2.

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2) $\int e^{-x^2-\frac{p^2}{4x^2}}\,dx = \frac{1}{2}e^{-p}\sqrt{\pi}$

Sur cette intégrale voyez: Laplace, Nouv. Bull. de la Soc. Philom. N. 43. — Id., Probab. L. 1. N°. 26. — Poisson, Nouv. Bull. de la Soc. Philom. N°. 50. — Id., P. 16. 215. N°. 8. — Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 23. — Cisa de Grésy, Mém. Turin. 1821. 209. II. § 37. — Cauchy, Sav. Etr. 1827. 124. Notes. N°. 2. — Von Schmidten, Cr. 5. 388. — Kummer, Cr. 17. 228. — Boole, L. 13. 111. — Bonnet, L. 14. 249. — Helmling, Transf. 27.

3) $\int e^{-p^2x^2-\frac{q^2}{x^2}}\,dx = \frac{1}{2p}e^{-2pq}\sqrt{\pi}$ Schlömilch, Gr. 9. 379. — Helmling, Transf. 27.

4) $\int e^{-\left(x-\frac{p}{x}\right)^{2b}}\,dx = \frac{1}{2b}\Gamma\left(\frac{1}{2b}\right)$ Boole, L. 13. 111.

5) $\int e^{-x^2+px}\,dx = \frac{1}{2}e^{\frac{1}{4}p^2}\sqrt{\pi}$ Cisa de Grésy, Mém. Turin. 1821. 209. II. N°. 39.

6) $\int e^{-(q^2x^2+pxi)}\,dx = e^{-\frac{p^2}{4q^2}}\sqrt{\frac{\pi}{2q}}$ Meyer, Int. Déf. 118; fautive selon Helmling, Transf. 6.

7) $\int e^{\left(\frac{x^2}{p^2}+\frac{q^2}{x^2}\right)ai}\,dx = \frac{1}{2}pe^{\frac{2qai}{p}+\frac{\pi i}{4}}\sqrt{\frac{\pi}{a}}$ Schlömilch, Stud. II. 24.

8) $\int e^{-x+2p\sqrt{x}}\,dx = pe^{p^2}\sqrt{\pi}-1$ V. T. 37. N°. 5.

9) $\int (e^{px}+e^{-px})\,e^{-x^2-\frac{1}{4}p^2}\,dx = \sqrt{\pi}$ Cauchy, Sav. Etr. 1827. 124. Note 2.

10) $\int (e^{-x}-1)^q\,e^{-px}\,dx = \frac{\Gamma(q+1)\,\Gamma(p)}{\Gamma(p+q+1)}$

11) $= \Delta^q\,.\,\frac{1}{p}$ pour q entier

(10, 11) Cauchy, P. 28. 147. P. 3. N°. 1.

12) $\int (e^{2px}+e^{-2px})\,e^{-x^2}\,dx = e^{p^2}\sqrt{\pi}$ Cauchy, Cours. Leç. 40.

13) $\int (e^{2px}+e^{-2px})\,e^{-q^2x^2}\,dx = \frac{\sqrt{\pi}}{2q}e^{\frac{p^2}{q^2}}$ Schlömilch, Stud. I. 12. — Helmling, Transf. 7.

14) $\int (e^{p\sqrt{x}}-e^{-p\sqrt{x}})\,e^{-x}\,dx = pe^{\frac{p^2}{4}}\sqrt{\pi}$ Helmling, Transform. 11.

1) $\int \frac{dx}{e^{2x}+1} = \frac{1}{2} l\, 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

2) $\int \frac{e^{px}-e^{-px}}{e^{\pi x}-1} dx = \frac{1}{p} - Cot.\, p$ Malmsten, Cr. 35. 55.

3) $\int \frac{e^{-qx}-e^{-px}}{1-e^{-x}} dx = Z'(p) - Z'(q)$ V. T. 5. N°. 8.

4) $\int \frac{1-e^{-px}}{1-e^{x}} dx = -A - Z'(1-p)$ V. T. 5. N°. 3.

5) $\int \frac{e^{-px}-e^{-qx}}{1-e^{x}} dx = Z'(1-p) - Z'(1-q)$ V. T. 5. N°. 4.

6) $\int \frac{dx}{1+e^{2px}} = \frac{1}{2p} l\, 2$ Lobatschewsky, Mém. Kasan. 1836. 1. II. N°. 20.

7) $\int \frac{e^{(q-p)x}+e^{-(q+p)x}}{1+e^{-2px}} dx = \frac{\pi}{2p} Sec. \frac{q\pi}{2p}$ V. T. 5. N°. 17.

8) $\int \frac{dx}{e^{px}+e^{-px}} = \frac{\pi}{4p}$ Poisson, P. 19. 404. N°. 77. — Raabe, Cr. 42. 348.

9) $\int \frac{e^{px}+e^{-px}}{e^{qx}+e^{-qx}} dx = \frac{\pi}{2q} Sec. \frac{p\pi}{2q}$

10) $\int \frac{e^{px}-e^{-px}}{e^{qx}-e^{-qx}} dx = \frac{\pi}{2q} Tang. \frac{p\pi}{2q}$

(9, 10), où $q > p$;

11) $\int \frac{p^{rx}+p^{-rx}}{q^{rx}+q^{-rx}} dx = \frac{\pi}{2r\, lq} Sec. \left(\frac{\pi}{2}\frac{lp}{lq}\right)$

12) $\int \frac{p^{rx}-p^{-rx}}{q^{rx}-q^{-rx}} dx = \frac{\pi}{2r\, lq} Tang. \left(\frac{\pi}{2}\frac{lp}{lq}\right)$

(11, 12), où $q > p$, $pq > 1$;

(9–12) Raabe, Int. 143. — Ohm, Ausw. 14.

13) $\int \frac{e^{px}-e^{-px}}{e^{2\pi x}-1} dx = \frac{1}{p} - \frac{1}{2} Cot. \frac{1}{2} p$ Malmsten, Cr. 35. 55.

14) $\int \frac{(e^{2px}+e^{-2px})(e^{2qx}+e^{-2qx})}{e^{\pi x}+e^{-\pi x}} dx = 2\frac{Cos.\, p \,.\, Cos.\, q}{Cos.\, 2p + Cos.\, 2q}$, $p < \frac{\pi}{2}$, $q < \frac{\pi}{2}$;

15) $\int \frac{(e^{2px}-e^{-2px})(e^{2qx}-e^{-2qx})}{e^{\pi x}+e^{-\pi x}} dx = 2\frac{Sin.\, p \,.\, Sin.\, q}{Cos.\, 2p + Cos.\, 2q}$

(14, 15) Poisson, P. 17. 612. N°. 21.

16) $\int \frac{e^{2px}+e^{-2px}}{e^{\pi x}+e^{-\pi x}} dx = \frac{1}{2} Sec.\, p$ Poisson, P. 18. 295. N°. 22. — Legendre, Exerc. 5. 45. — Malmsten, Cr. 38. 1 la trouve fautive $\frac{1}{2}$ Sec. $\frac{1}{2}$ p.

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17) $\int \frac{e^{2px} - e^{-2px}}{e^{\pi x} - e^{-\pi x}} dx = \frac{1}{2} Tang.\, p$ Poisson, P. 18. 295. N°. 22. — Legendre, Exerc. 5. 45. — Malmsten, Cr. 38. 1 la trouve fautive *Tang. p.*

18) $\int \frac{(e^{px} - e^{-px})(e^{qx} + e^{-qx})}{e^{\pi x} - e^{-\pi x}} dx = \frac{Sin.\, p}{Cos.\, p + Cos.\, q}, p < \pi$; Poisson, Mém. Inst. 1811. 163. N°. 26. — Id., P. 18. 295. N°. 21. — Plana, Mém. Turin. 1818. 7. Art. 4. N°. 20.

19) $\int \frac{(e^{px} + e^{-px})(e^{qx} + e^{-qx})}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} dx = 4 \frac{Cos.\, p\, .\, Cos.\, q}{Cos.\, 2p + Cos.\, 2q}$ ⎫ $, q < \pi, p < \pi$;

20) $\int \frac{(e^{px} - e^{-px})(e^{qx} - e^{-qx})}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} dx = 4 \frac{Sin.\, p\, .\, Sin.\, q}{Cos.\, 2p + Cos.\, 2q}$ ⎭ Poisson, P. 17. 612. N°. 21.

21) $\int \frac{e^{px} + e^{-px}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} = Sec.\, p$ ⎫ $, p^2 < \frac{1}{4}\pi^2$;

22) $\int \frac{e^{px} - e^{-px}}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} = Tang.\, p$ ⎭ Raabe, Cr. 42. 348.

23) $\int \left[(1 - e^{-x})^{-\frac{1}{2}} - 1\right] dx = 2\, l\, 2$ V. T. 15. N°. 1.

F. Expon.. Forme fract. à dén. polynôme. TABLE 39. Lim. 0 et ∞.

1) $\int \frac{e^x\, dx}{e^{2x} - 2e^x Cos.\, \lambda + 1} = \frac{\pi - \lambda}{2\, Sin.\, \lambda}$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

2) $\int \frac{e^{px} + e^{-px}}{e^x + e^{-x} + 2\, Cos.\, \lambda} dx = \frac{\pi\, Sin.\, p\lambda}{Sin.\, \lambda . Sin.\, p\pi}, p < 1$; Poisson, P. 18. 295. N°. 28.

3) $\int \frac{e^{(p+q)x} + e^{-(p+q)x} - e^{(p-q)x} - e^{(q-p)x}}{e^{2px} - 2 + e^{-2px}} dx = \frac{\pi}{2p} Tang. \frac{q\pi}{2p}$ ⎫ $, p > q$;

4) $\int \frac{e^{(p+q)x} + e^{-(p+q)x} + e^{(p-q)x} + e^{(q-p)x}}{e^{2px} + 2 + e^{-2px}} dx = \frac{\pi}{2p} Sec. \frac{q\pi}{2p}$ ⎭ Raabe, Int. 143.

5) $\int \frac{e^{px} + e^{-px}}{e^{qx} - 2\, Cos.\, \lambda + e^{-qx}} dx = \frac{\pi}{q} \frac{Sin.\left(p\frac{\pi - \lambda}{q}\right)}{Sin.\lambda .\, Sin.\frac{p\pi}{q}}$ V. T. 8. N°. 10.

6) $\int \frac{e^{px} + e^{-px}}{e^{qx} + 2\, Cos.\, \lambda + e^{-qx}} dx = \frac{\pi\, Sin.\frac{p\lambda}{q}}{q\, Sin.\lambda .\, Sin.\frac{p\pi}{q}}$ V. T. 8. N°. 12.

7) $\int \frac{e^{px} - 2\,Cos.\,\lambda + e^{-px}}{e^{qx} + 2\,Cos.\,\mu + e^{-qx}}\,dx = \frac{\pi\,Sin.\left(p\,\frac{\pi-\mu}{q}\right)}{q\,Sin.\,\mu\,.\,Sin.\,\frac{p\,\pi}{q}} - \frac{\pi-\mu}{q\,Sin.\,\mu}\,Cos.\,\lambda$ V. T. 8. N°. 9.

8) $\int \frac{dx}{(e^{p\sqrt{x}} + e^{-p\sqrt{x}})^2} = \frac{1}{2\,p^2}\,l\,2$ V. T. 38. N°. 6.

9) $\int \frac{dx}{(e^x + e^{-x})^{2p}} = \frac{\{\Gamma(p)\}^2}{4\,\Gamma(2\,p)}$ V. T. 5. N°. 24.

10) $\int \frac{e^{px}}{(e^{px} + 1)^3}\,dx = \frac{\pi}{8\,p}$ V. T. 38. N°. 6.

11) $\int \frac{e^{px} - 1}{(e^{px} + 1)^3}\,dx = \frac{1}{p}\,(1 - l\,2)$ V. T. 38. N°. 6.

12) $\int \frac{e^{-x}\,dx}{(e^x + e^{-x})^{2p+1}} = \frac{1}{p\,2^{2p+2}} + \frac{\{\Gamma(p)\}^2}{8\,\Gamma(2\,p)}$ V. T. 39. N°. 9.

13) $\int \frac{(e^{2px} - e^{-2px})\,(e^{\pi x} - e^{-\pi x})}{(e^{\pi x} + e^{-\pi x})^2}\,dx = \frac{p}{\pi}\,Sec.\,p\,,\ \ p < \frac{\pi}{2};$ V. T. 38. N°. 16.

14) $\int \frac{(e^{px} - e^{-px})\,(e^{qx} - e^{-qx})}{(e^{qx} + e^{-qx})^2}\,dx = \frac{p\,\pi}{2\,q^2}\,Sec.\,\frac{p\,\pi}{2\,q},\ \ q > p;$ V. T. 38. N°. 9.

15) $\int \frac{(p-\pi)\left\{e^{(\pi+p)x} - e^{-(\pi+p)x}\right\} + (p+\pi)\left\{e^{(\pi-p)x} - e^{(p-\pi)x}\right\}}{(e^{\pi x} - e^{-\pi x})^2}\,(e^{qx} - e^{-qx})\,dx = \frac{q\,Sin.\,p}{Cos.\,p + Cos.\,q},\ p + q < \pi;$ V. T. 38. N°. 18.

16) $\int \frac{e^{(q-p)x} + e^{(q-p)x}}{(e^x + e^{-x})^{p+q}}\,dx = \frac{1}{2}\,\mathrm{B}(p\,,q)$ Binet, P. 27. 123.

17) $\int \frac{e^{2px} + e^{-2px}}{(e^x + e^{-x})^{2q}}\,dx = \frac{1}{2}\,\mathrm{B}(q + p\,,\,q - p)$

18) $= \frac{\Gamma(q + p)\,\Gamma(q - p)}{2\,\Gamma(2\,q)}$

(17, 18) Meyer, Int. Déf. 312.

19) $\int \left\{e^{px} - \frac{1}{(1 + e^{-x})^p}\right\} e^{-(q+1)x}\,dx = \frac{q}{q - p + 1}\,\frac{\Gamma(q)\,\Gamma(p - q)}{\Gamma(p)}$ V. T. 22. N°. 16.

20) $\int \frac{e^x \pm Cos.\,\lambda}{(e^x + e^{-x} - 2\,Cos.\,\lambda)^2}\,dx = \frac{\pi - \lambda}{4\,Sin.\,\lambda} \mp \frac{1}{4}\,\frac{1}{1 - Cos.\,\lambda}$ V. T. 39. N°. 1.

21) $\int \frac{(e^{px} - e^{-px})\,(e^{qx} - e^{-qx})}{(e^{qx} + e^{-qx} + 2\,Cos.\,\lambda)^2}\,dx = \frac{-\,p\,\pi\,Sin.\,\frac{p\,\lambda}{q}}{q\,Sin.\,\lambda.\,Sin.\,\frac{p\,\pi}{q}},\ \ p < 1;$ V. T. 39. N°. 5.

1) $\int e^x \, dx = \infty$ Cauchy, Cours. Leç. 24.

2) $\int e^{pxi} \, dx = 0$ Poisson, P. 19. 404. N°. 69.

3) $\int e^{-x^2} \, dx = \sqrt{\pi}$ Poisson, Chal. 74. — Grunert, Gr. 2. 266.

4) $\int e^{-px^2} \, dx = \sqrt{\frac{\pi}{p}}$ Ohm, Ausw. 20.

5) $\int e^{x^2 i} \, dx = \frac{1+i}{2} \sqrt{2\pi}$ Cauchy, Lim. Imag. § 189. — Id., P. 19. 511.

6) $\int e^{px^2 i} \, dx = e^{\frac{1}{4}\pi i} \sqrt{\frac{\pi}{p}}$ Schaar, Mém. Brux. T. 25.

7) $\int e^{-x^2 i} \, dx = \frac{1-i}{2} \sqrt{2\pi}$ Cauchy, P. 19. 511.

8) $\int e^{-px^2 i} \, dx = e^{\frac{\pi i}{4}} \sqrt{\frac{\pi}{p}}$ Schlömilch, Stud. I. 13.

9) $\int e^{-x^2+2px} \, dx = e^{p^2} \sqrt{\pi}$ Poisson, Chaleur. 74. — Cisa de Grésy, Mém. Turin. 1821. 209. I. 39.

10) $\int e^{-x^2-2px} \, dx = e^{p^2} \sqrt{\pi}$ Cauchy, P. 19. 511.

11) $\int e^{-px^2-qx} \, dx = e^{\frac{q^2}{4p}} \sqrt{\frac{\pi}{p}}$ Cauchy, Exerc. 1827. p. 233. — Id., P. 19. 511.

12) $\int e^{-px^2+qx} \, dx = e^{\frac{q^2}{4p}} \sqrt{\frac{\pi}{p}}$ Ohm, Ausw. 20.

13) $\int e^{-(x^2+2px)i} \, dx = \frac{1+i}{\sqrt{2}} e^{-p^2 i} \sqrt{\pi}$ Cauchy, Lim. Imag. 190.

14) $\int e^{(px^2+qx)i} \, dx = e^{\left(\frac{\pi}{4}-\frac{q^2}{4p}\right)i} \sqrt{\frac{\pi}{p}}$ Schlömilch, Stud. I. 13.

15) $\qquad = (1+i) e^{-\frac{q^2 i}{4p}} \sqrt{\frac{\pi}{2p}}$

16) $\int e^{-(px^2+qx)i} \, dx = (1-i) e^{\frac{q^2 i}{4p}} \sqrt{\frac{\pi}{2p}}$

(15, 16) Cauchy, P. 19. 511.

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17) $\int e^{(px^2-qx)i}\,dx = (1+i)\,e^{-\frac{q^2 i}{p}}\sqrt{\frac{\pi}{2p}}$ Lejeune-Dirichlet, C. R. 8. 157.

18) $\int e^{-(x^2+p+2qx)}\,dx = e^{-p+q^2}\sqrt{\pi}$ Fourier, Chaleur. 364.

19) $\int e^{-(px^2+qx+r)}\,dx = e^{-r+\frac{q^2}{4p}}\sqrt{\frac{\pi}{p}}$, si la partie réelle de p est >0;

20) $\qquad = \infty$, si la partie réelle de p est <0;

(19, 20), où p, q et r sont imaginaires; Cauchy, Exerc. 1827. p. 283.

21) $\int e^{-px^2-\frac{q}{x^2}}\,dx = e^{-2\sqrt{pq}}\sqrt{\frac{\pi}{p}}$

22) $\int e^{-x^2-\frac{b^2}{4x^2}}\,dx = e^{-b}\sqrt{\pi}$

23) $\int e^{\left(px^2+\frac{q}{x^2}\right)i}\,dx = (1+i)\,e^{2i\sqrt{pq}}\sqrt{\frac{\pi}{2p}}$

24) $\int e^{-\left(px^2+\frac{q}{x^2}\right)i}\,dx = (1-i)\,e^{-2i\sqrt{pq}}\sqrt{\frac{\pi}{2p}}$

(21–24) Cauchy, P. 19. 511.

25) $\int e^{-(p+qi)^2\left\{x+\frac{r+si}{p+qi}\right\}^2}\,dx = \frac{\sqrt{\pi}}{p+qi}$; $p>q$;

26) $\int e^{-(qi-p)^2\left\{x-\frac{r+si}{p-qi}\right\}^2}\,dx = \frac{\sqrt{\pi}}{qi-p}$

27) $\int e^{-p\left\{x\pm\frac{r+si}{\sqrt{p}}\right\}^2}\,dx = \sqrt{\frac{\pi}{p}}$

(25–27) Cauchy, Exerc. 1827. p. 233.

28) $\int \frac{e^{-bx}}{1+e^{-ax}}\,dx = \frac{\pi}{a}\,Cosec.\,\frac{b\pi}{a}$ V. T. 22. N°. 9.

29) $\int \frac{e^{px}+e^{-px}}{e^{qx}+e^{-qx}}\,dx = \frac{\pi}{q}\,Sec.\,\frac{p\pi}{2q}$

30) $\int \frac{e^{px}-e^{-px}}{e^{qx}-e^{-qx}}\,dx = \frac{\pi}{q}\,Tang.\,\frac{p\pi}{2q}$

(29, 30) Ohm, Ausw. 14.

31) $\int \frac{e^{(p-q)x}}{(e^x+e^{-x})^{p+q}}\,dx = \frac{1}{2}\,\mathrm{B}(p,q)$ Binet, P. 27. 123.

32) $\int \frac{(1+e^{-x})^q-1}{(1+e^{-x})^{p+q}}\,dx = Z'(p+q)-Z'(q)$ V. T. 22. N°. 3.

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33) $\int\left(\frac{1}{1+e^{-x}}-\frac{1}{(1+e^{-x})^p}\right)dx = A + Z'(p)$ V. T. 22. N°. 17.

34) $\int\left(\frac{1}{(1+e^{-x})^q}-\frac{1}{(1+e^{-x})^p}\right)dx = Z'(p) - Z'(q)$ V. T. 22. N°. 18.

1) $\int_q^\infty \frac{dx}{e^x - p} = -\frac{1}{p}\, l(1 - p e^{-q})$ Hoppe, Cr. 40. 139.

2) $\int_0^1 e^{-x^q}\, dx = \frac{q\,.\,2q\,.\,3q\ldots.}{q+1\,.\,2q+1\,.\,3q+1\ldots.}$ Oettinger, Cr. 35. 13.

3) $\int_0^1 x^{-p^x}\, dx = \sum_1^\infty \frac{p^{n-1}}{n^n}$ Kummer, Cr. 17. 210.

4) $\int_0^1 e^{2a\sqrt{x}}\, dx = \frac{1}{a}\left(e^{2a} - \frac{1}{2a}e^{2a} + \frac{1}{2a}\right)$ Dienger, Cr. 46. 119.

5) $\int_0^1 p^{qx}\, dx = \frac{p^q - 1}{q\, l p}$ Meyer, Int. Déf. 98.

6) $\int_0^1 e^x\, dx = e - 1$

7) $\int_0^{2\pi} \frac{dx}{1 - p e^{xi}} = 2\pi,\ p < 1;$

(6, 7) Moigno, Int. 33, 138.

8) $\int_0^{2\pi} \frac{p e^{xi}\, dx}{p e^{xi} \pm q e^{ri}} = 0 \quad ;\ p < q;$

9) $\qquad = 2\pi\ ,\ p > q;$

(8, 9) Ohm, Ausw. 18.

10) $\int_0^q p^x\, dx = \frac{p^q - 1}{l p}$

11) $\int_{-1}^{+1} p^x\, dx = \frac{p^2 - 1}{p\, l p}$

(10, 11) Meyer, Int. Déf. 93.

12) $\int_{-\pi}^{\pi} e^{pxi}\,dx = 0$ Poisson, P. 19. 404. N°. 78.

13) $\int_{-\pi}^{\pi} (q + p e^{xi})^a\,dx = 2\pi q^a$

14) $\int_{-\pi}^{\pi} (p e^{xi})^a\,dx = 0$

15) $\int_{-\pi}^{\pi} \frac{dx}{(p e^{xi})^a} = 0$

16) $\int_{-\pi}^{\pi} (pe^{xi})^q\,dx = \frac{2}{q} p^q \operatorname{Sin.} q\pi\,,\ q < 1;$

17) $\int_{-\pi}^{\pi} \frac{dx}{(q e^{ri} + p e^{xi})^a} = 2\pi (q e^{ri})^a\,,\ p < q;$

18) $= 0 \qquad ,\ p > q;$

(13–18) Ohm, Ausw. 18.

19) $\int_{-\pi}^{\pi} e^{-axi} e^{pe^{xi}}\,dx = \frac{2\pi}{1^{a/1}} p^a$ Cauchy, Exerc. 1826. p. 205.

20) $\int_{-\pi}^{\pi} \frac{(e^{xi})^{a+1}\,dx}{\sqrt{(1 - 2 e^{xi} \operatorname{Cos.} \lambda + e^{2xi})}} = 0$ Dirksen, Ber. der Berlin. Acad. 1848. 120.

21) $\int_{-\infty}^{0} e^x\,dx = 1$ Cauchy, Cours. Leç. 24.

1) $\int \left(l \frac{1}{x}\right)^p dx = 1^{p/1}$, pour p entier; Euler, Calc. Int. 4. S. 3. 7.

2) $= \Gamma(p+1),\ \infty > p > -1$; c'est l'Intégrale Eulérienne de seconde espèce.

Voyez: Legendre, Exerc. 2. 54. — Id., Mém. Inst. 1809. 416. N°. 53. — Binet, P. 27. 123. — Cisa de Grésy, Mém. Turin. 1821. 209. I. N°. 16. — Schaar, Mém. Cour. Brux. T. 23. — Lejeune-Dirichlet, Cr. 15. 258.

3) $\int \left(l \frac{1}{x}\right)^2 dx = 2$ Plana, Cr. 17. 1.

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4) $\int \{1 + l(1+px)\}\, dx = \frac{1+p}{p}\, l(1+p)$ Dienger, Cr. 38. 331.

5) $\int lx \,.\, l(1-x)\, dx = 2 - \frac{1}{6}\pi^2$ V. T. 152. N°. 9, T. 160. N°. 9 et T. 42. N°. 2.

6) $\int llx \cdot dx = -$ A Mascheroni, Adn. p. 18.

7) $\int \{l((x))\}^p\, dx = (-1)^p\, \Gamma(p+1)\,,\ -\infty > p > -1\,;$ Ohm, Ausw. 14.

8) $\int \left(l\frac{1}{x}\right)^{p-1} dx\, ll\frac{1}{x} = \mathrm{Z}'(p)\, \Gamma(p)$ V. T. 377. N°. 1.

9) $\int l(x+q)\, dx = (1+q)\{l(1+q) - 1\} - q\,\{l(q) - 1\}$ Raabe, Cr. 25. 146.

1) $\int \frac{dx}{lx} = \mathrm{A} + l\,o$ Cisa de Grésy, Mém. Turin. 1821. 209. Art. 1. N°. 25, 27.

2) $= -\infty$ Legendre, Exerc. 3. 57.

3) $\int \frac{dx}{\left(l\frac{1}{x}\right)^p} = \frac{\pi}{\Gamma(p)}\, Cosec.\, p\,\pi$ V. T. 126. N°. 8.

4) $\int \frac{dx}{llx} = 0$ Mascheroni, Adn. p. 18.

5) $\int \frac{dx}{lp + lx} = \frac{1}{p}\, li.\, p$ Schlömilch, Gr. 5, 204.

6) $\int \frac{dx}{q - lx} = -e^q\, li.\,(e^{-q})$ V. T. 129. N°. 4.

7) $\int \frac{dx}{q + lx} = e^{-q}\, li.\,(e^q)$ V. T. 129. N°. 9.

8) $\int \frac{dx}{q^2 + (lx)^2} = \frac{1}{q}\left\{Ci.(q)\, Sin.\, q - Si.(q)\, Cos.\, q + \frac{\pi}{2}\, Cos.\, q\right\}$ V. T. 130. N°. 4.

9) $\int \frac{lx}{q^2 + (lx)^2}\, dx = Ci.(q)\, Cos.\, q + Si.(q)\, Sin.\, q - \frac{\pi}{2}\, Sin.\, q$ V. T. 130. N°. 5.

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10) $\int \frac{dx}{q^2-(lx)^2} = \frac{1}{2q}\left\{e^{-q}\,Ei.(q) - e^{q}\,Ei.(-q)\right\}$ V. T. 130. N°. 10.

11) $\int \frac{lx}{q^2-(lx)^2}\,dx = -\frac{1}{2}\left\{e^{-q}\,Ei.(q) + e^{q}\,Ei.(-q)\right\}$ V. T. 130. N°. 12.

12) $\int \frac{dx}{q^4-(lx)^4} = \frac{1}{4q^3}\left\{-e^{-q}\,Ei.(q) + e^{q}\,Ei.(-q) - 2\,Ci.(q)\,Sin.q + 2\,Si.(q)\,Cos.q - \pi\,Cos.q\right\}$ V. T. 132. N°. 1.

13) $\int \frac{lx}{q^4-(lx)^4}\,dx = \frac{1}{4q^2}\left\{e^{-q}\,Ei\,(q) + e^{q}\,Ei.(-q) - 2\,Ci.(q)\,Cos.q - 2\,Si.(q)\,Sin.q + \pi\,Sin.q\right\}$ V. T. 132. N°. 2.

14) $\int \frac{(lx)^2}{q^4-(lx)^4}\,dx = \frac{1}{4q}\left\{-e^{-q}\,Ei\,(q) + e^{q}\,Ei.(-q) + 2\,Ci.(q)\,Sin.q - 2\,Si.(q)\,Cos.q + \pi\,Cos.q\right\}$ V. T. 132. N°. 3.

15) $\int \frac{(lx)^3}{q^4-(lx)^4}\,dx = \frac{1}{4}\left\{e^{-q}\,Ei.(q) + e^{q}\,Ei.(-q) + 2\,Ci.(q)\,Cos.q + 2\,Si.(q)\,Sin.q - \pi\,Sin.q\right\}$ V. T. 132. N°. 4.

16) $\int \frac{dx}{(q-lx)^2} = -\frac{1}{q} - e^{q}\,Ei.(-q)$ V. T. 43. N°. 6.

17) $\int \frac{dx}{(q+lx)^2} = \frac{1}{q} + e^{-q}\,Ei.(q)$ V. T. 43. N°. 7.

18) $\int \frac{dx}{(lp+lx)^2} = -\frac{1}{lp} + \frac{1}{p}\,li.(p)$ V. T. 43. N°. 5.

F. Logarithm.. Forme irrat. TABLE 44. Lim. 0 et 1.

1) $\int dx\,\sqrt{l\frac{1}{x}} = \frac{1}{2}\sqrt{\pi}$ Euler, Calc. Int. 4. S. 3. 16. — Id., N. C. P. 16. 91. — Plana, Cr. 17 1.

2) $\int dx\left(l\frac{1}{x}\right)^{\frac{2a+1}{2}} = \frac{3^{a/2}}{2^{a-1}}\sqrt{\pi}$ Euler, Calc. Int. 4. S 3. 29. — Id. N. C. P. 16. 91.

3) $\int dx\,l\,l\,\sqrt[q]{\frac{1}{x}} = -A - lq$ V. T. 273. N°. 2.

4) $\int \frac{dx}{\sqrt{l\frac{1}{x}}} = \sqrt{\pi}$ Euler, Calc. Int. 4 S. 5. 211. — Id., C. P. 5. 44. — Id., N. C. P. 16. 91. — Legendre, Exerc. 2. 81. — Plana, Cr. 17. 1.

5) $\int \frac{dx}{\sqrt{\left(l\frac{1}{x}\right)^3}} = -2\sqrt{\pi}$ V. T. 126. N°. 3.

6) $\int \frac{dx}{\left(l\frac{1}{x}\right)^{a+\frac{1}{2}}} = \frac{(-2)^a \sqrt{\pi}}{1^{a/2}}$ V. T. 126. N°. 7.

7) $\int \frac{dx\, ll\frac{1}{x}}{\sqrt{l\frac{1}{x}}} = -(A + 2\,l\,2)\sqrt{\pi}$ V. T. 273. N°. 3.

8) $\int \frac{dx}{\sqrt{l\left(\sqrt[q]{\frac{1}{x}}\right)}}\, ll\left(\sqrt[q]{\frac{1}{x}}\right) = -(A + lq - 2\,l\,2)\sqrt{\pi q}$ V. T. 273. N°. 4.

F. Logarithm. TABLE 45. Limites diverses.

1) $\int_0^\infty lx\, l\frac{b^2+x^2}{a^2+x^2}\,dx = \pi(a-b) + \pi\, l\frac{b^b}{a^a}$ Schlömilch, Gr. 4. 306.

2) $\int_0^\infty dx\left(l\frac{1}{x}\right)^{\frac{2a-1}{2}} = \frac{1^{a/2}}{2^{a-1}}\sqrt{\pi}$ V. T. 142. N°. 8.

3) $\int_0^\infty dx\left(l\frac{1}{x}\right)^a = 0$ V. T. 142. N°. 9.

4) $\int_0^\infty dx\sqrt{l\left(\sqrt[p]{\frac{1}{x}}\right)} = \frac{1}{p}\sqrt{\frac{\pi}{p}}$ V. T. 142. N°. 7.

5) $\int_0^p \frac{dx}{lx} = li.(p)$ Schlömilch, Gr. 5. 204. — Mascheroni, Adn. 4 propose de l'appeler hyperlogarithme. — Voyez sur le Logarithme Integral: Soldner, Théorie et Tables d'une nouvelle fonction transcendante. Munich. 1809.

6) $\int_0^{e^p} \frac{dx}{lx} = \frac{1}{2}\,l\,a^2 + li.(p),\ p > 1$; Arndt, Gr. 10. 247.

7) $\int_1^e \frac{dx}{l\frac{1}{x}} = -\infty$

8) $\int_e^\infty \frac{dx}{l\frac{1}{x}} = -\infty$

(7, 8) Cisa de Grésy, Mém. Turin. 1821. 209. Art. 1. N°. 27.

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9) $\int_1^\infty \frac{dx}{l\frac{1}{x}} = -\infty$ Cisa de Grésy, Mém. Turin. 1821. 209. Art. 1. N°. 27.

10) $\int_1^{\frac{1}{e}} \frac{dx}{lx} = \infty$ V. T. 112. N°. 4.

11) $\int_0^{e^{-q}} \frac{dx}{lx+q} = -\infty$ V. T. 150. N°. 6.

12) $\int_{e^{-q}}^1 \frac{dx}{lx+q} = +\infty$ V. T. 149. N°. 15.

13) $\int_{\frac{1}{e}}^1 dx \sqrt{l\frac{1}{x}} = \frac{1}{2}\sqrt{\pi}$ V. T. 112. N°. 6.

14) $\int_1^e \frac{dx\, lx}{(1+lx)^2} = \frac{1}{2}e - 1$ V. T. 112. N°. 5.

15) $\int_0^{e^{-q}} \frac{dx}{lx} = A + lq + \sum_1^\infty (-1)^n \frac{q^n}{n\Gamma(n+1)} = Ei.(-q)$ V. T. 150. N°. 4.

16) $\int_0^{\sqrt[q]{\frac{1}{e}}} \frac{dx}{\sqrt{-(1+q\,l\,x)}} = \frac{1}{\sqrt[q]{e}}\sqrt{\frac{\pi}{p}}$ V. T. 150. N°. 1.

1) $\int Tang.\, x\, dx = \frac{1}{2}\, l\, 2$ Meyer, Int. Déf. 121.

2) $\int Tang.^a\, x\, dx = \sum_0^\infty \frac{(-1)^n}{a+2n+1}$

3) $\int Tang.^{2a}\, x\, dx = (-1)^a \frac{\pi}{4} + \sum_0^{a-1} \frac{(-1)^n}{2a-2n-1}$

(2–3: Arndt, Gr. 6. 484.)

4) $= (-1)^a \frac{\pi}{4} + (-1)^a \sum_1^a \frac{(-1)^n}{2n-1}$

5) $\int Tang.^{2a+1}\, x\, dx = (-1)^a \frac{1}{2} l\, 2 + (-1)^a \sum_1^a \frac{(-1)^n}{2n}$

(4–5: Cauchy, Cours. Leç. 32.)

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6) $\int Tang.^{2a+1} x\,dx = (-1)^a \frac{1}{2} l\,2 + \sum_0^{a-1} \frac{(-1)^n}{2a-2n}$ Arndt, Gr. 6. 434.

7) $\int Tang.^p\, x\,dx = \frac{1}{4}\left\{Z'\left(\frac{p+3}{4}\right) - Z'\left(\frac{p+1}{4}\right)\right\}$ V. T. 3. N°. 13.

8) $\int Tang.^p x.\, Sin.^2\, x\,dx = \frac{1+p}{8}\left\{Z'\left(\frac{p+3}{4}\right) - Z'\left(\frac{p+1}{4}\right)\right\} - \frac{1}{4}$ V. T. 46. N°. 7, 9.

9) $\int Tang.^p x.\, Cos.^2\, x\,dx = \frac{1-p}{8}\left\{Z'\left(\frac{p+3}{4}\right) - Z'\left(\frac{p+1}{4}\right)\right\} + \frac{1}{4}$ V. T. 4. N°. 18.

10) $\int Tang.^p x.\, Cos.\, 2x\,dx = \frac{1}{2} - \frac{p}{4}\left\{Z'\left(\frac{p+3}{4}\right) - Z'\left(\frac{p+1}{4}\right)\right\}$ V. T. 46. N°. 7, 9.

11) $\int Cos.^{p-1}\, 2x.\, Tang.\, x\,dx = -\frac{1}{4}\left\{Z'\left(\frac{p}{2}\right) - Z'\left(\frac{p+1}{2}\right)\right\}$ V. T. 3. N°. 1.

12) $\int (Cos.^{p-1}\, 2x - Sec.^p\, 2x)\, Cot.\, x\,dx = \pi\, Cot.\, p\pi$ V. T. 5. N°. 6.

13) $\int (Cos.^{p-1}\, 2x + Sec.^p\, 2x)\, Tang.\, x\,dx = \frac{1}{2}\pi\, Cosec.\, p\pi$ V. T. 5. N°. 1.

14) $\int (Sin.^{a+1}\, 2x - 1)\, Tang.\left(\frac{\pi}{4} + x\right) dx = -\frac{1}{2}\sum_0^a \frac{1}{n+1}$ V. T. 3. N°. 4.

15) $\int (Sin.^q\, 2x - Sin.^p\, 2x)\, Tang.\left(\frac{\pi}{4} + x\right) dx = \frac{1}{2}\left\{Z'(p+1) - Z'(q+1)\right\}$ V. T. 3. N°. 8.

16) $\int (Sin.^p\, 2x - Sin.^{1-p}\, 2x)\, Tang.\left(\frac{\pi}{4} + x\right) dx = \frac{1}{2}\pi\, Cot.\, p\pi$ V. T. 5. N°. 2.

17) $\int (Sin.^{p-1}\, 2x + Cosec.^p\, 2x)\, Tang.\left(\frac{\pi}{4} + x\right) dx = \frac{1}{2}\pi\, Cosec.\, p\pi$ V. T. 5. N°. 1.

18) $\int (Tang.^p\, x + Cot.^p\, x)\,dx = \frac{1}{2}\pi\, Sec.\, \frac{1}{2}p\pi$ V. T. 5. N°. 11.

19) $\int (Tang.^p\, x + Cot.^p\, x)\, Sin.\, 2x\,dx = \frac{1}{2}\frac{p\pi}{e^{\frac{1}{2}p\pi} - e^{-\frac{1}{2}p\pi}}$ V. T. 4. N°. 23.

1) $\int \frac{Cos.^q 2x}{Cos.^{q+2} x} dx = \frac{2^{2q}}{2q+1} \frac{\{\Gamma(q+1)\}^2}{\Gamma(2q+1)}$ V. T. 1. N°. 4.

2) $\int \frac{Cos.^b 2x \,.\, Sin.^{2a-1} x}{Cos.^{2a+b+1} x} dx = \frac{1}{2a} \frac{1^{a/1}\, 1^{b/1}}{1^{a+b/1}}$ V. T. 1. N°. 17.

3) $\int \frac{Cos.^b 2x \,.\, Sin.^{2a} x}{Cos.^{2a+b+2} x} dx = \frac{2^{b/2}}{(2a+1)^{b+1/2}}$ V. T. 1. N°. 18.

4) $\int \frac{Sin.^{2p-2} x}{Cos.^p 2x} dx = \frac{\Gamma(2p-1)\Gamma(1-p)}{2^{2p-1}\Gamma(p)}$ V. T. 4. N°. 19.

5) $\int \frac{1 - Tang.\, x}{Cos.\, 2x} Sin.^2 x\, dx = \frac{3}{4} l 2 - \frac{\pi}{8}$ V. T. 3. N°. 16.

6) $\int \frac{1 - Tang.^3 x}{Cos.\, 2x} Cos.^2 x\, dx = \frac{3}{4} l 2 + \frac{\pi}{8}$ V. T. 3. N°. 15.

7) $\int \frac{Tang.^{p-1} x - Cot.^{p-1} x}{Cos.\, 2x} dx = \frac{\pi}{2} Cot. \frac{p\pi}{2}$ V. T. 5. N°. 12.

8) $\int \frac{Cos.^{a+1} 2x - 1}{Tang.\, x} dx = -\frac{1}{2} \sum_0^a \frac{1}{n+1}$ V. T. 3. N°. 4.

9) $\int \frac{Cos.^q 2x - Cos.^p 2x}{Tang.\, x} dx = \frac{1}{2} \{Z'(p+1) - Z'(q+1)\}$ V. T. 3. N°. 8.

10) $\int \frac{Cos.^p 2x - Cos.^{1-p} 2x}{Tang.\, x} \frac{dx}{Cos.\, 2x} = \frac{1}{2}\pi\, Cot.\, p\pi$ V. T. 5. N°. 2.

11) $\int \frac{1 - Sec.^p 2x}{Tang.\, x} dx = \frac{1}{2} \{A + Z'(1-p)\}$ V. T. 5. N°. 3.

12) $\int \frac{Cos.^p 2x - Sec.^p 2x}{Tang.\, x} dx = -\frac{1}{2p} + \frac{\pi}{2} Cot.\, p\pi$ V. T. 5. N°. 5.

13) $\int \frac{Sin.^p 2x - 1}{Sin.^p 2x} Tang. \left(\frac{\pi}{4} + x\right) dx = \frac{1}{2} \{A + Z'(1-p)\}$ V. T. 5. N°. 3.

14) $\int \frac{Sin.^{2p} 2x - 1}{Sin.^p 2x} Tang. \left(\frac{\pi}{4} + x\right) dx = -\frac{1}{2p} + \frac{\pi}{2} Cot.\, p\pi$ V. T. 5. N°. 5.

15) $\int \frac{(Tang.^p x + Cot.^p x)}{Cos.\, 2x} Tang.\, x\, dx = -\frac{1}{p} + \frac{\pi}{2} Cot. \frac{1}{2} p\pi$ V. T. 5. N°. 14.

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16) $\int (Tang.^p x + Tang.^{-p} x)(Tang.^q x + Tang.^{-q} x) dx = 2\pi \frac{Cos. \frac{1}{2} p\pi . Cos. \frac{1}{2} q\pi}{Cos. p\pi + Cos. q\pi}, p<1, q<1;$ V. T. 5. N°. 10.

17) $\int (Tang.^p x - Tang.^{-p} x)(Tang.^q x - Tang.^{-q} x) dx = 2\pi \frac{Sin. \frac{1}{2} p\pi . Sin. \frac{1}{2} q\pi}{Cos. p\pi + Cos. q\pi}, p<1, q<1;$ V. T. 5. N°. 9.

18) $\int \frac{(Cos. x - Sin. x)^p}{Sin.^p x \, Sin. 2x} dx = -\frac{1}{2} \pi \, Cosec. p\pi$ V. T. 31. N°. 1.

19) $\int \frac{(Cos. x - Sin. x)^{1-p} Sin.^p x}{Cos.^3 x} dx = \frac{1-p}{2} p\pi \, Cosec. p\pi$ V. T. 31. N°. 5.

20) $\int \frac{(Tang.^p x - Cot.^p x)\,(Tang.^q x + Cot.^q x)}{Cos. 2x} dx = \frac{-\pi \, Sin. p\pi}{Cos. p\pi + Cos. q\pi}, p<1, q<1;$ V. T. 5. N°. 15.

21) $\int \left(\frac{Cos. x - Sin. x}{Cos. x}\right)^{p-1} \frac{dx}{Cos.^2 x} = \pi \, Cosec. p\pi$ V. T. 5. N°. 7.

22) $\int Sin. (p \, Tang. x) \frac{dx}{Sin. 2x} = \frac{1}{2} Si.(p)$ V. T. 192. N°. 5.

23) $\int Cos. (p \, Cot. x) \frac{dx}{Sin. 2x} = -\frac{1}{2} Ci.(p)$ V. T. 254. N°. 1.

24) $\int \frac{Cos. (q \, Tang. x) - Cos. (q \, Cot. x)}{Cos. 2x} dx = \frac{1}{2} \pi \, Sin. q$ V. T. 192. N°. 11.

F. Circ. Dir. rat. fract. à dén. binôme. TABLE 48. Lim. 0 et $\frac{\pi}{4}$.

1) $\int \frac{Tang. x}{1 + p \, Tang. x} dx = -\frac{1}{1+p^2} \left\{ l\frac{1+p}{\sqrt{2}} - \frac{1}{4} p\pi \right\}$ V. T. 6. N°. 1.

2) $\int \frac{dx}{1 - Sin. x . Cos. x} = \frac{2\pi}{3\sqrt{3}}$ V. T. 7. N°. 1.

3) $\int \frac{dx}{1 + Sin. x . Cos. x} = \frac{\pi}{3\sqrt{3}}$ V. T. 7. N°. 2.

4) $\int \frac{dx}{1 - Sin.^2 x . Cos.^2 x} = \frac{\pi}{2\sqrt{3}}$

5) $\int \frac{Sin. 2x}{1 - Sin.^2 x . Cos.^2 x} dx = \frac{\pi}{3\sqrt{3}}$

(4, 5) V. T. 48. N°. 2, 3.

6) $\int \frac{Sin.^2 x}{1 + 3 \, Sin.^2 x . Cos.^2 x} dx = \frac{1}{2} \pi$ V. T. 31. N°. 19.

7) $\int \frac{dx}{1 - 3\, Sin.^2 x.\, Cos.^2 x} = \frac{1}{2}\pi$ V. T. 7. N°. 19.

8) $\int \frac{Tang.\, x}{1 - Cos.\, \lambda.\, Sin.\, 2x}\, dx = \frac{\pi - \lambda}{2\, Tang.\, \lambda} - l\left(2\, Sin.\, \frac{1}{2}\lambda\right)$ V. T. 7. N°. 3, 5.

9) $\int \frac{Tang.^a x}{1 \pm Sin.\, 2x \sqrt{\frac{1}{2}}}\, dx = \frac{1}{8}\left\{Z'\left(\frac{a+7}{8}\right) - Z'\left(\frac{a+1}{8}\right) + Z'\left(\frac{a+5}{8}\right) - Z'\left(\frac{a+3}{8}\right)\right\} \mp$

$\mp \frac{1}{8}\sqrt{2}\left\{Z'\left(\frac{a+6}{8}\right) - Z'\left(\frac{a+2}{8}\right)\right\}$ V. T. 7. N°. 10.

10) $\int \frac{Tang.\, x}{1 - p\, Sin.\, 2x}\, dx = \frac{1}{2} l \{2(1-p)\} + \frac{p}{2} \frac{Arccos.\, (-p)}{Sin.\{Arccos.\, (-p)\}}$, pour $p < 1$;

11) $= \frac{1}{2} l\{2(p-1)\} - \frac{p}{2\sqrt{(p^2-1)}} l\{p + \sqrt{(p^2-1)}\}$, pour $p > 1$;

V. T. 7. N°. 11, 12.

12) $\int \frac{Tang.^c x\, dx}{1 + Sin.\, 2x.\, Cos.\, \frac{a\pi}{b}} = Cosec.\, \frac{a\pi}{b} \sum_0^{c-1} (-1)^{n-1} Sin.\, \frac{na\pi}{b}\left\{Z'\left(\frac{c+b+n}{2b}\right) - Z'\left(\frac{c+n}{2b}\right)\right\}$, pour $a+b$ impair;

13) $= Cosec.\, \frac{a\pi}{b} \sum_0^{\frac{1}{2}(c-1)} (-1)^{n-1} Sin.\, \frac{na\pi}{b}\left\{Z'\left(\frac{b+c-n}{b}\right) - Z'\left(\frac{c+n}{b}\right)\right\}$, pour $a+b$ pair;

V. T. 7. N°. 13, 14.

14) $\int \frac{Tang.^a x}{1 + Sin.\, x.\, Cos.\, x}\, dx = \frac{1}{3}\left\{Z'\left(\frac{a+2}{3}\right) - Z'\left(\frac{a+1}{3}\right)\right\}$ V. T. 7. N°. 8.

15) $\int \frac{Tang.^a x}{1 - Sin.\, x.\, Cos.\, x}\, dx = \frac{1}{6}\left\{Z'\left(\frac{a+5}{6}\right) - Z'\left(\frac{a+2}{6}\right) + Z'\left(\frac{a+4}{6}\right) - Z'\left(\frac{a+1}{6}\right)\right\}$ V. T. 7. N°. 9.

16) $\int \frac{Tang.^p x + Cot.^p x}{1 + Sin.\, 2x.\, Cos.\, \lambda}\, dx = \frac{\pi\, Sin.\, p\lambda}{Sin.\, p\pi\, .\, Sin\, \lambda}$ V. T. 7. N°. 7.

17) $\int \frac{1 - Tang.\, x}{1 - Sin.\, 2x.\, Cos.\, \lambda}\, dx = \frac{1}{Sin.\, \lambda} \sum_1^{\infty} \frac{Sin.\, n\lambda}{n(n+1)}$ V. T. 7. N°. 22.

18) $\int \frac{1 - Tang.\, x\, Cos.\, \lambda - Tang.^{a+1} x.\, Cos.\, \{(a+1)\lambda\} + Tang.^{a+2} x.\, Cos.\, a\lambda}{1 - Cos.\, \lambda.\, Sin.\, 2x}\, dx = \sum_1^a \frac{Cos.\, n\lambda}{n+1}$ V. T. 7. N°. 15.

19) $\int \frac{Sin.\, \lambda - Tang.^a x.\, Sin.\, \{(a+1)\lambda\} + Tang.^{a+1} x.\, Sin.\, a\lambda}{1 - Cos.\, \lambda.\, Sin.\, 2x}\, dx = \sum_1^a \frac{Sin.\, n\lambda}{n+1}$ V. T. 7. N°. 16.

F. Circ. Dir. rat. fract. à dén. composé. TABLE 49. Lim. 0 et $\frac{\pi}{4}$.

1) $\int \frac{Tang.^p x + Cot.^p x}{(1 + Sin.\, 2x.\, Cos.\, \lambda)^2} Sin.\, 2x\, dx = \frac{\pi}{Sin.\, p\pi.\, Sin.^3 \lambda} (p\, Sin.\, \lambda.\, Cos.\, p\lambda - Cos.\, \lambda.\, Sin.\, p\lambda)$ V. T. 8. N°. 1.

2) $\int \frac{Tang.^a x.\, Cos.^2 x}{(1 + Sin.\, x.\, Cos.\, x)^2} dx = 2\, \frac{1-a}{9} Z'\left(\frac{a+2}{3}\right) - \frac{2-a}{9} Z'\left(\frac{a+1}{3}\right) + \frac{a}{9} Z'\left(\frac{a}{3}\right) + \frac{1}{3}$ V. T. 8. N°. 2.

3) $\int \frac{Tang.^a x.\, Cos.^2 x}{(1 - Sin x.\, Cos.\, x)^2} dx = \frac{1-a}{9} \left\{ Z'\left(\frac{a+5}{6}\right) - Z'\left(\frac{a+2}{6}\right) \right\} - \frac{2-a}{18} \left\{ Z'\left(\frac{a+4}{6}\right) - Z'\left(\frac{a+1}{6}\right) \right\} +$

$+ \frac{a}{18} \left\{ Z'\left(\frac{a+3}{6}\right) - Z'\left(\frac{a}{b}\right) \right\} + \frac{1}{3}$ V. T. 8. N°. 3.

4) $\int \frac{Tang.^a x.\, Cos.^2 x}{(1 \pm Sin. x. Cos. x \sqrt{2})^2} dx = \frac{1-a}{8} \left\{ Z'\left(\frac{a+7}{8}\right) - Z'\left(\frac{a+3}{8}\right) \right\} - \frac{1}{8} \left\{ Z'\left(\frac{a+5}{8}\right) - Z'\left(\frac{a+1}{8}\right) \right\}$

$\pm \frac{2-a}{16} \sqrt{2} \left\{ Z'\left(\frac{a+6}{8}\right) - Z'\left(\frac{a+2}{8}\right) \right\} \mp \frac{a\sqrt{2}}{16} \left\{ Z'\left(\frac{a+4}{8}\right) - Z'\left(\frac{a}{8}\right) \right\} + \frac{1}{2}$ V. T. 8. N°. 4.

5) $\int \frac{Sin.^{p-1} 2x}{(Cos.\, x + Sin.\, x)^{2p}} dx = \frac{1}{2^{p+1}} \frac{\Gamma(p)\Gamma(\frac{1}{2})}{\Gamma(p + \frac{1}{2})}$ V. T. 4. N°. 3.

6) $\int \frac{Tang.^q x - Tang.^p x}{Cos.\, x - Sin.\, x} \frac{dx}{Cos.\, x} = Z'(1+p) - Z'(1+q)$ V. T. 3. N°. 8.

7) $\int \frac{Cot.^q x - Cot.^p x}{Cos.\, x - Sin.\, x} \frac{dx}{Cos.\, x} = Z'(1-p) - Z'(1-q)$ V. T. 5. N°. 4.

8) $\int \frac{Tang^{p-1} x + Cot.^p x}{Cos.\, x + Sin.\, x} \frac{dx}{Cos.\, x} = \pi\, Cosec.\, p\pi$ V. T. 5. N°. 1.

9) $\int \frac{Tang.^{p-1} x - Cot.^p x}{Cos.\, x - Sin.\, x} \frac{dx}{Cos.\, x} = \pi\, Cot.\, p\pi$ V. T. 5. N°. 6.

10) $\int \frac{Cot.^p x - 1}{Cos.\, x - Sin.\, x} \frac{dx}{Cos.\, x} = A - Z'(1-p)$ V. T. 5. N°. 3.

11) $\int \frac{Tang.^p x - Cot.^p x}{Cos.\, x - Sin.\, x} \frac{dx}{Cos.\, x} = \pi\, Cot.\, p\pi - \frac{1}{p}$ V. T. 5. N°. 5.

12) $\int \frac{Cos.\, 2x}{1 + Sin.\, 2x.\, Cos.\, \lambda} \frac{dx}{Cos.^2 x} = Cos.\, \lambda\, l\, \{2(1 + Cos.\, \lambda)\} + \frac{1}{2} Sin.\, \lambda - 1$ V. T. 7. N°. 6.

13) $\int \frac{Tang.^p x + Tang.^q x}{Tang.^{p+q} x + 1} \frac{dx}{Sin.\, 2x} = \frac{1}{2} \frac{\pi}{p+q} Sec. \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\}$ V. T. 31. N°. 17.

14) $\int \frac{Tang.^q x - Tang.^p x}{Tang.^{p+q} x - 1} \frac{dx}{Sin.\, 2x} = \frac{1}{2} \frac{\pi}{p+q} Tang. \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\}$ V. T. 31. N°. 18.

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15) $\int \frac{Tang.^q x - Tang.^p x}{Cos. x - Sin. x} \frac{dx}{Sin. x} = Z'(p) - Z'(q)$ V. T. 5. N°. 8.

16) $\int \frac{Tang.^p x - Tang.^{1-p} x}{Cos. x - Sin. x} \frac{dx}{Sin. x} = \pi \, Cot. p\pi$ V. T. 5. N°. 2.

17) $\int \frac{Tang.^q x - Cot.^q x}{Tang.^p x - Cot.^p x} \frac{dx}{Sin. 2x} = \frac{\pi}{4p} Tang. \frac{q\pi}{2p}$ V. T. 5. N°. 21.

18) $\int \frac{Tang.^q x + Cot.^q x}{Tang.^p x + Cot.^p x} \frac{dx}{Sin. 2x} = \frac{\pi}{4p} Sec. \frac{q\pi}{2p}$ V. T. 31. N°. 24.

19) $\int \frac{1}{Tang.^p x + Cot.^p x} \frac{dx}{Sin. 2x} = \frac{\pi}{4p}$ V. T. 5. N°. 23.

20) $\int \left\{ \frac{Sin. \lambda}{1 + Sin. 2x \, Cos. \lambda} - \frac{\lambda}{(Sin. x + Cos. x)^2} \right\} dx = 0$ V. T. 8. N°. 8.

21) $\int \frac{dx}{(Tang. x + Cot. x)^{2p} \, Sin. 2x} = \frac{\{\Gamma(p)\}^2}{4\Gamma(2p)}$ V. T. 5. N°. 24.

22) $\int \frac{Sin.^p x}{(Cos. x - Sin. x)^{p+1}} \frac{dx}{Cos. x} = -\pi \, Cosec. p\pi$ V. T. 4. N°. 8.

23) $\int \frac{Sin.^p x}{(Cos. x - Sin. x)^p} \frac{dx}{Cos.^2 x} = p\pi \, Cosec. p\pi$ V. T. 4. N°. 7.

24) $\int \frac{Sin.^p x}{(Cos. x - Sin. x)^p} \frac{dx}{Cos. 2x} = \frac{1}{2}\pi \, Cosec. p\pi$ V. T. 31. N°. 20.

25) $\int \frac{Sin.^p x}{(Cos. x - Sin. x)^{p-1}} \frac{dx}{Cos.^3 x} = \frac{1+p}{2} p\pi \, Cosec. p\pi$ V. T. 31. N°. 22.

26) $\int \frac{Sin.^p x}{(Cos. x - Sin. x)^p} \frac{dx}{Sin. 2x} = \frac{1}{2}\pi \, Cosec. p\pi$ V. T. 4. N°. 6.

27) $\int \frac{Tang.^{p-q} x + Cot.^{p-q} x}{(Tang. x + Cot. x)^{p+q}} \frac{dx}{Sin. 2x} = \frac{1}{4} B(p, q)$ V. T. 5. N°. 25.

28) $\int \frac{Tang.^{2p} x + Cot.^{2p} x}{(Tang. x + Cot. x)^{2q}} \frac{dx}{Sin. 2x} = \frac{\Gamma(p+q)\Gamma(q-p)}{q\Gamma(2q)}$ V. T. 5. N°. 26.

29) $\int \frac{Tang.^p x + Cot.^p x}{Tang.^q x + Cot.^q x + 2 Cos. \lambda} \frac{dx}{Sin. 2x} = \frac{\pi \, Sin. \frac{p\lambda}{q}}{2q \, Sin. \lambda . Sin. \frac{p\pi}{2q}}$ V. T. 8. N°. 12.

30) $\int \frac{Tang.^p x + Cot.^p x - 2 Cos. \lambda}{Tang.^q x + Cot.^q x - 2 Cos. \mu} \frac{dx}{Sin. 2x} = \frac{\pi \, Sin. \left(\frac{\pi - \mu}{q} p\right)}{2q \, Sin. \mu . Sin. \frac{p\pi}{q}} + \frac{\mu - \pi}{2q} \frac{Cos. \lambda}{Sin. \mu}$ V. T. 8. N°. 9.

1) $\int dx \sqrt{(1 - Tang.^4 x)} = \frac{\{\Gamma(\frac{1}{4})\}^2}{4\sqrt{2\pi}} - \frac{\pi\sqrt{2\pi}}{\{\Gamma(\frac{1}{4})\}^2}$ V. T. 12. N°. 9.

2) $\int \frac{Tang.^3 x}{\sqrt{Cos. 2x}} dx = \frac{1}{2}$ V. T. 13. N°. 7.

3) $\int \frac{dx \sqrt{Cos. 2x}}{Cos.^2 x} = \frac{\{\Gamma(\frac{1}{4})\}^2}{4\sqrt{2\pi}} - \frac{\pi\sqrt{2\pi}}{\{\Gamma(\frac{1}{4})\}^2}$ V. T. 12. N°. 9.

4) $\int \frac{dx \sqrt{Cos. 2x}}{Cos.^3 x} = \frac{1}{4}\pi$ V. T. 9. N°. 4.

5) $\int \frac{Cos.^{a-\frac{1}{2}} 2x}{Cos.^{2a+1} x} dx = \frac{(a+1)^{a/1}}{1^{a/1}} \frac{\pi}{2^{2a+1}}$ V. T. 9. N°. 5.

6) $\int \frac{Sin.^{2a-1} x}{Cos.^{2a+2} x} dx \sqrt{Cos. 2x} = \frac{2^{a-1/2}}{3^{a/2}}$ V. T. 9. N°. 6.

7) $\int \frac{Sin.^{2a} x}{Cos.^{2a+3} x} dx \sqrt{Cos. 2x} = \frac{3^{a-1/2}}{4^{a/2}} \frac{\pi}{4}$ V. T. 9. N°. 7.

8) $\int \frac{Sin.^{2a-1} x}{Cos.^{2a+2b-1} x} Cos.^{b-\frac{1}{2}} 2x\, dx = \frac{2^{a-1/2}\, 1^{b/2}}{1^{a+b/2}}$ V. T. 9. N°. 10.

9) $\int \frac{Sin.^{2a} x}{Cos.^{2a+2b} x} Cos.^{b-\frac{1}{2}} 2x\, dx = \frac{1^{a/2}\, 1^{b/2}}{1^{a+b/2}} \frac{\pi}{2^{a+b+1}}$ V. T. 9. N°. 8.

10) $\int \frac{Sin.^{2p} x}{Cos.^{p+\frac{1}{2}} 2x . Cos. x} dx = \frac{1}{2}\pi\, Sec.\, p\pi$ V. T. 12. N°. 17.

11) $\int \frac{(Cot. x - 1)^{p-\frac{1}{2}}}{Cos.^2 x} dx = \frac{2p+1}{2}\pi\, Sec.\, p\pi$ V. T. 32. N°. 3

12) $\int \frac{(Cot. x - 1)^{p-\frac{1}{2}}}{Sin. 2x} dx = \frac{1}{2}\pi\, Sec.\, p\pi$ V. T. 32. N°. 1.

13) $\int \frac{Tang.^{p-1} x + Tang.^{q-1} x}{Cos.^{\frac{p+q}{2}} 2x} Cos.^{p+q-2} x\, dx = \frac{1}{2} Cos. \left(\frac{q-p}{4}\pi\right) . Sec. \left(\frac{q+p}{4}\pi\right) B(\frac{1}{2}p, \frac{1}{2}q)$ V. T. 12. N°. 23.

14) $\int \frac{Tang.^{p-1} x - Tang.^{q-1} x}{Cos.^{\frac{p+q}{2}} 2x} Cos.^{p+q-2} x\, dx = \frac{1}{2} Sin. \left(\frac{q-p}{4}\pi\right) . Sec. \left(\frac{q+p}{4}\pi\right) B(\frac{1}{2}p, \frac{1}{2}q)$ V. T. 12. N°. 24.

15) $\int (\sqrt{Tang. x} + \sqrt{Cot. x})\, dx = \frac{1}{2}\pi\sqrt{2}$ V. T. 15. N°. 2.

16) $\int (Cos. 2x)^{a-\frac{1}{2}} . Cos. (p\, Tang. x) \frac{dx}{Cos.^{2a+1} x} = \frac{1^{a/2}}{2^{a+2}\, 1^{a/1}} \left\{1 + \sum_1^{\infty} (-1)^n \frac{(\frac{1}{2}p)^{2n}}{1^{n/1}(a+1)^{n/1}}\right\}$ V. T. 192. N°. 6.

1) $\int \frac{Cos.^{-\frac{1}{2}} 2x - 1}{Tang. x} dx = \frac{1}{2} l 2$ V. T. 15. N°. 5.

2) $\int \frac{dx}{Cos. x \sqrt{Cos. 2x}} = \frac{1}{2} \pi$ V. T. 12. N°. 10.

3) $\int \frac{Sin. x}{Cos.^2 x \sqrt{Cos. 2x}} dx = 1$ V. T. 12. N°. 11.

4) $\int \frac{Sin.^{2a-1} x}{Cos.^{2a} x \sqrt{Cos. 2x}} dx = \frac{2^{a-1/2}}{1^{a/2}}$ V. T. 12. N°. 13.

5) $\int \frac{Sin.^{2a} x}{Cos.^{2a+1} x \sqrt{Cos. 2x}} dx = \frac{3^{a-1/2}}{2^{a/2}} \frac{\pi}{2}$ V. T. 12. N°. 12.

6) $\int \frac{dx}{Cos.^2 x} \sqrt{\frac{Cos.^2 x - p. Sin.^2 x}{Cos. 2x}} = 1 - \sum_1^\infty \left\{\frac{1^{n-1/2}}{2^{n/2}}\right\}^2 (2n-1) p^{2n}$ V. T. 12. N°. 14.

7) $\int \frac{dx}{Cos.^2 x} \sqrt{\frac{Cos.^4 x - p^2 Sin.^4 x}{Cos. 2x}} = \frac{c F'(c) + b F'(b)}{(b+c)^2} + \frac{b-c}{(b+c)^2} \{E'(b) - E'(c)\}$,

où $2c^2 = \frac{(1-\sqrt{p})^2}{1+p}$, $2b^2 = \frac{(1+\sqrt{p})^2}{1+p}$; V. T. 13. N°. 8.

8) $\int \frac{Sin.^{p-\frac{1}{2}} 2x}{Cos.^p 2x. Cos. x} dx = \frac{2}{2p-1} \frac{\Gamma(p+\frac{1}{2}) \Gamma(1-p)}{\sqrt{\pi}} Sin. \left\{\frac{2p-1}{4} \pi\right\}$ V. T. 12. N°. 18.

9) $\int \frac{Sin.^{2a-1} x}{Cos.^{2a-2b+2} x. Cos.^{b-\frac{1}{2}} 2x} dx = (-1)^{b-1} \frac{2^{a/2}}{1^{b-1/2} 3^{a-b/2}} \frac{1}{2a}$ V. T. 12. N°. 19.

10) $\int \frac{Sin.^{2a} x}{Cos.^{2a-2b+3} x. Cos.^{b-\frac{1}{2}} 2x} dx = (-1)^{b-1} \frac{3^{a-1/2}}{1^{b-1/2} 4^{a-b/2}} \frac{\pi}{4}$ V. T. 12. N°. 21.

11) $\int \frac{(Cos. x - Sin. x)^{a-\frac{1}{2}}}{Cos.^{a+1} x} \frac{Tang.^b x}{\sqrt{Sin. x}} dx = \frac{1^{a/2} 1^{b/2}}{2^{a+b/2}}$ V. T. 15. N°. 8.

12) $\int \frac{(Cos. x - Sin. x)^{a-\frac{1}{2}}}{Cos.^{a+1} x \sqrt{Sin. x}} dx = \frac{1^{a/2}}{2^{a/2}} \pi$ V. T. 15. N°. 7.

13) $\int \frac{(Sin. x - Cos. x)^{p+\frac{1}{2}}}{Sin^{p+\frac{1}{2}} x. Cos^2 x} dx = \frac{2p+1}{2} \pi Sec. p\pi$ V. T. 11. N°. 1.

14) $\int \frac{(Sin. x - Cos. x)^{p-\frac{1}{2}}}{Sin.^{p+\frac{1}{2}} x. Cos. x} dx = \pi Sec. p\pi$ V. T. 11. N°. 2.

15) $\int \frac{1}{\sqrt[3]{Sin.^2 x. Cos. x}} \frac{dx}{\sqrt{Cos. 2x}} = \frac{3}{\sqrt[4]{3}} F'\left(Sin. \frac{\pi}{12}\right)$ V. T. 15. N°. 11.

16) $\int \frac{1}{\sqrt[3]{Sin.x.Cos.^2 x}} \frac{dx}{\sqrt{Cos.2x}} = \frac{1}{\sqrt[4]{3}} F'\left(Cos.\frac{\pi}{12}\right)$ V. T. 15. N°. 12.

17) $\int \frac{\sqrt[3]{Tang.x}}{\sqrt{Cos.2x}} \frac{dx}{Cos.x} = \frac{1-\sqrt{3}}{\sqrt[4]{3}} F'\left(Cos.\frac{\pi}{12}\right) + \frac{2\sqrt{3}}{\sqrt[4]{3}} E'\left(Cos.\frac{\pi}{12}\right)$ V. T. 12. N°. 15.

18) $\int \frac{\sqrt[3]{Tang.^2 x}}{\sqrt{Cos.2x}} \frac{dx}{Cos.x} = \frac{3\sqrt{3}}{\sqrt[4]{3}} E'\left(Sin.\frac{\pi}{12}\right) - \frac{3+2\sqrt{3}}{2\sqrt[4]{3}} F'\left(Sin.\frac{\pi}{12}\right)$ V. T. 12. N°. 16.

F. Circ. Dir. irrat. fract. à dén. à fact. binômes. TABLE 52. Lim. 0 et $\frac{\pi}{4}$.

1) $\int \frac{dx}{Cos.x\sqrt{Sin.x(Cos.x - Sin.x)}} = \pi$ V. T. 15. N°. 6.

2) $\int \frac{dx}{Cos.x\sqrt{Sin.x(Cos.x + p\,Sin.x)}} = \frac{2}{\sqrt{p}}\, l\{\sqrt{p} + \sqrt{(1+p)}\}$ V. T. 15. N°. 10.

3) $\int \frac{1}{a\,Cos.x - b\,Sin.x} \frac{dx}{\sqrt{Sin.x(Cos.x - Sin.x)}} = \frac{\pi}{\sqrt{a(a-b)}}$, b ni $= a$, ni $= 0$; V. T. 15. N°. 15.

4) $\int \frac{Sin.^a x}{Cos.^{a+1} x} \frac{dx}{\sqrt{Cos.x(Cos.x - Sin.x)}} = \frac{2^{a/2}}{3^{a/2}}\, 2$ V. T. 12. N°. 2.

5) $\int \frac{Sin.^a x}{Cos.^{a+1} x} \frac{dx}{\sqrt{Sin.x(Cos.x - Sin.x)}} = \frac{1^{a/2}}{2^{a/2}}\, \pi$ V. T. 15. N°. 9.

6) $\int \frac{dx}{Cos.x\sqrt{(Cos.^2 x + p\,Sin.^2 x)}} = \frac{1}{\sqrt{p}}\, l\{\sqrt{p} + \sqrt{(1+p)}\}$ V. T. 12. N°. 25.

7) $\int \frac{Tang.x}{\sqrt{(p\,Cos.^2 x + Sin.^2 x)}} \frac{dx}{\sqrt{Cos.2x}} = \frac{\pi}{2} - Arctg.\,p$ V. T. 16. N°. 6.

8) $\int \frac{\sqrt{Cot.x} - 1}{Cos.x - Sin.x} \frac{dx}{Cos.x} = l\,4$ V. T. 15. N°. 5.

9) $\int \frac{(1 - Tang.x)^{-\frac{1}{2}} - 1}{Sin.2x}\, dx = l\,2$ V. T. 15. N°. 1.

10) $\int \frac{1}{Tang.^2 x + Cot.^2 x} \frac{dx}{\sqrt{Cos.2x}} = \frac{\pi}{8}$ V. T. 16. N°. 3.

11) $\int \frac{Cot.^2 x}{Tang.^2 x + Cot.^2 x} \frac{dx}{\sqrt{Cos.2x}} = \frac{\pi}{8} + \frac{1}{4}\sqrt{2}.\, F'\left(Sin.\frac{\pi}{4}\right)$ V. T. 16. N°. 4.

12) $\int \frac{Sin^{p-\frac{1}{2}} x}{(Cos.x - Sin.x)^{p+\frac{1}{2}}} \frac{dx}{Cos.x} = \pi\, Sec.\, p\pi$ V. T. 12. N°. 6.

13) $\int \frac{Sin.^{p-\frac{1}{2}} x}{(Cos. x - Sin. x)^{p-\frac{1}{2}}} \frac{dx}{Cos.^2 x} = \frac{2p-1}{2} \pi \, Sec. \, p\pi$ V. T. 12. N°. 7.

14) $\int \frac{1}{(Cot. x - 1)^{p+\frac{1}{2}}} \frac{dx}{Cos.^2 x} = \frac{2p+1}{2} \pi \, Sec. \, p\pi$ V. T. 32. N°. 17.

15) $\int \frac{1}{(Cot. x - 1)^{p+\frac{1}{2}}} \frac{dx}{Sin. 2x} = \frac{1}{2} \pi \, Sec. \, p\pi$ V. T. 32. N°. 15.

16) $\int \frac{Sin.^{p-\frac{1}{2}} 2x}{(Cos. x - Sin. x)^{2p}} \frac{dx}{Cos. x} = \frac{2^{\frac{1}{2}-p}}{1-2p} \frac{\Gamma(p+\frac{1}{2}) \Gamma(1-p)}{\sqrt{\pi}}, \; p < \frac{1}{2}$; V. T. 12. N°. 4.

17) $\int \left(\frac{Sin. x}{Cos. x - Sin. x}\right)^p \frac{dx}{Cos. x \sqrt{Sin. x (Cos. x - Sin. x)}} = \pi \, Sec. \, p\pi, \; p < \frac{1}{2}$; V. T. 15. N°. 17.

18) $\int \frac{Sin.^{\frac{1}{2}q} x . Cos.^q x \, dx}{\left\{(Cos. x - Sin. x) \, Cos. (x+\lambda) . Cos. (x-\lambda)\right\}^{\frac{q+1}{2}}} = \frac{1}{q} Sin. q\lambda . Cosec. \lambda \, B\left(\frac{q+2}{2}, \frac{1-q}{2}\right)$ V. T. 16. N°. 8.

19) $\int \frac{Sin.^a x . Cos.^{1-\frac{1}{2}a} 2x}{(Cos.^2 x - p^2 Sin.^2 x)^{\frac{1}{2}a-1}} \frac{dx}{Cos.^4 x} = \frac{\Gamma\left(\frac{a+1}{2}\right) \Gamma\left(2-\frac{a}{2}\right)}{\sqrt{\{\pi(a-1)(a-3)(a-5)\}}} \left\{\frac{1+(a-3)p+p^2}{(1+p)^{a-3}} - \frac{1-(a-3)p+p^2}{(1-p)^{a-3}}\right\}$ V. T. 9. N°. 14.

F. Circ. Dir. rat. ent. à un fact. TABLE 53. Lim. 0 et $\frac{\pi}{2}$.

1) $\int Sin. \, bx \, dx = 0$, pour $b = 4a$;

2) $= \frac{1}{4a+1}$, pour $b = 4a+1$;

3) $= \frac{1}{2a+1}$, pour $b = 4a+2$;

4) $= \frac{1}{4a+3}$, pour $b = 4a+3$;

5) $\int Cos. \, bx \, dx = 0$, pour $b = 4a$;

6) $= \frac{1}{4a+1}$, pour $b = 4a+1$;

7) $= 0$, pour $b = 4a+2$;

8) $= \frac{-1}{4a+3}$, pour $b = 4a+3$;

(1–8) Meyer, Int. Déf. 97.

9) $\int Sin.\, x\, dx = 1$

10) $\int Cos.\, x\, dx = 1$

Meyer, Int. Déf. 97.

11) $\int Cos.^2\, x\, dx = \frac{1}{4}\pi$ Liouville, Cr. 13. 219.

12) $\int Sin.^{2a}\, x\, dx = \frac{1^{a/2}}{2^{a/2}}\frac{\pi}{2}$ Cauchy, Cours. Leç. 32. — Poisson, Chal. 78. — Dienger, Cr. 38. 331.

13) $\int Sin.^{2a+1}\, x\, dx = \frac{2^{a/2}}{3^{a/2}}$ Cauchy, Cours. Leç. 32. — Dienger, 38. 266. — Oettinger, Cr. 38. 162.

14) $= \frac{(1^{a/1})^2}{1^{2a+1/1}} 2^{2a}$

15) $= \frac{1^{a/1}}{1^{a+\frac{1}{2}/1}} \frac{1}{2}\sqrt{\pi}$

Lobatschewsky, Mém. Kasan. 1835. 211.

16) $\int Cos.^{2a}\, x\, dx = \frac{1^{a/2}}{2^{a/2}}\frac{\pi}{2}$ Cauchy, Cours. Leç. 32.

17) $\int Cos.^{2a+1}\, x\, dx = \frac{2^{a/2}}{3^{a/2}}$ Cauchy, Cours. Leç. 32. — Oettinger, Cr. 38. 162.

18) $\int Sin.^p\, x\, dx = 2^{p-2}\frac{\{\Gamma(\frac{1}{2}p)\}^2}{\Gamma(p)}$ Lobatschewsky, Mém. Kasan. 1835. 1.

19) $= \frac{\sqrt{\pi}}{2(\frac{1}{2}p)^{\frac{1}{2}/-1}}$ Lobatschewsky, Mém. Kasan. 1835. 211.

20) $\int Sin.^{p-1}\, x\, dx = \frac{1}{2}\sqrt{\pi}\frac{\Gamma(\frac{1}{2}p)}{\Gamma\left(\frac{1+p}{2}\right)}$ Raabe, Int. 222.

21) $\int Cos.^p\, x\, dx = \frac{\pi}{2^{p+1}}\frac{\Gamma(p+1)}{\{\Gamma(\frac{1}{2}p+1)\}^2}$ Lobatschewsky, Mém. Kasan. 1835. 211.

22) $= 2^{p-1}\frac{\{\Gamma(\frac{1}{2}p+1)\}^2}{\Gamma(p+1)}$ Serret, L. 8. 1.

23) $\int Cos.^{p-1}\, x\, dx = \frac{1}{2}\sqrt{\pi}\frac{\Gamma(\frac{1}{2}p)}{\Gamma\left(\frac{1+p}{2}\right)}$ Raabe, Int. 222.

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24) $\int Sin.^{2a+1} 2\,x\,dx = \frac{1^{a/1}.1^{a/1}}{1^{2a+1/1}} 2^{2a}$ Oettinger, Cr. 38. 162.

25) $\int Tang.^{2p-1} x\,dx = \frac{1}{2}\pi\, Cosec.\, p\,\pi\ ,\ 1 > p > 0$; Bonnet, L. 6. 238. — Oettinger, Cr. 38. 162.

26) $\int Tang.^{p} x\,dx = \frac{1}{2}\pi\, Sec.\, \frac{1}{2} p\,\pi\ ,\ 1 > p > 0$; Cauchy, Exerc. 1826. p. 205. — Schlömilch, Gr. 6. 200.

F. Circ. Dir. rat. ent.. Fact. $Sin.^a x$ et un autre. TABLE 54. Lim. 0 et $\frac{\pi}{2}$.

1) $\int Sin.^q x.\, Sin.\, \{(q+2)\,x\}\,dx = \frac{1}{q+1}\, Cos.\, \frac{q\pi}{2}$ Serret, L. 8. 1.

2) $\int Sin.^{q-1} x.\, Sin.\, q\,x\,dx = \frac{1}{1-q}\, Cos.\, \frac{q\,\pi}{2}$ Serret, L. 8. 489.

3) $\int Sin.^{2a+1} x.\, Sin.\{(2\,b+1)\,x\}\,dx = (-1)^b \frac{\pi}{2^{2a+2}} \frac{(a+b+2)^{a-b/1}}{1^{a-b/1}},\ a > b$; Jacobi, Cr. 15. 1.

4) $= 0,\quad a < b$; Ohm, Ausw. 13.

5) $\int Sin.^{2a} x.\, Sin.\, \{(2b+1)x\}\,dx = \frac{1^{2a/1}}{2^2-(2b+1)^2.\,4^2-(2b+1)^2 \ldots\ldots (2a)^2-(2b+1)^2} \frac{1}{2b+1}$ Ohm, Ausw. 13.

6) $\int Sin.^{2a} x.\, Sin.\, p x\, dx = \frac{1}{p} \frac{1^{2a/1}}{2^2-p^2.\,4^2-p^2 \ldots (2a)^2-p^2} \left\{1 - Cos.\frac{p\pi}{2}\left(1 - \frac{p^2}{1.2} - \frac{p^2.\,2^2-p^2}{1.\,2.\,3.\,4} - \ldots - \frac{p^2.\,2^2-p^2 \ldots (2a-2)^2-p^2}{1^{2a/1}}\right)\right\}$

7) $\int Sin.^{2a+1} x.\, Sin.\, p x\, dx = \frac{1}{p} Cos.\frac{p\pi}{2} \frac{1^{2a+1/1}}{1^2-p^2.\,3^2-p^2 \ldots (2a+1)^2-p^2} \left\{\frac{p^2}{1} + \frac{p^2}{1}\frac{1^2-p^2}{2\,.\,3} + \ldots + \frac{p^2.\,1-p^2 \ldots (2a-1)^2 p^2}{1^{2a+1/1}}\right\}$

Ces deux formules se trouvent chez Raabe, Int. 153.

8) $\int Sin.^q x.\, Cos.\, \{(q+2)\,x\}\,dx = \frac{-1}{q+1}\, Sin.\, \frac{q\,\pi}{2}$ Serret, L. 8. 1.

9) $\int Sin.^{q-2} x.\, Cos.\, q\,x\,d\,v = \frac{1}{q-1}\, Sin.\, \frac{q\,\pi}{2}$ Serret, L. 8. 489.

10) $\int Sin.^{2a} x.\, Cos.\, 2\,b\,x\,dx = (-1)^b \frac{\pi}{2^{2a+1}} \frac{(a+b+1)^{a-b/1}}{1^{a-b/1}},\ a > b$; Jacobi, Cr. 15. 1.

11) $= 0,\quad a < b$;

12) $\int Sin.^{2a+1} x.\, Cos.\, 2bx\,dx = \frac{1^{2a+1/1}}{1^2-(2b)^2.\,3^2-(2b)^2 \ldots\ldots (2a+1)^2-(2b)^2}$ } Ohm, Ausw. 13.

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13) $\int Sin.^{2a}x.Cos.px dx = \frac{1}{p} Sin. \frac{p\pi}{2} \frac{1^{2a/1}}{2^2-p^2.4^2-p^2....(2a)^2-p^2} \left\{1-\frac{p^2}{1.2}-\frac{p^2.2^2-p^2}{1.2.3.4}-....-\frac{p^2.2^2-p^2....(2a-2)^2-p^2}{1^{2a/1}}\right\}$

14) $\int Sin.^{2a+1}x.Cos.px dx = \frac{1^{2a+1/1}}{1^2-p^2.3^2-p^2...(2a+1)^2-p^2} \left\{1-\frac{1}{p} Sin. \frac{p\pi}{2}\left(\frac{p^2}{1}+\frac{p^2 1^2-p^2}{1\ 2.3}+....+\frac{p^2.1^2-p^2....(2a-1)^2-p^2}{1^{2a+1/1}}\right)\right\}$

Sur ces deux formules voyez Raabe, Int. 153.

15) $\int Sin.^p x. Cos. \left\{p\left(\frac{\pi}{2}-x\right)\right\} dx = \frac{\pi}{2p+1}$ Cauchy, Exerc. 1826. p. 255.

1) $\int Cos.^{q-1} x. Sin. \{(q+1)x\} dx = \frac{1}{q}$ Serret, L. 8. 1. — Id., L. 8. 489. — Kummer, Cr. 20. 1.

2) $\int Cos.^a x. Sin. ax\, dx = \frac{1}{2^{a+1}} \sum_{1}^{a} \frac{2^a}{a}$ Serret, L. 8. 1.

3) $\int Cos.^q x. Sin. \{(q+2b)x\} dx = (q+2b) \sum_{1}^{b} (-1)^{n-1}\, 2^{2n-2} \frac{(q+b+1)^{n-1/1}(b-n+1)^{n-1/1}}{(q+1)^{2n/1}}$ Serret, L. 8. 1.

4) $\int Cos.^{2a} x. Sin. px dx = \frac{1}{p} \frac{1^{2a/1}}{2^2-p^2.4^2-p^2...(2a)^2-p^2} \left\{1-Cos. \frac{p\pi}{2}-\frac{p^2}{1.2}-\frac{p^2.2^2-p^2}{1.2.3.4}-....-\frac{p^2.2^2-p^2....(2a-2)^2-p^2}{1^{2a/1}}\right\}$

5) $\int Cos.^{2a+1}x. Sin. px dx = \frac{1}{p} \frac{1^{2a+1/1}}{1^2-p^2.3^2-p^2....(2a+1)^2-p^2} \left\{p\, Sin. \frac{p\pi}{2}-\frac{p^2}{1}-\frac{p^2.1^2-p^2}{1.2.3}-....-\frac{p^2.1^2-p^2....(2a-1)^2-p^2}{1^{2a+1/1}}\right\}$

Raabe, Int. 153. déduit ces deux formules.

6) $\int Cos.^p x. Cos. qx\, dx = \frac{\pi}{2^{p+1}} \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+q}{2}+1\right)\Gamma\left(\frac{p-q}{2}+1\right)}$ Cauchy, Lim. Imag. 124. — Catalan, L. 5. 110. — Serret, L. 8. 1. — Id., L. 8. 489. — Kummer, Cr. 17. 210. — Id., Cr. 20. 1. — Lobatschewsky, Mém. Kasan. 1835. 211. — Schlömilch, Stud. I. 24.

7) $= \frac{\pi}{(p+1)\, 2^{p+1}\, B\left(\frac{p+q}{2}+1, \frac{p-q}{2}+1\right)}$ Serret, L. 8. 1. — Binet, P. 27. 123.

8) $\int Cos.^q x. Cos. \{(q+2b)x\} dx = 0$ Poisson, P. 19. 404. N°. 76. — Id., Conn. des Temps. 1836. p. 1. — Serret, L. 8. 1.

9) $\int Cos.^q x. \{Cos. (q-2b) x\} dx = \frac{\pi}{2^{q+1}} \frac{(q-b+1)^{b/1}}{1^{b/1}}, \; q > b-1$; Poisson, P. 19. 404. N°. 76. — Binet, P. 27. 123. — Serret, L. 8. 1. — Jacobi, Cr. 15. 1.

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10) $\int Cos.^a x . Cos. \{(a+2p)x\} dx = \frac{Sin. p\pi}{2^{a+1}} \frac{1^{a/1}\Gamma(p)}{\Gamma(a+p+1)}$ Kummer, Cr. 20. 1.

11) $\int Cos.^p x . Cos. \{(2b-p)x\} dx = \frac{\pi}{2^{p+1}} \frac{\Gamma(p+1)}{1^{b/1}\Gamma(1+p-b)}$ Cauchy, Bull. d. Sc. Math. de Férussac. 1825. N°. 250. — Hill, Cr. 7. 102.

12) $\int Cos.^a x . Cos. 2bx\, dx = \frac{\pi}{2^{a+1}} \frac{1^{a/1}}{\Gamma(\frac{1}{2}a+b+1)\Gamma(\frac{1}{2}a-b+1)}$ Lobatschewsky, Mém. Kasan. 1835. 211.

13) $\int Cos.^q x . Cos. qx\, dx = \frac{\pi}{2^{q+1}}$ Serret, L. 8. 1 — Id., L. 8. 489. — Lobatschewsky, Mém. Kasan. 1835. 211. — Poisson. P. 19. 404. N°. 76. (la trouve faut.) — Id., Conn. des Temps. 1836. p. 1.

14) $\int Cos.^{q-1} x . Cos. \{(q+1)x\} dx = 0$ Cauchy, Exerc. 1826. p. 205. — Serret, L. 8. 1. — Id., L. 8. 489. — Kummer, Cr. 20. 1. — Lindmann, Stockh. Handl. 1850. II.

15) $\int Cos.^{a+b} x . Cos. \{(a-b)x\} dx = \frac{\pi}{2^{a+b+1}} \frac{1^{a+b/1}}{1^{a/1}\,1^{b/1}}$ Cauchy, Lim. Imag. 123. — Oettinger, Cr. 38. 216.

16) $\int Cos.^{2a} x . Cos. 2bx\, dx = \frac{\pi}{2^{2a+1}} \frac{1^{2a/1}}{1^{a+b/1}\,1^{a-b/1}}$ Cauchy, Exerc. de Math. 2. 368. — Oettinger, Cr. 38. 216.

17) $= \frac{\pi}{2^{2a+1}} \frac{(a+b+1)^{a-b/1}}{1^{a-b/1}}$

18) $\int Cos.^{2a+1} x . Cos. \{(2b+1)x\} dx = \frac{\pi}{2^{2a+2}} \frac{(a+b+2)^{a-b/1}}{1^{a-b/1}}$

(17–18) Jacobi, Cr. 15. 1.

19) $\int Cos.^{2a} x . Cos. px\, dx = \frac{1^{2a/1}}{2^2-p^2 \,.\, 4^2-p^2 \ldots (2a)^2-p^2} \frac{1}{p} Sin. \frac{1}{2} p\pi$

20) $\int Cos.^{2a+1} x . Cos. px\, dx = \frac{1^{2a+1/1}}{1^2-p^2 \,.\, 3^2-p^2 \ldots (2a+1)^2-p^2} Cos. \frac{1}{2} p\pi$

(19–20) Raabe, Int. 153.

21) $\int Cos.^{p-2b} x . Cos. px\, dx = \frac{-\Gamma(b+p)}{1^{b/1}\Gamma(p)} \frac{\pi}{2^{p+2b-1}} \sum_0^\infty \frac{b^{n/-1}}{(p+b-1)^{n/-1}} \binom{b}{n}$ Schlömilch, Cr. 33. 353.

F. Circ. Dir. rat. ent.. Produit de deux puissances. TABLE 56. Lim 0 et $\frac{\pi}{2}$.

1) $\int Sin.^{2a} x . Cos.^{2b} x\, dx = \frac{1^{a/2}\,1^{b/2}}{2^{a+b/2}} \frac{\pi}{2}$ Jacobi, Cr. 15. 1.

2) $= \frac{\Gamma(a+\frac{1}{2})\Gamma(b+\frac{1}{2})}{2\Gamma(a+b+1)}$ Schlömilch, Gr. 4. 316.

3) $\int Sin.^{2a} x . Cos.^{2b+1} x \, dx = \frac{1}{2a+1} \frac{1^{a/2} \, 2^{b/2}}{3^{a+b/2}}$ Oettinger, Cr. 38. 162.

4) $\int Sin.^{2a+1} x . Cos.^{2b} x \, dx = \frac{1^{a/2} \, 1^{b/2}}{3^{a+b/2}}$ Ohm, Ausw. 49.

5) $\int Sin.^{2a+1} x . Cos.^{2b+1} x \, dx = \frac{1^{a+1/1} \, 1^{b/1}}{1^{a+b+1/1}} \frac{1}{2(a+1)}$ Oettinger, Cr. 38. 162. — Lobatschewsky, Mém. Kasan. 1835. 211.

6) $\int Sin.^{2p-1} x . Cos.^{1-2p} x \, dx = \frac{1}{2} \pi \, Cosec. \, p \pi$ Bonnet., L. 6. 238. — Oettinger, Cr. 38. 162. — Id., Cr. 38. 216.

7) $\int Sin.^{p-1} x . Cos.^{q-1} x \, dx = \frac{\Gamma(\frac{1}{2} p) \, \Gamma(\frac{1}{2} q)}{2 \, \Gamma\left(\frac{p+q}{2}\right)}$ Raabe, Int. 22?.

8) $\int Sin.^{a-1} x . Cos.^{2b+1} x \, dx = \frac{2^{b/2}}{a^{b+2/2}}$

9) $\int Cos.^{a-1} x . Sin.^{2b+1} x \, dx = \frac{2^{b/2}}{a^{b+2/2}}$

(8, 9) Oettinger, Cr. 38. 162.

10) $\int Cos.^{a+b} x . Tang.^{2a+1} x \, dx = \frac{1^{a/1} \, 1^{b/1}}{1^{a+b/1}} \frac{1}{2b}$

11) $\int Cos.^{a+1} x . Tang.^{2a+1} x \, dx = \frac{1}{2(a+1)}$

12) $\int Cos.^{2a+1} x . Tang.^{2a+1} x \, dx = \frac{1^{a/1} \, 1^{a/1}}{2 \, . \, 1^{2a+1/1}}$

13) $\int Cos.^{2a+2b-2} x . Tang.^{2a-1} x \, dx = \frac{1^{a-1/1} \, 1^{b-1/1}}{2 \, . \, 1^{a+b-1/1}}$

(10–13) Oettinger, Cr. 38. 162.

14) $\int Cos.^{2a-2} x . Tang.^{p-1} x \, dx = \frac{\Gamma(\frac{1}{2} p) \, \Gamma(a - \frac{1}{2} p)}{2 \, \Gamma(a)}$ V. T. 21. N°. 9.

15) $\int Cos.^{2p-1} x . Tang.^{p-q} x \, dx = \frac{\Gamma\left(\frac{p-q+1}{2}\right) \Gamma\left(\frac{p+q}{2}\right)}{2 \, \Gamma(p + \frac{1}{2})}$ V. T. 27. N°. 10.

16) $\int Sin.^{a+b+1} x . Cos.^{a-b+1} x \, dx = \frac{\Gamma\left(\frac{a+b}{2} + 1\right) \Gamma\left(\frac{a+b}{2} - 1\right)}{2 . 1^{a+1/1}}$ Lobatschewsky, Mém. Kasan. 1835. 211.

1) $\int Sin.^{2a-1} x.\, Cos.^{2a-b-1} x.\, Sin.\, bx\, dx = 0\,,\quad b > 2a\,;$

2) $\int Sin.^{2a} x.\, Cos.^{b-2a-2} x.\, Cos.\, bx\, dx = 0\,,\quad b > 2a+1\,;$

Cauchy, P. 28. 147. I. § 3.

3) $\int Sin.^{2a-1} x.\, Cos.^{2b-2a-1} x.\, Cos.\, 2bx\, dx = \frac{\Gamma(\frac{1}{2})\,\Gamma(b-a+\frac{1}{2})\,\Gamma(a)\,\Gamma(b-a)}{2\,\Gamma(b)\,\Gamma(b+\frac{1}{2})\,\Gamma(\frac{1}{2}-a)}$ Kummer, Cr. 17. 210. — Schlömilch, Stud. I. 24.

4) $= (-1)^a \frac{1^{2a-1/1}\, 1^{2b-2a-1/1}}{1^{2b-1/1}}$

5) $\int Sin.^{2a-2b-1} x.\, Cos.^{2a-1} x.\, Cos.\, 2bx\, dx = (-1)^{b-a} \frac{1^{2a-1/1}\, 1^{2b-2a-1/1}}{1^{2b-1/1}}$

Oettinger, Cr. 38. 216.

6) $\int Sin.^{p-q-1} x.\, Cos.^{q-1} x.\, Sin.\, px\, dx = \frac{1^{q-1/1}\, 1^{p-q-1/1}}{1^{p-1/1}}\, Sin.\left\{\frac{p-q}{2}\pi\right\}$

7) $\int Sin.^{p-q-1} x.\, Cos.^{q-1} x.\, Cos.\, px\, dx = \frac{1^{q-1/1}\, 1^{p-q-1/1}}{1^{p-1/1}}\, Cos.\left\{\frac{p-q}{2}\pi\right\}$

, où p et q des fractions seulement; Oettinger, Cr. 38. 216.

8) $\int Sin.^{p-1} x.\, Cos.^{q-1} x.\, Cos.\, \{(p+q)\,x\}\, dx = B(p,q)\, Cos.\, \frac{q\pi}{2}$ Serret, L. 8. 1.

9) $= \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}\, Cos.\, \frac{p\pi}{2}$, $2 > p > 0\,;$

10) $\int Sin.^{p-1} x.\, Cos.^{q-1} x.\, Sin.\, \{(p+q)\,x\}\, dx = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}\, Sin.\, \frac{p\pi}{2}$

Serret, L. 8. 489. — Kummer, Cr. 17. 210. — Id., Cr. 20. 1. — Schlömilch, Stud. I. 24.

11) $= \frac{\Gamma(q)}{\Gamma(p+q)\,\Gamma(1-p)}\, \frac{\pi}{2\, Cos.\, \frac{1}{2} p\pi}$

12) $= B(p,q)\, Sin.\, \frac{q\pi}{2}$

Serret, L. 8. 1.

13) $\int Sin.^{2a-1} x.\, Cos.^{2b-1} x.\, Cos.\, \{2(a+b)\,x\}\, dx = (-1)^a \frac{1^{2a-1/1}\, 1^{2b-1/1}}{1^{2a+2b-1/1}}$, où p et q des fractions seulement;

14) $\int Sin.^{b-1} x.\, Cos.^{a-b-1} x.\, Cos.\, ax\, dx = \frac{1^{a-b-1/1}}{1^{a-1/1}\, 1^{b-1/1}}\, \frac{\pi}{2}\, Cosec.\, \frac{p\pi}{2}$

15) $\int Sin.^{p-1} x.\, Cos.^{q-p-1} x.\, Cos.\, qx\, dx = \frac{1^{p-1/1}\, 1^{q-p-1/1}}{1^{q-1/1}}\, Cos.\, \frac{p\pi}{2}$

16) $\int Sin.^{p-1} x.\, Cos.^{q-1} x.\, Cos.\, \{(p+q)\,x\}\, dx = \frac{1^{p-1/1}\, 1^{q-1/1}}{1^{p+q-1/1}}\, Cos.\, \frac{p\pi}{2}$

Oettinger, Cr. 38. 216.

17) $\int Sin.^{2p-1} x . Cos.^{2q-2p-1} x . Cos. 2 q x d x = \frac{(2-2p)^{q-1/2}\, 1^{q-p/2}\, 1^{p/2}}{1^{2q-1/1}} \frac{\pi}{2} Cot. p \pi$

18) $\int Sin.^{2p-1} x . Cos.^{2q-1} x . Cos. \{2(p+q)x\} dx = \frac{(2-2p)^{p+q-1/2}\, 1^{p/2}\, 1^{q/2}}{1^{2p+2q-1/1}} \frac{\pi}{2} Cot. p \pi$

, où p et q des fractions seulement; Oettinger, Cr. 38. 216.

19) $\int Cos.^{a} x . Sin. b x . Sin. x d x = \frac{b \pi 1^{a/1}}{2^{a+2} \Gamma\left(\frac{a+b+3}{2}\right) \Gamma\left(\frac{a-b+3}{2}\right)}$ Lobatschewsky, Mém. Kasan. 1835. 211.

20) $\int Cos.^{a} x . Sin. a x . Sin. 2 b x d x = \frac{\pi}{2^{a+2}} \frac{a^{b/-1}}{1^{b/1}}$

21) $\int Cos.^{a} x . Cos. a x . Cos. 2 b x d x = \frac{\pi}{2^{a+2}} \frac{a^{b/-1}}{1^{b/1}}$

Poisson, P. 19. 404. N°. 76. — Id., Conn. des Temps. 1836. 1.

22) $\int (2 Cos. x)^{a-1} . Cos. \{(a+1)x\} . Cos. 2 b x d x = \frac{\pi}{4} \frac{(a-b+1)^{b-1/1}}{1^{b-1/1}}$ Kummer, Cr. 17. 210.

F. Circ. Dir. rat. ent.. Fact. *Tang.*^{a}x et autres. TABLE 58. Lim. 0 et $\frac{\pi}{2}$.

1) $\int Cos.^{p+2b} x . Sin. p x . Tang. x \, dx = \frac{\Gamma(b+p)}{1^{b/1}\Gamma(p)} \cdot \frac{\pi}{2^{p+2b-1}} \sum_{0}^{\infty} \binom{b}{n} \frac{b^{n/-1}}{(p+b-1)^{n/-1}}$ Schlömilch, Cr. 33. 353.

2) $\int Cos.^{p-2} x . Tang.^{b} x . Sin. p x d x = 0$ V. T. 58. N°. 4, 7.

3) $\int Cos.^{p-2} x . Tang.^{b} x . Cos. p x d x = 0$ V. T. 58. N°. 5, 6.

4) $\int Cos.^{p+b-1} x . Tang.^{c-1} x . Cos. p x . Sin. \{(b+1)x\} dx = (-1)^{\frac{c}{2}} \frac{\pi}{2^{p+b-1}} \frac{\Gamma(b+p)}{1^{b/1}\Gamma(p)} \sum_{0}^{\infty} (-2)^{n} \binom{c-1}{n} \frac{b^{n/-1}}{(p+b-1)^{n/-1}}$

5) $\int Cos.^{p+b-1} x . Tang.^{c} x . Cos. p x . Cos. \{(b+1) x\} dx = (-1)^{\frac{c}{2}} \frac{\pi}{2^{p+b-1}} \frac{\Gamma(b+p)}{1^{b/1}\Gamma(p)} \sum_{0}^{\infty} (-2)^{n} \binom{c}{n} \frac{b^{n/-1}}{(p+b-1)^{n/-1}}$

6) $\int Cos.^{p+b-1} x . Tang.^{c} x . Sin. p x . Sin. \{(b+1)x\} d x = (-1)^{\frac{c}{2}} \frac{\pi}{2^{p+b-1}} \frac{\Gamma(b+p)}{1^{b/1}\Gamma(p)} \sum_{0}^{\infty} (-2)^{n} \binom{c}{n} \frac{b^{n/-1}}{(p+b-1)^{n/-1}}$

7) $\int Cos.^{p+b-1} x . Tang.^{c-1} x . Sin. p x . Cos. \{(b+1)x\} dx = (-1)^{\frac{c}{2}+1} \frac{\pi}{2^{p+b-1}} \frac{\Gamma(b+p)}{1^{b/1}\Gamma(p)} \sum_{0}^{\infty} (-2)^{n} \binom{c-1}{n} \frac{b^{n/-1}}{(p+b-1)^{n/-1}}$

Sur ces 4 formules voyez: Schlömilch, Cr. 33. 353.

1) $\int Cos.(p\,Tang.x)\,dx = \frac{1}{2}\pi e^{-p}$ Serret, L. 8. 489. — Dienger, Gr. 10. 341.

2) $\int Sin.(p\,Tang.x)\,dx = \frac{1}{2}\{e^{-p}Ei.(p) - e^{p}Ei.(-p)\}$ V. T. 204. N°. 7.

3) $\int Cos.^2(p\,Tang.x)\,dx = \frac{1}{4}\pi(1+e^{-2p})$ V. T. 205. N°. 22.

4) $\int Sin.^2(p\,Tang.x)\,dx = \frac{1}{4}\pi(1-e^{-2p})$ V. T. 205. N°. 21.

5) $\int Sin.(p\,Tang.x).Tang.x\,dx = \frac{1}{2}\pi e^{-p}$ V. T. 204. N°. 3.

6) $\int Cos.(p\,Tang.x).Tang.x\,dx = -\frac{1}{2}\{e^{-p}Ei.(p) + e^{p}Ei.(-p)\}$ V. T. 204. N°. 8.

7) $\int Tang.(p\,Tang.x).Tang.x\,dx = \frac{\pi}{e^{2q}+1}$ V. T. 204. N°. 9.

8) $\int Sin.(p\,Tang.x).Sin.2x\,dx = \frac{1}{2}p\pi e^{-p}$ V. T. 208. N°. 3.

9) $\int Cos.(p\,Tang.x).Cos.^2x\,dx = \frac{1+p}{4}\pi e^{-p}$ V. T. 208. N°. 7.

10) $\int Cos.(p\,Tang.x).Sin.^2x\,dx = \frac{1-p}{4}\pi e^{-p}$ V. T. 208. N°. 8.

11) $\int Cos.(p\,Tang.x).Cos.2x\,dx = \frac{1}{2}p\pi e^{-p}$ V. T. 59. N°. 9, 10.

12) $\int Sin.(p\,Tang.x).Tang.^{2a+1}x\,dx = (-1)^a\frac{\pi}{2}e^{-p}$ V. T. 205. N°. 27.

13) $\int Cos.(p\,Tang.x).Tang.^{2a}x\,dx = (-1)^a\frac{\pi}{2}e^{-p}$ V. T. 205. N°. 26.

14) $\int Cos.\left(p\,Tang.\frac{x}{q}\right).Tang.^2x\,dx = -\frac{p}{2}e^{-pq}$ V. T. 205. N°. 12.

15) $\int Cot.\left(p\,Tang.\frac{x}{q}\right).Tang.x\,dx = \frac{\pi q}{e^{2pq}-1}$ V. T. 205. N°. 16.

16) $\int Cos.^{2a-1}x.Cos.(2\,Tang.x\sqrt{c})dx = \frac{\sqrt{\pi}}{2\Gamma(a+\frac{1}{2})}\{\Gamma(a)\psi(1-a,c)+\Gamma(-a)c^a\psi(1+a,c)\}$ Kummer, Cr. 17. 228.

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17) $\int Cos.^{p-1} x . Sin. \{(p+1) x\} Sin. (c \, Tang. x) \, dx = \frac{\pi}{2 \Gamma (p+1)} c^p e^{-c}$ } Kummer, Cr. 17. 228.

18) $\int Cos.^{p-1} x . Cos. \{(p+1) x\} Cos. (c \, Tang. x) \, dx = \frac{\pi}{2 \Gamma (p+1)} c^p e^{-c}$ }

19) $\int \{ Cos. (q \, Tang. x) + Tang. x . Sin. (q \, Tang. x) \} \, dx = \pi e^{-q}$ V. T. 204. N°. 18.

F.Circ.Dir.rat.ent.comp.à autre arg. monôme. TABLE 60. Lim. 0 et $\frac{\pi}{2}$.

1) $\int Cos. 2qx . Cos. (2p \, Cos. x) dx = \frac{1}{2q} Sin. q\pi \left\{ 1 + \sum_1^\infty (-1)^n \frac{p^{2n}}{1^2 - q^2 . 2^2 - q^2 \ldots . n^2 - q^2} \right\}$ Kummer, Cr. 17. 210.

2) $\int Sin. (a \, Sin. x) . Sin. 2x \, dx = -\frac{2}{a}$ V. T. 192. N°. 1.

3) $\int Sin. (a \, Cos. x) . Sin. 2x \, dx = -\frac{2}{a}$ V. T. 192. N°. 1.

4) $\int Sin. (p \, Cos. x) . Tang. x \, dx = \sum_1^\infty \frac{1}{2n+1} \frac{p^{2n-1}}{1^{2n-1/1}}$ V. T. 192. N°. 5.

5) $\int Sin (p \, Cot. x) . Tang. x \, dx = \frac{\pi}{2} (1 - e^{-q})$ V. T. 212. N°. 4.

6) $\int Tang. (p \, Cot. x) . Tang. x \, dx = \frac{\pi}{2} \frac{e^p - e^{-p}}{e^p + e^{-p}}$ V. T. 212. N°. 5.

7) $\int Sin. (p \, Cot. q x) . Tang.^{2a-1} x \, dx = (-1)^a \frac{\pi}{2} e^{-p}$ V. T. 212. N°. 14.

8) $\int Cos. (p \, Cot. q x) . Tang.^{2a} x \, dx = (-1)^a \frac{\pi}{2} e^{-p}$ V. T. 212. N°. 15.

9) $\int Sin. (a \, Cos. x) . Cos.^q x . Tang. x \, dx = \frac{1-q}{(lq)^2 + a^2} \frac{a}{q}$ V. T. 192. N°. 4.

10) $\int Sin. (\frac{1}{2} p \pi - q \, Tang. x) . Tang.^{p-1} x \, dx = \frac{1}{2} \pi e^{-q}$ V. T. 204. N°. 14.

11) $\int Cos. (\frac{1}{2} p \pi - q \, Tang. x) . Tang.^p x \, dx = \frac{1}{2} \pi e^{-q}$ V. T. 204. N°. 15.

12) $\int \{1 - Cos.(p\, Cot.\, x)\}\, Tang.^2\, x\, dx = \frac{\pi}{2}(e^{-p} + p - 1)$ V. T. 212. N°. 13.

13) $\int \{Cos.(a\, Cot.\, x) - Cos.(b\, Cot.\, x)\}\, Tang.^2\, x\, dx = \frac{\pi}{2}(e^{-b} - e^{-a}) + \frac{b-a}{2}\pi$ V. T. 212. N°. 7.

F. Circ. Dir. rat. ent. comp. à arg. binôme. TABLE 61. Lim. 0 et $\frac{\pi}{2}$.

1) $\int Cos.^{p-2} x.\, Cos.(p\, Tang.\, x - p x)\, dx = \frac{\pi}{\Gamma(p+1)}\left(\frac{p}{e}\right)^p$ Legendre, Exerc. 3. 40.

2) $\int Cos.^{p-1} x.\, Cos.\{c\, Tang.\, x - (p+1)x\}\, dx = \frac{\pi}{\Gamma(p+1)} c^p e^{-c}$

3) $\int Cos.^{p-1} x.\, Cos.\{c\, Tang.\, x + (p+1)x\}\, dx = 0$

4) $\int Cos.^{p-1} x.\, Cos.\{c\, Tang.\, x + (p-1)x\}\, dx = \frac{\pi}{2p} e^{-c}$

(2–4) Kummer, Cr. 17. 228.

5) $\int Cos.^{p-2} x.\, Cos.\{p x - c\, Tang.\, x\}\, dx = \frac{\pi}{\Gamma(p)} e^{-c} c^{p-1}$ Lobatschewsky, Mém. Kasan. 1835. 211.

6) $\int Cos.^{p-1} x.\, Cos.\{c\, Tang.\, x + b x\}\, dx = \frac{\pi e^{-c}\Gamma(p)}{2^p \Gamma\left(\frac{p-b+1}{2}\right)\Gamma\left(\frac{p+b+1}{2}\right)} \varphi\left(\frac{b-p+1}{2}, 1-p, 2c\right) - \frac{\pi c^p e^{-c} Cos.\left\{\frac{p-b}{2}\pi\right\}}{\Gamma(p+1)\, Sin.\, p\pi} \varphi\left(\frac{b+p+1}{2}, 1+p, 2c\right)$

7) $\int Cos.^{p-b-1} x.\, Cos.\{c\, Tang.\, x + (p+b-1)x\}\, dx = \frac{\pi}{\Gamma(1-p)} e^{-c} 2^{p-b} c^{-p} \sum_0^\infty (-1)^n \frac{p^{n/1}\, b^{n/1}}{1^{n/1}(2c)^n}$

De ces deux intégrales voyez: Kummer, Cr. 17. 228.

8) $\int Cos.(2x - 2\, Tang.\, x)\, dx = \frac{2\pi}{e^2}$ V. T. 209. N°. 19.

9) $\int Sin.\left(\frac{1}{2}p\pi - p\, Tang.\, x\right).\, Tang.^{a-1}\, x\, dx = \frac{1}{2}\pi e^{-p}$ V. T. 205. N°. 24.

10) $\int Cos.\left(\frac{1}{2}p\pi - p\, Tang.\, x\right).\, Tang.^{a}\, x\, dx = \frac{1}{2}\pi e^{-p}$ V. T. 205. N°. 25.

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11) $\int Sin.^{p-1} x . Cos.^{q-1} x . Cos. \{c\, Tang.\, x + (p+q)x\}\, dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} Cos. \frac{p\pi}{2} \varphi(p, 1-q, c) +$

$+ c^q\, Cos. \frac{p\pi}{2} \Gamma(-q)\varphi(p+q, 1+q, c)$

12) $\int Sin.^{p-1} x . Cos.^{q-1} x . Sin. \{c\, Tang.\, x + (p+q)x\}\, dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} Sin. \frac{p\pi}{2} \varphi(p, 1-q, c) +$

$+ c^q\, Sin. \frac{p\pi}{2} \Gamma(-q)\varphi(p+q, 1+q, c)$

13) $\int Sin.^{p-1} x . Cos.^{q-1} x . Sin. \left\{c\, Tang.\, x + (p+q)x - \frac{p\pi}{2}\right\} dx = 0$

14) $\int Sin.^{p-1} x . Cos.^{q-1} x . Sin. \{c\, Tang.\, x - (p+q)x\}\, dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} Sin. \frac{p\pi}{2} \varphi(p, 1-q, -c) -$

$- c^q\, Sin. \left\{\left(\frac{p}{2}+q\right)\pi\right\} \Gamma(-q)\varphi(p+q, 1-q, -c)$

15) $\int Sin.^{p-1} x . Cos.^{q-1} x . Cos. \{c\, Tang.\, x - (p+q)x\}\, dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} Cos. \frac{p\pi}{2} \varphi(p, 1-q, -c) +$

$+ c^q\, Cos. \left\{\left(\frac{p}{2}+q\right)\pi\right\} \Gamma(-q)\varphi(p+q, 1-q, -c)$

16) $\int Sin.^{p-1} x . Cos.^{q-1} x . Sin \{c\, Tang.\, x - (p+q)x + (\tfrac{1}{2}p+q)\pi\}\, dx = \frac{\pi\,\Gamma(p)}{\Gamma(1-q)\Gamma(p+q)} \varphi(p, 1-q, c)$

17) $\int Sin.^{p-1} x . Cos.^{q-1} x . Sin. \{c\, Tang.\, x - (p+q)x + \tfrac{1}{2}p\pi\}\, dx = \frac{\pi}{\Gamma(q+1)} c^q \varphi(p+q, 1+q, -c)$

Kummer, Cr. 17. 228, a déduit les formules 11 à 17.

F. Circ. Dir. rat. fract. à num. mon. et dén. $Sin.^a x, Cos.^a x$. TABLE 62. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{Sin.\, px . Cos.^{p-1} x}{Sin.\, x} dx = \frac{1}{2}\pi$ Cauchy, Exerc. 1826. p. 205. — Serret, L. 8. 1. — Id., L. 8. 489. — Liouville, Cr. 13. 219. — Kummer, Cr. 17. 228. — Schlömilch, Cr. 33. 353. — Id., Gr. 6. 200. — Id., Stud. I. 15.

2) $\int \frac{Sin.\,(2k+1)x}{Sin.\, x} dx = \frac{1}{2}\pi$, pour $k = \infty$; Schlömilch, Beitr. 1. § 4.

3) $\int \frac{Cos.^{2p-1} x}{Sin.^{2p-1} x} dx = \frac{1}{2}\pi\, Cosec.\, p\pi$, $1 > p > 0$; Oettinger, Cr. 38. 162.

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4) $\int \frac{Cos.^{2a-1} x}{Sin.^{2b} x} dx = \frac{(-1)^b}{2a} \frac{2^{a/2}}{1^{b/2}\, 1^{a-b/2}}$ V. T. 12. N°. 19.

5) $\int \frac{Cos.^{2a} x}{Sin.^{2b} x} dx = (-1)^b \frac{\pi}{4} \frac{1^{a/2}}{1^{b/2}\, 2^{a-b/2}}$ V. T. 12. N°. 21.

6) $\int \frac{Cos.^{2p-2} x}{Sin.^{2p-1} x} dx = \frac{\Gamma(2p-1)\Gamma(1-p)}{2^{2p-1}\Gamma(p)}$ V. T. 4. N°. 19.

7) $\int \frac{Sin.^{2a-1} x}{Cos.^{2b} x} dx = \frac{(-1)^b}{2a} \frac{2^{a/2}}{1^{b/2}\, 1^{a-b/2}}$ V. T. 12. N°. 19.

8) $\int \frac{Sin.^{2a} x}{Cos.^{2b} x} dx = (-1)^b \frac{\pi}{4} \frac{1^{a/2}}{1^{b/2}\, 2^{a-b/2}}$ V. T. 12. N°. 21.

9) $\int \frac{Sin.^{2p-2} x}{Cos.^{2p-1} x} dx = \frac{\Gamma(2p-1)\Gamma(1-p)}{2^{2p-1}\Gamma(p)}$ V. T. 4. N°. 19.

10) $\int \frac{Sin.\{(2-p)x\}}{Sin.^p x} dx = \frac{1}{1-p} Cos. \frac{1}{2} p\pi, \; 2 > p > 0;$

11) $\int \frac{Cos.\{(2-p)x\}}{Sin.^p x} dx = \frac{1}{1-p} Sin. \frac{1}{2} p\pi, \; 1 > p > 0;$

(10, 11) Schlömilch, Gr. 6. 200.

F. Circ. Dir. rat. fract. à num. mon. et autre dén. mon. TABLE 63. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{Sin.\, qx}{Tang.x} dx = 2\pi q^2$ V. T. 354. N°. 3.

2) $\int \frac{Sin.\, 4ax}{Tang.\, x} dx = \frac{2\pi}{ak} \sum_1^{k-1} Cos. \frac{2an\pi}{k}\, l\, Sin. \frac{n\pi}{2k}$, où $k = \infty$; V. T. 356. N°. 4.

3) $\int \frac{Cos.^{a-1} x.\, Sin.\{(a+1)x\}}{Tang.\, x} dx = \frac{1}{2}\pi$ Lindmann, Stockh. Handl. 1850.

4) $\int \frac{dx}{Tang.^p x} = \frac{1}{2}\pi\, Sec.\, p\pi, \; 1 > p > 0;$ Schlömilch, Gr. 6. 200.

5) $\int \frac{Sin.\, 2x}{Tang.^p x} dx = \frac{1}{2} p\pi\, Cosec. \frac{p\pi}{2}, \; 2 > p > 0;$

6) $\int \frac{Cos.\, 2x}{Tang.^p x} dx = \frac{1}{2} p\pi\, Sec. \frac{p\pi}{2}, \; 1 > p > 0;$

(5, 6) Schlömilch, Gr. 6. 200. — Id., Stud. I. 15.

7) $\int \frac{Sin.^{2a-2} x}{Tang.^{p-1} x} dx = \frac{\Gamma(\frac{1}{2}p)\Gamma(a-\frac{1}{2}p)}{2\Gamma(a)}$ V. T. 21. N°. 9.

8) $\int \frac{Sin.^{2p-1} x}{Tang.^{p-1} x} dx = \frac{\Gamma\left(\frac{p-q+1}{2}\right)\Gamma\left(\frac{p+q}{2}\right)}{2\,\Gamma(p+\frac{1}{2})}$ V. T. 27. N°. 10.

9) $\int \frac{Sin. p x. Cos.^{p-2} x}{Tang.^q x} dx = \frac{\Gamma(p+q-1)}{2\,\Gamma(p)\,\Gamma(q)} \pi\, Cosec. \frac{1}{2} q \pi, \; 2 > q > 0;$

10) $\int \frac{Cos. p x. Cos.^{p-2} x}{Tang.^q x} dx = \frac{\Gamma(p+q-1)}{2\,\Gamma(p)\,\Gamma(q)} \pi\, Sec. \frac{1}{2} q \pi, \; 1 > q > 0;$

(9, 10) Schlömilch, Gr. 6. 200. — Id., Cr. 33. 353. — Id., Stud. I. 15.

11) $\int \frac{Cos.^2 x}{Cos. 2x} dx = 0$ V. T. 21. N°. 7.

12) $\int \frac{Sin.^2 x}{Cos. 2x} dx = -\frac{1}{4}\pi$ V. T. 24. N°. 14.

13) $\int \frac{Cos.^2 x}{Cos. 2x} dx = \frac{1}{4}\pi$ V. T. 24. N°. 13.

14) $\int \frac{Tang.^{p-1} x}{Cos. 2x} dx = \frac{1}{2}\pi\, Cot. \frac{1}{2} p \pi$ V. T. 19. N°. 9.

15) $\int \frac{dx}{Cos. 2x\, Tang.^{p-1} x} = -\frac{1}{2}\pi\, Cot. \frac{1}{2} p \pi$ V. T. 19. N°. 9.

F. Circ. Dir. rat. fract. à num. binôme et dén. monôme. TABLE 64. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{Sin.^{p-1} x - Sin.^{1-p} x}{Cos. x} dx = \frac{1}{2}\pi\, Cot. \frac{1}{2} p \pi$ V. T. 5. N°. 12.

2) $\int \frac{Sin.^p x - Cosec.^p x}{Cos. x} dx = -\frac{1}{2}\pi\, Tang. \frac{1}{2} p \pi$ V. T. 5. N°. 13.

3) $\int \frac{Sin.^p x - Sin.^q x}{Cos. x} dx = \frac{1}{2}\left\{Z'\left(\frac{q+1}{2}\right) - Z'\left(\frac{p+1}{2}\right)\right\}$ V. T. 3. N°. 14.

4) $\int \frac{Cos.^{p-1} x - Sec.^{p-1} x}{Sin. x} dx = \frac{1}{2}\pi\, Cot. \frac{1}{2} p \pi$ V. T. 5. N°. 12.

5) $\int (Sec. x - 1)^p\, Tang. x\, dx = -\pi\, Cosec. p \pi$ V. T. 31. N°. 1

6) $\int (Sec. x - 1)^{1-p}\, Sin. 2x\, dx = (1-p)\, p \pi\, Cosec. p \pi$ V. T. 31. N°. 5.

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7) $\int (Cosec.\, x - 1)^{1-p}\, Sin.\, 2\, x\, d\, x = (1-p)\, p\, \pi\, Cosec.\, p\, \pi$ V: T. 31. N°. 5.

8) $\int \frac{a + b\, Tang.^2\, x}{Cos.\, 2\, x}\, d\, x = \frac{a-b}{4}\pi$ V. T. 24. N°. 9.

9) $\int \frac{(Cosec.\, x - 1)^p}{Tang.\, x}\, d\, x = -\pi\, Cosec.\, p\, \pi$ V. T. 31. N°. 1.

10) $\int \frac{Tang.^{p-1}\, x - Tang.^{1-p}\, x}{Cos.\, 2\, x}\, d\, x = \pi\, Cot.\, \frac{1}{2}\, p\, \pi$ V. T. 47. N°. 7 et T. 92. N°. 1.

11) $\int \frac{Sin.^{p-1}\, x + Sin.^{q-1}\, x}{Cos^{p+q-1}\, x}\, d\, x = \frac{1}{2}\, Cos.\left(\frac{q-p}{4}\pi\right).\, Sec.\left(\frac{q+p}{4}\pi\right) B(\tfrac{1}{2}p, \tfrac{1}{2}q)$ V. T. 12. N°. 23.

12) $\int \frac{Sin.^{p-1}\, x - Sin.^{q-1}\, x}{Cos.^{p+q-1}\, x}\, d\, x = \frac{1}{2}\, Sin.\left(\frac{q-p}{4}\pi\right).\, Cosec.\left(\frac{q+p}{4}\pi\right) B(\tfrac{1}{2}p, \tfrac{1}{2}q)$ V. T. 12. N°. 24.

13) $\int \frac{Cos^{p-1}\, x + Cos.^{q-1}\, x}{Sin.^{p+q-1}\, x}\, d\, x = \frac{1}{2}\, Cos.\left(\frac{q-p}{4}\pi\right).\, Sec.\left(\frac{q+p}{4}\pi\right) B(\tfrac{1}{2}p, \tfrac{1}{2}q)$ V. T. 12. N°. 23.

14) $\int \frac{Cos.^{p-1}\, x - Cos.^{q-1}\, x}{Cos^{p+q-1}\, x}\, d\, x = \frac{1}{2}\, Sin.\left(\frac{q-p}{4}\pi\right).\, Cosec.\left(\frac{q+p}{4}\pi\right) B(\tfrac{1}{2}p, \tfrac{1}{2}q)$ V. T. 12. N°. 24.

15) $\int (Tang.^p\, x - Cot.^p\, x)\, (Tang.^q\, x + Cot.^q\, x)\, \frac{d\, x}{Cos.\, 2\, x} = \frac{-2\, \pi\, Sin.\, p\, \pi}{Cos\, p\, \pi + Cos.\, q\, \pi}, p < 1$; V. T. 47. N°. 20 et T. 92. N°. 4.

16) $\int (Tang.^p\, x + Cot^p\, x)\, (Tang.^q\, x + Cot.^q\, x)\, d\, x = \frac{4\, \pi\, Cos.\, \frac{1}{2}\, p\, \pi\, .\, Cos.\, \frac{1}{2}\, q\, \pi}{Cos.\, p\, \pi + Cos.\, q\, \pi}$ V. T. 47. N°. 16 et T. 92. N°. 2.

17) $\int (Tang.^p\, x - Cot.^p\, x)\, (Tang.^q\, x - Cot.^q\, x)\, d\, x = \frac{4\, \pi\, Sin.\, \frac{1}{2}\, p\, \pi\, .\, Sin.\, \frac{1}{2}\, q\, \pi}{Cos.\, p\, \pi + Cos.\, q\, \pi}$ V. T. 47. N°. 17 et T. 92. N°. 3.

F. Circ. Dir. rat. fract. à dén. binôme du 1^{er} degré. TABLE 65. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{d\, x}{2 - Sin.\, x} = \frac{2\, \pi}{3\sqrt{3}}$ V. T. 7 N°. 1.

2) $\int \frac{d\, x}{2 + Sin.\, x} = \frac{\pi}{3\sqrt{3}}$ V. T. 7. N°. 2.

3) $\int \frac{d\, x}{1 - Sin.\, x.\, Cos.\, x} = \frac{4\, \pi}{3\sqrt{3}}$ V. T. 48. N°. 2 et T. 92. N°. 5.

4) $\int \frac{d\, x}{1 + Sin.\, x.\, Cos.\, x} = \frac{2\, \pi}{3\sqrt{3}}$ V. T. 25. N°. 1.

5) $\int \frac{dx}{1 - Sin. x. Cos. \lambda} = (\pi - \lambda) \, Cosec. \lambda$ V. T. 7. N°. 3.

6) $\int \frac{dx}{1 + Sin. x. Cos. \lambda} = \lambda \, Cosec. \lambda$ V. T. 7. N°. 4.

7) $\int \frac{Sin. x}{Sin. x \pm q \, Cos. x} dx = \frac{q}{1+q^2} \left\{ \frac{\pi}{2q} \pm l \, q \right\}$ V. T. 24. N°. 3, 4.

8) $\int \frac{Cos. x}{Sin. x \pm q \, Cos. x} dx = \frac{1}{1+q^2} \left\{ \pm \frac{q\pi}{2} - l \, q \right\}$ V. T. 24. N°. 1, 2.

9) $\int \frac{dx}{p + q \, Cos. x} = \frac{1}{\sqrt{(p^2 - q^2)}} Arccos. \frac{q}{p}$, pour $q < p$; Lobatto, Int. 53.

10) $= \frac{1}{\sqrt{(q^2 - p^2)}} l \frac{q + \sqrt{(q^2 - p^2)}}{q}$, pour $q > p$;

11) $= \frac{1}{p}$, pour $q = p$;

12) $\int \frac{dx}{-p + q \, Cos. x} = - \frac{1}{\sqrt{(q^2 - p^2)}} l \frac{\sqrt{(q^2 - p^2)} - q}{p}$, pour $p < q$; (val. princ.)

13) $= -\infty$, pour $p = q$;

(10–13: Björling, Gr. 21. 26.)

14) $\int \frac{Tang.^p x}{1 + Sin. 2x. Cos. \lambda} dx = \frac{\pi}{Sin. p\pi} \frac{Sin. p\lambda}{Sin. \lambda}$, $\lambda^2 < \pi^2, p^2 < 1$; V. T. 25. N°. 5.

15) $\int \frac{Tang.^{p-1} x}{1 + Sin. x \, Cos. x} dx = \frac{2\pi}{\sqrt{3}} Cosec. p\pi. Sin. \left\{ \frac{1-p}{2} \pi \right\}$, $1 > p > 0$; V. T. 25. N°. 4.

16) $\int \frac{Cos. x - Sin. x}{Cos. x + Sin. x} dx = 0$ V. T. 24. N°. 5.

17) $\int \frac{q \, Sin. x - Cos. x}{Sin. x + q \, Cos. x} dx = l \, q$ V. T. 24. N°. 6.

F. Circ. Dir. rat. fract. à dén. binôme de plus haut degré. TABLE 66. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{Sin.^4 x}{1 + p \, Sin.^2 x} dx = \frac{\pi}{2p^2 \sqrt{(1+p)}} + \frac{p-2}{4p^2} \pi$ Ramus, Danske Afh. 6. 265.

2) $\int \frac{dx}{1 + p^2 \, Tang.^2 x} = \frac{\pi}{2} \frac{1}{1+p}$ Mösta, Gr. 10. 449.

3) $\int \frac{Tang.^2 x}{1 + p^2 Tang.^2 x} dx = \frac{\pi}{2p(p+1)}$ V. T. 24. N°. 7.

4) $\int \frac{dx}{4 - Sin.^2 x} = \frac{\pi}{2\sqrt{3}}$ V. T. 65. N°. 1, 2.

5) $\int \frac{Sin. x}{4 - Sin.^2 x} dx = \frac{\pi}{3\sqrt{3}}$ V. T. 65. N°. 1, 2.

6) $\int \frac{dx}{1 - Sin.^2 x . Cos.^2 x} = \frac{\pi}{\sqrt{3}}$ V. T. 65. N°. 3, 4.

7) $\int \frac{Sin. 2x}{1 - Sin.^2 x . Cos.^2 x} dx = \frac{2\pi}{3\sqrt{3}}$ V. T. 65. N°. 3, 4.

8) $\int \frac{dx}{1 - Sin.^2 x . Cos.^2 \lambda} = \frac{1}{2} \pi \, Cosec. \lambda$ V. T. 65. N°. 5 6.

9) $\int \frac{Sin. x}{1 - Sin.^2 x . Cos.^2 \lambda} dx = \frac{1}{2} (\pi - 2\lambda) \, Cosec. \lambda$ V. T. 65. N°. 5, 6.

10) $\int \frac{Cos.^2 x}{1 - Sin.^2 x . Cos.^2 x} dx = \pi$ V. T. 25. N°. 16.

11) $\int \frac{dx}{p^2 + Tang.^2 x} = \frac{\pi}{2p(p+1)}$ V. T. 24. N°. 7.

12) $\int \frac{Cos.^p x + Cos.^q x}{Cos.^{p+q} x + 1} Tang. x \, dx = \frac{\pi}{p+q} Sec. \left\{ \frac{q-p}{p+q} \frac{\pi}{2} \right\}$ V. T. 31. N°. 17.

13) $\int \frac{Cos.^p x - Cos.^q x}{Cos.^{p+q} x - 1} Tang. x \, dx = \frac{\pi}{p+q} Tang. \left\{ \frac{q-p}{q+p} \frac{\pi}{2} \right\}$ V. T. 31. N°. 18.

14) $\int \frac{Tang. x}{Cos.^p x + Sec.^p x} dx = \frac{1}{4p} \pi$ V. T. 5. N°. 23.

15) $\int \frac{Cos.^2 x . Tang.^{p-1} x}{1 - 3 Sin.^2 x . Cos.^2 x} dx = \frac{\pi}{\sqrt{3}} Cosec. \tfrac{1}{2} p\pi . Sin. \left\{ \frac{2-p}{6} \pi \right\}, \; 4 > p > 0$; V. T. 25. N°. 15.

16) $\int \frac{Cos.^2 x}{Sin.^2 \lambda + Cos.^2 \lambda . Cos.^2 x} dx = \frac{\pi}{2} \frac{1 - Sin. \lambda}{Cos.^2 \lambda}$

17) $\int \frac{Cos. x}{Sin.^2 \lambda + Cos.^2 \lambda . Cos.^2 x} dx = - Sec. \lambda \, l \, Tang. \tfrac{1}{2} \lambda$

(16, 17) Bonnet, L. 17. 265.

18) $\int \frac{dx}{p^2 Cos.^2 x + q^2 Sin.^2 x} = \frac{\pi}{2pq}$ Catalan, L. 6. 340. — Tortolini, Cr. 34. 101.

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19) $\int \frac{Tang.^r\, x}{p^2\, Cos.^2\, x + q^2\, Sin.^2\, x}\, dx = \frac{1}{2}\pi\, p^{r-1}\, q^{-r-1}\, Sec.\, \frac{1}{2} r\pi$ Schlömilch, Höh. An. 85.

20) $\int \frac{Cos.^a\, x.\, Cos.\, a\, x}{Cos.^2\, x + b^2\, Sin.^2\, x}\, dx = \frac{\pi}{2}\, \frac{b^{a-1}}{(b+1)^a}$ Serret, L. 8. 489.

21) $\int \frac{Sin.\, 2\, x}{a^2\, Sin.^2\, x + Cos.^2\, x}\, dx = \frac{2}{1+a^2}\, l\, a$ V. T. 24. N°. 12.

22) $\int \frac{Sin.\, 2\, x}{Sin.^2\, x + a^2\, Cos.^2\, x}\, dx = \frac{-2}{1+a^2}\, l\, a$ V. T. 24. N°. 12.

F. Circ. Dir. rat. fract. à dén., puiss. de binômes. TABLE 67. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{Tang.\, x}{(Sec.\, x - 1)^p}\, dx = \pi\, Cosec.\, p\,\pi$ V. T. 31. N°. 20.

2) $\int \frac{Sin.\, 2\, x}{(Sec.\, x - 1)^p}\, dx = (1-p)\, p\, \pi\, Cosec.\, p\,\pi$ V. T. 31. N°. 21.

3) $\int \frac{Sin.\, 2\, x}{(Cosec.\, x - 1)^p}\, dx = (1-p)\, p\, \pi\, Cosec.\, p\,\pi$ V. T. 31. N°. 21.

4) $\int \frac{Sin.\, 2\, x.\, Cos.\, x}{(1 - Cos.^2\, \lambda.\, Sin.^2\, x)^2}\, dx = \frac{\pi - 2\lambda}{Sin.\, 2\lambda.\, Cos.\, \lambda}$ V. T. 66. N°. 9.

5) $\int \frac{Cos.^2\, x.\, Tang.^{p+1}\, x}{(1 + Sin.\, 2\, x.\, Cos.\, \lambda)^2}\, dx = \frac{\pi}{2\, Sin.\, p\,\pi}\, \frac{p\, Sin.\, \lambda.\, Cos.\, p\,\lambda - Cos.\, \lambda.\, Sin.\, p\,\lambda}{Sin.^3\, \lambda}$ V. T. 26. N°. 1.

6) $\int \frac{Sin.^{1-p}\, x.\, Cos.^p\, x}{(Cos.\, x + Sin.\, x)^3}\, dx = \frac{1-p}{2}\, p\,\pi\, Cosec.\, p\,\pi$ V. T. 18. N°. 22.

7) $\int \frac{dx}{(p^2\, Cos.^2\, x + q^2\, Sin.^2\, x)^2} = \frac{\pi}{4}\, \frac{p^2 + q^2}{p^3\, q^3}$ Tortolini, Cr. 34. 101.

8) $\int \frac{Sin.^2\, x}{(p^2\, Cos.^2\, x + q^2\, Sin.^2\, x)^2}\, dx = \frac{\pi}{4}\, \frac{1}{p\, q^3}$ Grunert, Cr. 8. 146. — Tortolini, Cr. 34. 101.

9) $\int \frac{Cos.^2\, x}{(p^2\, Cos.^2\, x + q^2\, Sin.^2\, x)^2}\, dx = \frac{\pi}{4}\, \frac{1}{p^3\, q}$

10) $\int \frac{dx}{(p^2\, Cos.^2\, x + q^2\, Sin.^2\, x)^3} = \frac{\pi}{16}\, \frac{3\, p^4 + 2\, p^2\, q^2 + 3\, q^4}{p^5\, q^5}$

11) $\int \frac{Sin.^2\, x}{(p^2\, Cos.^2\, x + q^2\, Sin.^2\, x)^3}\, dx = \frac{\pi}{16}\, \frac{3\, p^2 + q^2}{p^3\, q^5}$

(9–11) Tortolini, Cr. 34. 101.

12) $\int \frac{Cos.^2 x}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^3} dx = \frac{\pi}{16} \frac{p^2 + 3q^2}{p^5 q^3}$

13) $\int \frac{dx}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^4} = \frac{\pi}{32} \frac{5p^6 + 7p^4 q^2 + 7p^2 q^4 + 5q^6}{p^7 q^7}$

14) $\int \frac{Sin.^2 x}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^4} dx = \frac{\pi}{32} \frac{5p^4 + 6p^2 q^2 + q^4}{p^5 q^7}$

15) $\int \frac{Cos.^2 x}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^4} dx = \frac{\pi}{32} \frac{p^4 + 6p^2 q^2 + 5q^4}{p^7 q^5}$ Tortolini, Cr. 34. 101.

16) $\int \frac{Sin.^4 x}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^4} dx = \frac{\pi}{32} \frac{5p^2 + q^2}{p^3 q^7}$

17) $\int \frac{Cos.^4 x}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^4} dx = \frac{\pi}{32} \frac{p^2 + 5q^2}{p^7 q^3}$

18) $\int \frac{Sin.^2 x . Cos.^2 x}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^4} dx = \frac{\pi}{32} \frac{p^2 + q^2}{p^5 q^5}$

19) $\int \frac{Cos. 2x}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^2} dx = \frac{\pi}{4} \frac{q^2 - p^2}{p^3 q^3}$ V. T. 67. N°. 8, 9.

20) $\int \frac{Cos. 2x}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^3} dx = \frac{3\pi}{16} \frac{q^4 - p^4}{p^5 q^5}$ V. T. 67. N°. 11, 12.

21) $\int \frac{Cos. 2x}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^4} dx = \frac{\pi}{32} \frac{5q^6 - p^2 q^4 + p^4 q^2 - 5p^6}{p^7 q^7}$ V. T. 67. N°. 14, 15.

22) $\int \frac{Cos.^{2r-1} x . Sin.^{2s-1} x}{(p^2 Cos.^2 x + q^2 Sin.^2 x)^{r+s}} dx = \frac{\Gamma(r)\Gamma(s)}{\Gamma(r+s)} \frac{1}{2 p^{2r} q^{2s}}$ Schlömilch, Höh. Anal. 85.

23) $\int \frac{Sin.^{p-1} 2x}{(Sin. x + Cos. x)^{2p}} dx = \frac{1}{2^p} \frac{\Gamma(p)\Gamma(\frac{1}{2})}{\Gamma(p+\frac{1}{2})}$ V. T. 49. N°. 5. et T. 92. N°. 9.

24) $\int \frac{dx}{(Tang. x + Cot. x)^2} = \frac{1}{16}\pi$ V. T. 21. N°. 5.

F. Circ. Dir. rat. fract. à dén., prod. de mon. et bin. TABLE 68. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{Tang.^p x}{Sin. x + Cos. x} \frac{dx}{Sin. x} = \pi \, Cosec. p\pi$ V. T. 22. N°. 1.

2) $\int \frac{Tang.^{p-1} x + Cot.^p x}{Sin. x + Cos. x} \frac{dx}{Cos. x} = \pi \, Cosec. p\pi$ V. T. 22. N°. 4.

3) $\int \frac{1}{1+Sin.\,x.\,Cos.\,x}\frac{dx}{Tang^{p-1}x} = \frac{2\pi}{\sqrt{3}}\,Cosec.\,p\pi.\,Sin.\left\{\frac{1-p}{2}\pi\right\},\ 1>p>0;$ V. T. 25. N°. 4.

4) $\int \frac{Sin.^2 x}{1-3\,Sin.^2 x.\,Cos.^2 x}\frac{dx}{Tang^{p-1}x} = \frac{\pi}{\sqrt{3}}\,Cosec.\,\frac{1}{2}p\pi.\,Sin.\left\{\frac{2-p}{6}\pi\right\},\ 4>p>0;$ V. T. 25. N°. 15.

5) $\int \frac{Sin.^2 x}{p^2\,Cos.^2 x+Sin.^2 x}\frac{dx}{Cos.\,2x} = -\frac{1}{2}\frac{p\pi}{1+p^2}$ V. T. 24. N°. 14.

6) $\int \frac{Cos.^2 x}{p^2\,Cos.^2 x+Sin.^2 x}\frac{dx}{Cos.\,2x} = \frac{1}{2p}\frac{\pi}{1+p^2}$ V. T. 24. N°. 13.

7) $\int \frac{Sin.^2 x}{Cos.^2 x+p^2\,Sin.^2 x}\frac{dx}{Cos.\,2x} = -\frac{1}{2p}\frac{\pi}{1+p^2}$ V. T. 24. N°. 13.

8) $\int \frac{Cos.^2 x}{Cos.^2 x+p^2\,Sin.^2 x}\frac{dx}{Cos.\,2x} = \frac{1}{2}\frac{p\pi}{1+p^2}$ V. T. 24. N°. 14.

9) $\int \frac{Cot^p x}{1-(2q-q^2)\,Cos.^2 x}dx = \frac{1}{(1-q)^{p+1}}\frac{\pi}{2}\,Sec.\,\frac{1}{2}p\pi,\ p^2<1;$

10) $\int \frac{Cot^{p-1} x}{1-(2q-q^2)\,Cos.^2 x}dx = \frac{1}{(1-q)^p}\frac{\pi}{2}\,Cosec.\,\frac{1}{2}p\pi,\ 2>p>0;$

11) $\int \frac{Cot^p x}{1-q\,Cos.^2 x}dx = \frac{1}{\sqrt{(1-q)^{p+1}}}\frac{\pi}{2}\,Sec.\,\frac{1}{2}p\pi,\ p^2<1;$

(9–11): $q^2<1;$ Schlömilch, Stud. I. 15.

12) $\int \frac{1}{1+Sin.\,2x.\,Cos.\,\lambda}\frac{dx}{Tang^p x} = \frac{\pi}{Sin.\,p\pi}\frac{Sin.\,p\lambda}{Sin.\,\lambda},\ \lambda^2<\pi^2,\ p^2<1;$ V. T. 25. N°. 5.

13) $\int \frac{Tang^p x+Cot^p x}{1+Cos.\,\lambda.\,Sin.\,2x}dx = \frac{2\pi}{Sin.\,p\pi}\frac{Sin.\,p\lambda}{Sin.\,\lambda},\ p^2<1;$ V. T. 48. N°. 16 et T. 92. N°. 7.

14) $\int \frac{1}{Sin^p x+Cosec^p x}\frac{dx}{Tang.\,x} = \frac{\pi}{4p}$ V. T. 5. N°. 23.

15) $\int \frac{Sin.^p x+Sin.^q x}{Sin^{p+q}x+1}\frac{dx}{Tang.\,x} = \frac{\pi}{p+q}\,Sec.\left\{\frac{q-p}{q+p}\frac{\pi}{2}\right\}$ V. T. 31. N°. 17.

16) $\int \frac{Sin^p x-Sin^q x}{1-Sin^{p+q}x}\frac{dx}{Tang.\,x} = \frac{\pi}{p+q}\,Tang.\left\{\frac{q-p}{q+p}\frac{\pi}{2}\right\}$ V. T. 31. N°. 18.

17) $\int \frac{Cos^p x+Sec^p x}{Cos^q x+Sec^q x}\,Tang.\,x\,dx = \frac{\pi}{2q}\,Sec.\,\frac{p\pi}{2q}$ V. T. 31. N°. 24.

18) $\int \frac{Cot^p x}{Tang^q x-Cot^q x}\frac{dx}{Sin.\,2x} = -\frac{\pi}{4q}\,Tang.\,\frac{p\pi}{2q}$ V. T. 22. N°. 13.

19) $\int \frac{1}{Tang.^p x + Cot.^p x} \frac{dx}{Sin. 2x} = \frac{\pi}{4p}$ V. T. 49. N°. 19 et T. 92. N°. 10.

20) $\int \frac{Tang.^q x + Tang.^p x}{Tang.^{p+q} x + 1} \frac{dx}{Sin. 2x} = \frac{\pi}{p+q} Sec. \left\{\frac{q-p}{q+p} \frac{\pi}{2}\right\}$ V. T. 49. N°. 13 et T. 92. N°. 11.

21) $\int \frac{Tang.^q x - Tang.^p x}{Tang.^{p+q} x - 1} \frac{dx}{Sin. 2x} = \frac{\pi}{p+q} Tang. \left\{\frac{q-p}{q+p} \frac{\pi}{2}\right\}$ V. T. 49. N°. 14 et T. 92. N°. 12.

22) $\int \frac{Tang.^q x + Cot.^q x}{Tang.^p x + Cot.^p x} \frac{dx}{Sin. 2x} = \frac{\pi}{2p} Sec. \frac{q\pi}{2p}$ V. T. 22. N°. 14.

23) $\int \frac{Tang.^q x - Cot.^q x}{Tang.^p x - Cot.^p x} \frac{dx}{Sin. 2x} = \frac{\pi}{2p} Tang. \frac{q\pi}{2p}$ V. T. 22. N°. 15.

24) $\int \frac{Sin.^2 x}{(1 + Sin. 2x . Cos. \lambda)^2} \frac{dx}{Tang.^{p+1} x} = \frac{\pi}{2 Sin. p\pi} \frac{p Sin. \lambda . Cos. p\lambda - Cos. \lambda . Sin. p\lambda}{Sin.^3 \lambda}$ V. T. 26. N°. 1.

25) $\int \left(\frac{Tang.^p x - Cot.^p x}{Cos. x - Sin. x}\right)^q dx = 2(1 - 2p\pi Cot. 2p\pi)$ V. T. 28. N°. 7.

26) $\int \frac{Tang.^p x}{(1 + Tang. x)^3} \frac{dx}{Sin. 2x} = \frac{1+p}{4} p\pi Cosec. p\pi$ V. T. 18. N°. 23.

27) $\int \frac{Cos.^{2a} x}{(1 - q Cos.^2 x)^{a+1}} \frac{dx}{Tang.^p x} = \frac{(p+1)^{a/2}}{2^{a/2}} \frac{\pi Sec. \frac{1}{2} p\pi}{2(1-a)^{\frac{1}{2}(p+1)+a+1}}, p^2 < 1, q^2 < 1$; Schlömilch, Stud. I. 15.

28) $\int \frac{1}{(Cosec. x - 1)^p} \frac{dx}{Tang. x} = \pi Cosec. p\pi$ V. T. 31. N°. 20.

29) $\int \frac{(1 + Tang. x)^q - 1}{(1 + Tang. x)^{p+q}} \frac{dx}{Sin 2x} = \frac{1}{2} \{Z'(p+q) - Z'(q)\}$ V. T. 22. N°. 3.

30) $\int \{(1 + Tang. x)^{-q} - (1 + Tang. x)^{-p}\} \frac{dx}{Sin. 2x} = \frac{1}{4} \{Z'(p) - Z'(q)\}$ V. T. 22. N°. 18.

31) $\int \left\{Cot.^p x - \frac{1}{(1 + Tang. x)^p}\right\} \frac{Tang.^{q+2} x}{Sin.^2 x} dx = \frac{q}{q-p+1} \frac{\Gamma(q)\Gamma(p-q)}{\Gamma(p)}$ V. T. 22. N°. 16.

F. Circ. Dir. rat. fract. à dén. binôme. TABLE 69. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{1 - q Cos. 2x}{1 - 2q Cos. 2x + q^2} Tang.^p x\, dx = \frac{1}{4} \pi Sec. \frac{1}{2} p\pi \left\{1 + \left(\frac{1-q}{1+q}\right)^p\right\}$, pour $q^2 < 1$; Cauchy, Lim. Imag. 116.

2) $= \frac{1}{4} \pi Sec. \frac{1}{2} p\pi \left\{1 - \left(\frac{q-1}{q+1}\right)^p\right\}$, pour $q^2 > 1$; Cauchy, Lim. Imag. 122.

3) $\int \frac{q\, Sin. 2x}{1-2q\, Cos. 2x+q^2}\, Tang.^p x\, dx = \frac{1}{4}\pi\, Cosec. \tfrac{1}{2}p\pi \left\{1-\left(\frac{1-q}{1+q}\right)^p\right\}$, pour $q^2 < 1$; Cauchy, Lim. Imag. 117.

4) $= \frac{1}{4}\pi\, Cosec. \tfrac{1}{2}p\pi \left\{1+\left(\frac{q-1}{q+1}\right)^p\right\}$, pour $q^2 > 1$; Cauchy, Lim. Imag. 122.

5) $\int \frac{Cos.^a x . Cos. a x}{1-2q\, Cos. 2x+q^2}\, dx = \frac{\pi}{2(1-q^2)} \left(\frac{1+q}{2}\right)^a$

6) $\int \frac{Cos.^a x . Sin. a x . Sin. 2x}{1-2q\, Cos. 2x+q^2}\, dx = \frac{\pi}{4q}\left\{\left(\frac{1+q}{2}\right)^a - \frac{1}{2^a}\right\}$

(5, 6): $a > 0$, $p^2 < 1$; Poisson, P. 19. 404. N°. 76.

7) $\int \frac{dx}{1-c^2(a^2 Sin.^2 x + b^2 Cos.^2 x)} = \frac{\pi}{2\sqrt{(1-a^2c^2)(1-b^2c^2)}}$ Plana, Cr. 17. 345.

8) $\int \frac{dx}{a+b\, Sin.^2 x + c\, Cos.^2 x} = \frac{\pi}{2\sqrt{(a+b)(a+c)}}$ Lobatto, Int. 53.

9) $\int \frac{Cos.^3 x}{1+2\, Cos. \lambda . Sin. x + Sin.^2 x}\, dx = Cos. \lambda\, l\{2(1+Cos. \lambda)\} + \tfrac{1}{2}\lambda\, Sin. \lambda$ V. T. 7. N°. 6.

10) $\int \frac{Tang.^p x}{Tang.^q x + Cot.^q x - 2\, Cos. \lambda}\, \frac{dx}{Sin. 2x} = \frac{\pi\, Sin. \left\{p\frac{\pi-\lambda}{q}\right\}}{2q\, Sin. \lambda . Sin. \frac{p\pi}{q}}$ V. T. 26. N°. 10.

11) $\int \frac{Tang.^p x + Cot.^p x - 2\, Cos. \mu}{Tang.^q x + Cot.^q x - 2\, Cos. \lambda}\, \frac{dx}{Sin. 2x} = \frac{\pi\, Sin. \left\{p\frac{\pi-\lambda}{q}\right\}}{q\, Sin. \lambda . Sin. \frac{p\pi}{q}} + \frac{\lambda-\pi}{q}\, \frac{Cos. \mu}{Sin. \lambda}$ V. T. 26. N°. 9.

12) $\int \frac{Sin.^p x - 2\, Cos. \lambda + Cosec.^p x}{Sin.^q x - 2\, Cos. \mu + Cosec.^q x}\, \frac{dx}{Tang. x} = \frac{\pi\, Sin. \left\{\frac{\pi-\mu}{q}p\right\}}{q\, Sin. \mu . Sin. \frac{p\pi}{q}} + \frac{\mu-\pi}{q}\, \frac{Cos. \lambda}{Sin. \mu}$ V. T. 8. N°. 9.

13) $\int \frac{Sin.^p x + Cosec.^p x}{Sin.^q x + 2\, Cos. \lambda + Cosec.^q x}\, \frac{dx}{Tang. x} = \frac{\pi\, Sin. \frac{p\lambda}{q}}{q\, Sin. \lambda . Sin. \frac{p\pi}{q}}$ V. T. 8. N°. 12.

14) $\int \frac{Sin. 2x . Sin. \{(2k+1)x\}}{1-2p\, Cos. 2x+p^2}\, \frac{dx}{Cos. x} = 0$, $p^2 < 1$, pour $k = \infty$; Schlömilch, Beitr. II. 1.

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15) $\int \frac{Sin.\,2x\; Cos.\,\{(2k+1)x\}}{1-2p\,Cos.\,2x+p^2}\,\frac{dx}{Sin.\,x} = 0$, $p^2 < 1$, pour $k=\infty$; Schlömilch, Beitr. II. 1.

16) $\int \frac{dx}{\{1+q^2(1-p^2 Sin.^2 x)\}(1-p^2 Sin.^2 x)} = \frac{\pi}{2\sqrt{(1-p^2)}} - \frac{\pi q^2}{2\sqrt{\{(1+q^2)(1-p^2q^2+q^2)\}}}$ Roberts, L. 11. 157.

F. Circ. Dir. rat. fract. comp. à arg. *Tang. x*. TABLE 70. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{Sin.\,(a\,Tang.\,x)}{Sin.\,x.\,Cos.\,x}\,dx = \frac{1}{2}\pi$ Lobatschewsky, Mém. Kasan. 1835. 211. la trouve faut. $= \pi$.

2) $\int Sin.\,(q\,Tang.\,x)\,\frac{dx}{Tang.\,x} = \frac{1}{2}\pi\,(1-e^{-q})$ V. T. 212. N°. 4.

3) $\int Tang.\,(q\,Tang.\,x)\,\frac{dx}{Tang.\,x} = \frac{1}{2}\pi\,\frac{e^q-e^{-q}}{e^q+e^{-q}}$ V. T. 212. N°. 5.

4) $\int Sin.\,(q\,Tang.\,x)\,\frac{dx}{Cos.\,2x.\,Tang.\,x} = \frac{1}{2}\pi\,(1-Cos.\,q)$ V. T. 212. N°. 11.

5) $\int Sin.\,(q\,Tang.\,x)\,\frac{Tang.\,x}{Cos.\,2x}\,dx = -\frac{1}{2}\pi\,Cos.\,p$ V. T. 204. N°. 22.

6) $\int Cos.\,(q\,Tang.\,x)\,\frac{dx}{Cos.\,2x} = \frac{1}{2}\pi\,Sin.\,p$ V. T. 204. N°. 21.

7) $\int Sin.\,(q\,Tang.\,x)\,\frac{dx}{Cos.\,2x} = Ci.\,(q)\,Sin.\,q - Si.\,(q)\,Cos.\,q$ V. T. 206. N°. 9.

8) $\int Cos.\,(q\,Tang.\,x)\,\frac{Tang.\,x}{Cos.\,2x}\,dx = Ci.\,(q)\,Cos.\,q + Si.\,(q)\,Sin.\,q$ V. T. 206. N°. 10.

9) $\int Sin.\,(q\,Tang.\,x).\,Sin.\,(\pi\,Tang.\,x)\,\frac{dx}{Cos.\,2x} = \frac{\pi}{2}\,Sin.\,q$, pour $0<q<\pi$;

10) $= 0$, pour $q>\pi$; } V. T. 204. N°. 23, 24.

11) $\int Tang.\,(p\,Tang.\,x)\,\frac{Tang.\,x}{Cos.\,2x}\,dx = -\frac{1}{2}\pi$ V. T. 206. N°. 15.

12) $\int \frac{Tang.\,x}{Tang.\,(q\,Tang.\,x)}\,dx = \frac{\pi}{e^{2q}-1}$ V. T. 204. N°. 10.

13) $\int \frac{Tang.\,x}{Tang.\,(p\,Tang.\,x)}\,\frac{dx}{Cos.\,2x} = \frac{1}{2}\pi$ V. T. 206. N°. 19.

14) $\int \frac{Tang.\, x}{Sin.\,(p\, Tang.\, x)} \frac{dx}{Cos.\, 2x} = 0$ V. T. 206 N°. 20.

15) $\int Cos.^2\,(p\, Tang.\, x) \frac{dx}{Cos.\, 2x} = \frac{1}{4}\pi\, Sin.\, 2p$ V. T. 206. N°. 21.

16) $\int Sin.\,(p\, Tang.\, x) \frac{dx}{Tang.^{2a-1} x} = (-1)^a \frac{\pi}{2} e^{-p}$ V. T. 212. N°. 14.

17) $\int Cos.\,(p\, Tang.\, x) \frac{dx}{Tang.^{2a} x} = (-1)^a \frac{\pi}{2} e^{-p}$ V. T. 212. N°. 15.

18) $\int Cos.\,(p\, Tang.\, x) \left(\frac{Cos.\, x}{Cos.\, 2x}\right)^2 dx = \frac{\pi}{4}(Sin.\, p - p\, Cos.\, p)$ V. T. 208. N°. 17.

19) $\int \frac{1 - Cos.\,(p\, Tang.\, x)}{Tang.^2\, x} dx = \frac{\pi}{2}(e^{-p} + p - 1)$ V. T. 212. N°. 13.

20) $\int \frac{Cos.\,(Tang.\, x) - Cos.^2\, x - 1}{Tang.\, x} dx = -A$ V. T. 212. N°. 6.

21) $\int \frac{Cos.\,(a\, Tang.\, x) - Cos.\,(b\, Tang.\, x)}{Tang.^2\, x} dx = \frac{\pi}{2}(e^{-b} - e^{-a}) + \frac{b-a}{2}\pi$ V. T. 212. N°. 7.

22) $\int \frac{Cos.^{p-1}\, x}{Sin.\, x} Sin.\,(a\, Tang.\, x + px)\, dx = \frac{1}{2}\pi$ Kummer, Cr. 17. 228. — Id., Cr. 20. 1.

23) $\int \frac{Sin.\, px - Sin.\,(px - a\, Tang.\, x)}{Sin.\, x} Cos.^{p-1} x\, dx = \frac{1}{2}\pi$ Lobatschewsky, Mém. Kasan. 1835. 211, trouve fautivement: π.

F. Circ. Dir. rat. fract. comp. à autre arg. TABLE 71. Lim. 0 et $\frac{\pi}{2}$.

1) $\int Sin.\,(q\, Cot.\, x) \frac{dx}{Tang.\, x} = \frac{1}{2}\pi\, e^{-q}$ V. T. 204. N°. 3.

2) $\int Cos.\,(q\, Cot.\, x) \frac{dx}{Tang.\, x} = -\frac{1}{2}\{e^{-q} Ei.(q) + e^{q} Ei.(-q)\}$ V. T. 204. N°. 8.

3) $\int Tang.\,(q\, Cot.\, x) \frac{dx}{Tang.\, x} = \frac{\pi}{e^{2q}+1}$ V. T. 204. N°. 9.

4) $\int Cot.\,(q\, Cot.\, x) \frac{dx}{Tang.\, x} = \frac{\pi}{e^{2q}-1}$ V. T. 204. N°. 10.

5) $\int Sin.(q\ Cot.\ x)\ \frac{Tang.\ x\ d\ x}{Cos.\ 2\ x} = \frac{1}{2}\pi\,(Cos.\ q - 1)$ V. T. 212. N°. 17.

6) $\int Sin.(q\ Cot.\ x)\ \frac{d\ x}{Cos.\ 2\ x.\ Tang.\ x} = \frac{1}{2}\pi\ Cos.\ q$ V. T. 204. N°. 22.

7) $\int Cos.(q\ Cot.\ x)\ \frac{d\ x}{Cos.\ 2\ x.\ Tang.\ x} = - Ci.(q).\ Cos.\ q - Si.(q).\ Sin.\ q$ V. T. 206. N°. 10.

8) $\int Sin.(q\ Cot.\ x)\ \frac{d\ x}{Tang.^{2b+1}\ x} = (-1)^b\ \frac{1}{2}\pi\ e^{-q}$ V. T. 205. N°. 27.

9) $\int Cos.(q\ Cot.\ x)\ \frac{d\ x}{Tang.^{2b}\ x} = (-1)^b\ \frac{1}{2}\pi\ e^{-q}$ V. T. 205. N°. 26.

10) $\int Cos.(q\ Cot.\ x)\ \frac{d\ x}{Tang.^2\ x} = -\frac{1}{2}\ q\ e^{-q}$ V. T. 205. N°. 12.

11) $\int Cos.(q\ Cot.\ x)\ \left(\frac{Sin.\ x}{Cos.\ 2\ x}\right)^2 d\ x = \frac{\pi}{4}(Sin.\ p - p\ Cos.\ p)$ V. T. 208. N°. 17.

12) $\int Cot.(q\ Cot.\ x)\ \frac{d\ x}{Tang.\ x} = \frac{\pi}{e^q - 1}$ V. T. 205. N°. 18.

13) $\int Cot.(q\ Cot.\ x)\ \frac{d\ x}{Tang.\ x.\ Cos.\ 2\ x} = -\frac{1}{2}\pi$ V. T. 206. N°. 19.

14) $\int Cosec.(q\ Cot.\ x)\ \frac{d\ x}{Tang.\ x.\ Cos.\ 2\ x} = 0$ V. T. 206. N°. 20.

15) $\int Cos.^2(q\ Cot.\ x)\ \frac{d\ x}{Cos.\ 2\ x} = -\frac{1}{4}\pi\ Sin.\ 2\ p$ V. T. 206. N°. 21.

16) $\int Sin.(p\ Sin.\ x)\ \frac{d\ x}{Tang.\ x} = \sum_1^\infty \frac{p^{2n-1}}{1^{2n-1/1}}\ \frac{1}{2\ n + 1}$ V. T. 192. N°. 5.

17) $\int Sin.^q\ x.\ Sin.(a\ Sin.\ x)\ \frac{d\ x}{Tang.\ x} = \frac{1 - q}{(l\ q)^2 + a^2}\ \frac{a}{q}$ V. T. 192. N°. 4.

18) $\int Sin.(p\ Cosec.\ x).\ Sin.(p\ Cot.\ x)\ \frac{d\ x}{Cos.\ x} = \pi\ Sin.\ p$ V. T. 192. N°. 10.

19) $\int Sin.(p\ Sec.\ x).\ Sin.(p\ Tang.\ x)\ \frac{d\ x}{Sin.\ x} = \pi\ Sin.\ p$ V. T. 192. N°. 10.

20) $\int Sin.(\frac{1}{2}\ p\ \pi - q\ Cot.\ x)\ \frac{d\ x}{Tang.^{p-1}\ x} = \frac{1}{2}\pi\ e^{-q}$ V. T. 205. N°. 24.

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21) $\int Cos.(\frac{1}{2}p\pi - q\,Cot.\,x)\frac{dx}{Tang.^p x} = \frac{1}{2}\pi e^{-q}$ V. T. 205. N°. 25.

22) $\int \frac{Cos.(q\,Sin.\,x) - Cos.(q\,Cosec.\,x)}{Cos.\,x}dx = \frac{1}{2}\pi\,Sin.\,q$ V. T. 192. N°. 11.

23) $\int \frac{Cos.(q\,Cos.\,x) - Cos.(q\,Sec.\,x)}{Sin.\,x}dx = \frac{1}{2}\pi\,Sin.\,q$ V. T. 192. N°. 11.

24) $\int \{Sin.(q\,Cot.\,x) + Tang.\,x.\,Cos.(q\,Cot.\,x)\}\frac{dx}{Tang.\,x} = \pi e^{-q}$ V. T. 204. N°. 15.

F. Circ. Dir. irrat. ent. TABLE 72. Lim. 0 et $\frac{\pi}{2}$.

1) $\int dx\sqrt{(1-q^2 Sin.^2 x)} = 1 - \sum_1^\infty \left\{\frac{1^{n-1/2}}{n^{n/2}}\right\}^2 (2n-1)q^{2n},\ q<1;$ V. T. 12. N°. 14.

2) $= E'(q)$, la Fonction elliptique complète de seconde espèce. Legendre, Exerc. 1. 138.

3) $\int Sin.\,x\,dx\sqrt{(1-q^2 Sin.^2 x)} = \frac{1}{2}\left\{1 + \frac{1-q^2}{2q}\,l\,\frac{1+q}{1-q}\right\},\ q<1;$ Catalan, L. 4. 323. — Lobatto, L. 5. 113. — Dienger, Cr. 46. 119. — Grunert, Gr. 4. 113.

4) $\int Sin.^2 x\,dx\sqrt{(1-q^2 Sin.^2 x)} = \frac{2q^2-1}{3q^2}E'(q) + \frac{1-q^2}{3q^2}F'(q),\ q<1;$

5) $\int Cos.^2 x\,dx\sqrt{(1-q^2 Sin.^2 x)} = \frac{1+q^2}{3q^2}E'(q) - \frac{1-q^2}{3q^2}F'(q),\ q<1;$

(4, 5) Legendre, Exerc. 1. 138.

6) $\int Sin.^3 x\,dx\sqrt{(1-q^2 Sin.^2 x)} = \frac{3q^2-1}{8q^2} + \frac{1-q^2}{16}\frac{1+3q^2}{q^4}\,l\,\frac{1+q}{1-q},\ q<1;$

Lobatto, L. 5. 113. — Grunert, Gr. 4. 113. — Catalan, L. 4. 323. la trouve faut. $= \frac{3q-2}{q^2} + \ldots$

7) $\int dx\sqrt{(1-Sin.^2\lambda.\,Sin.^2 x)} = Cos.^2\lambda.\,Cosec.\,\lambda.\,E'(Sin.\,\lambda)$ Lobatschewsky, Mém. Kasan. 1835. 1.

8) $\int Sin.\,x.\,Cos.^2 x\,dx\sqrt{(1-q^2 Sin.^2 x)} = \frac{1}{4} + \frac{1-q^2}{8q^2} + \frac{(1-q^2)^2}{16q^3}\,l\,\frac{1-q}{1+q},\ q<1;$ Dienger, Cr. 46. 119.

9) $\int Tang.^2 x\,dx\sqrt{(1-q^2 Sin.^2 x)} = \infty,\ q<1;$ Legendre, Exerc. 1. 138.

10) $\int dx\sqrt{(1-q^2 Sin.^2 x)^3} = \frac{4-2q^2}{3}E'(q) - \frac{1-q^2}{3}F'(q),\ q<1;$ Legendre, Exerc. 1. 138. — Poisson, L. 2. 184.

11) $\int dx \sqrt{(1-p^2 Cos.^2 x)} = 1 - \sum_1^\infty \left\{\frac{1^{n-1/2}}{n^{n/2}}\right\}^2 (2n-1) p^{2n}$ V. T. 12. N°. 14.

12) $\int Sin.^p x . Cos.^{3-p} x (1 - q^2 Sin.^2 x)^{1-\frac{1}{2}p} dx = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(2-\frac{p}{2}\right)}{q^3\sqrt{\{\pi(p-1)(p-3)(p-5)\}}}$
$\left\{\frac{1+(p-3)q+q^2}{(1+q)^{p-3}} - \frac{1-(p-3)q+q^2}{(1-q)^{p-3}}\right\}$ V. T. 9. N°. 14.

13) $\int Tang.^{2a+1} x . Cos.^{a+\frac{1}{2}b} x \, dx = \frac{2^{a/2}}{b^{a+1/2}}$

14) $\int Tang.^{2a+1} x . Cos.^{a+b+\frac{1}{2}} x \, dx = \frac{1^{b+1/2}\, 1^{a/1}}{(2b+1)^{a+b+1/2}} 2^a$

15) $\int Tang.^2 x \, dx \sqrt{Cos.^3 x} = \frac{1}{3}$

(13–15) Oettinger, Cr. 38. 162.

16) $\int Sin.^{\frac{2a}{b}-1} x . Cos.^{2c-\frac{2a}{b}-1} x . Cos. 2cx \, dx = \frac{\pi}{2 b^{2c-1}} Cosec. \frac{a\pi}{b} \frac{(b-2a)^{2c-1/b}}{1^{2c-1/1}}$

17) $\int Sin.^{\frac{2a}{b}-1} x . Cos.^{\frac{2c}{d}-\frac{2a}{b}-1} x . Cos. \frac{2cx}{d} dx = \pi Sec. \frac{a\pi}{b} \frac{d.2d.3d.......}{2c.2c+d.2c+2d...} \frac{2bd.\ 2bd+bc...}{2bd-2ad.2bd-2ad+bc...}$

Voyez de ces deux formules: Oettinger, Cr. 38. 216.

18) $\int Sin.^{a-1} x \, dx \sqrt{(1+Cos. 2x)} = \frac{1}{a}\sqrt{2}$

19) $\int Sin.^{a-1} x (1+Cos. 2x)^{b+\frac{1}{2}} dx = \frac{1^{\frac{1}{2}a/1}\, 1^{b/1}}{1^{\frac{1}{2}a+b|1}} \frac{2^b}{a} \sqrt{2}$

(18–19) Oettinger, Cr. 38. 162.

20) $\int dx \sqrt[3]{Sin. x} = \frac{1-\sqrt{3}}{\sqrt[4]{3}} F'\left(Cos. \frac{\pi}{12}\right) + \frac{2\sqrt{3}}{\sqrt[4]{3}} E'\left(Cos. \frac{\pi}{12}\right)$ V. T. 12. N°. 15.

21) $\int dx \sqrt[3]{Sin.^2 x} = \frac{3\sqrt{3}}{\sqrt[4]{3}} E'\left(Sin. \frac{\pi}{12}\right) - \frac{3+2\sqrt{3}}{2\sqrt[4]{3}} F'\left(Sin. \frac{\pi}{12}\right)$ V. T. 12. N°. 16.

22) $\int dx \sqrt[3]{Cos. x} = \frac{1-\sqrt{3}}{\sqrt[4]{3}} F'\left(Cos. \frac{\pi}{12}\right) + \frac{2\sqrt{3}}{\sqrt[4]{3}} E'\left(Cos. \frac{\pi}{12}\right)$ V. T. 12. N°. 15.

23) $\int dx \sqrt[3]{Cos.^2 x} = \frac{3\sqrt{3}}{\sqrt[4]{3}} E'\left(Sin. \frac{\pi}{12}\right) - \frac{3+2\sqrt{3}}{2\sqrt[4]{3}} F'\left(Sin. \frac{\pi}{12}\right)$ V. T. 12. N°. 16.

1) $\int \frac{Sin.^{p-\frac{1}{2}} x}{Cos.^{2p-1} x} dx = \frac{2^{\frac{1}{2}-p}}{2p-1} \frac{\Gamma(p+\frac{1}{2})\Gamma(1-p)}{\sqrt{\pi}} Sin. \left\{\frac{2p-1}{4}\pi\right\}$, $p < 1$; V. T. 12. N°. 18.

2) $\int \frac{Cos.^{p-\frac{1}{2}} x}{Sin.^{2p-1} x} dx = \frac{2^{\frac{1}{2}-p}}{2p-1} \frac{\Gamma(p+\frac{1}{2})\Gamma(1-p)}{\sqrt{\pi}} Sin. \left\{\frac{2p-1}{4}\pi\right\}$, $p < 1$; V. T. 12. N°. 18.

3) $\int \frac{dx}{\sqrt{Cos. x}} = \sqrt{2}. F'\left(Sin. \frac{\pi}{4}\right)$ V. T. 13. N°. 6.

4) $\int (\sqrt{Tang. x} + \sqrt{Cot. x})\, dx = \pi \sqrt{2}$ V. T. 50. N°. 15 et T. 92. N°. 13.

5) $\int \frac{Cos. x - Sin. x}{\sqrt[4]{Cos.^3 2x}} dx = 0$ V. T. 27. N°. 14.

6) $\int \frac{Cos. x + Sin. x}{Cos. 2x \sqrt{Sin. 2x}} dx = 0$ V. T. 28. N°. 2.

7) $\int \frac{dx}{\sqrt[3]{Sin. x}} = \frac{1}{\sqrt[4]{3}} F'\left(Cos. \frac{\pi}{12}\right)$ V. T. 15. N°. 12.

8) $\int \frac{dx}{\sqrt[3]{Sin.^2 x}} = \frac{3}{\sqrt[4]{3}} F'\left(Sin. \frac{\pi}{12}\right)$ V. T. 15. N°. 11.

9) $\int \frac{dx}{\sqrt[3]{Cos. x}} = \frac{1}{\sqrt[4]{3}} F'\left(Cos. \frac{\pi}{12}\right)$ V. T. 15. N°. 12.

10) $\int \frac{dx}{\sqrt[3]{Cos.^2 x}} = \frac{3}{\sqrt[4]{3}} F'\left(Sin. \frac{\pi}{12}\right)$ V. T. 15. N°. 11.

11) $\int \frac{dx}{Tang. x} \sqrt[q]{Tang.^2 x} = \frac{1}{2}\pi\, Cosec. \frac{\pi}{q}$ V. T. 32. N°. 13.

12) $\int \frac{Tang. x}{\sqrt[q]{Tang.^2 x}} dx = \frac{1}{2}\pi\, Cosec. \frac{\pi}{q}$ V. T. 32. N°. 13.

13) $\int dx \sqrt{\frac{1-p^2 Sin.^2 x}{Sin. x}} = \frac{2c F'(c) + 2b F'(b)}{(b+c)^2} + 2\frac{b-c}{(b+c)^2}\left\{E'(b) - E'(c)\right\}$, où $2c^2 = \frac{(1-\sqrt{p})^2}{1+p}$ et $2b^2 = \frac{(1+\sqrt{p})^2}{1+p}$; Legendre, Exerc. 6. 308.

14) $\int \frac{dx}{Cos.^2 x} \sqrt{(1-p^2 Sin. x)} = \infty$ Legendre, Exerc. 1. 138.

15) $\int (Sec. x - 1)^{p+\frac{1}{2}} Sin. x\, dx = \frac{2p+1}{2}\pi\, Sec. p\pi$ V. T. 32. N°. 3.

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16) $\int (Sec.\, x - 1)^{p-\frac{1}{2}}\, Tang.\, x\, dx = \pi\, Sec.\, p\pi$ V. T. 32. N°. 1.

17) $\int (Cosec.\, x - 1)^{p+\frac{1}{2}}\, Cos.\, x\, dx = \frac{2p+1}{2}\pi\, Sec.\, p\pi$ V. T. 32. N°. 3.

18) $\int (Cosec.\, x - 1)^{p-\frac{1}{2}}\, \frac{dx}{Tang.\, x} = \pi\, Sec.\, p\pi$ V. T. 32. N°. 1.

F. Circ. Dir. irrat. fract. à dén. binôme du 1er degré. TABLE 74. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{dx}{\sqrt{(a + b\, Cos.\, x)}} = \frac{2}{\sqrt{(a+b)}} F'\left(\sqrt{\frac{2b}{a+b}}\right)$

2) $\int \frac{dx}{\sqrt{(a - b\, Cos.\, x)}} = \frac{2}{\sqrt{(a+b)}} \left\{F'\left(\sqrt{\frac{2b}{a+b}}\right) - F\left(\frac{1}{4}\pi, \sqrt{\frac{2b}{a+b}}\right)\right\}$

3) $\int \frac{Cos.\, x}{\sqrt{(a - b\, Cos.\, x)}}\, dx = \frac{2}{b\sqrt{(a+b)}} \left\{a F'\left(\sqrt{\frac{2b}{a+b}}\right) - F\left(\frac{1}{4}\pi, \sqrt{\frac{2b}{a+b}}\right)\right\}$
$- (a+b)\left\{E'\left(\sqrt{\frac{2b}{a+b}}\right) - E\left(\frac{1}{4}\pi, \sqrt{\frac{2b}{a+b}}\right)\right\}$

4) $\int \frac{Cos.\, x}{\sqrt{(a+b\, Cos.\, x)}}\, dx = \frac{2}{b\sqrt{(a+b)}} \left\{(a+b) E'\left(\sqrt{\frac{2b}{a+b}}\right) - a F'\left(\sqrt{\frac{2b}{a+b}}\right)\right\}$

(1–4): $a > b > 0$; Dienger, Gr. 13. 424.

5) $\int \frac{dx}{\sqrt{(3 \pm Cos.\, 2x)}} = \frac{1}{2} F'\left(Sin.\, \frac{\pi}{4}\right)$ V. T. 13. N°. 6.

6) $\int \frac{Sin.^2\, x}{\sqrt{(3 + Cos.\, 2x)}}\, dx = \frac{\{\Gamma(\frac{1}{4})\}^2}{8\sqrt{\pi}} - \frac{\pi\sqrt{\pi}}{\{\Gamma(\frac{1}{4})\}^2}$ V. T. 13. N°. 9.

7) $\int \frac{Sin.^3\, x}{\sqrt{(3 - Cos.\, 2x)}}\, dx = \frac{1}{4}\sqrt{2}$ V. T. 13. N°. 7.

8) $\int \frac{Tang.^{p+\frac{1}{2}}\, x}{(Cos.\, x + Sin.\, x)^2}\, dx = \frac{2p+1}{2}\pi\, Sec.\, p\pi$ V. T. 27. N°. 4.

9) $\int \frac{Sin.\, x}{(Sec.\, x - 1)^{p+\frac{1}{2}}}\, dx = \frac{2p+1}{2}\pi\, Sec.\, p\pi$ V. T. 32. N°. 17.

10) $\int \frac{Tang.\, x}{(Sec.\, x - 1)^{p-\frac{1}{2}}}\, dx = \pi\, Sec.\, p\pi$ V. T. 32. N°. 15.

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11) $\int \frac{Cos. x}{(Cosec. x - 1)^{p+\frac{1}{2}}} dx = \frac{2p+1}{2} \pi \, Sec. p\pi$ V. T. 32. N°. 17.

12) $\int \frac{Sin.^{p+\frac{1}{2}} x}{(1 - Sin. x)^{p-\frac{1}{2}} Tang. x} dx = \frac{1-2p}{2} \pi \, Sec. p\pi$ V. T. 12. N°. 8.

F. Circ. Dir. irrat. fract. à dén. binôme du 2d degré. TABLE 75. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{dx}{\sqrt{(1 + Sin.^2 x)}} = \frac{1}{2} \sqrt{2} \,.\, F'\left(Sin. \frac{\pi}{4}\right)$ V. T. 13. N°. 6.

2) $\int \frac{Cos.^2 x}{\sqrt{(1 + Sin.^2 x)}} dx = \frac{\{\Gamma(\frac{1}{4})\}^2}{4\sqrt{2\pi}} - \frac{\pi\sqrt{2\pi}}{\{\Gamma(\frac{1}{4})\}^2}$ V. T. 12. N°. 9.

3) $\int \frac{Sin.^2 x}{\sqrt{(1 + Sin.^2 x)}} dx = \frac{1}{2}\sqrt{2} \,.\, F'\left(Sin. \frac{\pi}{4}\right) + \frac{\pi\sqrt{2\pi}}{\{\Gamma(\frac{1}{4})\}^2} - \frac{\{\Gamma(\frac{1}{4})\}^2}{4\sqrt{2\pi}}$ V. T. 75. N°. 1, 2.

4) $\int \frac{Sin.^3 x}{\sqrt{(1 + Sin.^2 x)}} dx = \frac{1}{2}$ V. T. 13. N°. 7.

5) $\int \frac{Sin. x}{\sqrt{(p^2 + Sin.^2 x)}} dx = \frac{\pi}{2} - Arctang. p$ V. T. 16. N°. 6.

6) $\int \frac{Sin. x}{\sqrt{(1 + p^2 Sin.^2 x)}} dx = \frac{1}{p} Arctang. p$

7) $\int \frac{Sin.^3 x}{\sqrt{(1 + p^2 Sin.^2 x)^3}} dx = \frac{1}{p^3} \left\{ -\frac{p}{1+p^2} + Arctang. p \right\}$

(6, 7) $p^2 < 1$; Dienger, Cr. 46. 119.

8) $\int \frac{Cos. x}{\sqrt{(p^2 + Cos.^2 x)}} dx = \frac{\pi}{2} - Arctang. p$ V. T. 16. N°. 6.

9) $\int \frac{dx}{\sqrt{(1 - p^2 Sin.^2 x)}} = F'(p)$, la Fonction elliptique complète de première espèce; Legendre, Exerc. 1. 138.

10) $\int \frac{Sin. x}{\sqrt{(1 - p^2 Sin.^2 x)}} dx = \frac{1}{2p} l \frac{1+p}{1-p}$, $p^2 < 1$; Dienger, Cr. 46. 119.

11) $\int \frac{Sin.^2 x}{\sqrt{(1 - p^2 Sin.^2 x)}} dx = \frac{1}{p^2} \{F'(p) - E'(p)\}$

12) $\int \frac{Cos.^2 x}{\sqrt{(1 - p^2 Sin.^2 x)}} dx = F'(p) - \frac{1}{p^2}\{F'(p) - E'(p)\}$

(11, 12) $p^2 < 1$; Legendre, Exerc. 1. 138. — Dienger, Cr. 46. 119.

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13) $\int \frac{Tang.^2 x}{\sqrt{(1-p^2 Sin.^2 x)}} dx = \infty$ Legendre, Exerc. 1. 138.

14) $\int \frac{Cos. 2x}{\sqrt{(1-p^2 Sin.^2 x)}} dx = F'(p) - \frac{2}{p^2}\{F'(p) - E'(p)\}$ V. T. 75. N°. 11, 12.

15) $\int \frac{Sin.^4 x}{\sqrt{(1-p^2 Sin.^2 x)}} dx = \frac{2}{3}\frac{1+p^2}{p^4}\{F'(p) - E'(p)\} - \frac{1}{3p^2} F'(p)$ Dienger, Cr. 46. 119. — Poisson, L. 2. 184.

16) $\int \frac{Sin. x. Cos.^2 x}{\sqrt{(1-p^2 Sin.^2 x)}} dx = \frac{1}{2p^2} + \frac{1-p^2}{4p^3} l \frac{1-p}{1+p}$ Dienger, Cr. 46. 119.

17) $\int \frac{Tang.^2 \frac{1}{2} x}{\sqrt{(1-p^2 Sin.^2 x)}} dx = 2\sqrt{(1-p^2)} + F'(p) - 2E'(p)$

18) $\int \frac{dx}{\sqrt{(1-p^2 Sin.^2 x)^3}} = \frac{1}{1-p^2} E'(p)$

(17, 18) $, p < 1$; Legendre, Exerc. 1. 138.

19) $\int \frac{Sin. x}{\sqrt{(1-p^2 Sin.^2 x)^3}} dx = \frac{1}{1-p^2}$ Plana, Cr. 17. 345.

20) $\int \frac{Sin.^2 x}{\sqrt{(1-p^2 Sin.^2 x)^3}} dx = \frac{1}{p^2(3-p^2)} E'(p) - \frac{1}{p^2} F'(p)$ Roberts, L. 11. 157.

21) $\int \frac{2 Sin.^2 x - 1}{\sqrt{(1-p\, Sin.^2 x)}} dx = \left(\frac{2}{p^2} - 1\right) F'(p) - \frac{2}{p^2} E'(p)$ Ramus, Danske Afh. 6. 265.

22) $\int \frac{Sin.^6 x}{\sqrt{(1-p^2 Sin.^2 2x)}} dx = \frac{3}{8p^2}\{E'(p) - F'(p)\} + \frac{1}{2} F'(p)$ V. T. 28. N°. 22.

23) $\int \frac{Sin.^6 x. Cos.^4 x}{\sqrt{(1-p^2 Sin.^2 2x)}} dx = \frac{2p^2+1}{8p^2} E'(p) - \frac{1-p^2}{8p^2} F'(p)$ V. T. 28. N°. 23.

24) $\int \frac{Sin. x. Cos. x}{\sqrt{(Cos.^2 \mu - Sin.^2 \lambda. Sin.^2 x)}} dx = \frac{Cos. \mu}{Sin.^2 \lambda} - \frac{1}{Sin.^2 \lambda}\sqrt{(Cos.^2 \mu + Sin^2 \lambda)}$ Poisson, Chaleur. § 216.

25) $\int dx \sqrt{\frac{1-p^2 Sin.^4 x}{1+Sin.^2 x}} = \frac{cF'(c)+bF'(b)}{(b+c)^2} + \frac{b-c}{(b+c)^2}\{E'(b) - E'(c)\}$, où $2c^2 = \frac{(1-\sqrt{p})^2}{1+p}$, $2b^2 = \frac{(1+\sqrt{p})^2}{1+p}$; V. T. 13. N°. 8.

26) $\int \frac{Tang.^{2a} x. Cos.^2 x}{(1+Sec.^2 x)^{b+\frac{1}{2}}} dx = 2^{a-b+\frac{1}{2}} \frac{\Gamma(a+\frac{1}{2})\Gamma(b-a)}{\Gamma(b+\frac{1}{2})}$ V. T. 27. N°. 12.

1) $\int \frac{dx}{Cos.^2 x \sqrt{(1-p^2 Sin.^2 x)}} = \infty$ ⎫ $, p < 1$; Legendre, Exerc. 1. 138.

2) $\int \frac{dx}{Cos.^2 \frac{1}{2} x \sqrt{(1-p^2 Sin.^2 x)}} = 2\sqrt{(1-p^2)} + 2 F'(p) - 2 E'(p)$ ⎭

3) $\int \frac{dx}{Sin.^2 x \sqrt{(1-p^2 Sin.^2 x)}} = F'(p) - E'(p) - \sqrt{(1-p^2)}$ Roberts, L. 11. 157.

4) $\int \frac{Sin. x}{1 - p\, Sin. x} \frac{dx}{\sqrt{(1-p^2 Sin.^2 x)}} = \frac{1}{p^3} E'(p) + \frac{1}{(1-p^2)p} F'(p)$ ⎫ Dienger, Gr. 11. 88.

5) $\int \frac{1}{1 - p\, Sin. x} \frac{dx}{\sqrt{(1-p^2 Sin.^2 x)}} = \frac{1}{p^2} E'(p) + \frac{p}{1-p^2}$ ⎭

6) $\int \frac{dx}{\sqrt{\{Cos. x (Cos. x + p^2 Sin. x)(Cos. x + q^2 Sin. x)(Cos. x + r^2 Sin. x)\}}} = \frac{2}{\sqrt{(p^2-r^2)}} F\left(\varphi, \sqrt{\frac{p^2-q^2}{p^2-r^2}}\right)$ V. T. 28. N°. 14.

7) $\int \frac{dx}{\sqrt{\{Cos. x (p^2 Cos. x + Sin. x)(q^2 Cos. x + Sin. x)(r^2 Cos. x + Sin. x)\}}} = \frac{2}{\sqrt{(p^2-r^2)}} F\left(\varphi, \sqrt{\frac{p^2-q^2}{p^2-r^2}}\right)$ V. T. 28. N°. 15.

Dans les formules 4, 5, on a $Cos. \varphi = \frac{r}{p}$, où $p > q > r$;

8) $\int \frac{dx}{\sqrt{\{Cos. x (p^2 Cos. x + l^2 Sin. x)(q^2 Cos. x + m^2 Sin. x)(r^2 Cos. x + n^2 Sin. x)\}}} = \frac{\pi}{2 p m n\, Sin. \varphi}$

$F\left(\varphi, \frac{n}{m}\sqrt{\frac{p^2 m^2 - q^2 l^2}{p^2 n^2 - r^2 l^2}}\right)$, où $Cos.^2 \varphi = \frac{r l}{p n}$, $pm > ql, pn > rl$; V. T. 28. N°. 16.

9) $\int \frac{dx}{\sqrt{\{Sin. x (p^2 Cos. x + l^2 Sin. x)(q^2 Cos. x + m^2 Sin. x)(r^2 Cos. x + n^2 Sin. x)\}}} = \frac{\pi}{2 l q r\, Sin. \varphi}$

$F\left(\varphi, \frac{r}{p}\sqrt{\frac{p^2 m^2 - q^2 l^2}{p^2 n^2 - r^2 l^2}}\right)$, où $Cos.^2 \varphi = \frac{pn}{rl}$, $ql > pm, rl > pn$; V. T. 28. N°. 16.

10) $\int \frac{Sin.^{p-\frac{1}{2}} x}{Cos. x + Sin. x} \frac{dx}{Cos.^{p+\frac{1}{2}} x} = \pi\, Sec. p\pi$ V. T. 27. N°. 2.

11) $\int \frac{1}{Cos. x + Sin. x} \frac{dx}{Cos. x. Tang.^{p-\frac{1}{2}} x} = \pi\, Sec. p\pi$ V. T. 28. N°. 5.

12) $\int \frac{1}{(Cos. x + Sin. x)^2} \frac{dx}{Tang.^{p+\frac{1}{2}} x} = \frac{2p+1}{2} \pi\, Sec. p\pi$ V. T. 27. N°. 4.

13) $\int \frac{Sin.^{\frac{1}{2}(p+q-1)} x}{(Cos. x + Sin. x)^{p+\frac{1}{2}}} \frac{dx}{Cos.^{\frac{1}{2}(q+1)} x} = \frac{\Gamma\left(\frac{p+q}{2}\right)\Gamma\left(\frac{p-q+1}{2}\right)}{\Gamma(p+\frac{1}{2})}$ V. T. 27. N°. 8.

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14) $\int \frac{1}{(Cosec.\, x - 1)^{p-\frac{1}{2}}} \frac{dx}{Tang.\, x} = \pi\, Sec.\, p\, \pi$ V. T. 32. N°. 15.

15) $\int \frac{Sin.^{q+1} x}{(1 - p^2\, Sin.^2 x)^{\frac{q+1}{2}}} \frac{dx}{Cos.^q x} = \frac{(1-p)^{-q} - (1+p)^{-q}}{4pq}\, B\left(\frac{q+2}{2}, \frac{1-q}{2}\right)$ V. T. 16. N°. 7.

16) $\int \frac{Sin.^{q+1} x}{(Cos.^2 \lambda - Sin.^2 x.\, Sin.^2 \lambda)^{\frac{q+1}{2}}} \frac{dx}{Cos.^q x} = \frac{Sin.\, q\lambda}{2q Sin.\lambda}\, B\left(\frac{2+q}{2}, \frac{1-q}{2}\right)$ V. T. 16. N°. 8.

17) $\int \frac{1}{Sin.\, x + Cos.\, x} \frac{dx}{\sqrt{Sin.\, 2x}} = \frac{\pi}{2}\sqrt{2}$ V. T. 28. N°. 1.

18) $\int \frac{Sin.\, x.\, Cos.\, x}{1 - Sin.^2\lambda.\, Sin.^2 x} \frac{dx}{\sqrt{(Cos.^2\mu - Sin.^2\lambda.\, Sin.^2 x)}} = \frac{1}{Sin.^2\lambda.\, Sin.^2\mu}\left\{\frac{\pi}{2} - \mu - Arccos.\left(\frac{Sin.\mu}{Cos.\lambda}\right)\right\}$

19) $\int \frac{Cos.^2 x}{1 - Sin.^2\lambda.\, Sin.^2 x} \frac{dx}{\sqrt{(Cos.^2\mu - Sin.^2\lambda.\, Sin.^2 x)}} = Sec.\, \mu\, F'\left(\frac{Sin.\lambda}{Cos.\mu}\right) -$

$- \frac{Cos.\lambda}{Sin.^2\lambda.\, Sin.\mu}\left\{F'\left(\frac{Sin.\lambda}{Cos.\mu}\right) E\left(\frac{Sin.\lambda}{Cos.\mu}, \frac{\pi}{2} - \mu\right) - E'\left(\frac{Sin.\lambda}{Cos.\mu}\right) F\left(\frac{Sin.\lambda}{Cos.\mu}, \frac{\pi}{2} - \mu\right)\right\}$

(18, 19: Poisson, Chaleur. 216.)

20) $\int \frac{Cos.^2 x}{1 + p^2\, Sin.^2 x} \frac{dx}{\sqrt{(1 - q^2\, Sin.^2 x)}} = \frac{Tang.\, \mu}{2\sqrt{(Cos.^2\mu + q^2\, Sin.^2\mu)}}$

$\left[\pi + 2F'(q)F\{\sqrt{(1-q^2)}, \mu\} - 2F'(q)E\{\sqrt{(1-q^2)}, \mu\} - 4E'(q)F\{\sqrt{(1-q^2)}, \mu\}\right]$, où $p = Cot.\mu$; Catalan, L. 6. 419.

21) $\int \frac{Sin.\, x}{\sqrt{\left(\frac{Cos.^2 x}{r^2} + \frac{Sin.^2 x}{q^2}\right)}} \frac{dx}{\sqrt{\left(\frac{Cos.^2 x}{r^2} + \frac{Sin.^2 x}{p^2}\right)}} = \frac{pqr}{\sqrt{(r^2 - p^2)}} F\left\{Arccos.\frac{p}{r}, \sqrt{\frac{r^2 - q^2}{r^2 - p^2}}\right\}$, $r > q > p$; Jacobi, Cr. 10. 101.

F. Circ. Dir. irrat. fract. comp. TABLE 77. Lim. 0 et $\frac{\pi}{2}$.

1) $\int Sni.\left(\frac{1}{k} Sec.\, x\right) \frac{dx}{\sqrt{Cos^3.\, x}} = \left(Cos.\frac{1}{k} + Sin.\frac{1}{k}\right)\sqrt{\frac{k\pi}{4}}$

2) $\int Cos.\left(\frac{1}{k} Sec.\, x\right) \frac{dx}{\sqrt{Cos.^3 x}} = \left(Cos.\frac{1}{k} - Sin.\frac{1}{k}\right)\sqrt{\frac{k\pi}{4}}$

(1, 2:) , pour $k = \infty$; Poisson, Mém. Ac. 1816. 71. N°. 40.

3) $\int Sin.\, (p\, Tang.\, x) \frac{dx}{Cos.\, x\sqrt{Sin.\, 2x}} = \frac{1}{2}\sqrt{\frac{\pi}{p}}$ V. T. 224. N°. 4.

4) $\int Cos.\, (p\, Tang.\, x) \frac{dx}{Cos.\, x\sqrt{Sin.\, 2x}} = \frac{1}{2}\sqrt{\frac{\pi}{p}}$ V. T. 224. N° 5.

1) $\int Sin.x\,dx = 2$
2) $\int Sin.^3 x\,dx = \frac{4}{3}$
Poisson, Chaleur. 82.

3) $\int Sin.^{2a+1} x\,dx = \frac{(1^{a/1})^2}{2^{2a/1}} 2^{2a}$ Oettinger, Cr. 38. 162.

4) $\int Sin.^a x\,dx = \frac{\pi}{2^a} \frac{\Gamma(a+1)}{\left\{\Gamma\left(\frac{a}{2}-1\right)\right\}^2}$ Lobatschewsky, Mém. Kasan. 1835. 211.

5) $\int Sin.^{p-1} x\,dx = \frac{\Gamma(\frac{1}{2}p)}{\Gamma\left(\frac{1+p}{2}\right)} \sqrt{\pi}$ V. T. 53. N°. 20.

6) $= \frac{\left\{\Gamma(\frac{1}{2}p)\right\}^2}{\Gamma(p)} 2^{p-1}$ Lobatschewsky, Mém. Kasan. 1835. 1.

7) $\int Cos.^{2a+1} x\,dx = 0$
8) $\int Cos.^{2a} x\,dx = \frac{1^{a/2}}{2^{a/2}} \pi$
Cauchy, Exerc. 1826. p. 205.

9) $\int Sin.^2 px\,dx = \frac{1}{2}\pi$
10) $\int Cos.^2 px\,dx = \frac{1}{2}\pi$
Euler, Calc. Int. 4. S. 4. 94. — Poisson, Chal. 92.

11) $\int Sin.\,ax.\,Sin.\,bx\,dx = 0$
12) $\int Cos.\,ax.\,Cos.\,bx\,dx = 0$
, $a >$ ou $< b$; Poisson, Chal. 92.

13) $\int Sin.px.Sin.ax\,dx = (-1)^{a-1} \frac{a\,Sin.p\pi}{a^2-p^2}$
14) $\int Cos.px.\,Sin.\,x\,dx = \frac{\pi\,Cos.p\pi}{1-p^2} + \frac{2p\,Sin.p\pi}{(1-p^2)^2}$
15) $\int Cos.px.Cos.ax\,dx = (-1)^a \frac{p\,Sin.p\pi}{a^2-p^2}$
Schlömilch, Beitr. I. § 8.

16) $\int Sin.^q x . Sin. q x dx = \frac{\pi}{2^q} Sin. \frac{q\pi}{2}$

17) $\int Sin.^q x . Cos. q x dx = \frac{\pi}{2^q} Cos. \frac{q\pi}{2}$

} Lobatschewsky, Mém. Kasan. 1835. 211.

18) $\int Sin.^q x . Sin. p x dx = \frac{\pi}{2^q} \frac{Sin. \frac{1}{2} p\pi \Gamma(q+1)}{\Gamma\left(\frac{p+q}{2}+1\right)\Gamma\left(\frac{q-p}{2}+1\right)}$

19) $\int Sin.^q x . Cos. p x dx = \frac{\pi}{2^q} \frac{Cos. \frac{1}{2} p\pi \Gamma(q+1)}{\Gamma\left(\frac{p+q}{2}+1\right)\Gamma\left(\frac{q-p}{2}+1\right)}$

} Kummer, Cr. 17. 210. — Lobatschewsky, Mém. Kasan. 1835. 211.

20) $\int Sin.^{q-1} x . Sin. p x dx = 2^{q-1} Sin. \frac{1}{2} p\pi \frac{\Gamma\left(\frac{q-p}{2}\right)\Gamma\left(\frac{q+p}{2}\right)\Gamma(q)}{\Gamma(q-p)\Gamma(q+p)}$

21) $\int Sin.^{q-1} x . Cos. p x dx = 2^{q-1} Cos. \frac{1}{2} p\pi \frac{\Gamma\left(\frac{q-p}{2}\right)\Gamma\left(\frac{q+p}{2}\right)\Gamma(q)}{\Gamma(q-p)\Gamma(q+p)}$

22) $\int Sin.^{q-1} x . Cos. \left\{p\left(\frac{\pi}{2}-x\right)\right\} dx = 2^{q-1} \frac{\Gamma\left(\frac{q-p}{2}\right)\Gamma\left(\frac{q+p}{2}\right)\Gamma(q)}{\Gamma(q-p)\Gamma(q+p)}$

} Serret, L. 8. 1.

23) $\int Cos.^{2a-1} x . Sin.^{2b-1} x dx = 0$ Poisson, Chaleur. 80.

24) $\int Sin.^b x . Cos.^{2a} x dx = \frac{\Gamma\left(\frac{b+1}{2}\right)\Gamma\left(\frac{2a+1}{2}\right)}{\Gamma\left(a+\frac{b}{2}+1\right)}$ Lobatschewsky, Mém. Kasan. 1836. 1.

25) $\int Cos.^{a+b} \frac{1}{2} x . Cos. \frac{1}{2} b x . Cos. \frac{1}{2} a x dx = \frac{\pi}{2^{a+b+1}} \left\{2 + \sum_1^\infty \binom{a}{n}\binom{b}{n}\right\}$

26) $\int Cos.^a \frac{1}{2} x . Cos. \frac{1}{2} a x dx = \frac{\pi}{2^a}$

} Smaasen, Cr. 42. 222.

F. Circ. Dir. rat. ent. trinôme. TABLE 79. Lim. 0 et π.

1) $\int (1 + p^2 - 2 p \, Cos. x) \, dx = (1 + p^2) \pi$

2) $\int (1 + p^2 - 2 p \, Cos. x) \, Cos. x \, dx = - p \pi$

3) $\int (1 + p^2 - 2 p \, Cos. x)^2 \, dx = (1 + 4 p^2 + p^4) \pi$

4) $\int (1 + p^2 - 2 p \, Cos. x)^2 \, Cos. x \, dx = - 2 p (1 + p^2) \pi$

5) $\int (1 + p^2 - 2 p \, Cos. x)^2 \, Cos. 2 x \, dx = p^2 \pi$

(1–5) Euler, Calc. Int. 4. S. 4. 23.

6) $\int (1 + p^2 - 2 p \, Cos. x)^a \, Cos. a x \, dx = (-1)^a p^a \pi$ Euler, Calc. Int. 4. S. 4. 30. — Legendre, Exerc. 3. 64.

7) $\int (1 + p^2 - 2 p \, Cos. x)^a \, dx = \pi \sum_0^a \binom{a}{n}^2 p^{2n}$ Euler, Calc. Int. 4. S. 4. 31, 67. — Legendre, Exerc. 3. 64.

8) $\int (1 + p^2 - 2 p Cos. x)^a Cos. b x dx = \pi (-p)^b \frac{a^{b/1}}{1^{b/1}} \left\{ 1 + \binom{a}{1} \frac{a-b}{b+1} p^2 + \binom{a}{2} \frac{a-b . a-b-1}{b+1 . b+2} p^4 + \dots \right\}$ Legendre, Exerc. 3. 63.

F. Circ. Dir. rat. ent. composée. TABLE 80. Lim. 0 et π.

1) $\int Cos. (a \, Sin. x) \, dx = \pi \sum_0^\infty \frac{(-a^2)^n}{(2^{n+1/2})^2}$ Fourier, Chal. 314.

2) $\int Cos. (q \, Cos. x) \, Sin. x \, dx = \frac{2}{q} Sin. q$

3) $\int Cos. (q \, Cos. x) \, Sin.^3 x \, dx = \frac{4}{q^3} (Sin. q - q \, Cos. q)$

(2–3) Poisson, Chal. 82.

4) $\int Cos. (q \, Sin. x) \, Cos. \{(2 b + 1) x\} \, dx = 0$

5) $\int Sin. (q \, Sin. x) \, Sin. 2 b x \, dx = 0$

6) $\int Cos. (q \, Sin. x) \, Cos. 2 b x \, dx = \left(\frac{q}{2}\right)^{2b} \frac{\pi}{1^{2b/1}} \left[1 + \sum_1^\infty (-1)^n \frac{(\frac{1}{2} q)^{2n}}{1^{n/1} (2 b + 1)^{n/1}} \right]$

7) $\int Sin. (q \, Sin. x) Sin. \{(2 b + 1) x\} \, dx = \left(\frac{q}{2}\right)^{2b+1} \frac{\pi}{1^{2b+1/1}} \left[1 + \sum_1^\infty (-1)^n \frac{(\frac{1}{2} q)^{2n}}{1^{n/1} (2 b + 2)^{n/1}} \right]$

(4–7) Poisson, Conn. des Temps. 1836. 1. — Lefort, L. 11. 142.

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8) $\int Cos.\left\{a(x-q\,Sin.\,x)\right\}dx = \frac{\pi(\frac{1}{2}aq)^a}{1^{a/1}}\left\{1+\sum_1^\infty(-1)^n\frac{(\frac{1}{2}aq)^{2n}}{1^{n/1}(1+a)^{n/1}}\right\}$

9) $\int(1-q\,Cos.x)^2 Cos.\left\{a(x-q\,Sin.x)\right\}dx = -\frac{\pi}{a}\frac{(\frac{1}{2}aq)^a}{1^{a/1}}\left\{1+\sum_1^\infty(-1)^n(\tfrac{1}{2}aq)^{2n}\frac{a+2n}{1^{n/1}(1+a)^{n/1}}\right\}$

Bessel, Abh. Berlin. 1816. 49. — Id., ib. 1824. 1. — Poisson, Conn. des Temps. 1836. 1. — Lefort, L. 11. 142.

10) $\int\frac{Sin.\left\{a(x-q\,Sin.x)\right\}}{(1-q\,Cos.\,x)^2}Sin.\,x\,dx = \frac{\pi}{2}\frac{a^2(\frac{1}{2}aq)^{a-1}}{1^{a/1}}\left\{1+\sum_1^\infty(-1)^n\frac{(\frac{1}{2}aq)^{2n}}{1^{n/1}(1+a)^{n/1}}\right\}$

11) $\int\frac{q-Cos.x}{(1-q\,Cos.x)^2}Cos.\left\{a(x-q\,Sin.x)\right\}dx = -\frac{\pi}{a}\frac{(\frac{1}{2}aq)^a}{1^{a/1}}\left\{1+\sum_1^\infty(-1)^n(\tfrac{1}{2}aq)^{2n}\frac{a+2n}{1^{n/1}(1+a)^{n/1}}\right\}$

Poisson, Conn. des Temps. 1836. 1.

F. Circ. Dir. rat. fract. à dén. monôme. TABLE 81. Lim. 0 et π.

1) $\int\frac{dx}{Cos.\,x} = 0$ V. T. 19. N°. 13.

2) $\int\frac{Sin.\,x}{Cos.^2\,x}dx = \infty$ Schlömilch, Int. 24.

3) $\int Sin.^{2a+1}x\,\frac{dx}{Tang.\frac{1}{2}x} = (-1)^a\binom{-\frac{1}{2}}{a}$ Raabe, Cr. 25. 160.

F. Circ. Dir. rat. fract. à dén. binôme du 1^er^ degré. TABLE 82. Lim. 0 et π.

1) $\int\frac{dx}{2-Sin.\,x} = \frac{4\pi}{3\sqrt{3}}$ V. T. 65. N°. 3.

2) $\int\frac{dx}{2+Sin.\,x} = \frac{2\pi}{3\sqrt{3}}$ V. T. 65. N°. 4.

3) $\int\frac{dx}{1-Sin.\,x.\,Cos.\frac{2a\pi}{b}} = \frac{1}{b}Cosec.\frac{2a\pi}{b}\sum_1^{b-1}Sin.\frac{2na\pi}{b}.\,Cot.\frac{n\pi}{b}$ V. T. 25. N°. 14.

4) $\int\frac{dx}{1-Sin.\,x.\,Cos.\,\lambda} = 2(\pi-\lambda)\,Cosec.\,\lambda$ V. T. 25. N°. 2.

5) $\int\frac{dx}{1+Sin.\,x.\,Cos.\,\lambda} = 2\lambda\,Cosec.\,\lambda$ V. T. 25. N°. 3.

6) $\int\frac{dx}{p+q\,Cos.\,x} = \frac{\pi}{\sqrt{(p^2-q^2)}},\ p^2>q^2;$ Euler, Calc. Int. T. 4. S. 6. 22. — Schlömilch, Cr. 33. 268. — Ramus, Danske Afh. 6. 265. — Björling, Gr. 21. 26.

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7) $\int \frac{dx}{p + q\,Cos.\,x} = 0$ (valeur princ.) , $p^2 < q^2$;

8) $\int \frac{dx}{-p+q\,Cos.\,x} = \frac{-\pi}{\sqrt{(p^2-q^2)}}$, $p^2 > q^2$;

9) $= 0$ (valeur princ.) , $p^2 < q^2$; — Björling, Gr. 21. 26.

10) $\int \frac{Sin.^a x}{p+q\,Cos.\,x}\,dx = \frac{2\sqrt{\pi}}{a(p^2-q^2)^{\frac{a+1}{2}}} \frac{\Gamma\left(\frac{a+1}{2}\right)}{\Gamma\left(\frac{a}{2}\right)}$ Lobatschewsky, Mém. Kasan. 1835. 1.

11) $\int \frac{Cos.\,a\,x}{1+p\,Cos.\,x}\,dx = \frac{\pi}{\sqrt{(1-p^2)}} \left\{\frac{\sqrt{(1-p^2)}-1}{p}\right\}^a$, $p < 1$;

12) $\int \frac{Cos.\,a\,x}{1-p\,Cos.\,x}\,dx = \frac{\pi}{\sqrt{(1-p^2)}} \left\{\frac{1-\sqrt{(1-p^2)}}{p}\right\}^a$ — Ohm, Ausw. 26.

13) $\int \frac{dx}{a-b\,i\,Cos.\,x} = \frac{\pi}{\sqrt{(a^2+b^2)}}$, $1 > p > 0$; Bonnet, L 17. 265.

14) $\int \frac{dx}{(1+Sin.\lambda.\,Cos.\,x)^{a+1}} = \frac{1^{a/2}}{1^{a/1}}\pi \sum_0^\infty \left\{(-1)^n \frac{(n+1)^{n/1}}{(2a-1)^{n/-2}} \binom{a}{2n} \frac{1}{2^n} Sec.^{2(a-n)+1}\lambda\right\}$

15) $\int \frac{Cos.^a x}{(1+Sin.\lambda.\,Cos.\,x)^{a+1}}\,dx = \frac{1^{a/2}}{1^{a/1}} \frac{(-1)^a \pi}{Sin.^{a+1}\lambda} \sum_0^\infty \left\{(-1)^n \frac{(n+1)^{n/1}}{(2a-1)^{n/-2}} \binom{a}{2n} \frac{1}{2^n} Tang.^{2(a-n)+1}\lambda\right\}$ — Schlömilch, Cr. 33. 268.

16) $\int \frac{Sin.^a x}{(p+q\,Cos.\,x)^{a+1}}\,dx = \frac{1^{a-1/1}}{2^{a-2}(p^2-q^2)^{\frac{a+1}{2}}\left\{\Gamma\left(\frac{a}{2}\right)\right\}^2} \frac{\pi}{a}$ Lobatschewsky, Mém. Kasan. 1835. 1.

17) $= \frac{\Gamma\left(\frac{a+1}{2}\right)\sqrt{\pi}}{(p^2-q^2)^{\frac{a+1}{2}}\,\Gamma\left(\frac{1}{2}a+1\right)}$

18) $\int \frac{Sin.^a x}{(p+q\,Cos.\,x)^b}\,dx = \frac{\Gamma\left(\frac{a+1}{2}\right)\sqrt{\pi}}{p^b\,\Gamma\left(\frac{a}{2}+1\right)} \sum_0^\infty \frac{(\frac{1}{2}n-\frac{1}{2})^{n/-1}\,(\frac{1}{2}n)^{n/1}}{(\frac{1}{2}a+1)^{n/1}\,1^{n/1}} \left(\frac{q}{p}\right)^{2n}$ — Lobatschewsky, Mém. Kasan. 1836. 1.

19) $= \frac{\left\{\Gamma\left(\frac{a+1}{2}\right)\right\}^3}{1^{a/1}} \frac{a+1}{(p^2-q^2)^{\frac{a+1}{2}}}$ Schlömilch, Höh. Anal. 85.

20) $\int \frac{dx}{1 \pm Sin.\,x.\,Cos.\,x\sqrt{3}} = 2\pi$ V. T. 30. N°. 15.

1) $\int \frac{Sin.\, x}{1 + Cos.^2 x} d x = -\frac{1}{2}\pi$ Grunert, Gr. 4. 118.

2) $\int \frac{d x}{4 - Sin.^2 x} = \frac{\pi}{2\sqrt{3}}$ V. T. 66. N°. 6.

3) $\int \frac{Sin.\, x}{4 - Sin.^2 x} d x = \frac{\pi}{3\sqrt{3}}$ V. T. 66. N°. 7.

4) $\int \frac{d x}{4 - 3\, Sin.^2 x} = \frac{1}{2}\pi$ V. T. 30. N°. 16.

5) $\int \frac{d x}{1 - Cos.^2 \lambda . Sin.^2 x} = \pi\, Cosec.\, \lambda$ V. T. 82. N°. 4, 5.

6) $\int \frac{Sin.\, x}{1 - Cos.^2 \lambda . Sin.^2 x} d x = 2(\pi - 2\lambda)\, Cosec.\, 2\lambda$ V. T. 82. N°. 4, 5.

7) $\int \frac{d x}{p^2\, Sin.^2 x + q^2\, Cos.^2 x} = \frac{\pi}{p q}$ Grunert, Cr. 8. 146. — Lobatto, Cr. 11. 169.

8) $\int \frac{d x}{(p^2\, Sin.^2 x + q^2\, Cos.^2 x)^2} = \frac{p^2 + q^2}{2 p^3 q^3}\pi$ V. T. 83. N°. 7, 9.

9) $\int \frac{Sin.^2 x}{(p^2\, Sin.^2 x + q^2\, Cos.^2 x)^2} d x = \frac{\pi}{2 p^3 q}$ Grunert, Cr. 8. 146.

10) $\int \frac{Cos.^2 x}{(p^2\, Sin.^2 x + q^2\, Cos.^2 x)^2} d x = \frac{\pi}{2 p q^3}$ V. T. 83. N°. 7, 9.

11) $\int \frac{Cos.\, 2 x}{(p^2\, Sin.^2 x + q^2\, Cos.^2 x)^2} d x = \frac{p^2 - q^2}{2 p^3 q^3}\pi$ V. T. 82. N°. 9, 10.

12) $\int \frac{Cot.^q x}{1 - (2 p - p^2)\, Cos.^2 x} d x = \frac{1}{(1-p)^{q+1}} \frac{\pi}{2} Sec.\, \frac{1}{2} q \pi , \; q^2 < 1 ;$

13) $\int \frac{Tang.^{1-q} x}{1 - (2 p - p^2)\, Cos.^2 x} d x = \frac{1}{(1-p)^q} \frac{\pi}{2} Cosec.\, \frac{1}{2} q \pi , \; 0 < q < 2 ;$

(12, 13) $, \; p^2 < 1;$ Meyer, Int. Déf. 376.

14) $\int \frac{Cos.\, a x}{1 - p^2\, Sin.\, \frac{1}{2} x} d x = \frac{(-1)^a \pi}{\sqrt{(1 - p^2)}} \left\{ \frac{1 - \sqrt{(1 - p^2)}}{p} \right\}^{2a}$ Plana, Mém. Turin. 1820. 389. N° 4.

1) $\int \frac{dx}{1+p^2-2p\,Cos.x} = \frac{\pi}{1-p^2}, p^2<1;$

2) $= \frac{\pi}{p^2-1}, p^2>1;$

Euler, Calc. Int. 4. S. 4. 22. — Schlömilch, Beitr. II. § 1.

3) $\int \frac{Cos.\,ax}{1+p^2-2p\,Cos.x}dx = \frac{\pi p^a}{1-p^2}, p^2<1;$

4) $= \frac{\pi p^{-a}}{p^2-1}, p^2>1;$

Euler, Calc. Int. 4. S. 4. 22, 45. — Legendre, Exerc. 3. 61. — Poisson, P. 19. 404. N°. 75. — Plana, Mém. Turin. 1817. 7. Art. 2, 14. — Schlömilch, Beitr. II. § 1.

5) $\int \frac{Sin.\,ax.\,Sin.x}{1+p^2-2p\,Cos.x}dx = \frac{1}{2}\pi p^{a-1}, p^2<1;$

6) $= \frac{1}{2}\pi\frac{1}{p^{1+a}}, p^2>1;$

Poisson, P. 19. 404. N°. 95. — Schlömilch, Beitr. II. § 1.

7) $\int \frac{Cos.\,ax.\,Cos.x}{1+p^2-2p\,Cos.x}dx = \frac{\pi}{2}\frac{1+p^2}{1-p^2}p^{a-1}, p^2<1;$

8) $= \frac{\pi}{2}\frac{p^2+1}{p^2-1}\frac{1}{p^{a+1}}, p^2>1;$

Bierens de Haan, Gr. 13. 193.

9) $\int \frac{dx}{1+p^2+2p\,Cos.x} = \frac{\pi}{1-p^2}$

10) $\int \frac{Cos.x}{1+p^2+2p\,Cos.x}dx = \frac{p\pi}{p^2-1}$

$, p<1;$ Raabe, Int. 161.

11) $\int \frac{Cos.\,kx}{1+p^2-2p\,Cos.x}dx = 0$

12) $\int \frac{Sin.\,kx.\,Tang.x}{1+p^2-2p\,Cos.x}dx = \infty$

, pour $k=\infty$; Meyer, Int. Déf. 220; il trouve pour 12) faut. $=0$; encore k y doit être de la forme $2k+1$.

13) $\int \frac{dx}{1-p\,Cos.\lambda - p\,i\,Sin.\lambda.\,Cos.x} = \frac{\pi}{\sqrt{(1-2p\,Cos.\lambda+p^2)}}, 0<p<1;$ Bonnet, L. 17. 265.

14) $\int \frac{Sin.\,2ax.\,Sin.x}{1+p^2-2p\,Cos.\,2x}dx = 0, p^2<1 \text{ et } p^2>1;$

15) $\int \frac{Sin.\{(2a-1)x\}.\,Sin.x}{1+p^2-2p\,Cos.\,2x}dx = \frac{\pi}{2}\frac{p^a}{1+p}, p^2<1;$

16) $= \frac{\pi}{2}\frac{p^{-a}}{1+p}, p^2>1;$

Bierens de Haan, Gr. 13. 193.

17) $\int \frac{Sin.\{(2a-1)x\}.Sin.2x}{1+p^2-2p\,Cos.2x}dx = 0$, $p^2<1$ et $p^2>1$;

18) $\int \frac{Cos.\{(2a-1)x\}}{1+p^2-2p\,Cos.2x}dx = 0$

19) $\int \frac{Cos.2ax.Cos.x}{1+p^2-2p\,Cos.2x}dx = 0$

20) $\int \frac{Cos.\{(2a-1)x\}.Cos.x}{1+p^2-2p\,Cos.2x}dx = \frac{\pi}{2}\frac{p^a}{1-p}$, $p^2<1$;

21) $= \frac{\pi}{2}\frac{p^{-a}}{p-1}$, $p^2>1$;

22) $\int \frac{Cos.\{(2a-1)x\}.Cos.2x}{1+p^2-2p\,Cos.2x}dx = 0$, $p^2<1$ et $p^2>1$;

Bierens de Haan, Gr. 13. 193.

23) $\int \frac{1-p\,Cos.2x}{1+p^2-2p\,Cos.2x}dx = \pi$ Smaasen, Cr. 42. 222.

24) $\int \frac{Cos.x\,Cos.\{(2c+1)x\}}{1+(a\,Sin.x+b)^2}dx = \frac{\pi}{a}Cos.\left\{(2c+1)\,Arctg.\sqrt{\frac{q}{2}}\right\}.Tang.^{2c+1}\left\{\frac{1}{2}\,Arccos.\sqrt{\frac{q}{2b^2}}\right\}$

25) $\int \frac{Cos.x.Cos.2cx}{1+(a\,Sin.x+b)^2}dx = -\frac{\pi}{a}Sin.\left\{2c\,Arctg.\sqrt{\frac{q}{2}}\right\}.Tang.^{2c}\left\{\frac{1}{2}\,Arccos.\sqrt{\frac{q}{2b^2}}\right\}$

De ces deux formules, où $q=-(1+a^2-b^2)+\sqrt{\{(1+a^2-b^2)^2+4b^2\}}$, voyez: Legendre, Exerc. 5. 121.

26) $\int \frac{1-p^b\,Cos.bx}{1-2p^b\,Cos.bx+p^{2b}}Cos.^c\tfrac{1}{2}x.Cos.\tfrac{1}{2}cx\,dx = \frac{\pi}{2^{c+1}}\sum_1^\infty \binom{c}{nb}p^{nb}$ Smaasen, Cr. 42. 222.

F. Circ. Dir. rat. fract. à dén. d'un fact. trinôme et d'autres. TABLE 85. Lim. 0 et π.

1) $\int \frac{Sin.x}{1+p^2-2p\,Cos.x}\frac{dx}{Tang.\frac{1}{2}x} = \frac{\pi}{1-p}$, $p^2<1$;

2) $= \frac{\pi}{p-1}$, $p^2>1$;

3) $\int \frac{Sin.x.Tang.\frac{1}{2}x}{1+p^2-2p\,Cos.x}dx = \frac{\pi}{1+p}$, $p^2<1$ et $p^2>1$;

4) $\int \frac{1}{1+p^2-2p\,Cos.x}\frac{dx}{Cos.x} = \infty$, $p^2<1$;

5) $= \infty$, $p^2>1$;

Schlömilch, Beitr. II. § 1. Il trouve faut.:

pour 4) $\frac{2\pi p}{1-p^4}$, $p^2<1$;

pour 5) $\frac{2\pi p}{p^4-1}$, $p^2>1$.

6) $\int \frac{dx}{(1+p^2-2p\,Cos.\,x)^2} = \frac{1+p^2}{(1-p^2)^3}\pi\,,\ p^2<1\,;$

7) $= \frac{1+p^2}{(p^2-1)^3}\pi\,,\ p^2>1\,;$

8) $\int \frac{dx}{(1+p^2-2p\,Cos.\,x)^3} = \frac{1+4p^2+p^4}{(1-p^2)^5}\pi\,,\ p^2<1\,;$

9) $= \frac{1+4p^2+p^4}{(p^2-1)^5}\pi\,,\ p^2>1\,;$

10) $\int \frac{dx}{(1+p^2-2p\,Cos.\,x)^4} = \frac{1+9p^2+9p^4+p^6}{(1-p^2)^7}\,,\ p^2<1\,;$

11) $= \frac{1+9p^2+9p^4+p^6}{(p^2-1)^7}\,,\ p^2>1\,;$

Euler, Calc. Int. 4. S. 4. 22.

12) $\int \frac{dx}{(1+p^2-2p\,Cos.\,x)^{c+1}} = \frac{\pi}{(1-p^2)^{2c+1}}\sum_0^c \binom{c}{n}^2 a^{2n}\,,\ p^2<1\,;$

13) $= \frac{\pi}{(p^2-1)^{2c+1}}\sum_0^c \binom{c}{n}^2 a^{2n}\,,\ p^2>1\,;$

Euler, Calc. Int. 4. S. 4. 31, 67. — Legendre, Exerc. 3. 62.

14) $\int \frac{Cos.\,ax}{(1+p^2-2p\,Cos.\,x)^2}dx = \frac{\pi p^a}{(1-p^2)^3}\{a+1-(a-1)p^2\}\,,\ p^2<1\,;$

15) $= \frac{\pi p^{-a}}{(p^2-1)^3}\{(a+1)p^2-(a-1)\}\,,\ p^2>1\,;$

Euler, Calc. Int. 4. S. 4. 22, 50. — Legendre, Exerc. 3. 62.

16) $\int \frac{Cos.\,ax}{(1+p^2-2p\,Cos.\,x)^3}dx = \binom{a+2}{2}\frac{\pi p^a}{(1-p^2)^5}\left\{1-\frac{a-2}{a+1}2p^2+\frac{a-2}{a+1}\frac{a-1}{a+2}p^4\right\},\ p^2<1\,;$

17) $= \binom{a+2}{2}\frac{\pi p^{-a}}{(p^2-1)^5}\left\{p^4-\frac{a-2}{a+1}2p^2+\frac{a-2}{a+1}\frac{a-1}{a+2}\right\},\ p^2>1\,;$

Euler, Calc. Int. 4. S. 4. 22, 56. — Legendre, Exerc. 3. 62.

18) $\int \frac{Cos.\,ax}{(1+p^2-2p\,Cos.\,x)^4}dx = \binom{a+3}{3}\frac{\pi p^a}{(1-p^2)^7}$

$\left\{1-\frac{a-3}{a+1}3p^2+\frac{a-3}{a+1}\frac{a-2}{a+2}3p^4-\frac{a-3}{a+1}\frac{a-2}{a+2}\frac{a-1}{a+3}p^6\right\},\ p^2<1\,;$

Euler, Calc. Int. 4. S. 4. 22.

19) $\int \frac{Cos.\, a\, x}{(1 + p^2 - 2\, p\, Cos.\, x)^4}\, dx = \binom{a+3}{3} \frac{\pi\, p^{-a}}{(p^2 - 1)^7}$
$\left\{p^6 - \frac{a-3}{a+1} 3\, p^4 + \frac{a-3}{a+1}\frac{a-2}{a+2} 3\, p^2 - \frac{a-3}{a+1}\frac{a-2}{a+2}\frac{a-1}{a+3}\right\}, p^2 > 1;$ Euler, Calc. Int. 4. S. 4. 22.

20) $\int \frac{Cos.\, a\, x}{(1 + p^2 - 2\, p\, Cos.\, x)^{c+1}}\, dx = \binom{a+c}{c} \frac{\pi\, p^a}{(1 - p^2)^{2c+1}}$
$\left\{1 - \binom{c}{1}\frac{a-c}{a+1} p^2 + \binom{c}{2}\frac{a-c}{a+1}\frac{a-c+1}{a+2} p^4 - \ldots\right\}, p^2 < 1;$

21) $\int \frac{Cos.\, a\, x}{(1 + p^2 - 2\, p\, Cos.\, x)^{c+1}}\, dx = \binom{a+c}{c} \frac{\pi\, p^{-a}}{(p^2 - 1)^{2c+1}}$
$\left\{p^{2c} - \binom{c}{1}\frac{a-c}{a+1} p^{2c-2} + \binom{c}{2}\frac{a-c}{a+1}\frac{a-c+1}{a+2} p^{2c-4} - \ldots\right\}, p^2 > 1;$

Euler, Calc. Int. T. 4. S. 4. 81. — Legendre, Exerc. 3. 62.

22) $\int \frac{Cos.\, a\, x}{(1 + p^2 - 2\, p\, Cos.\, x)^{a+1}}\, dx = \frac{\pi\, p^a}{(1 - p^2)^{2a+1}} \frac{(a+1)^{a/1}}{1^{a/1}}, p^2 < 1;$

23) $= \frac{\pi\, p^a}{(p^2 - 1)^{2a+1}} \frac{(a+1)^{a/1}}{1^{a/1}}, p^2 > 1;$

Euler, Calc. Int. T. 4. S. 4. 30. — Legendre, Exerc. 3. 62.

24) $\int \frac{Sin.^{2a}\, x}{(1 - 2\, p\, Cos.\, x + p^2)^{2a}}\, dx = \frac{1^{2a/2}}{2^{2a/2}}\, \pi, p^2 < 1;$

25) $= \frac{1^{2a/2}}{2^{2a/2}} \frac{\pi}{p^{2a}}, p^2 > 1;$

de Morgan, Definite Integrals. (Enc. Metrop.).

26) $\int \frac{Cos.\, a\, x}{(1 + p - 2\, Cos.\, x\, \sqrt{p})^b}\, dx = \frac{\pi}{1^{b-1/1}\, p^{\frac{1}{2}a}} \frac{d^{b-1}}{d\, p^{b-1}} \cdot \frac{p^{a+b-1}}{(1-p)^b}$ Boole, Phil. Trans. 1844.

27) $\int \frac{1}{1 + p^2 - 2\, p\, Cos.\, x} \frac{dx}{1 + q^2 - 2\, q\, Cos.\, x} = \frac{\pi}{(1 - p^2)(1 - q^2)} \frac{1 + pq}{1 - pq}, p^2 < 1, q^2 < 1;$

28) $= \frac{\pi}{(p^2 - 1)(q^2 - 1)} \frac{pq + 1}{pq - 1}, p^2 > 1, q^2 > 1;$

29) $\int \frac{Sin.^2\, x}{1 + p^2 - 2\, p\, Cos.\, x} \frac{dx}{1 + q^2 - 2\, q\, Cos.\, x} = \frac{\pi}{2} \frac{1}{1 - p\, q}, p^2 < 1, q^2 < 1;$

30) $= \frac{\pi}{2} \frac{1}{p\, q - 1}, p^2 > 1, q^2 > 1;$

Schlömilch, Beitr. II. 2.

31) $\int \frac{Cos.\, a x}{(1-2p_1\, Cos.x+p_1{}^2)^l (1-2p_2\, Cos.x+p_2{}^2)^m \ldots \text{(à } h \text{ facteurs)}} dx = \frac{\pi}{\Gamma(l)\Gamma(m)\ldots} \frac{d^{l-1}}{dy_1{}^{l-1}} \frac{d^{m-1}}{dy_2{}^{m-1}} \ldots$

$$\frac{y_1{}^{l-1} y_2{}^{m-1} \ldots}{(1-y_1)^l (1-y_2)^m \ldots} \times \left\{ Y_1 \left(\frac{y_1}{p_1}\right)^{h+a-1} + Y_2 \left(\frac{y_2}{p_2}\right)^{h+a-1} + \ldots \right\}$$

, où les fonctions $Y_q = \frac{\left(1-\frac{y_1}{p_1}\right)^2 \left(1-\frac{y_2}{p_2}\right)^2 \ldots \left(1-\frac{y_h}{p^h}\right)^2}{\left(1-\frac{y_q}{p_q}\right)^2 \times \left(\frac{y_q}{p_q}-\frac{y_1}{p_1}\right)\left(\frac{y_q}{p_q}-\frac{y_2}{p_2}\right) \ldots \left(\frac{y_q}{p_q}-\frac{y_h}{p_h}\right)}$

Après la différentiation mettez $p_1{}^2, p_2{}^2 \ldots p_h{}^2$ au lieu de $y_1, y_2, \ldots y_h$.

Voyez de cette intégrale: Boole, Phil. Trans. 1844.

32) $\int \frac{Cos.\, 2kx}{1-2p\, Cos.\, x+p^2} \frac{dx}{Cos.\, x} = 0$, pour $k=\infty$; Schlömilch. Beitr. II. § 1.

F. Circ. Dir. irrat. fract. TABLE 86. Lim. 0 et π.

1) $\int \frac{Cos^2. \frac{1}{2} x}{\sqrt{(1-p^2\, Cos.^2\, x)}} dx = F(p)$ Raabe, Cr. 25. 160.

2) $\int \frac{Sin.\, x}{\sqrt{(1-Cos.\, x)}} dx = \sqrt{2}$

3) $\int \frac{Sin.\, x}{\sqrt{(1+p^2-2p\, Cos.\, x)}} dx = 2,\ p^2<1;$

4) $= \frac{2}{p},\ p^2>1;$

(2–4) Poisson, Mém. Ac. 1823. 571. N°. 12.

5) $\int \frac{Cos.\, x}{\sqrt{(1+p^2-2p\, Cos.\, x)}} dx = \frac{2}{p}\{F'(p)-E'(p)\}, p<1;$ Ramus, Danske Afh. 6. 365.

6) $\int \frac{Cos.\, ax}{\sqrt{(1+p^2-2p\, Cos.\, x)}} dx = \frac{1^{a/2}}{2^{a/2}} \pi p^a \sum_1^\infty \frac{1^{n/2}(2a+1)^{n/2}}{2^{n/2}(2a+2)^{n/2}} p^{2n},\ p<1;$

7) $= \frac{1^{a/2}}{2^{a/2}} \frac{\pi p^a}{\sqrt{(1-p^2)}} \sum_0^\infty \frac{(1^{n/2})^2}{2^{n/2}(2a+2)^{n/2}} \left(\frac{p^2}{p^2-1}\right)^n, p<\sqrt{\tfrac{1}{2}};$

(6–7) Schlömilch, Stud. II. 7.

8) $\int \frac{Sin.\, x}{\sqrt{(1+p^2-2p\, Cos.\, x)^3}} dx = \frac{2}{1-p^2}, p^2<1;$

9) $\int \quad = \frac{2}{p(p^2-1)}, p^2>1;$

(8–9) Poisson, P. 19. 145. N°. 4. — Id., Chaleur. 107, 178.

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10) $\int \frac{Sin.\, x}{\sqrt{(1 + p^2 - 2\,p\, Cos.\, x)^3}}\, dx = \frac{2}{p^2 - 1}\,,\ p^2 = 1\,;$ Meyer, Int. Déf. 279.

11) $\int \frac{p - Cos.\, x}{\sqrt{(1 + p^2 - 2\,p\, Cos.\, x)^3}}\, Sin.\, x\, dx = 0\ ,\ p^2 < 1\,;$

12) $= \frac{2}{p^2}\,,\ p^2 > 1\,;$

Poisson, Mém. Ac. 1823. 571. N°. 12.

13) $\int \frac{dx}{\sqrt{(1 + p^2 - 2\,p\, Cos.\, 2\,x)}} = \frac{2}{1 + p}\, E'\left(\frac{2\sqrt{p}}{1 + p}\right)$ Smaasen, Cr. 42. 222.

14) $\int \frac{dx}{\sqrt{(p^2 - q^2\, Cos.\, x)^3}} = \frac{2}{\sqrt{(p^2 + q)}}\ \frac{1}{p^2\, q}\, E'\left(\frac{2\,q}{p^2 + q}\right)$

15) $\int \frac{Cos.\, x}{\sqrt{(p^2 - q^2\, Cos.\, x)^3}}\, dx = \frac{2}{q\sqrt{(p^2 + q)}} \left\{ - F'\left(\frac{2\,q}{p^2 + q}\right) + \frac{p^2}{p^2 - q}\, E'\left(\frac{2\,q}{p^2 + q}\right) \right\}$

Plana, Mém. Turin. 1820. 389.

16) $\int \frac{Sin.^{2a}\, x}{\sqrt{(1 - p^2\, Sin.^2\, x)}}\, dx = \frac{1^{a/2}}{2^{a/2}}\, \pi \sum_1^{\infty} \frac{1^{n/2}\,(2\,a + 1)^{n\,2}}{2^{n/2}\,(2\,a + 2)^{n\,2}}\, p^{2n}\,,\ p < 1\,;$

17) $= \frac{1^{a/2}}{2^{a/2}}\ \frac{\pi}{\sqrt{(1 - p^2)}} \sum_0^{\infty} \frac{(1^{n/2})^2}{2^{n/2}(2\,a + 2)^{n/2}} \left(\frac{p^2}{p^2 - 1}\right)^n,\ p < \sqrt{\tfrac{1}{2}}\,;$

Schlömilch, Stud. II. 7.

F.Circ.Dir.rat.ent. TABLE 87. Lim. 0 et 2 π.

1) $\int Sin.^{2a+1}\, x\, dx = 0$

2) $\int Sin.^{2a+1}\, x.\ Cos.\, b\, x\, dx = 0$

3) $\int Sin.^{2a}\, x.\ Cos.\, x\, dx = 0$

4) $\int (Cos.\, x - Cos.\, a\, x)\, dx = 0$

5) $\int (Cos.\, x - Cos.\, a\, x)\, Cos.\, x\, dx = \pi$

6) $\int (Cos.\, x - Cos.\, a\, x)\, Cos.\, a\, x\, dx = -\pi$

Raabe, Cr. 23. 105.

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7) $\int (Cos.\, x - Cos.\, a x)\, Cos.\, b x\, d x = 0$, où $b > 1$ n'est pas facteur de a. Raabe; Cr. 23. 105.

8) $\int Sin.^{2a} x\, d x = \frac{1^{a/2}}{2^{a/2}} 2\pi$ Schubert, Samml. 118.

9) $\int Cos.\, \{a(x - q\, Sin.\, x)\}.\, Cos.\, x\, d x = \frac{2\pi (\frac{1}{2} a q)^a}{q \,.\, 1^{a/1}} \left\{1 + \sum_1^\infty (-1)^n \frac{(\frac{1}{2} a q)^{2n}}{1^{n/1} (1+a)^{n/1}}\right\}$

10) $\int Cos.(ax - p\, Cos.x - q\, Sin.x) dx = 2\pi Cos.\left(a Arctg. \frac{q}{p}\right) \frac{(p^2+q^2)^{\frac{1}{2}a}}{2^a \,.\, 1^{a/1}} \left\{1 + \sum_1^\infty \frac{(-1)^n}{1^{n/1}(1+a)^{n/1}} \left(\frac{p^2+q^2}{4}\right)^n\right\}$

11) $\int Cos.(p Cos.x + q Sin.x).Cos.2ax dx = 2\pi Cos.\left(2a Arctg. \frac{q}{p}\right) \frac{(p^2+q^2)^a}{2^{2a} \,.\, 1^{2a/1}} \left\{1 + \sum_1^\infty (-1)^n \frac{(p^2+q^2)^n}{2^{2n} . 1^{n/1} (1+2a)^{n/1}}\right\}$

12) $\int Cos.\, (p\, Cos.\, x + q\, Sin.\, x).\, Cos.\, \{(2a+1) x\}\, d x = 0$

13) $\int Sin.\, (p\, Cos.\, x + q\, Sin.\, x).\, Sin.\, 2 a x\, d x = 0$

14) $\int Sin.(p Cos.x + q Sin.x).Sin.\{(2a-1)x\} dx = 2\pi Cos.\left\{(2a-1) Arctg. \frac{q}{p}\right\} \frac{\sqrt{(p^2+q^2)^{2a-1}}}{2^{2a-1}\, 1^{2a-1/1}} \left\{1 + \sum_1^\infty (-1)^n \frac{(p^2+q^2)^n}{2^{2n} 1^{n/1} (2a)^{n/1}}\right\}$

15) $\int Cos.\, (p\, Sin.\, x).\, Cos.^{2a} x\, d x = \frac{1^{a/2}}{2^{a-1}\, 1^{a/1}} \left\{1 + \sum_1^\infty \frac{(-1)^n}{1^{n/1} (a+1)^{n/1}} \left(\frac{p}{2}\right)^{2n}\right\}$

Sur les Integrales 9 à 15 voyez: Bessel, Abhandl. Berlin. 1824. 1.

F. Circ. Dir. rat. fract. à dén. monôme et binôme. TABLE 88. Lim. 0 et 2π.

1) $\int Sin.^{2a+1} x \frac{d x}{Tang.\, \frac{1}{2} x} = \frac{1^{a/2}}{2^{a/2}} 2\pi$ Raabe, Cr. 23. 105.

2) $\int \frac{Sin.\, a x.\, Sin.\, x}{1 + Cos.\, x} dx = (-1)^a\, 2\pi$ V. T. 356. N°. 5.

3) $\int \frac{Sin.\, a x}{1 - p\, Cos.\, x} dx = 0$

4) $\int \frac{Cos.\, a x}{1 - p\, Cos.\, x} dx = \frac{2\pi}{\sqrt{(1-p^2)}} \left\{\frac{1 - \sqrt{(1-p^2)}}{p}\right\}^a$

(3 et 4) $p < 1$; Raabe, Int. 172. — Ohm, Ausw. 26.

5) $\int \frac{Sin.\, a x}{1 + p\, Cos.\, x} dx = 0$, $p < 1$; Ohm, Ausw. 26.

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6) $\int \frac{Cos.\, a x}{1 + p\, Cos.\, x} d x = \frac{(-1)^a 2\pi}{\sqrt{(1-p^2)}} \left\{\frac{1 - \sqrt{(1-p^2)}}{p}\right\}^a$, $p < 1$; Ohm, Ausw. 26.

7) $\int \frac{d x}{1 - Cos.^2 \lambda .\, Sin.^2 x} = 2\pi\, Cosec.\, \lambda$. V. T. 30. N°. 17.

8) $\int \frac{Sin.\, x}{1 - Cos.^2 \lambda .\, Sin.^2 x} d x = 4(\pi - 2\lambda)\, Cosec.\, 2\lambda$ V. T. 30. N°. 17.

9) $\int \frac{d x}{p + q\, Cos.\, x} = \frac{2\pi}{\sqrt{(p^2 - q^2)}}$, $p^2 > q^2$;

10) $= 0$ (val. princ.), $p^2 < q^2$;

11) $\int \frac{d x}{-p + q\, Cos.\, x} = \frac{-2\pi}{\sqrt{(p^2 - q^2)}}$, $p^2 > q^2$;

12) $= 0$ (val. princ.), $p^2 < q^2$;

(9–12) Björling, Gr. 20. 26.

13) $\int \frac{Cos.\, x - Cos.\, a x}{Tang.\, \frac{1}{2} x} d x = 0$ Raabe, Cr. 23. 105.

14) $\int \frac{Sin.\, a x}{(1 - p\, Cos.\, x)(1 - q\, Cos.\, x)(1 - r\, Cos.\, x) \ldots\ldots} dx = 0$, pour toute $p, q, r, \ldots < 1$; Raabe, Int. 172.

F. Circ. Dir. rat. fract. à dén. trinôme de *Cos.* TABLE 89. Lim. 0 et 2π.

1) $\int \frac{d x}{1 + p^2 - 2p\, Cos\, x} = \frac{2\pi}{1 - p^2}$, $p < 1$;

2) $= \frac{2\pi}{p^2 - 1}$, $p > 1$;

(1–2) Bierens de Haan, Gr. 13. 193. — Ohm, Ausw. 26.

3) $\int \frac{Sin.\, x}{1 + p^2 - 2p\, Cos.\, x} d x = 0$

4) $\int \frac{Sin.\, a x}{1 + p^2 - 2p\, Cos.\, x} d x = 0$

(3–4) $p < 1$; Raabe, Int. 172.

5) $\int \frac{Cos.\, x}{1 + p^2 - 2p\, Cos.\, x} = \frac{2\pi p}{1 - p^2}$, $p < 1$;

6) $= \frac{2}{p} \frac{\pi}{p^2 - 1}$, $p > 1$;

(5–6) Bierens de Haan, Gr. 13. 193.

7) $\int \frac{Cos.\, a\, x}{1+p^2-2p\, Cos.\, x} dx = \frac{2\pi p^a}{1-p^2},\ p<1;$

8) $= \frac{2\pi p^{-a}}{p^2-1},\ p>1;$

Raabe, Int. 172. — Bierens de Haan, Gr. 13. 193.

9) $\int \frac{Sin.\, a\, x.\, Sin.\, x}{1+p^2-2p\, Cos.\, x} dx = \pi p^{a-1},\ p<1;$

10) $= \frac{\pi}{p^{a-1}},\ p>1;$

11) $\int \frac{Cos.\, a\, x.\, Cos.\, x}{1+p^2-2p\, Cos.\, x} dx = \pi p^{a-1} \frac{1+p^2}{1-p^2},\ p<1;$

12) $= \frac{\pi}{p^{a-1}} \frac{p^2+1}{p^2-1},\ p>1;$

Bierens de Haan, Gr. 13. 193.

13) $\int \frac{Sin.\, a\, x - p\, Sin.\, \{(a+1)x\}}{1+p^2-2p\, Cos.\, x} dx = 0$

14) $\int \frac{Cos.\, a\, x - p\, Cos.\, \{(a+1)x\}}{1+p^2-2p\, Cos.\, x} dx = 2\pi p^a$

$,\ p<1;$ Ohm, Ausw. 26. — Raabe, Int. 172.

15) $\int \frac{Cos.\, x}{e^a+e^{-a}-2\, Cos.\, x} dx = 2\pi a \frac{e^{-a}}{e^a-e^{-a}}$ Poisson, P. 17. 612. N°. 20.

16) $\int \frac{1-p\, Cos.\, x+p\, i\, Sin.\, x}{1-2p\, Cos.\, x+p^2} = 2\pi,\ p<1;$ Moigno, Calc. Int. 138.

F. Circ. Dir. rat. fract. à dén. trinôme de *Sin.* et *Cos.* TABLE 90. Lim. 0 et 2π.

1) $\int \frac{dx}{a+b\, Cos.\, x+c\, Sin.\, x} = \frac{2\pi}{\sqrt{(a^2-b^2-c^2)}},\ a^2>b^2+c^2;$

2) $= 0 \qquad ,\ a^2<b^2+c^2;$

Dienger, Gr. 12. 409.

3) $= 0$ (val. princ.) $,\ a^2<b^2+c^2;$

4) $= \infty \qquad ,\ a^2=b^2+c^2;$

5) $\int \frac{dx}{-a+b\, Cos.\, x+c\, Sin.\, x} = \frac{-2\pi}{\sqrt{(a^2-b^2-c^2)}},\ a^2>b^2+c^2;$

6) $= 0$ (val. princ.) $,\ a^2<b^2+c^2;$

7) $= -\infty \qquad ,\ a^2=b^2+c^2;$

Björling, Gr. 21. 26.

8) $\int \frac{dx}{1 - p\,Cos.\,x - p\,i\,Sin.\,x} = 2\pi$, $p < 1$; Moigno, Calc. Int. 138.

9) $\int \frac{dx}{a + bi\,Cos.\,x + ci\,Sin.\,x} = \frac{2\pi}{\sqrt{(a^2 + b^2 + c^2)}}$ Jacobi, L. 10. 229.

10) $\int \frac{dx}{1 - (p + qi)\,Cos.\,x - (r + si)\,Sin.\,x} = 0$, $(ps - qr)^2 > q^2 + s^2$;

11) $= \frac{2\pi}{\sqrt{(1 - GH)}}$, $(ps - qr)^2 < q^2 + s^2$;

12) $\int \frac{Cos.\,x}{1 - (p + qi)\,Cos.\,x - (r + si)\,Sin.\,x}\,dx = -\frac{2\pi}{G}$

13) $\int \frac{Sin.\,x}{1 - (p + qi)\,Cos.\,x - (r + si)\,Sin.\,x}\,dx = \frac{2\pi i}{G}$

(12, 13): $(ps - qr)^2 > q^2 + s^2$;

14) $\int \frac{Sin.\,ax}{1 - (p + qi)\,Cos.\,x - (r + si)\,Sin.\,x}\,dx =$

$= \frac{\pi i}{\sqrt{(1 - GH)}} \frac{\{1 + \sqrt{(1 - GH)}\}^a - \{1 - \sqrt{(1 - GH)}\}^a}{G^a}$, $(ps - qr)^2 > q^2 + s^2$;

15) $\int \frac{Sin.\,ax}{1 - (p + qi)\,Cos.\,x - (r + si)\,Sin.\,x}\,dx = \frac{\pi i}{\sqrt{(1 - GH)}} \frac{G^a - H^a}{\{1 + \sqrt{(1 - GH)}\}^a}$, $(ps - qr)^2 < q^2 + s^2$;

16) $\int \frac{Cos.\,ax}{1 - (p + qi)\,Cos.\,x - (r + si)\,Sin.\,x}\,dx =$

$= \frac{-\pi}{\sqrt{(1 - GH)}} \frac{\{1 + \sqrt{(1 - GH)}\}^a - \{1 - \sqrt{(1 - GH)}\}^a}{G^a}$, $(ps - qr)^2 > q^2 + s^2$;

17) $\int \frac{Cos.\,ax}{1 - (p + qi)\,Cos.\,x - (r + si)\,Sin.\,x}\,dx = \frac{\pi}{\sqrt{(1 - GH)}} \frac{G^a + H^a}{\{1 + \sqrt{(1 - GH)}\}^a}$, $(ps - qr)^2 < q^2 + s^2$;

18) $\int \frac{dx}{p + qi - (r + si)\,Cos.\,x - (t + ui)\,Sin.\,x} = 0$, $(ru - st)^2 > (ps - qr)^2 + (pu - qt)^2$;

Dans les formules (10) à (18), trouvées par Jacobi, Cr. 32. 8, on a p, q, r, s réels, $(ps - qr)^2 \gtrless q^2 + s^2$, $ps - qr > 0$, a entier et > 0, $G = p + s + (q - r)i$, $H = p - s + (q + r)i$, $\sqrt{(1 - GH)}$ positive.

19) $\int \frac{dx}{(a + b\,Cos.\,x + c\,Sin.\,x)^2} = \frac{2a\pi}{\sqrt{(a^2 - b^2 - c^2)^3}}$, $a^2 > b^2 + c^2$;

20) $= 0$, $a^2 < b^2 + c^2$;

(19, 20): Dienger, Gr. 12. 409.

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21) $\int \frac{dx}{(a + b\,Cos.\,x + c\,Sin.\,x)^3} = \frac{2a^2 + b^2 + c^2}{\sqrt{(a^2 - b^2 - c^2)^5}} \frac{\pi}{2}, \quad a^2 > b^2 + c^2;$

22) $= 0, \quad a^2 < b^2 + c^2;$

23) $\int \frac{dx}{(a + b\,Cos.\,x + c\,Sin.\,x)^4} = \frac{2a^2 + 3b^2 + 3c^2}{\sqrt{(a^2 - b^2 - c^2)^7}} \frac{\pi}{2}, \quad a^2 > b^2 + c^2;$

24) $= 0, \quad a^2 < b^2 + c^2;$

25) $\int \frac{dx}{(a + bi\,Cos.\,x + ci\,Sin.\,x)^2} = \frac{2a\pi}{\sqrt{(a^2 + b^2 + c^2)^3}}$

26) $\int \frac{dx}{(r - q\,Cos.\,\lambda + qi\,Sin.\,\lambda.\,Cos.\,x)^2} = 2\pi \frac{r - q\,Cos.\,\lambda}{\sqrt{(r^2 - 2rq\,Cos.\,\lambda + q^2)}}$

Dienger, Gr. 12. 409. (21–26)

F. Circ. Dir. irrat. fract. TABLE 91. Lim. 0 et 2 π.

1) $\int \frac{dx}{\sqrt{(a - b\,Cos.\,x)}} = \frac{4}{\sqrt{(a+b)}} F'\left(\sqrt{\frac{2b}{a+b}}\right)$

2) $\int \frac{Cos.\,x}{\sqrt{(a - b\,Cos.\,x)}} dx = \frac{4}{b\sqrt{(a+b)}} \left\{ aF'\left(\sqrt{\frac{2b}{a+b}}\right) - (a+b)E'\left(\sqrt{\frac{2b}{a+b}}\right) \right\}$

3) $\int \frac{dx}{\sqrt{(a + b\,Cos.\,x)}} = \frac{4}{\sqrt{(a+b)}} F'\left(\sqrt{\frac{2b}{a+b}}\right)$

4) $\int \frac{Cos.\,x}{\sqrt{(a + b\,Cos.\,x)}} dx = \frac{4}{b\sqrt{(a+b)}} \left\{ (a+b)E'\left(\sqrt{\frac{2b}{a+b}}\right) - aF'\left(\sqrt{\frac{2b}{a+b}}\right) \right\}$

5) $\int \frac{dx}{\sqrt{(a + b\,Cos.\,x)^3}} = \frac{4\sqrt{(a+b)}}{a^2 - b^2} E'\left(\sqrt{\frac{2b}{a+b}}\right)$

Dienger, Gr. 13. 424. (1–5)

F. Circ. Dir. fract. TABLE 92. Lim. $\frac{\pi}{4}$ et $\frac{\pi}{2}$.

1) $\int \frac{Tang.^{p-1}\,x - Tang.^{1-p}\,x}{Cos.\,2x} dx = \frac{1}{2}\pi\,Cot.\,\frac{1}{2}p\pi$ V. T. 5. N°. 12.

2) $\int (Tang.^p\,x + Cot.^p\,x)(Tang.^q\,x + Cot.^q\,x)\,dx = 2\pi \frac{Cos.\,\frac{1}{2}p\pi.\,Cos.\,\frac{1}{2}q\pi}{Cos.\,p\pi + Cos.\,q\pi}$ V. T. 5. N°. 10.

3) $\int (Tang.^p\,x - Cot.^p\,x)(Tang.^q\,x - Cot.^q\,x)\,dx = 2\pi \frac{Sin.\,\frac{1}{2}p\pi.\,Sin.\,\frac{1}{2}q\pi}{Cos.\,p\pi + Cos.\,q\pi}$ V. T. 5. N°. 9.

4) $\int (Tang.^p\, x - Cot.^p\, x)\,(Tang.^q\, x + Cot.^q\, x)\,\frac{dx}{Cos.\,2x} = \frac{-\pi\, Sin.\, p\pi}{Cos.\, p\pi + Cos.\, q\pi}$ V. T. 5. N°. 15.

5) $\int \frac{dx}{1 - Sin.\, x.\, Cos.\, x} = \frac{2\pi}{3\sqrt{3}}$ V. T. 7. N°. 1.

6) $\int \frac{dx}{1 + Sin.\, x.\, Cos.\, x} = \frac{\pi}{3\sqrt{3}}$ V. T. 7. N°. 2.

7) $\int \frac{Tang.^p\, x + Cot.^p\, x}{1 + Sin.\, 2x.\, Cos.\, \lambda}\, dx = \frac{\pi\, Sin.\, p\lambda}{Sin.\, p\pi.\, Sin.\, \lambda}$, $p < 1$; V. T. 7. N°. 7.

8) $\int \frac{dx}{1 - 3\, Sin.^2\, x.\, Cos.^2\, x} = \frac{1}{2}\pi$ V. T. 7. N°. 19.

9) $\int \frac{Sin.^{p-1}\, 2x}{(Cos.\, x + Sin.\, x)^{2p}}\, dx = \frac{1}{2^{p+1}}\, \frac{\Gamma(p)\,\Gamma(\frac{1}{2})}{\Gamma(p + \frac{1}{2})}$ V. T. 4. N°. 3.

10) $\int \frac{1}{Tang.^p\, x + Cot.^p\, x}\, \frac{dx}{Sin.\, 2x} = \frac{\pi}{8p}$ V. T. 5. N°. 23.

11) $\int \frac{Tang.^p\, x + Tang.^q\, x}{Tang.^{p+q}\, x + 1}\, \frac{dx}{Sin.\, 2x} = \frac{1}{2}\, \frac{\pi}{p+q}\, Sec. \left\{\frac{q-p}{q+p}\, \frac{\pi}{2}\right\}$ V. T. 31. N°. 17.

12) $\int \frac{Tang.^p\, x - Tang.^q\, x}{Tang.^{p+q}\, x - 1}\, \frac{dx}{Sin.\, 2x} = \frac{1}{2}\, \frac{\pi}{p+q}\, Tang. \left\{\frac{q-p}{q+p}\, \frac{\pi}{2}\right\}$ V. T. 31. N°. 18.

13) $\int (\sqrt{Tang.\, x} + \sqrt{Cot.\, x})\, dx = \frac{1}{2}\pi\sqrt{2}$ V. T. 15. N°. 2.

F. Circ. Dir. TABLE 93. Lim. $-\frac{\pi}{2}$ et $\frac{\pi}{2}$.

1) $\int Cos.^{b-1}\, x.\, Cos.\, ax\, dx = 2^{b-2}\, \frac{\Gamma\left(\frac{b-a}{2}\right)\Gamma\left(\frac{a+b}{2}\right)\Gamma(b)}{\Gamma(b-a)\,\Gamma(b+a)}$ Serret, L. 8. 1.

2) $\int \qquad = \frac{\pi\,\Gamma(b)}{2^{b-1}\,\Gamma\left(\frac{a+b}{2} - 1\right)\Gamma\left(\frac{b-a}{2} - 1\right)}$

3) $\int Cos.^b\, x.\, Sin.\, ax\, dx = 0$

(2, 3) Kummer, Cr. 17. 210.

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4) $\int Cos.^b x . Cos. (\frac{1}{2} a \pi - a x) \, dx = \frac{\pi \Gamma (b+1) Cos. \frac{1}{2} a \pi}{2^b \Gamma \left(\frac{a+b}{2}+1\right) \Gamma \left(\frac{b-a}{2}+1\right)}$

5) $\int Cos.^b x . Sin. (\frac{1}{2} a \pi - a x) \, dx = \frac{\pi \Gamma (b+1) Sin. \frac{1}{2} a \pi}{2^b \Gamma \left(\frac{a+b}{2}+1\right) \Gamma \left(\frac{b-a}{2}+1\right)}$ } Kummer, Cr. 17. 210.

6) $\int Cos.^p x . Cos. \{q(x-\lambda)\} \, dx = \frac{\pi Cos. p \lambda}{2^p} \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+q}{2}+1\right) \Gamma\left(\frac{q-p}{2}+1\right)}$ Lobatschewsky, Mém. Kasan. 1835. 211.

7) $\int \frac{dx}{1 \pm Sin. x . Cos. x} = \frac{2\pi}{\sqrt{3}}$ V. T. 30. N°. 10, 11.

8) $\int \frac{dx}{1 \pm Sin. x . Cos. x \sqrt{3}} = 2\pi$ V. T. 30. N°. 15.

9) $\int \frac{Sin.^2 x}{(p^2 Sin.^2 x + q^2 Cos.^2 x)^2} \, dx = \frac{\pi}{2 p^3 q}$ Grunert, Cr. 8. 146.

F. Circ. Dir. TABLE 94. Lim. $p\pi$ et $q\pi$.

1) $\int_{-\frac{1}{2}\pi}^{0} \frac{Sin.^2 x}{(p^2 Sin.^2 x + q^2 Cos.^2 x)^2} \, dx = \frac{\pi}{4 p^3 q}$ Grunert, Cr. 8. 146.

2) $\int_{-2\pi}^{0} \frac{Sin. \{(a+\frac{1}{2})x\}}{Sin. \frac{1}{2} x} \, dx = 2\pi$ Lobatschewsky, Mém. Kasan. 1835. 211.

3) $\int_{0}^{\frac{4\pi}{3}} \frac{Sin. x}{1 + Cos.^2 x} \, dx = \frac{3}{4}\pi - Arctang. \sqrt{2}$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 2.

4) $\int_{-\pi}^{\pi} Cos.^a x . Cos. \{b(x-\lambda)\} \, dx = \frac{\pi}{2^a} \frac{1+(-1)^{a+b}}{\Gamma\left(\frac{a+b}{2}+1\right) \Gamma\left(\frac{a-b}{2}+1\right)} 1^{a/1} Cos. b \lambda$ Lobatschewsky, Mém. Kasan. 1835. 211.

5) $\int_{-\frac{\pi}{2}}^{2\pi} Cos. x \, dx \sqrt{(1 - q^2 Cos.^2 x)} = -\frac{1}{2} \left\{1 + \frac{q^2-1}{2q} l \frac{1-q}{1+q}\right\}$ Rogner, Material.

6) $\int_{0}^{2h\pi} \frac{dx}{a + b Cos. x + c Sin. x} = \frac{2 h \pi}{\sqrt{(a^2 - b^2 - c^2)}}, \; a^2 > b^2 + c^2;$

7) $= 0, \; a^2 < b^2 + c^2;$ } Dienger, Gr. 12. 409.

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8) $\int_{2k\pi}^{2(h+k)\pi} \frac{dx}{a + b\,Cos.\,x + c\,Sin.\,x} = \frac{2h\pi}{\sqrt{(a^2 - b^2 - c^2)}}$, $a^2 > b^2 + c^2$;

9) $= 0$, $a^2 < b^2 + c^2$;

10) $\int_0^{2h\pi} \frac{dx}{(a + b\,Cos.\,x + c\,Sin.\,x)^2} = \frac{2ah\pi}{\sqrt{(a^2 - b^2 - c^2)^3}}$, $a^2 > b^2 + c^2$;

11) $= 0$, $a^2 < b^2 + c^2$;

12) $\int_0^{2h\pi} \frac{dx}{(a + b\,Cos.\,x + c\,Sin.\,x)^3} = \frac{2a^2 + b^2 + c^2}{\sqrt{(a^2 - b^2 - c^2)^5}} h\pi$, $a^2 > b^2 + c^2$;

13) $= 0$, $a^2 < b^2 + c^2$;

14) $\int_0^{2h\pi} \frac{dx}{(a + b\,Cos.\,x + c\,Sin.\,x)^4} = \frac{2a^2 + 3(b^2 + c^2)}{\sqrt{(a^2 - b^2 - c^2)^7}} h\pi$, $a^2 > b^2 + c^2$;

15) $= 0$, $a^2 < b^2 + c^2$;

16) $\int_0^{2h\pi} \frac{dx}{a + (b\,Cos.\,x + c\,Sin.\,x)\,i} = \frac{2h\pi}{\sqrt{(a^2 + b^2 + c^2)}}$

17) $\int_0^{2h\pi} \frac{dx}{\{a + (b\,Cos.\,x + c\,Sin.\,x)\,i\}^2} = \frac{2ah\pi}{\sqrt{(a^2 + b^2 + c^2)^3}}$

Dienger, Gr. 12. 409.

18) $\int_{\frac{a\pi}{b}}^{\frac{a+1}{b}\pi} Sin.\,bx\,dx = (-1)^a \frac{2}{b}$ Ohm, Ausw. 55.

1) $\int Cos.\,qx\,dx = \frac{1}{q} Sin.\,q$

2) $\int Sin.\,qx\,dx = \frac{1}{q}(1 - Cos.\,q)$

, $q^2 < \pi^2$; Dienger, Cr. 28. 331.

3) $\int Sin.^2 (2\pi x)\,dx = \frac{1}{2}$

4) $\int Cos.^2 (2\pi x)\,dx = \frac{1}{2}$

Abria, L. 4. 248.

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5) $\int Sin.\, 2\, a x.\, Cos.\, (2 \pi x)\, d x = 0$ Abria, L. 4. 248.

6) $\int \frac{Sin\, (2 a \pi x)}{Tang.\, \pi x}\, d x = 1$ V. T. 301. N°. 5.

7) $\int \frac{Cos.(2 a \pi x)}{Tang.\, \pi x}\, d x = \infty$ V. T. 301. N°. 4.

8) $\int Sin.(2\, a \pi x)\, d x = 0$ Kummer, Cr. 35. 1.

9) $\int Cos.\, (p \sqrt{x})\, d x = \frac{1}{p^2} (p\, Sin.\, p + Cos.\, p - 1)$ Dienger, Cr. 46. 119.

F. Circ. Dir. rat. à un fact. TABLE 96. Lim. 0 et ∞.

1) $\int Sin.\, x\, d x = \alpha$, où α indéterminé; Meyer, Int. Déf. 98.

2) $\int \quad = 1$

3) $\int Cos.\, x\, d x = 0$

(2, 3) Raabe, Cr. 15. 355. — Id., Cr. 16. 219. — Boncompagni, Cr. 25. 74.

4) $\int \quad = \alpha$, où α indéterminé; Meyer, Int. Déf. 98. — Lejeune-Dirichlet, Cr. 17. 57.

5) $\int Sin.\, p x\, d x = \frac{1}{p}$

6) $\int Cos.\, p x\, d x = 0$

(5, 6) Poisson, P. 16. 215. N°. 2. — Cisa de Grésy, Mém. Turin. 1821. 209. II. Art. 53. — Plana, Mém. Brux. T. 10. — Oettinger, Cr. 38. 216.

7) $\int Sin.\, (x^2)\, d x = \frac{1}{4} \sqrt{2 \pi}$

8) $\int Cos.\, (x^2)\, d x = \frac{1}{4} \sqrt{2 \pi}$

(7, 8) Cauchy, Sav. Etr. 1827. 599. P. I. § 6.

9) $\int Sin.\, \left(\frac{x^2}{2 q^2}\right) d x = \frac{1}{2} q \sqrt{\pi}$

10) $\int Cos.\, \left(\frac{x^2}{2 q^2}\right) d x = \frac{1}{2} q \sqrt{\pi}$

(9, 10) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 4. — Schlömilch, Stud. I. 13. — Helmling, Transf. 81. — Mascheroni, Adn. 57. les trouve fautiv. $= q \sqrt{\pi}$.

11) $\int Sin.^{2a} x\,dx = \infty$

12) $\int Sin.^{2a+1} x\,dx = \frac{2^{a/2}}{3^{a/2}}$

} Raabe, Int. 149.

13) $\int Cos.^{2a} x\,dx = \infty$

14) $\int Cos.^{2a+1} x\,dx = 0$

} Raabe, Int. 152.

1) $\int Sin.^{2a} x . Sin. p x\,dx = \frac{1}{p} \frac{1^{2a/1}}{2^2 - p^2 . 4^2 - p^2 \ldots\ldots (2a)^2 - p^2}$

, où p tout nombre, hors la forme $2k$, où $k \overline{\overline{<}} a$;

2) $\int Sin.^{2a} x . Cos. p x\,dx = 0$

3) $\int Sin.^{2a+1} x . Sin. p x\,dx = 0$

4) $\int Sin.^{2a+1} x . Cos. p x\,dx = \frac{1^{2a+1/1}}{1^2 - p^2 . 3^2 - p^2 \ldots (2a+1)^2 - p^2}$

, où p tout nombre, hors la forme $2k+1$, où $k \overline{\overline{<}} a$;

} Raabe, Int. 150.

5) $\int Cos.^{2a} x . Sin. p x\,dx = \frac{1}{p} \frac{1^{2a/1}}{2^2 - p^2 . 4^2 - p^2 \ldots (2a)^2 - p^2}$

$\left\{1 - \frac{p^2}{1.2} - \frac{p^2 . 2^2 - p^2}{1.2.3.4} - \ldots - \frac{p^2 . 2^2 - p^2 \ldots (2a-2)^2 - p^2}{1^{2a/1}}\right\}$

, où p tout nombre, hors la forme $2k$, où $k \overline{\overline{<}} a$;

6) $\int Cos.^{2a} x . Cos. p x\,dx = 0$

7) $\int Cos.^{2a+1} x . Sin. p x\,dx = \frac{1^{2a+1/1}}{1^2 - p^2 . 3^2 - p^2 \ldots (2a+1)^2 - p^2}$

$\left\{- \frac{p^2}{1} - \frac{p^2 . 1^2 - p^2}{1.2.3} - \ldots - \frac{p^2 . 1^2 - p^2 \ldots (2a-1)^2 - p^2}{1^{2a+1/1}}\right\}$

, où p tout nombre, hors la forme $2k+1$, où $k \overline{\overline{<}} a$;

} Raabe, Int. 153.

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8) $\int Cos.^{2a+1} x . Cos. p x \, dx = 0$ Raabe, Int. 153.

9) $\int Sin. (x^2). Cos. 2 q x \, dx = \frac{1}{4} \{Cos. (q^2) - Sin. (q^2)\} \sqrt{2 \pi}$

10) $\int Cos. (x^2). Cos. 2 q x \, dx = \frac{1}{4} \{Cos. (q^2) + Sin. (q^2)\} \sqrt{2 \pi}$

11) $\int Sin. (q^2 + x^2). Cos. 2 q x \, dx = \frac{1}{4} \sqrt{2 \pi}$

12) $\int Cos. (q^2 + x^2). Cos. 2 q x \, dx = \frac{1}{4} \sqrt{2 \pi}$

13) $\int \{Cos. (x^2) + Sin. (x^2)\} Cos. 2 q x \, dx = \frac{1}{2} Cos. (q^2) \sqrt{2 \pi}$

14) $\int \{Cos. (x^2) - Sin. (x^2)\} Cos. 2 q x \, dx = \frac{1}{2} Sin. (q^2) \sqrt{2 \pi}$

(9–14) Cauchy, Sav. Etr. 1827. 124. Note 2.

15) $\int Sin. (a x^2). Cos. b x \, dx = \frac{1}{2} \left\{ Cos. \left(\frac{b^2}{4 a}\right) - Sin. \left(\frac{b^2}{4 a}\right) \right\} \sqrt{\frac{\pi}{2 a}}$

16) $\int Cos. (a x^2). Cos. b x \, dx = \frac{1}{2} \left\{ Cos. \left(\frac{b^2}{4 a}\right) + Sin. \left(\frac{b^2}{4 a}\right) \right\} \sqrt{\frac{\pi}{2 a}}$

(15–16) Raabe, Int. 168.

1) $\int Sin. \left(x^2 - q + \frac{q^2}{4 x^2}\right) dx = \frac{1}{4} \sqrt{2 \pi}$

2) $\int Cos. \left(x^2 - q + \frac{q^2}{4 x^2}\right) dx = \frac{1}{4} \sqrt{2 \pi}$

3) $\int Sin. \left(x^2 + \frac{q^2}{4 x^2}\right) dx = \frac{1}{4} (Cos. q + Sin. q) \sqrt{2 \pi}$

4) $\int Cos. \left(x^2 + \frac{q^2}{4 x^2}\right) dx = \frac{1}{4} (Cos. q - Sin. q) \sqrt{2 \pi}$

5) $\int \left\{ Sin. \left(x^2 + \frac{q^2}{4 x^2}\right) + Cos. \left(x^2 + \frac{q^2}{4 x^2}\right) \right\} dx = \frac{1}{2} Cos. q \sqrt{2 \pi}$

6) $\int \left\{ Sin. \left(x^2 + \frac{q^2}{4 x^2}\right) - Cos. \left(x^2 + \frac{q^2}{4 x^2}\right) \right\} dx = \frac{1}{2} Sin. q \sqrt{2 \pi}$

(1–6) Cauchy, Sav. Etr. 1827, 124. Note 2.

7) $\int \frac{Sin.\, b\, x}{Sin.\, a\, x}\, d\, x = 0\;,\; b < a$; Cisa de Grésy, Mém. Turin. 1821. 209. II. 59.

8) $\int \frac{Sin.\, x}{1 - p\, Cos.\, x}\, d\, x = 0$

9) $\int \frac{Sin.\, a\, x}{1 - p\, Cos.\, x}\, dx = \frac{1}{\sqrt{(1-p^2)}} \left[\left\{\frac{1+\sqrt{(1-p^2)}}{p}\right\}^a - \left\{\frac{1-\sqrt{(1-p^2)}}{p}\right\}^a\right] l\, \frac{\sqrt{(1+p)}+\sqrt{(1-p)}}{2\sqrt{(1-p)}}$
$- \frac{1}{\sqrt{(1-p^2)}} \sum_1^{a-1} \frac{1}{n-a} \left[\left\{\frac{1+\sqrt{(1-p^2)}}{p}\right\}^n - \left\{\frac{1-\sqrt{(1-p^2)}}{p}\right\}^n\right]$

10) $\int \frac{Cos.\, a\, x}{1 - p\, Cos.\, x}\, d\, x = \infty$

(8–10: Raabe, Int. 189.)

11) $\int \frac{Cos.\, x}{a^2 + 4\, k^2\, Sin.^2 \frac{x}{2\, k}}\, d\, x = \frac{\pi}{2\, a}\, e^{-a}$, où $k = \infty$;

12) $\int \frac{Sin.\, \frac{x}{k}.\, Sin.\, x}{a^2 + 4\, k^2\, Sin.^2 \frac{x}{2\, k}}\, d\, x = \frac{\pi}{2\, k}\, e^{-a}$, où $k = \infty$;

(11–12: Poisson, P. 19. 404. N°. 75.)

F. Circ. Dir. irrat. TABLE 99. Lim. 0 et ∞.

1) $\int Sin.\, x\, d\, x\, \sqrt{Sin.\, 2\, q\, x} = \frac{1}{2}\left(Sin.\, \frac{1}{2}\, q + Cos.\, \frac{1}{2}\, q\right)\sqrt{q\, \pi}$

2) $\int Cos.\, x\, d\, x\, \sqrt{Sin.\, 2\, q\, x} = \frac{1}{2}\left(Sin.\, \frac{1}{2}\, q - Cos.\, \frac{1}{2}\, q\right)\sqrt{q\, \pi}$

(1–2: Cauchy, Sav. Etr. 1827. 124. Note 3.)

3) $\int Sin.\, x\, d\, x\, \sqrt{Cos.\, 2\, q\, x} = \sum_0^\infty (-1)^n \frac{1}{(2n+1)^{2n/1}}\, (2\, q)^{2n}$

4) $\int Cos.\, x\, d\, x\, \sqrt{Cos.\, 2\, q\, x} = \sum_0^\infty (-1)^n \frac{1}{(2n+2)^{2n+1/1}}\, (2\, q)^{2n+1}$

(3–4: Cauchy, Sav. Etr. 1827. 124. Note 2.)

5) $\int \frac{Cos.\, a\, x}{\sqrt{(1 - p^2\, Sin.^2\, x)}}\, d\, x = 0$

6) $\int \frac{Cos.\, a\, x}{(1 - p^2\, Sin.^2\, x)^{\frac{2\, b + 1}{2}}}\, d\, x = 0$

(5–6: $p^2 < 1$; Raabe, Int. 196.)

1) $\int Sin.(x^2)\,dx = \frac{1}{2}\sqrt{2\pi}$

2) $\int Cos.(x^2)\,dx = \frac{1}{2}\sqrt{2\pi}$

Fourier, Chaleur. 407. — Lejeune-Dirichlet, Cr. 17. 57. — Id., Abh. Berlin. 1835.

3) $\int Sin.(px^2)\,dx = \sqrt{\frac{\pi}{2p}}$

4) $\int Cos.(px^2)\,dx = \sqrt{\frac{\pi}{2p}}$

Schlömilch, Stud. I. 13.

5) $\int Sin.\left(\frac{\pi x^2}{2}\right)dx = 1$

6) $\int Cos.\left(\frac{\pi x^2}{2}\right)dx = 1$

Abria, L. 4. 248.

7) $\int Cos.\{(x+p)^2\}\,dx = \sqrt{\frac{\pi}{2}}$ Lejeune-Dirichlet, Abh. Berlin. 1835. — Id., Cr. 17. 57.

8) $\int Sin.\left\{\left(px - \frac{q}{x}\right)^2\right\}dx = \frac{1}{p}\sqrt{\frac{\pi}{2}}$

9) $\int Cos.\left\{\left(px - \frac{q}{x}\right)^2\right\}dx = \frac{1}{p}\sqrt{\frac{\pi}{2}}$

10) $\int Sin.\left(p^2x^2 + \frac{q^2}{x^2}\right)dx = Sin.\left(\frac{1}{4}\pi + 2pq\right)\frac{\sqrt{\pi}}{p}$

11) $\int Cos.\left(p^2x^2 + \frac{q^2}{x^2}\right)dx = Cos.\left(\frac{1}{4}\pi + 2pq\right)\frac{\sqrt{\pi}}{p}$

Ohm, Ausw. 24.

12) $\int Sin.(px^2 + qx)\,dx = Sin.\left(\frac{1}{4}\pi - \frac{q^2}{4p}\right)\sqrt{\frac{\pi}{p}}$

13) $\int Cos.(px^2 + qx)\,dx = Cos.\left(\frac{1}{4}\pi - \frac{q^2}{4p}\right)\sqrt{\frac{\pi}{p}}$

14) $\int Sin.(px^2 + qx + r)\,dx = Sin.\left(\frac{1}{4}\pi - \frac{q^2 - 4pr}{4p}\right)\sqrt{\frac{\pi}{p}}$

15) $\int Cos.(px^2 + qx + r)\,dx = Cos.\left(\frac{1}{4}\pi - \frac{q^2 - 4pr}{4p}\right)\sqrt{\frac{\pi}{p}}$

Ohm, Ausw. 25.

1) $\int Sin.(q x^2). Cos. p x\, dx = Sin.\left(\frac{\pi}{4} - \frac{p^2}{4q^2}\right)\sqrt{\frac{\pi}{q}}$

2) $\int Cos.(q x^2). Cos. p x\, dx = Sin.\left(\frac{\pi}{4} + \frac{p^2}{4q^2}\right)\sqrt{\frac{\pi}{q}}$

Fourier, Chaleur. 407.

3) $\int Sin.(x^2). Cos. 2 p x\, dx = \frac{1}{2}\{Cos.(p^2) - Sin.(p^2)\}\sqrt{2\pi}$

4) $\int Cos.(x^2). Cos. 2 p x\, dx = \frac{1}{2}\{Cos.(p^2) + Sin.(p^2)\}\sqrt{2\pi}$

Lejeune-Dirichlet, Cr. 17. 57. — Id., Abh. Berlin. 1835. — Schlömilch, Stud. II. 9.

5) $\int Sin.\left(\frac{x^2}{4q}\right). Cos. p x\, dx = \{Cos.(p^2 q) - Sin.(p^2 q)\}\sqrt{2 q \pi}$

6) $\int Cos.\left(\frac{x^2}{4q}\right). Cos. p x\, dx = \{Cos.(p^2 q) + Sin.(p^2 q)\}\sqrt{2 q \pi}$

Schlömilch, Stud. II. 9.

7) $\int Sin.(p x^2). Sin. q x\, dx = 0$

8) $\int Cos.(p x^2). Sin. q x\, dx = 0$

Ohm, Ausw. 25.

9) $\int Sin.(p^2 + x^2). Sin. 2 p x\, dx = 0$ Lejeune-Dirichlet, Cr. 17. 57. — Schlömilch, Stud. II. 9.

10) $\int Cos.(p^2 + x^2). Sin. 2 p x\, dx = 0$

11) $\int Sin.(p^2 + x^2). Cos. 2 p x\, dx = \sqrt{\frac{\pi}{2}}$

12) $\int Cos.(p^2 + x^2). Cos. 2 p x\, dx = \sqrt{\frac{\pi}{2}}$

Schlömilch, Stud. II. 9.

F. Circ. Dir. rat. ent. TABLE 102. Lim. $\frac{\pi}{2}$ et ∞.

1) $\int Cos. x\, dx = -1$

2) $\int Cos.^{2a} x\, dx = \infty$

3) $\int Cos.^{2a+1} x\, dx = -\frac{2^{a/2}}{3^{a/2}}$

4) $\int Sin.^{2a} x\,dx = \infty$

5) $\int Sin.^{2a+1} x\,dx = 0$

6) $\int Sin.^{2a} x .\, Sin.\, p x\,dx = \frac{1}{p} Cos. \frac{1}{2} p \pi \frac{1^{2a/1}}{2^2 - p^2 . 4^2 - p^2 \ldots . (2a)^2 - p^2}$

$\left\{1 - \frac{p^2}{1.2} - \frac{p^2 . 2^2 - p^2}{1.2.3.4} - \ldots . - \frac{p^2 . 2^2 - p^2 \ldots . (2a-2)^2 - p^2}{1^{2a/1}}\right\}$

7) $\int Sin.^{2a} x .\, Cos.\, p x\,dx = -\frac{1}{p} Sin. \frac{1}{2} p \pi \frac{1^{2a/1}}{2^2 - p^2 . 4^2 - p^2 \ldots . (2a)^2 - p^2}$

$\left\{1 - \frac{p^2}{1.2} - \frac{p^2 . 2^2 - p^2}{1.2.3.4} - \ldots . - \frac{p^2 . 2^2 - p^2 \ldots . (2a-2)^2 - p^2}{1^{2a/1}}\right\}$

8) $\int Sin.^{2a+1} x .\, Sin.\, p x\,dx = \frac{1}{p} Cos. \frac{1}{2} p \pi \frac{1^{2a+1/1}}{1^2 - p^2 . 3^2 - p^2 \ldots . (2a+1)^2 - p^2}$

$\left\{- \frac{p^2}{1} - \frac{p^2 . 1^2 - p^2}{1.2.3} - \ldots . - \frac{p^2 . 1^2 - p^2 \ldots . (2a-1)^2 - p^2}{1^{2a+1/1}}\right\}$

9) $\int Sin.^{2a+1} x .\, Cos.\, p x\,dx = -\frac{1}{p} Sin. \frac{1}{2} p \pi \frac{1^{2a+1/1}}{1^2 - p^2 . 3^2 - p^2 \ldots . (2a+1)^2 - p^2}$

$\left\{- \frac{p^2}{1} - \frac{p^2 . 1^2 - p^2}{1.2.3} - \ldots . - \frac{p^2 . 1^2 - p^2 \ldots . (2a-1)^2 - p^2}{1^{2a+1/1}}\right\}$

10) $\int Cos.^{2a} x .\, Sin.\, p x\,dx = \frac{1}{p} Cos. \frac{1}{2} p \pi \frac{1^{2a/1}}{2^2 - p^2 . 4^2 - p^2 \ldots . (2a)^2 - p^2}$

11) $\int Cos.^{2a} x .\, Cos.\, p x\,dx = -\frac{1}{p} Sin. \frac{1}{2} p \pi \frac{1^{2a/1}}{2^2 - p^2 . 4^2 - p^2 \ldots . (2a)^2 - p^2}$

12) $\int Cos.^{2a+1} x .\, Sin.\, p x\,dx = Sin. \frac{1}{2} p \pi \frac{1^{2a+1/1}}{1^2 - p^2 . 3^2 - p^2 \ldots . (2a+1)^2 - p^2}$

13) $\int Cos.^{2a+1} x .\, Cos.\, p x\,dx = - Cos. \frac{1}{2} p \pi \frac{1^{2a+1/1}}{1^2 - p^2 . 3^2 - p^2 \ldots . (2a+1)^2 - p^2}$

Toutes ces formules sont déduites par Raabe, Int. 253.

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$$1)\int_0^a Sin.^2\frac{2b\pi x}{a}dx=\frac{1}{2}a$$

$$2)\int_0^a Cos.^2\frac{2b\pi x}{a}dx=\frac{1}{2}a$$

$$3)\int_0^a Sin.\frac{2b\pi x}{a}\,.\,Cos.\frac{2b\pi x}{a}dx=0$$

$$4)\int_0^a Sin.\frac{2b\pi x}{a}\,.\,Sin.\frac{2c\pi x}{a}dx=0$$

$$5)\int_0^a Sin.\frac{2b\pi x}{a}\,.\,Cos.\frac{2c\pi x}{a}dx=0$$

$$6)\int_0^a Cos.\frac{2b\pi x}{a}\,.\,Cos.\frac{2c\pi x}{a}dx=0$$

$$7)\int_0^a Cos.\frac{2b\pi x}{a}\,.\,Sin.\frac{2c\pi x}{a}dx=0$$

(4 à 7) $, b > c;$ (1 à 7) Poisson, Chaleur. 134.

$$8)\int_0^{\frac{1}{2}\pi}\frac{Sin.\,px}{Sin.\,x}dx=\frac{1}{2}\pi\,,\ p<\pi;$$ Ohm, Ausw. 67.

$$9)\int_0^{\frac{1}{2}Arccos.p}dx\sqrt{\frac{Cos.\,2x-p}{Cos.\,2x-1}}=2\pi\left\{1-\sqrt{\frac{1+p}{2}}\right\}$$ Catalan, L. 6. 419.

$$10)\int_0^\lambda\frac{dx}{Cos.^2x\sqrt{(1-p^2Sin.^2x)}}=\frac{1}{1-p^2}\{Tang.\lambda\sqrt{(1-p^2Sin.^2\lambda)}+(1-p^2)F(p,\lambda)-E(p,\lambda)\}$$

$$11)\int_0^\lambda\frac{dx}{\sqrt{(1-p^2Sin.^2x)^3}}=\frac{1}{1-p^2}\left\{E(p,\lambda)-\frac{p^2\,Sin.\lambda.\,Cos.\lambda}{\sqrt{(1-p^2\,Sin.^2\lambda)}}\right\}$$

(10, 11) Catalan, L. 4. 323.

F. Circ. Dir. irrat. fract. TABLE 104. Lim. 0 et λ.

Les formules 1 à 15 de cette Table sont déduites par Legendre, Exerc. Supplém. Tome I, aux numéros indiqués; on y a partout:

$$c=\frac{Sin.\lambda}{Sin.\mu},\quad Cos.\,\nu=Cos.\lambda.\,Cos.\mu\,,\quad Tang.\,\theta=Sin.\lambda.\,Cot.\mu\,,\quad Cot.\,\varphi=Sin.\mu.\,Cot.\lambda.$$

$$1)\int\frac{Sin.\,x}{\sqrt{(Cos.^2 x - Cos.^2\lambda)}}\,dx = \frac{1}{2}\,l\,\frac{1+Sin.\,\lambda}{1-Sin.\,\lambda}$$

$$2)\int\frac{Sin.^3 x}{\sqrt{(Cos.^2 x - Cos.^2\lambda)}}\,dx = \frac{1+Sin.^2\lambda}{4}\,l\,\frac{1+Sin.\,\lambda}{1-Sin.\,\lambda} - \frac{1}{2}\,Sin.\,\lambda$$

N°. 11.

$$3)\int\frac{Sin.\,x.\,Cos.\,x}{\sqrt{\{(Cos.^2 x - Cos.^2\lambda)(1-Cos.^2\mu.\,Cos.^2 x)\}}}\,dx = \theta\,Sec.\,\mu$$

$$4)\int\frac{Sin.\,x.\,Cos.^3 x}{\sqrt{\{(Cos.^2 x - Cos.^2\lambda)(1-Cos.^2\mu.\,Cos.^2 x)\}}}\,dx = \frac{1+Cos.^2\nu}{2\,Cos.^2\mu}\,\theta - \frac{Sin.\,\mu.\,Sin.\,\lambda}{2\,Cos.^2\mu}$$

$$5)\int\frac{Sin.\,x}{\sqrt{\{(Cos.^2 x - Cos.^2\lambda)(1-Cos.^2\mu.\,Cos.^2 x)\}}}\,\frac{dx}{Cos.\,x} = \varphi\,Sec.\,\lambda$$

$$6)\int\frac{Sin.\,x}{\sqrt{\{(Cos.^2 x - Cos.^2\lambda)(1-Cos.^2\mu.\,Cos.^2 x)\}}}\,\frac{dx}{Cos.^3 x} = \frac{1+Cos.^2\nu}{2\,Cos.^3\lambda}\,\varphi + \frac{Sin.\,\mu.\,Sin.\,\lambda}{2\,Cos.^2\lambda}$$

N°. 40.

$$7)\int\frac{Sin.\,x}{\sqrt{\{(Cos.^2 x - Cos.^2\lambda)(1-Cos.^2\mu.\,Cos.^2 x)\}}}\,dx = \mathrm{F}(Sin.\,\mu,\varphi)$$

$$8)\int\frac{Sin.\,x}{\sqrt{\{(Cos.^2 x - Cos.^2\lambda)(1-Cos.^2\mu.\,Cos.^2 x)\}}}\,\frac{dx}{Cos.^2 x} = Sec.^2\lambda.\,\mathrm{E}(Sin.\,\mu,\varphi)$$

$$9)\int\frac{Sin.\,x.\,Cos.^2 x}{\sqrt{\{(Cos.^2 x - Cos.^2\lambda)(1-Cos.^2\mu.\,Cos.^2 x)\}}}\,dx = Sec.^2\mu.\,\mathrm{E}(Sin.\,\mu,\varphi) - \frac{Sin.\,\lambda.\,Sin.\,\mu}{Cos.^2\mu}$$

$$10)\int\frac{dx}{\sqrt{(Cos.^2 x - Sin.^2\lambda)}} = \mathrm{F}'(Sin.\,\lambda)$$

$$11)\int\frac{dx}{Cos.^2 x\sqrt{(Cos.^2 x - Sin.^2\lambda)}} = Sec.^2\lambda.\,\mathrm{E}'(Sin.\,\lambda)$$

$$12)\int\frac{Cos.^2 x}{\sqrt{(Cos.^2 x - Sin.^2\lambda)}}\,dx = \mathrm{E}'(Sin.\,\lambda)$$

N°. 41.

$$13)\int\frac{dx}{\sqrt{\{(Cos.^2 x - Cos.^2\lambda)(1-Cos.^2\mu.\,Cos.^2 x)\}}} = Cosec.\,\nu.\,\mathrm{F}'(c)$$

$$14)\int\frac{dx}{Cos.^2 x\sqrt{\{(Cos.^2 x - Cos.^2\lambda)(1-Cos.^2\mu.\,Cos.^2 x)\}}} = \frac{Cos.^2\mu}{Sin.\,\nu}\,\mathrm{F}'(c) + \frac{Sin.\,\mu}{Cos.^2\lambda}\,\mathrm{E}'(c)$$

$$15)\int\frac{Cos.^2 x}{\sqrt{\{(Cos.^2 x - Cos.^2\lambda)(1-Cos.^2\mu.\,Cos.^2 x)\}}}\,dx = \frac{Cos.^2\lambda}{Sin.\,\nu}\,\mathrm{F}'(c) + \frac{Cos.\,\lambda}{Cos.\,\nu}\{\mathrm{F}'(c)\mathrm{E}(c,\nu) - \mathrm{E}'(c)\mathrm{F}(c,\nu)\}$$

N°. 42.

$$16)\int\frac{Cos.\,\tfrac{1}{2}x}{1+p^2-2p\,Cos.\,x}\,\frac{dx}{\sqrt{\{2(Cos.\,x - Cos.\,\lambda)\}}} = \frac{\pi}{2(1-p)\sqrt{(1-2p\,Cos.\,\lambda+p^2)}}$$

Vieille, Exerc. 164.

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1) $\int Sin. x. Cos. x\, dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{16} (Sin.^2 \mu - Sin.^2 \lambda)^2$

2) $\int Sin.^3 x. Cos. x\, dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{32} (Sin.^2 \mu - Sin.^2 \lambda)^2 (Sin.^2 \lambda + Sin.^2 \mu)$

3) $\int Sin.^{2a+1} x. Cos. x\, dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} =$

$$= \frac{\pi}{4} \frac{(Sin.^2 \mu - Sin.^2 \lambda)^2}{Sin.^4 \mu} Sin.^{2a+4} \mu \sum_0^\infty (-1)^n \binom{a}{n} \frac{1^{n/2}}{4^{n/2}} \frac{(Sin.^2 \mu - Sin.^2 \lambda)^n}{Sin.^{2n} \mu}$$

Voyez de ces formules: Legendre, Exerc. Suppl. N°. 6.

F. Circ. Dir. irrat. fract. à dén. rat. TABLE 106. Lim. λ et μ.

Toutes les formules de cette Table sont déduites par Legendre, Exerc. Suppl. Tome I, les formules 1 à 9 dans le N°. 5, les formules 10 à 16 dans le N°. 7; on a dans ces formules:

$$k = \frac{Sin.^2 \mu - Sin.^2 \lambda}{Sin.^2 \mu} \qquad h = \frac{Cos.^2 \lambda - Cos.^2 \mu}{Cos.^2 \lambda}$$

1) $\int \frac{Cos. x}{Sin. x} dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{4} (Sin. \mu - Sin. \lambda)^2$

2) $\int \frac{Cos. x}{Sin.^3 x} dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{4} \frac{(Sin. \mu - Sin. \lambda)^2}{Sin. \lambda. Sin \mu}$

3) $\int \frac{Cos. x}{Sin.^5 x} dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{16} \frac{(Sin.^2 \mu - Sin.^2 \lambda)^2}{Sin.^3 \lambda. Sin.^3 \mu}$

4) $\int \frac{Cos. x}{Sin^7. x} dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{32} \frac{(Sin.^2 \mu - Sin.^2 \lambda)^2}{Sin.^5 \lambda. Sin.^5 \mu} (Sin.^2 \lambda + Sin.^2 \mu)$

5) $\int \frac{Cos. x\, dx}{Sin.^{2a+1} x} \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin^2 \mu - Sin.^2 x)\}} = \frac{\pi k^2 Sin. \mu}{4 Sin.^{2a+1} \lambda} \sum_0^\infty (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}} k^n$

6) $\int \frac{Sin. x}{Cos. x} dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{4} (Cos. \lambda - Cos. \mu)^2$

7) $\int \frac{Sin. x}{Cos.^3 x} dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{4} \frac{(Cos. \lambda - Cos. \mu)^2}{Cos. \lambda. Cos. \mu}$

8) $\int \frac{Sin. x}{Cos.^5 x} dx \sqrt{\{(Sin.^2 x - Sin^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{16} \frac{(Cos.^2 \lambda - Cos.^2 \mu)^2}{Cos.^3 \lambda. Cos.^3 \mu}$

9) $\int \frac{Sin. x}{Cos.^{2a+1} x} dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi h^2 Cos. \lambda}{4 Cos.^{2a-1} \mu} \sum_0^\infty (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}} h^n$

10) $$\int \frac{dx}{Sin\, x.\, Cos.\, x} \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{2} \{1 - Cos.(\mu - \lambda)\}$$

11) $$\int \frac{dx}{Sin.^3 x.\, Cos.\, x} \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi\, Sin.^2 (\mu - \lambda)}{4\, Sin.\, \lambda.\, Sin.\, \mu}$$

12) $$\int \frac{dx}{Sin.^{2a+1} x. Cos. x} \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \int_\lambda^\mu \frac{dx}{Sin.^{2a-1} x. Cos. x} \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}}$$
$$+ \frac{\pi k^2\, Sin.\, \mu}{4\, Sin.^{2a-1} \lambda} \sum_0^\infty (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}} k^n$$

13) $$\int \frac{dx}{Sin. x. Cos.^3 x} \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi\, Sin.^2 (\mu - \lambda)}{4\, Cos.\, \lambda.\, Cos.\, \mu}$$

14) $$\int \frac{dx}{Sin. x. Cos.^{2a+1} x} \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \int_\lambda^\mu \frac{dx}{Sin. x. Cos.^{2a-1} x} \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}}$$
$$+ \frac{\pi h^2\, Cos.\, \lambda}{4\, Cos.^{2a-1} \mu} \sum_0^\infty (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}} h^n$$

15) $$\int \frac{Sin.^3 x}{Cos.\, x} dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \frac{\pi}{2} (Cos.\, \lambda - Cos.\, \mu)^2 - \frac{\pi}{16} (Sin.^2 \mu - Sin.^2 \lambda)^2$$

16) $$\int \frac{Sin.^{2a+1} x}{Cos.\, x} dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}} = \int_\lambda^\mu \frac{Sin.^{2a-1} x}{Cos.\, x} dx \sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}}$$
$$- \frac{\pi}{4} k^2\, Sin.^{2a} \mu \sum_0^\infty (-1)^n \binom{a-2}{n} \frac{3^{n/2}}{4^{n+1/2}} k^n$$

F. Circ. Dir. irrat. fract. à dén. irrat. TABLE 107. Lim. λ et μ.

Toutes les formules de cette Table sont trouvées par Legendre, Exerc. Suppl. I, aux numéros indiqués; on a dans ces intégrales partout:

$$k = \frac{Sin.^2 \mu - Sin.^2 \lambda}{Sin.^2 \mu}, \quad h = \frac{Cos.^2 \lambda - Cos.^2 \mu}{Cos.^2 \lambda}, \quad Cos.\, \theta = \frac{Cos.\, \mu}{Cos.\, \lambda}.$$

1) $$\int \frac{Sin.\, x.\, Cos.\, x}{\sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}}} dx = \frac{1}{2} \pi$$

2) $$\int \frac{Sin.^3 x.\, Cos.\, x}{\sqrt{\{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)\}}} dx = \frac{1}{4} \pi (Sin.^2 \lambda + Sin.^2 \mu)$$

(1, 2) N°. 2.

$$3)\int\frac{\mathrm{Sin.}^5 x.\,\mathrm{Cos.}\,x}{\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{1}{2}\pi\left(\frac{1.3}{2.4}\mathrm{Sin.}^4\lambda+\frac{1}{2}\mathrm{Sin.}^2\lambda.\,\mathrm{Sin.}^2\mu+\frac{1.3}{2.4}\mathrm{Sin.}^4\mu\right)$$

$$4)\int\frac{\mathrm{Sin.}^{2a+1} x.\,\mathrm{Cos.}\,x}{\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{1}{2}\pi\,\mathrm{Sin.}^{2a}\mu\sum_0^\infty(-1)^n\binom{a}{n}\frac{1^{n/2}}{2^{n/2}}k^n$$

N°. 2.

$$5)\int\frac{\mathrm{Cos.}\,x}{\mathrm{Sin.}x\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{\pi}{2\,\mathrm{Sin.}\,\lambda.\,\mathrm{Sin.}\,\mu}$$

$$6)\int\frac{\mathrm{Cos.}\,x}{\mathrm{Sin.}^3x\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{\pi}{4\mathrm{Sin.}^3\lambda.\,\mathrm{Sin.}^3\mu}(\mathrm{Sin.}^2\lambda+\mathrm{Sin.}^2\mu)$$

$$7)\int\frac{\mathrm{Cos.}\,x}{\mathrm{Sin.}^{2a+1}x\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{\pi}{2\,\mathrm{Sin.}^{2a+1}\lambda.\,\mathrm{Sin.}\mu}\sum_0^\infty(-1)^n\binom{a}{n}\frac{1^{n/2}}{2^{n/2}}k^n$$

N°. 3.

$$8)\int\frac{\mathrm{Sin.}\,x.\,\mathrm{Cos.}^3x}{\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{\pi}{4}(\mathrm{Cos.}^2\lambda+\mathrm{Cos.}^2\mu)$$

$$9)\int\frac{\mathrm{Sin.}\,x.\,\mathrm{Cos.}^{2a+1}x}{\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{1}{2}\pi\,\mathrm{Cos.}^{2a}\lambda\sum_0^\infty(-1)^n\binom{a}{n}\frac{1^{n/2}}{2^{n/2}}h^n$$

$$10)\int\frac{\mathrm{Sin.}\,x}{\mathrm{Cos.}\,x\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{\pi}{2\,\mathrm{Cos.}\,\lambda.\,\mathrm{Cos.}\,\mu}$$

$$11)\int\frac{\mathrm{Sin.}\,x}{\mathrm{Cos.}^3x\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{\pi}{4\,\mathrm{Cos}^3\lambda.\,\mathrm{Cos.}^3\mu}(\mathrm{Cos.}^2\lambda+\mathrm{Cos.}^2\mu)$$

$$12)\int\frac{\mathrm{Sin.}\,x}{\mathrm{Cos.}^{2a+1}x\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{\pi}{2\,\mathrm{Cos.}^{2a+1}\mu.\,\mathrm{Cos.}\,\lambda}\sum_0^\infty(-1)^n\binom{a}{n}\frac{1^{n/2}}{2^{n/2}}h^n$$

N°. 4.

$$13)\int\frac{dx}{\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}=\frac{1}{\mathrm{Cos.}\,\lambda.\,\mathrm{Sin.}\,\mu}\mathrm{F}'\left(\frac{\mathrm{Sin.}\,\theta}{\mathrm{Sin.}\,\mu}\right)$$

$$14)\int\frac{dx}{\mathrm{Sin.}^2x\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}=\frac{1}{\mathrm{Cos.}\lambda.\,\mathrm{Sin.}\,\mu}\mathrm{F}'\left(\frac{\mathrm{Sin.}\,\theta}{\mathrm{Sin.}\mu}\right)+\frac{\mathrm{Cos.}\,\lambda}{\mathrm{Sin.}^2\lambda.\,\mathrm{Sin.}\mu}\mathrm{E}'\left(\frac{\mathrm{Sin.}\,\theta}{\mathrm{Sin.}\mu}\right)$$

$$15)\int\frac{dx}{\mathrm{Cos.}^2x\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}=\frac{1}{\mathrm{Cos.}\lambda.\,\mathrm{Sin.}\mu}\mathrm{F}'\left(\frac{\mathrm{Sin.}\,\theta}{\mathrm{Sin.}\,\mu}\right)+\frac{\mathrm{Sin.}\,\mu}{\mathrm{Cos.}\lambda.\,\mathrm{Cos.}^2\mu}\mathrm{E}'\left(\frac{\mathrm{Sin.}\,\theta}{\mathrm{Sin.}\,\mu}\right)$$

N°. 9.

$$16)\int\frac{\mathrm{Sin.}^2x}{\sqrt{\{(\mathrm{Sin.}^2x-\mathrm{Sin.}^2\lambda)(\mathrm{Sin.}^2\mu-\mathrm{Sin.}^2x)\}}}\,dx=\frac{\mathrm{Sin.}\,\mu}{\mathrm{Cos.}\,\lambda}\mathrm{F}'\left(\frac{\mathrm{Sin.}\,\theta}{\mathrm{Sin.}\,\mu}\right)+$$

$$+\,\mathrm{E}'\left(\frac{\mathrm{Sin.}\,\theta}{\mathrm{Sin.}\,\mu}\right)\mathrm{F}\left(\frac{\mathrm{Sin.}\,\theta}{\mathrm{Sin.}\,\mu},\mu\right)-\mathrm{F}'\left(\frac{\mathrm{Sin.}\,\theta}{\mathrm{Sin.}\,\mu}\right)\mathrm{E}\left(\frac{\mathrm{Sin.}\,\theta}{\mathrm{Sin.}\,\mu},\mu\right)$$

N°. 10.

$$17)\int\frac{Sin.^4 x}{\sqrt{\{(Sin.^2 x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2 x)\}}}dx=\frac{1+Sin.^2\lambda+Sin.^2\mu}{2}\Big[E'\left(\frac{Sin.\theta}{Sin.\mu}\right)F'\left(\frac{Sin.\theta}{Sin.\mu},\mu\right)-$$
$$-F'\left(\frac{Sin.\theta}{Sin.\mu}\right)E\left(\frac{Sin.\theta}{Sin.\mu},\mu\right)+\frac{1+Sin.^2\mu}{2\,Cos.\lambda}Sin.\mu\,F'\left(\frac{Sin.\theta}{Sin.\mu}\right)-\frac{Sin\,\mu.\,Cos.\lambda}{2}E'\left(\frac{Sin.\theta}{Sin.\mu}\right)\Big]$$

$$18)\int dx\sqrt{\frac{Sin.^2 x-Sin.^2\lambda}{Sin.^2\mu-Sin.^2 x}}=\frac{Sin.^2\mu-Sin.^2\lambda}{Sin.\mu.Cos.\lambda}F'\left(\frac{Sin.\theta}{Sin.\mu}\right)+E'\left(\frac{Sin.\theta}{Sin.\mu}\right)F\left(\frac{Sin.\theta}{Sin.\mu},\mu\right)-F'\left(\frac{Sin.\theta}{Sin.\mu}\right)E\left(\frac{Sin.\theta}{Sin.\mu},\mu\right)$$

$$19)\int dx\sqrt{\frac{Sin.^2\mu-Sin.^2 x}{Sin.^2 x-Sin.^2\lambda}}=F'\left(\frac{Sin.\theta}{Sin.\mu}\right)E\left(\frac{Sin.\theta}{Sin.\mu},\mu\right)-E'\left(\frac{Sin.\theta}{Sin.\mu}\right)F\left(\frac{Sin.\theta}{Sin.\mu},\mu\right)$$

N°. 10.

$$20)\int\frac{Cos.x}{\sqrt{\{(Sin.^2 x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2 x)\}}}dx=\frac{1}{Sin.\mu}F'\left(\sqrt{\frac{Sin.^2\mu-Sin.^2\lambda}{Sin.^2\mu}}\right)$$

$$21)\int\frac{Sin.^2 x.\,Cos.x}{\sqrt{\{(Sin.^2 x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2 x)\}}}dx=\frac{1}{Sin.\mu}E'\left(\sqrt{\frac{Sin.^2\mu-Sin.^2\lambda}{Sin.^2\mu}}\right)$$

$$22)\int\frac{Cos.x}{Sin.^2 x\sqrt{\{(Sin.^2 x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2 x)\}}}dx=\frac{1}{Sin.^2\lambda.\,Sin.^2\mu}E'\left(\sqrt{\frac{Sin.^2\mu-Sin.^2\lambda}{Sin.^2\mu}}\right)$$

$$23)\int\frac{dx}{Cos.x\sqrt{\{(Sin.^2 x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2 x)\}}}=\frac{1}{Sin.\mu.Cos.^2\mu}\Pi'\left\{\frac{Sin.^2\mu-Sin.^2\lambda}{Sin.^2\mu},Tang.^2\mu,\sqrt{\frac{Sin.^2\mu-Sin.^2\lambda}{Sin.^2\mu}}\right\}$$

N°. 50.

F. Circul. Inverses. TABLE 108. Lim. 0 et 1.

$$1)\int Arctang.\,x\,dx=\frac{1}{4}\pi-\frac{1}{2}l\,2$$

$$2)\int Arctang.\,px\,dx=Arctang.p-\frac{1}{2p}l(1+p^2)$$

$$3)\int Arcsin.\,x\,dx=\frac{1}{2}\pi-1$$

$$4)\int Arcsin.\,p\,x\,dx=Arcsin.\,p+\frac{1}{p}\sqrt{(1-p^2)}-\frac{1}{p}$$

$$5)\int Arctang.(xe^{pi})dx=\frac{1}{4}\pi-\frac{1}{2}pSin.p-\frac{1}{2}Cos.pl(2Cos.p)+\frac{i}{4}\left\{l\frac{1+Sin.p}{1-Sin.p}+2Sin.pl(2Cos.p)-pCos.p\right\},p\leqq\tfrac{1}{4}\pi^2;$$

$$6)\int Arcsin.(xe^{pi})dx=Arcsin.\left(\frac{Cos.p}{\sqrt{(1+Sin.p)}}\right)-Cos.p+\left(Cos.\frac{\pi+2p}{4}-iSin.\frac{\pi+2p}{4}\right)\sqrt{(2Sin.p)}+iSin.p+$$
$$+il\left\{Sin.\frac{1}{2}p+\sqrt{(1+Sin.p)}\right\},\ p\leqq\tfrac{1}{2}\pi;$$

Sur les intégrales 1 à 6 voyez: Dienger, Cr. 38. 331.

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7) $\int Arccot.\, x\, dx = \frac{1}{4}\pi + \frac{1}{2} l\, 2$ V. T. 108. N°. 1.

8) $\int Arcsin.\, (\sqrt{x})\, dx = \frac{\pi}{4}$ V. T. 9. N°. 4.

9) $\int Arccos.\, (\sqrt{x})\, dx = \frac{\pi}{4}$ V. T. 9. N°. 4.

10) $\int (Arccot.\, x)^2\, dx = -\frac{\pi}{16} + \frac{3}{4}\pi\, l\, 2 - \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 27.

F. Circul. Inverses. TABLE 109. Lim. 0 et ∞.

1) $\int Arctang.\, x\, dx = \infty$ V. T. 109. N°. 2.

2) $\int Arccot.\, x\, dx = \frac{1}{2}\pi\, l\, 2$ V. T. 264. N°. 2.

3) $\int (Arccos.\, x)^2\, dx = \frac{1}{2}\pi\, l\, 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

4) $\int (Arccot.\, x)^2\, dx = \pi\, l\, 2$ Cauchy, Lim. Imag. Add § 31. — Id., Sav. Etr. 1827. 599. P. 2. § 5. — Mosta, Gr. 10. 449.

5) $\int (Arccot.\, x)^p\, dx = p\left(\frac{\pi}{2}\right)^{p-1}\left\{1 - \sum_1^\infty \frac{2}{p+2m-1} \sum_1^\infty \frac{1}{(2n)^{2m}}\right\}$ V. T. 265. N°. 26.

6) $\int Arctang.\, px.\, Arctang.\, qx\, dx = \infty$ V. T. 109. N°. 1, 10.

7) $\int Arctang.\, x.\, Arccot.\, x\, dx = \frac{\pi - 4}{4}\pi\, l\, 2$ V. T. 109. N°. 2, 4.

8) $\int Arctang.\, x.\, Arccot.\, \frac{x}{q}\, dx = \frac{\pi^2 q}{2} l\, 2 + \frac{\pi q}{2} l\, 2 - \frac{1+q}{2}\pi\, l\, (1+q)$ V. T. 109. N°. 2, 11.

9) $\int Arctang.\, \frac{x}{q}\, Arccot.\, x\, dx = \frac{\pi^2}{2} l\, 2 + \frac{\pi q}{2} l\, 2 - \frac{1+q}{2}\pi\, l\, (1+q)$ V. T. 109. N°. 2, 11.

10) $\int Arctang.\, px.\, Arccot.\, qx\, dx = \frac{\pi}{2}\left\{\frac{\pi}{2q} l\, 2 + \frac{1}{p} l \frac{q}{p+q} + \frac{1}{q} l \frac{p+q}{p}\right\}$ V. T. 109. N°. 2, 12.

11) $\int Arccot.\, x.\, Arccot.\, qx\, dx = \frac{\pi}{2}\left\{\frac{1+q}{q} l\, (1+q) - l\, q\right\}$ V. T. 264. N°. 13.

12) $\int Arccot.\, px.\, Arccot.\, qx\, dx = \frac{\pi}{2}\left\{\frac{1}{p} l\left(1 + \frac{p}{q}\right) + \frac{1}{q} l\left(1 + \frac{q}{p}\right)\right\}$ V. T. 264. N°. 14.

1) $\int_1^\infty Arctang.\, x\, dx = \infty$ V. T. 108. N°. 1 et T. 109. N°. 1.

2) $\int_1^\infty Arccot.\, x\, dx = \frac{\pi - 1}{2} l\, 2 - \frac{\pi}{4}$ V. T. 108. N°. 7 et T. 109. N°. 2.

3) $\int_1^\infty (Arccot.\, x)^2\, dx = \frac{\pi^2}{16} + \frac{\pi}{4} l\, 2 + \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 7.

4) $\int_1^\infty (Arccot.\, x)^p\, dx = \left(\frac{\pi}{4}\right)^p + \frac{p}{2}\left(\frac{\pi}{2}\right)^{p-1} \left\{1 - \sum_1^\infty \frac{4}{p + 2m} \sum_1^\infty \frac{1}{(4n)^{2m}}\right\}$ V. T. 270. N°. 11.

5) $\int_0^p Arcsin.\left(\frac{x}{p}\right) dx = \frac{\pi - 2}{2} p$ V. T. 34. N°. 7.

6) $\int_0^p Arccos.\left(\frac{x}{p}\right) dx = p$ V. T. 34. N°. 7.

7) $\int_0^p Arcsin.\left(\sqrt{\frac{x}{p}}\right) dx = \frac{1}{4} p\, \pi$ V. T. 34. N°. 8.

8) $\int_0^p Arccos.\left(\sqrt{\frac{x}{p}}\right) dx = \frac{1}{4} p\, \pi$ V. T. 34. N°. 8.

1) $\int_0^1 B'(x)\, dx = \frac{(-1)^{a-1}}{2a+2} B_{2a+1}$

2) $\int_0^1 B''(x)\, dx = 0$

3) $\int_0^1 \{B'(x)\}^2\, dx = \frac{1^{2a+1/1}}{(2a+2)^{2a+3/2}} B_{4a+3} + \left(\frac{1}{2a+2} B_{2a+1}\right)^2$

4) $\int_0^1 \{B''(x)\}^2\, dx = \frac{1^{2a/1}}{(2a+1)^{2a+2/1}} B_{4a+1}$

(1)–(4): Raabe, Cr. 42. 348.

5) $\int_0^1 dx\, li.(x) = -l\, 2$ V. T. 300. N°. 1.

NB. Les deuxième et troisième parties de ces Tables paraîtront dans le cours de l'année; on y joindra la préface, qui doit donner divers renseignements, et que l'on devra consulter par conséquence, en faisant usage de ces Tables. Il suffit d'observer ici, qu'en général les premières lettres *a*, *b*, *c* etc. désignent des nombres entiers, et que les lettres *p*, *q*, *r* etc. représentent au contraire des quantités quelconques, fractionnaires et irrationnelles. Toujours les lettres ne valent que pour des valeurs positives, à moins que le contraire ne soit expressément énoncé.

SUR L'IMPRIMERIE DE W. J. KRÖBER.

TABLES

D'INTÉGRALES DÉFINIES

PAR

D. BIERENS DE HAAN.

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DEUXIÈME PARTIE.

AMSTERDAM,
C. G. VAN DER POST.
1857.

1) $\int e^{ax}\, x\, dx = \frac{1}{a^3}\{(a-1)e^a + 1\}$ }
2) $\int e^{-x^2}\, x\, dx = \frac{e-1}{2e}$ } Dienger, Cr. 46. 119.

3) $\int e^{-x^2}\, x^3\, dx = \frac{e-2}{2e}$ V. T. 115. N°. 2.

4) $\int \frac{e^{-x}\, dx}{x} = \infty$ Cisa de Grésy, Mém. Turin. 1821. 209. I. 27.

5) $\int \frac{e^x\, x\, dx}{(1+x)^2} = \frac{1}{2}e - 1$ Rogner, Mater.

6) $\int e^{-x}\, dx \sqrt{x} = \frac{1}{2}\sqrt{\pi}$ Plana, Cr. 17. 1.

7) $\int \frac{e^{-x}\, dx}{\sqrt{x}} = \frac{2 + e\sqrt{\pi}}{4e}$ V. T. 115. N°. 6.

8) $\int \left(e^{1-\frac{1}{x}} - x^q\right) \frac{dx}{x(1-x)} = Z'(q)$ Lejeune-Dirichlet, Cr. 15. 258. — Stern, Cr. 21. 377. — Grunert, Gr. 2. 266.

9) $\int \left(\frac{b e^{1-\frac{1}{x}}}{1-x} - \frac{x^a}{1-x^{\frac{1}{b}}}\right)\frac{dx}{x} = \sum_1^b Z'\left(a + \frac{n-1}{n}\right)$

10) $\int \left(\frac{b e^{1-x^{-b}}}{1-x^b} - \frac{x^{ab}}{1-x}\right)\frac{dx}{x} = \frac{1}{b}\sum_1^b Z'\left(a + \frac{n-1}{n}\right)$

11) $\int \left(\frac{b e^{1-x^{-b}}}{1-x^b} - \frac{e^{1-\frac{1}{x}}}{1-x}\right)\frac{dx}{x} = \frac{1}{b}\frac{d}{da}\, l\, \frac{\Gamma(a)\Gamma\left(a+\frac{1}{b}\right)\ldots\Gamma\left(a+\frac{b-1}{b}\right)}{\Gamma(ab)}$

(9–11) Lejeune-Dirichlet, Cr. 15. 258. — Grunert, Gr. 2. 266. — Arndt, Gr. 10. 253.

12) $\int \frac{e^{px} - 2px - e^{-px}}{e^{\pi x} - e^{-\pi x}}\frac{dx}{x} = p\sqrt{\pi} - l\,\frac{\Gamma\left(\frac{\pi+p}{2\pi}\right)}{\Gamma\left(\frac{\pi-p}{2\pi}\right)}$, $p < \pi$; Malmsten, Cr. 38. 1.

F. Algébr. rat. ent. Expon. monôme e^{ax}. TABLE 113. Lim. 0 et ∞.

1) $\int e^{-ax}\, x\, dx = \frac{1}{a^2}$ V. T. 152. N°. 1.

2) $\int e^{-x} x^{a-1} dx = 1^{a-1/1} = 1.2.3.....a-1$

3) $\int e^{-x} x^{p-1} dx = \Gamma(p)$

Euler, Calc. Int. 4. S. 5. 129. — Legendre, Exerc. 3. 31. — Poisson, P. 19. 404. N°. 68. — Binet, P. 27. 123. — Liouville, Cr. 11. 1. — Oettinger, Cr. 35. 13. — Lejeune-Dirichlet, Cr. 15. 258. — Schaar, Mém. Cour. Brux. T. 22. — Lobatschewsky, Mém. Kasan. 1835. 211. — Id., ib. 1836. 1. II. form. (12).

$\infty > p > -1$; C'est la fonction Eulérienne de seconde espèce.

4) $\int e^{-qx} x^{a-1} dx = \frac{1^{a-1/1}}{q^a}$ Euler, Calc. Int. 4. S. 5. 131. — Lejeune-Dirichlet, Cr. 15. 258. — Oettinger, Cr. 35. 13. — Grunert, Gr. 2. 266.

5) $\int e^{-qx} x^{p-1} dx = \frac{\Gamma(p)}{q^p}$ Cauchy, Cours. Leç. 32. — Kummer, Cr. 17. 228. — Serret, L. 8. 1.

6) $\int e^{-cx} x^{a+p-1} dx = \frac{p^{a/1}}{c^{a+p}} \Gamma(p)$ Schlömilch, Stud. I. 1.

7) $\int r^{-x} x^{p-1} dx = \frac{\Gamma(p)}{(lr)^p}$, $\infty > p > -1$, $r > 1$; Lejeune-Dirichlet, Cr. 4. 94.

8) $\int x^x e^{-x} dx = e^{-k} k^k \sqrt{2k\pi}$, pour $k = \infty$; Liouville, L. 11. 464.

9) $\int \{e^x x^{q-1} - e^{-px}(1-e^{-x})^{q-1}\} dx = \frac{\Gamma(p+q)-\Gamma(p)}{q} \frac{\Gamma(1+q)}{\Gamma(p+q)}$ Schaar, Mém. Cour. Brux. T. 23.

10) $\int e^{-px}(1-e^{-qx})^a x^b dx = (-1)^b 1^{b/1} \sum_0^\infty \binom{a}{n} \frac{(-1)^n}{(p+nq)^{b+1}}$ V. T. 151. N°. 8.

11) $\int e^{-qx}(x+x^2)^{p-1} dx = \Gamma(2p-1) q^{1-2p} e^{\frac{1}{8}q} \psi\left(\frac{3}{2}-p, \frac{1}{16}q^2\right) + \frac{\Gamma(1-2p)\Gamma(p)}{\Gamma(1-p)} e^{\frac{1}{8}q} \psi\left(\frac{1}{2}+p, \frac{1}{16}q^2\right)$ Kummer, Cr. 17. 228.

12) $\int e^{-axi} x^{p-1} dx = \frac{\Gamma(p)}{a^p} e^{-\frac{1}{2}p\pi i}$ Moigno, Calc. Int. 132.

13) $\int e^{axi} x^{p-1} dx = \frac{\Gamma(p)}{a^p} e^{\frac{1}{2}p\pi i}$, $1 > p > 0$; Lejeune-Dirichlet, C. R. 8. 157. — Schlömilch, St. I. 13.

14) $\int e^{-(1+qi)x} x^{p-1} dx = \frac{\Gamma(p)}{(1+qi)^p}$ Schlömilch, Stud. I. 13.

15) $\int e^{-(a+bi)x} x^{p-1} dx = \frac{\Gamma(p)}{(a^2+b^2)^{\frac{1}{2}p}} e^{pi\,Arctang.\frac{b}{a}}$ Moigno, Calc. Int. 132.

16) $\int e^{-(p+qi)x} x^a dx = \frac{1^{a/1}}{(p+qi)^{a+1}}$, où il y a faut.: $(p+qi)^a$ Meyer, Int. Déf. 117.

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17) $\int e^{-(p+qi)x} x^{r-1}\, dx = \frac{\Gamma(r)}{(p+qi)^r}$ Cauchy, Cours. Leç. 39. — Lejeune-Dirichlet, Cr. 4. 94.

18) $= \frac{\Gamma(r)}{\sqrt{(p^2+q^2)^r}} e^{-ri\,Arctang.\frac{q}{p}}$ Serret, L. 8. 1.

19) $\int (1-e^{-x})\, x e^{-x}\, dx = \frac{3}{4}$ V. T. 151. N°. 12.

F. Algébr. rat. ent. Expon. monôme e^{ax^b} pour b spécial. TABLE 114. Lim. 0 et ∞.

1) $\int e^{-x^2} x^2\, dx = \frac{1}{4}\sqrt{\pi}$

2) $\int e^{-x^2} x^4\, dx = \frac{3}{8}\sqrt{\pi}$

3) $\int e^{-x^2} x^6\, dx = \frac{15}{16}\sqrt{\pi}$

(1–3) Kramp, Réfr. 3. N°. 70. — Boncompagni, Cr. 25. 74.

4) $\int e^{-x^2} x^{2a+1}\, dx = 3^{a-2/1}$ Oettinger, Cr. 35. 13.

5) $\int \qquad = 0$ (fautif) Boncompagni, Cr. 25. 74.

6) $\int e^{-x^2} x^{2a}\, dx = \frac{1}{2}\Gamma\left(\frac{2a+1}{2}\right)$ Legendre, Exerc. 3. 29.

7) $\int \qquad = \frac{1^{a/2}}{2^{a+1}}\sqrt{\pi}$ Kramp, Réfr. 3. N°. 70. — Laplace, Mém. Inst. 1809. 253. § 3. — Boncompagni, Cr. 25. 74. — Oettinger, Cr. 35. 13.

8) $\int e^{-px^2} x^{2a}\, dx = \frac{1^{a/2}}{(2p)^a}\frac{1}{2}\sqrt{\frac{\pi}{p}}$ Schlömilch, Gr. 5. 90. — Id., Gr. 5. 100, — Id., Beitr. III. 14. — Id., Stud. 1. 12.

9) $\int e^{-px^2} x^{2a+1}\, dx = \frac{1}{(2p)^{a+1}} 1^{a/1}$ Schlömilch, Beitr. III. 14.

10) $\int e^{-p^2x^2} x\, dx = \frac{1}{2p^2}$ Schlömilch, Gr. 9. 379.

11) $\int e^{-px^2} x^2\, dx = \frac{1}{4p}\sqrt{\frac{\pi}{p}}$ Ohm, Ausw. 20.

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F. Algébr. rat. ent. Expon. monôme e^{ax^b} pour b spécial. **TABLE 114 suite.** Lim. 0 et ∞.

12) $\int e^{-px^2} x^4 \, dx = \frac{3}{8p^2} \sqrt{\frac{\pi}{p}}$ V. T. 114. N°. 11.

13) $\int r^{-p^2 x^2} x^{2a} \, dx = \frac{1^{a/2}}{2^{a+1} (p \sqrt{l r})^{2a+1}} \sqrt{\pi}$ Meyer, Int. déf. 116.

14) $\int e^{px^2 i} x^{2q-1} \, dx = \frac{\Gamma(q)}{2 p^q} e^{\frac{1}{2}\pi q i}$ Schaar, Mém. Brux. T. 24.

15) $\int e^{-x^4} x^2 \, dx = \frac{\pi^{\frac{3}{4}}}{4 \sqrt{(2 \pi_1 \sqrt{2})}}$, où $\pi_1 = 1,31102877714605987$ Laplace, Mém. Acad. 1782. 1. § 5.

16) $\int e^{-x^6} x^2 \, dx = \frac{1}{6} \sqrt{\pi}$

17) $\int e^{-x^6} x^3 \, dx = \frac{1}{6} \frac{3.6.9\ldots}{2.5.8\ldots} = \frac{1^{-\frac{1}{3}/1}}{6}$

(16–17) Oettinger, Cr. 35. 13.

F. Algébr. rat. ent. Expon. monôme e^{ax^b} pour b général. **TABLE 115.** Lim. 0 et ∞.

1) $\int e^{-x^b} x^{a-1} \, dx = \frac{1}{a} 1^{\frac{a}{b}/1}$

2) $\int e^{-x^b} x^{ab-1} \, dx = \frac{1}{ab} 1^{a/1}$

(1–2) Kramp, Réfr. 3. N. 62, 64. — Oettinger, Cr. 35. 13.

3) $\int e^{-x^p} x^{p-1} \, dx = \frac{1}{p}$

4) $\int e^{-x^p} x^{2p-1} \, dx = \frac{1}{p}$

5) $\int e^{-x^{2p}} x^{p-1} \, dx = \frac{1}{2p} \sqrt{\pi}$

(3–5) Kramp, Réfr. 3. N°. 65, 66.

6) $\int e^{-x^a} x^b \, dx = \frac{1}{a} \Gamma \frac{b+1}{a}$ Legendre, Mém. Inst. 1809. 416. N°. 81. — Id., Exerc. 2. 81.

7) $\int e^{-x^a} x^{\frac{1}{2}a-1} \, dx = \frac{1}{a} \sqrt{\pi}$ Laplace, Mém. Acad. 1782. 1. § 5.

8) $\int e^{-x^{ac}} x^{ab-1} \, dx = \frac{1}{ab} 1^{\frac{b}{c}/1}$ Kramp, Réfr. 3. N°. 68.

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9) $\int e^{-qx^a} x^{ap-1}\, dx = \frac{1}{a q^p}\Gamma(p)$ Boncompagni, Cr. 25. 74.

10) $\int e^{-c^b x^b} x^{a-1}\, dx = \frac{1}{a c^a} e^{c^b} 1^{\frac{a}{b}/1}$ Oettinger, Cr. 35. 13.

F. Algébr. rat. ent. Expon. monôme d'autre forme. TABLE 116. Lim. 0 et ∞ .

1) $\int e^{-x^2+px} x\, dx = \frac{1}{2}(-1 + \frac{1}{2} p e^{\frac{1}{4}p^2} \sqrt{\pi})$ V. T. 37. N°. 5.

2) $\int \left(e^{px} - e^{-px}\right) e^{-x^2} x\, dx = \frac{1}{2} p e^{\frac{1}{4}p^2} \sqrt{\pi}$ V. T. 37. N°. 9.

3) $\int e^{-(x+\frac{q}{x})} x^{p-1}\, dx = \Gamma(p)\, \psi(1-p, q) + \Gamma(-p)\, q^p\, \psi(1+p, q),\ q > 0;$ Kummer, Cr. 17. 228.

4) $\int e^{-(ax^q+bx)} x^{pq-1}\, dx = \frac{\Gamma(p)}{q a^p}\left(1 + \frac{b}{a}\right)^{-p}$ Boncompagni, Cr. 25. 74. Elle ne vaut que pour $q = 1$.

5) $\int e^{-\frac{1}{x^6}} x\, dx = \frac{1}{2} 1^{\frac{1}{3}/1}$

6) $\int e^{-\frac{1}{x^6}} x^2\, dx = \frac{1}{3}\sqrt{\pi}$

7) $\int e^{-\frac{1}{x^a}} x^{b-1}\, dx = \frac{1}{b} 1^{-\frac{b}{a}/1}$

(5–7) Oettinger, Cr. 35. 13.

8) $\int e^{-q(x^2+\frac{1}{x^2})} x^{2a}\, dx = \frac{1}{2} e^{-2q} \sqrt{\frac{\pi}{q}} \sum_0^\infty (-1)^n \frac{(a-n+1)^{2n/1}}{2^n\, 1^{n/1}} \left(\frac{1}{2q}\right)^n$ Cauchy, P. 28. 147. P. 2. — Id., Exerc. 1826. p. 54.

9) $\int e^{-xe^{qi}} x^{a-1}\, dx = e^{-pqi}\Gamma(p)$ Serret, L. 8. 1.

10) $\int \left\{\frac{d^n}{dy^n}\left(y\, e^{-x^2 y^2}\right)\right\} e^{-x^2}\, dx = \frac{1}{2}\sqrt{\pi}\, \frac{d^n}{dy^n}\left(\frac{y}{\sqrt{(1+y^2)}}\right)$ Schlömilch, Gr. 4. 364.

11) $\int e^{-x^{\frac{2a}{1+2b}}} x^{a-1}\, dx = \frac{1+2b}{a \cdot 2^{b+1}} 1^{b/2} \sqrt{\pi}$ Kramp, Réfr. 3. 67.

12) $\int (e^{-x} - 1)^a e^{-px} x^{b-1}\, dx = 1^{b/1} \Delta^a (p^{-b})$ Cauchy, P. 28. 147. P. 3. § 1.

1) $\int \frac{x}{e^x+1} dx = \frac{1}{12}\pi^2$ V. T. 152. N°. 3.

2) $\int \frac{x^3}{e^x+1} dx = \frac{7}{120}\pi^4$ V. T. 154. N°. 10.

3) $\int \frac{x^5}{e^x+1} dx = \frac{31}{256}\pi^6$ V. T. 155. N°. 2.

4) $\int \frac{x^7}{e^x+1} dx = \frac{127}{1680}\pi^8$ V. T. 155. N°. 9.

5) $\int \frac{x}{e^x-1} dx = \frac{1}{6}\pi^2$ Cauchy, Mém. Paris. 1823. 603. — Id., Sav. Etr. 1827. 599. P. 2. § 5.

6) $\int \frac{x^3}{e^x-1} dx = \frac{1}{15}\pi^4$

7) $\int \frac{x^5}{e^x-1} dx = \frac{8}{63}\pi^6$

(6, 7) Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

8) $\int \frac{x}{e^{2x}-1} dx = \frac{1}{24}\pi^2$ V. T. 152. N°. 14.

9) $\int \frac{x}{e^{2x}+1} dx = \frac{1}{48}\pi^2$ V. T. 152. N°. 12.

10) $\int \frac{x^3}{e^{2x}+1} dx = \frac{7}{1920}\pi^4$ V. T. 154. N°. 13.

11) $\int \frac{x^{p+1}}{e^x-q} dx = \frac{p+1}{q}\Gamma(p+1)\sum_0^\infty \frac{q^n}{n^{p+2}}$ Hoppe, Cr. 40. 139.

12) $\int \frac{x^{2a}}{e^x+1} dx = \frac{2^{2a}-1}{2^{2a}} 1^{2a/1} \sum_0^\infty \frac{1}{n^{2a+1}}$ V. T. 157. N°. 2.

13) $\int \frac{x^{2a-1}}{e^x+1} dx = \frac{2^{2a-1}-1}{2a}\pi^{2a} B_{2a-1}$ V. T. 157. N°. 5.

14) $\int \frac{x^{2a}}{e^x-1} dx = 1^{2a/1} \sum_0^\infty \frac{1}{n^{2a+1}}$ V. T. 157. N°. 3.

15) $\int \frac{x^{2a-1}}{e^x-1} dx = \frac{2^{2a-1}\pi^{2a}}{2a} B_{2a-1}$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Raabe, Cr. 42. 348.

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16) $\int \frac{x^{p-1}}{e^x+1} dx = \Gamma(p) \sum_0^\infty \frac{(-1)^n}{(n+1)^p}$ V. T. 157. N°. 8.

17) $\int \frac{x^{p-1}}{e^x-1} dx = \Gamma(p) \sum_0^\infty \frac{1}{(n+1)^p}$ V. T. 157. N°. 9.

18) $\int \frac{x}{e^{2\pi x}-1} dx = \frac{1}{24}$

19) $\int \frac{x^3}{e^{2\pi x}-1} dx = \frac{1}{240}$

} Poisson, Mém. Inst. 1811. 163. N°. 40.

20) $\int \frac{x}{e^{\frac{2\pi x}{a}}-1} dx = \frac{1}{24} a^2$ Cauchy, Mém. Ac. 1823. 603.

21) $\int \frac{x^{2a-1}}{e^{\pi x}+1} dx = \frac{2^{2a-1}-1}{2a} B_{2a-1}$ Schlömilch, Gr. 3. 9.

22) $\int \frac{x^{2a-1}}{e^{\pi x}-1} dx = \frac{2^{2a-1}}{2a} B_{2a-1}$ Malmsten, Cr. 35. 55.

23) $\int \frac{x^{2a+1}}{e^{2\pi x}-1} dx = \frac{1}{4a} B_{2a-1}$ Binet, P. 27. 123. — Plana, Mém. Turin. 1820. 1. — Malmsten, Cr. 35. 55. — Schlömilch, Gr. 3. 9. — Id., Gr. 12. 130.

F. Algébr. rat. ent. monôme. Expon. binôme $e^{ax} \pm 1$ en dén. } Numér. alg. et exp. TABLE 118. Lim. 0 et ∞.

1) $\int \frac{x e^{-x}}{e^x-1} dx = \frac{1}{6}\pi^2 - 1$ V. T. 152. N°. 3.

2) $\int \frac{e^{-2x} x}{1+e^{-x}} dx = -\frac{1}{12}\pi^2 + 1$ V. T. 152. N°. 4.

3) $\int \frac{e^{-3x} x}{1+e^{-x}} dx = \frac{1}{12}\pi^2 - \frac{3}{4}$ V. T. 152. N°. 5.

4) $\int \frac{e^{-x} x^{p+1}}{1-q e^{-x}} dx = \frac{p+1}{q} \Gamma(p+1) \sum_1^\infty \frac{q^n}{n^{p+2}}$ V. T. 157. N°. 10.

5) $\int \frac{1-e^{-px}}{1-e^{-x}} e^{-x} x^{a-1} dx = 1^{a/1} \sum_1^b \frac{1}{n^a}$ V. T. 157. N°. 14.

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6) $\int \frac{e^{-ax} x^{p-1}}{1+e^{-x}} dx = \Gamma(p) \sum_0^\infty \frac{(-1)^n}{(a+n)^p}$ V. T. 157. N°. 11.

7) $\int \frac{e^{-px} x}{1-e^{-x}} dx = \sum_0^\infty \frac{1}{(n+p)^2}$

8) $\int \frac{e^{-ax} x^{p-1}}{1-e^{x}} dx = -\Gamma(p) \sum_1^\infty \frac{1}{(a+n)^p}$

(7, 8) Binet, P. 27. 123.

9) $\int \frac{e^{-px} x^r}{1-e^{-qx}} dx = 1^{r/1} \sum_0^\infty \frac{1}{(p+nq)^{r+1}}$ V. T. 158. N°. 10.

10) $\int \frac{e^{-x}+1}{e^x-1} x\, dx = -1 + 2 \sum_1^\infty \frac{1}{n^2}$ Euler, N. C. P. 14. 129.

11) $\int \frac{1-e^{-x}}{1+e^{-3x}} x e^{-x} dx = \frac{2}{27} \pi^3$ V. T. 152. N°. 15.

12) $\int \frac{e^{-ax}+e^{(a-b)x}}{1-e^{-bx}} x\, dx = \left(\frac{\pi}{b}\right)^2 Cosec.^2 \frac{a\pi}{b}$ V. T. 152. N°. 21.

13) $\int \frac{e^{-qx}-e^{(q-p)x}}{1-e^{-px}} x^2 dx = \left(\frac{\pi}{p\, Sin. \frac{q\pi}{p}}\right)^3 2\, Cos. \frac{q\pi}{p}$ V. T. 154. N°. 7.

14) $\int \frac{e^{-qx}+e^{(q-p)x}}{1-e^{-px}} x^3 dx = \left(\frac{\pi}{p\, Sin. \frac{q\pi}{p}}\right)^4 \left(2 + 4\, Cos.^2 \frac{q\pi}{p}\right)$ V. T. 154. N° 8.

15) $\int \frac{e^{\pi x}+1}{e^{\pi x}-1} x^{2a-1} dx = \frac{2^{2a-1}}{a} B_{2a-1}$ Schlömilch, Gr. 1. 360.

16) $\int \frac{1-e^{(c-b)x}}{1-e^{-bx}} e^{-\frac{1}{2}cx} x^{2a} dx = \frac{(-1)^{a+1}}{b} (2\pi)^{2a+1} \sum_1^b B''\left(\frac{n}{2b}\right) Sin. \frac{nc\pi}{b}$ V. T. 164. N°. 6.

17) $\int \frac{1+e^{(c-b)x}}{1+e^{-bx}} e^{-\frac{1}{2}cx} x^{2a} dx = \frac{(-1)^{a+1}}{b} (2\pi)^{2a+1} \sum_1^b B''\left(\frac{2n-1}{4b}\right) Sin. \left(\frac{2n-1}{2b} c\pi\right)$ V. T. 164. N°. 5.

18) $\int e^{-px} (e^{-x}-1)^c \left(p + \frac{ce^{-x}}{e^{-x}-1}\right) x^q dx = \Gamma(q) \Delta^c (p^{-q})$ Cauchy, P. 28. 147. P. III. § 1.

F. Algébr. rat. ent. monôme. Exp. binôme $(e^{ax} \pm 1)^2$ en dénom.	TABLE 119.	Lim. 0 et ∞.

1) $\int \frac{e^x - e^{-x} + 2}{(e^x - 1)^2} x^2 \, dx = -1 + 2 \sum_1^\infty \frac{1}{n^2}$ V. T. 118. N°. 10.

2) $\int \frac{e^x - 2}{(e^x - 1)^2} x^2 \, dx = \frac{1}{3} \pi^2$ V. T. 117. N°. 4.

3) $\int \frac{e^x x^q}{(e^x + 1)^2} dx = \Gamma(q) \sum_0^\infty \frac{(-1)^n}{(1+n)^q}$ V. T. 117. N°. 15.

4) $\int \frac{e^x x^q}{(e^x - 1)^2} dx = \Gamma(q) \sum_0^\infty \frac{1}{(1+n)^q}$ V. T. 117. N°. 16.

5) $\int \frac{e^x x^2}{(e^x + 1)^2} dx = \frac{1}{6} \pi^2$ V. T. 117. N°. 1.

6) $\int \frac{e^{-x} x^{q+1}}{(p e^{-x} - 1)^2} dx = \frac{q}{(q+1)p} \Gamma(q) \sum_1^\infty \frac{p^n}{n^{q+1}}$ V. T. 117. N°. 10.

7) $\int \frac{(a-1) e^{-x} + a}{(1 + e^{-x})^2} e^{-ax} x^p \, dx = p \Gamma(p) \sum_0^\infty \frac{(-1)^n}{(a+n)^p}$ V. T. 118. N°. 6.

8) $\int \frac{(a+1) e^{-x} + a}{(1 - e^x)^2} e^{-ax} x^p \, dx = p \Gamma(p) \sum_1^\infty \frac{-1}{(a+n)^p}$ V. T. 118. N°. 8.

9) $\int \frac{e^{\pi x} x^{2a}}{(e^{\pi x} + 1)^2} dx = \frac{2^{2a-1} - 1}{\pi} B_{2a-1}$ V. T. 117. N°. 20.

10) $\int \frac{e^{\pi x} x^{2a}}{(e^{\pi x} - 1)^2} dx = \frac{2^{2a-1}}{\pi} B_{2a-1}$ V. T. 117. N°. 21.

F. Algébr. rat. ent. monôme. Exp. bin. $(e^{ax} \pm e^{-ax})$ en dén.	Numér. algébr.	TABLE 120.	Lim. 0 et ∞.

1) $\int \frac{x}{e^x + e^{-x}} dx = -\sum_1^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 187. N°. 2.

2) $= \frac{1}{2} \pi l 2 - 2 L \left(\frac{\pi}{4}\right)$ Lobatschewsky, Mém. Kasan. 1836. 1. I. form. (103).

3) $\int \frac{x^2}{e^x + e^{-x}} dx = \frac{1}{16} \pi^3$ V. T. 154. N°. 1.

4) $\int \frac{x^4}{e^x + e^{-x}} dx = \frac{5}{64} \pi^5$ V. T. 155. N°. 1.

5) $\int \frac{x^6}{e^x + e^{-x}} dx = \frac{61}{256} \pi^7$ V. T. 155. N°. 8.

6) $\int \frac{x}{e^x - e^{-x}} dx = \frac{3}{4} \frac{1}{6} \pi^2$

7) $\int \frac{x^3}{e^x - e^{-x}} dx = \frac{15}{8} \frac{1}{30} \pi^4$ } Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

8) $\int \frac{x^5}{e^x - e^{-x}} dx = \frac{63}{16} \frac{1}{42} \pi^6$

9) $\int \frac{x^7}{e^x - e^{-x}} dx = \frac{17}{32} \pi^8$ V. T. 155. N°. 10

10) $\int \frac{x^{2a}}{e^x + e^{-x}} dx = \frac{1}{2} (-1)^{a+1} (2\pi)^{2a+1} \mathrm{B}'' \left(\frac{1}{4}\right)$ Raabe, Cr. 42. 348.

11) $\int \frac{x^q}{e^x + e^{-x}} dx = \Gamma(q+1) \sum_0^\infty \frac{(-1)^n}{(2n+1)^{q+1}}$ V. T. 187. N°. 7.

12) $\int \frac{x^{2a+1}}{e^x - e^{-x}} dx = \frac{1}{4} (-1)^{a+1} (2\pi)^{2a+2} \mathrm{B}' \left(\frac{1}{2}\right)$ Raabe, Cr. 42. 348.

13) $\int \frac{x^{2a}}{e^x - e^{-x}} dx = \frac{2^{2a+1} - 1}{2^{2a+1}} 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 158. N°. 4.

14) $\int \frac{x^{2a}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} dx = \frac{1}{2} \mathrm{B}_{2a}$ Schlömilch, Gr. 1. 360. — Id., Gr. 12. 130. — Id., Beitr. II. § 6.

15) $\int \frac{x^{2a}}{e^{qx} + e^{-qx}} dx = \frac{(-1)^{a+1}}{2} \left(\frac{2\pi}{q}\right)^{2a+1} \mathrm{B}'' \left(\frac{1}{4}\right)$ Raabe, Cr. 42. 348.

16) $\int \frac{x^{2a}}{e^{\frac{1}{2}bx} + e^{-\frac{1}{2}bx}} dx = \frac{(-1)^{a+1}}{b} (4\pi)^{2a+1} \sum_1^b (-1)^{n-1} \mathrm{B}'' \left(\frac{2n-1}{4b}\right)$ V. T. 164. N°. 4.

17) $\int \frac{x}{e^{ax} - e^{-ax}} dx = \frac{\pi^2}{8a^2}$ V. T. 152. N°. 18.

18) $\int \frac{x^{2a-1}}{e^{\pi x} - e^{-\pi x}} dx = \frac{2^{2a} - 1}{4a} \mathrm{B}_{2a-1}$ Schlömilch, Beitr. II. § 6. — Id., Gr. 12. 130.

19) $\int \frac{x^{2a-1}}{e^{qx} - e^{-qx}} dx = \frac{(-1)^a}{4} \left(\frac{2\pi}{q}\right)^{2a} \mathrm{B}' \left(\tfrac{1}{2}\right)$ Raabe, Cr. 42. 348.

20) $\int \frac{x^{2a-1}}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} dx = \frac{2^{2a} - 1}{2a} 2^{2a} \mathrm{B}_{2a-1}$ Schlömilch, Gr. 3. 9.

1) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} x\,dx = \infty$ V. T. 153. N°. 10.

2) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} x\,dx = \infty$ V. T. 153. N°. 11.

3) $\int \frac{e^x - e^{-x}}{e^{px} + e^{-px}} x\,dx = \frac{\pi^2}{4p^2} Sin. \frac{\pi}{2p}. Sec.^2 \frac{\pi}{2p}, p < 1;$ V. T. 152. N°. 19.

4) $\int \frac{e^x + e^{-x}}{e^{px} - e^{-px}} x\,dx = \frac{\pi^2}{4p^2} Sec.^2 \frac{\pi}{2p}, p < 1;$ V. T. 152. N°. 20.

5) $\int \frac{e^{qx} + e^{-qx}}{e^{px} + e^{-px}} x\,dx = -\frac{\pi^2}{4p^2} Sin. \frac{q\pi}{2p}. Sec.^2 \frac{q\pi}{2p}, p < q;$ V. T. 153. N°. 12.

6) $\int \frac{e^{qx} - e^{-qx}}{e^{px} - e^{-px}} x\,dx = \frac{\pi^2}{4p^2} Sec.^2 \frac{q\pi}{2p}, p < q;$ V. T. 153. N°. 13.

7) $\int \frac{e^x + e^{-x}}{e^{3x} + e^{-3x}} x^2\,dx = \frac{3}{64}\pi^3 \sqrt{2}$ V. T. 154. N°. 2.

8) $\int \frac{e^{qx} + e^{-qx}}{e^{px} + e^{-px}} x^2\,dx = \frac{\pi^3}{8p^3} \left\{3\, Sec.^3 \frac{q\pi}{2p} - Sec. \frac{q\pi}{2p}\right\}, p < q;$ V. T. 154. N°. 5.

9) $\int \frac{e^{qx} - e^{-qx}}{e^{px} - e^{-px}} x^2\,dx = \frac{3\pi^3}{8p^3} Sin. \frac{q\pi}{2p}. Sec.^3 \frac{q\pi}{2q}, p < q;$ V. T. 154. N°. 6.

10) $\int \frac{e^x - e^{-x}}{e^{3x} - e^{-3x}} x^2\,dx = \frac{\pi^3}{81\sqrt{3}}$ V. T. 154. N°. 2.

11) $\int \frac{e^{2x} - e^{-2x}}{e^{3x} - e^{-3x}} x^2\,dx = \frac{\pi^3}{9\sqrt{3}}$ V. T. 154. N°. 3.

12) $\int \frac{e^{-x}}{e^{\frac{1}{2}x} - e^{-\frac{1}{2}x}} x\,dx = 2\pi^2$ V. T. 163. N°. 18.

13) $\int \frac{e^{px} + e^{-px}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} x^{2a}\,dx = \frac{d^{2a}}{dp^{2a}}. Sec.\, p$

14) $\int \frac{e^{px} - e^{-px}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} x^{2a+1}\,dx = \frac{d^{2a+1}}{dp^{2a+1}}. Sec.\, p$ } $, 2p < \pi;$

15) $\int \frac{e^{-\frac{1}{2}x}}{e^{\frac{1}{2}x} + e^{-\frac{1}{2}x}} x^{2a+1}\,dx = -\frac{1}{2}\frac{(2\pi)^{2a+1}}{2a+2} B_{2a+1} + \frac{1}{2}(-1)^{a+1}(2\pi)^{2a+2} B'(\tfrac{1}{2})$

16) $\int \frac{e^{px} - e^{-px}}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} x^{2a}\,dx = \frac{d^{2a}}{dp^{2a}}. Tang.\, p, p < \frac{1}{2}\pi;$

} Raabe, Cr. 42. 448. (13–16)

17) $\int \frac{e^{px} + e^{-px}}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} x^{2a+1}\, dx = \frac{d^{2a+1}}{d\, p^{2a+1}} . \, Tang.\, p \;, p < \frac{1}{2}\pi;$

18) $\int \frac{e^{-\frac{1}{2}x}}{e^{\frac{1}{2}x} - e^{-\frac{1}{2}x}} x^{2a+1}\, dx = \frac{1}{2} \frac{(2\pi)^{2a+2}}{2a+2} \mathrm{B}_{2a+1}$

} Raabe, Cr. 42. 348.

19) $\int \frac{e^{bx} + e^{-bx}}{e^{x} + e^{-x}} x^{2a}\, dx = \frac{(-1)^{a+1}}{b} \left(\frac{2\pi}{b}\right)^{2a+1} \sum_{1}^{b} (-1)^{n-1} \mathrm{B}'' \left(\frac{2n-1}{4b}\right) Sin. \left\{\frac{2n-1}{2} b\pi\right\}$ V. T. 158. N°. 3.

20) $\int \frac{e^{(c-b)x} + e^{(b-c)x}}{e^{cx} + e^{-cx}} x^{2a}\, dx = \frac{(-1)^{a+1}}{c} (2\pi)^{2a+1} \sum_{1}^{c} \mathrm{B}'' \left(\frac{2n-1}{4c}\right) Sin. \left\{\frac{2n-1}{2c} b\pi\right\}$ V. T. 158. N°. 12.

21) $\int \frac{e^{bx} - e^{-bx}}{e^{x} - e^{-x}} x^{2a}\, dx = \frac{(-1)^{a+1}}{b} \left(\frac{2\pi}{b}\right)^{2a+1} \sum_{1}^{b} (-1)^{n-1} \mathrm{B}'' \left(\frac{n}{2b}\right) Sin.\, n b\pi$ V. T. 158. N°. 9.

22) $\int \frac{e^{(c-b)x} - e^{(b-c)x}}{e^{cx} - e^{-cx}} x^{2a}\, dx = \frac{(-1)^{a+1}}{c} (2\pi)^{2a+1} \sum_{1}^{c-1} \mathrm{B}'' \left(\frac{n}{2c}\right) Sin. \frac{nb\pi}{c}$ V. T. 158. N°. 11.

F. Algébr. rat. ent. monôme. Exp. bin. $(e^{ax} \pm e^{-ax})^2$ en dén. TABLE 122. Lim. 0 et ∞.

1) $\int \frac{x}{(e^{x} - e^{-x})^2}\, dx = \frac{1}{4} l\, 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Lobatschewsky, Mém. Kasan. 1836. I. I. form. (100).

2) $\int \frac{x}{(e^{px} + e^{-px})^2}\, dx = \frac{1}{4p^2} l\, 2$ Lobatschewsky, Mém. Kasan. 1836. I. I. form. (100), II. form. (20).

3) $\int \frac{e^{px} - e^{-px}}{(e^{px} + e^{-px})^2} x\, dx = \frac{\pi}{4p^2}$ V. T. 38. N°. 8.

4) $\int \frac{e^{x} - e^{-x}}{(e^{x} + e^{-x})^{2p+1}} x\, dx = \frac{\{\Gamma(p)\}^2}{8p\,\Gamma(2p)}, p < 1;$ V. T. 39. N°. 9.

5) $\int \frac{e^{px} - e^{-px}}{(e^{px} + e^{-px})^3} x^2\, dx = \frac{1}{4p^3} l\, 2$ V. T. 122. N°. 2.

6) $\int \frac{e^{ax} + e^{-ax}}{(e^{ax} - e^{-ax})^2} x^2\, dx = \frac{\pi}{4a^3}$ V. T. 120. N°. 17.

7) $\int \frac{(p+q)(e^{(p+q)x} - e^{-(p+q)x}) + (p-q)(e^{(p-q)x} - e^{(q-p)x})}{(e^{px} + e^{-px})^2} x^2\, dx = \frac{\pi^2}{2p^2} Sin. \frac{q\pi}{2p} . Sec.^2 \frac{q\pi}{2p}, p < q;$ V. T. 121. N°. 5.

8) $\int \frac{(p-q)(e^{(p-q)x} - e^{(q-p)x}) - (p+q)(e^{(p+q)x} - e^{-(p+q)x})}{(e^{px} - e^{-px})^2} x^2\, dx = \frac{\pi^2}{2p^2} Sec.^2 \frac{q\pi}{2p}, p < q;$ V. T. 121. N°. 6.

F. Algébr. rat. ent. monôme. Exp.bin.$(e^{ax} \pm e^{-ax})^2$ en dén.	TABLE 122 suite.	Lim. 0 et ∞.

9) $\displaystyle\int \frac{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}}{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x})^2} x^{2a+1}\, dx = \frac{2a+1}{\pi} B_{2a}$ V. T. 120. N°. 14.

10) $\displaystyle\int \frac{e^{\pi x} - e^{-\pi x}}{(e^{\pi x} + e^{-\pi x})^2} x^{2a+1}\, dx = \frac{2a+1}{\pi} \frac{2^{2a+1}-1}{(2\pi)^{2a+1}} 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 120. N°. 13.

11) $\displaystyle\int \frac{x^{2a}}{(e^{\pi x} - e^{-\pi x})^2}\, dx = \frac{1}{4\pi} B_{2a-1}$ V. T. 117. N°. 22.

12) $\displaystyle\int \frac{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}{(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})^2} x^{2a}\, dx = \frac{2^{2a}-1}{\pi} 2^{2a+1} B_{2a-1}$ V. T. 120. N°. 20.

13) $\displaystyle\int \frac{e^{\pi x} + e^{-\pi x}}{(e^{\pi x} - e^{-\pi x})^2} x^{2a}\, dx = \frac{2^{2a}-1}{2\pi} B_{2a-1}$ V. T. 120. N°. 18.

14) $\displaystyle\int \frac{e^{qx} - e^{-qx}}{(e^{qx} + e^{-qx})^2} x^{2a+1}\, dx = \frac{2a+1}{2q} (-1)^{a+1} \left(\frac{2\pi}{q}\right)^{2a+1} B''\left(\frac{1}{4}\right)$ V. T. 120. N°. 15.

15) $\displaystyle\int \frac{e^{qx} + e^{-qx}}{(e^{qx} - e^{-qx})^2} x^{2a}\, dx = \frac{a}{2q} (-1)^a \left(\frac{2\pi}{q}\right)^{2a} B'\left(\frac{1}{2}\right)$ V. T. 120. N°. 19.

16) $\displaystyle\int \frac{e^{x} - e^{-x}}{(e^{x} + e^{-x})^2} x^{p}\, dx = p\,\Gamma(p) \sum_0^\infty \frac{(-1)^n}{(2n+1)^p}$ V. T. 120. N°. 11.

F. Algébr. rat. ent. binôme. Expon. binôme en dénom.	TABLE 123.	Lim. 0 et ∞.

1) $\displaystyle\int \frac{(1+xi)^{2a} - (1-xi)^{2a}}{i} \frac{dx}{e^{\pi x}+1} = \frac{1}{2a+1}$

2) $\displaystyle\int \frac{(1+xi)^{2a-1} - (1-xi)^{2a-1}}{i} \frac{dx}{e^{\pi x}+1} = \frac{1}{2a}\left\{1 + (-1)^a 2^{2a} B_{2a-1}\right\}$ Schlömilch, Gr. 3. 9. (1–3)

3) $\displaystyle\int \frac{(1+xi)^{2a} - (1-xi)^{2a}}{i} \frac{dx}{e^{2\pi x}-1} = \frac{1}{2}\frac{2a-1}{2a+1}$

4) $\displaystyle\int \frac{(1+xi)^{2a-1} - (1-xi)^{2a-1}}{i} \frac{dx}{e^{2\pi x}-1} = \frac{a-1}{2a} + (-1)^{a+1} \frac{1}{2a} B_{2a-1}$ Malmsten, Cr. 35. 55. — Schlömilch, Gr. 3. 9.

5) $\displaystyle\int \frac{(1+xi)^{2a-1} - (1-xi)^{2a-1}}{i} \frac{dx}{e^{\pi x}-1} = 2a - 1 + (-1)^a \frac{2^{2a-1}-1}{2a} B_{2a-1}$ Malmsten, Cr. 35. 55.

6) $\displaystyle\int \frac{(1+xi)^{2a} - (1-xi)^{2a}}{i} \frac{dx}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} = (-1)^{a+1} B_{2a} + 1$ Schlömilch, Gr. 3. 9.

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7) $\int \frac{(1+xi)^{2a-1}-(1-xi)^{2a-1}}{i} \frac{dx}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}} = 1$ Schlömilch, Gr. 3. 9.

8) $\int \frac{(1+xi)^{2a-1}+(1-xi)^{2a-1}}{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}} dx = (-1)^{a+1} \frac{2^{2a}-1}{2a} 2^{2a} B_{2a+1}$ Schlömilch, Gr. 1. 360.

F. Algébr. rat. ent. Expon. polyn. en dén. } Numér. alg. TABLE 124. Lim. 0 et ∞.

1) $\int \frac{x}{e^x+e^{-x}-1} dx = \frac{4}{27}\pi^2$ V. T. 153. N°. 8.

2) $\int \frac{x}{e^{2x}+e^{-2x}-1} dx = \frac{1}{27}\pi^2$ V. T. 153. N°. 7.

3) $\int \frac{x}{e^x+e^{-x}-2\,Cos.\,2\lambda} dx = \frac{\lambda l 2 - L(\lambda)}{Sin.\,\lambda.\,Cos.\,\lambda}$ Lobatschewsky, Mém. Kasan. 1836. 1. II. form. (31).

4) $\int \frac{x}{e^{2x}+e^{-2x}-2\,Cos.\,2\lambda} dx = \frac{1}{2\,Sin.\,2\lambda}\left\{(\frac{1}{2}\pi-\lambda)\,l\,2 - L(\frac{1}{2}\pi-\lambda)\right\}$ Lobatschewsky, Mém. Kasan. 1836. 1. I. form. (95).

5) $\int \frac{x^2}{e^x+e^{-x}+1} dx = \frac{8}{243}\pi^3\sqrt{3}$ V. T. 156. N°. 1.

6) $\int \frac{x^2}{e^x+e^{-x}-1} dx = \frac{10}{243}\pi^3\sqrt{3}$ V. T. 156. N°. 2.

7) $\int \frac{x^2}{e^x+e^{-x}+2\,Cos.\,\lambda} dx = \frac{\lambda}{2\,Sin.\,\lambda}\,\frac{\pi^2-\lambda^2}{3}$ V. T. 156. N°. 3.

8) $\int \frac{x^2}{e^x+e^{-x}-2\,Cos.\,\lambda} dx = \frac{2\lambda}{Sin.\,\lambda}\left\{\frac{1}{6}\pi^2-\frac{1}{4}\pi\lambda+\frac{1}{12}\lambda^2\right\}$ V. T. 156. N°. 4.

9) $\int \frac{x^4}{e^x+e^{-x}-2\,Cos.\,\lambda} dx = \frac{\lambda}{Sin.\,\lambda}\,\frac{\pi^2-\lambda^2}{3}\,\frac{7\pi^2-3\lambda^2}{5}$ V. T. 156. N°. 5.

10) $\int \frac{x^{2a}}{e^x+e^{-x}-2\,Cos.\,2p\pi} dx = \frac{(-1)^{a+1}}{2\,Sin.\,2p\pi}(2\pi)^{2a+1}B''(p)\,,\ p<1;$ Raabe, Cr. 42. 348.

11) $\int \frac{x^{2a}}{e^x+e^{-x}+1} dx = \frac{(-1)^{a+1}}{\sqrt{3}}(2\pi)^{2a+1}B''\left(\frac{1}{3}\right)$ V. T. 159. N°. 1.

12) $\int \frac{x^{2a}}{e^x+e^{-x}-1} dx = \frac{(-1)^{a+1}}{\sqrt{3}}(2\pi)^{2a+1}B''\left(\frac{1}{6}\right)$ V. T. 159. N°. 2.

1) $\int \frac{1+2e^{-x}}{e^x+e^{-x}+1} x\,dx = \frac{1}{9}\pi^2$ V. T. 153. N°. 1.

2) $\int \frac{1-2e^{-x}}{e^x+e^{-x}-1} x\,dx = \frac{1}{18}\pi^2$ V. T. 153. N°. 2.

3) $\int \frac{e^x \operatorname{Cos.}\lambda - 1}{e^{2x}+1-2e^x \operatorname{Cos.}\lambda} x\,dx = \frac{1}{6}\pi^2 - \frac{1}{2}\pi\lambda + \frac{1}{4}\lambda^2$ Cauchy, Sav. Etr. 1827. 599. P. 1. § 5.

4) $\int \frac{e^x - e^{-x}}{e^{2x}+e^{-2x}+2p} x\,dx = \frac{\pi}{2\sqrt{\{2(p-1)\}}}\, l \frac{\sqrt{(p-1)}+\sqrt{(p+1)}-\sqrt{2}}{\sqrt{(p-1)}-\sqrt{(p+1)}+\sqrt{2}}$

5) $\int \frac{e^x - e^{-x}}{e^{2x}+e^{-2x}+e^{2p}+e^{-2p}} x\,dx = \frac{p}{2}\frac{\pi}{e^p - e^{-p}}$

6) $\int \frac{e^x - e^{-x}}{e^{2x}+e^{-2x}+2\operatorname{Cos.}2\lambda} x\,dx = \frac{1}{4}\pi\lambda \operatorname{Cosec.}\lambda$

} Lobatschewsky, Mém. Kasan. 1835. 1.

7) $\int \frac{e^x + e^{-x}}{e^{2x}+e^{-2x}-2\operatorname{Cos.}2\lambda} x\,dx = \frac{1}{\operatorname{Sin.}\lambda}\left\{\frac{1}{2}\pi\, l\, 2 - \mathrm{L}\left(\frac{\lambda}{2}\right) - \mathrm{L}\left(\frac{2\pi-\lambda}{2}\right)\right\}$ Lobatschewsky, Mém. Kasan. 1836. 1. I. form. (103), II form. (33).

8) $\int \frac{e^x}{e^{2x}+1-2e^x \operatorname{Cos.}\lambda} x^2\,dx = \left(\frac{1}{3}\pi^2\lambda + \frac{1}{2}\pi\lambda^2 + \frac{1}{6}\lambda^3\right)\operatorname{Cosec.}\lambda$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

9) $\int \frac{\operatorname{Cos.}2p\pi - e^{-x}}{e^x+e^{-x}-2\operatorname{Cos.}2p\pi} x^{2a+1}\,dx = \frac{(-1)^a}{2}(2\pi)^{2a+2}\mathrm{B}'(p) + \frac{(2\pi)^{2a+2}}{2(2a+2)}\mathrm{B}_{2a+1}, p<1;$ Raabe, Cr. 42. 348.

10) $\int \frac{\operatorname{Cos.}\lambda - p e^{-x}}{e^x+p^2e^{-x}-2p\operatorname{Cos.}\lambda} e^{(1-q)x} x^{r-1}\,dx = \Gamma(r)\sum_1^\infty \frac{p^{n-1}\operatorname{Cos.}n\lambda}{(q+n-1)^r}$ V. T. 159. N°. 7.

11) $\int \frac{e^x - e^{-x}}{(e^x - 2\operatorname{Cos.}\lambda + e^{-x})^2} x\,dx = \frac{\pi-\lambda}{2\operatorname{Sin.}\lambda}$ V. T. 39. N°. 1.

12) $\int \frac{e^x - e^{-x}}{(e^x+e^{-x}-1)^2} x^2\,dx = \frac{8}{27}\pi^3$ V. T. 124. N°. 1.

13) $\int \frac{e^x - e^{-x}}{(e^x+e^{-x}+1)^2} x^3\,dx = \frac{8}{81}\pi^3\sqrt{3}$ V. T. 124. N°. 5.

14) $\int \frac{e^x - e^{-x}}{(e^x+e^x-1)^2} x^3\,dx = \frac{10}{81}\pi^3\sqrt{3}$ V. T. 124. N°. 6.

15) $\int \frac{e^x - e^{-x}}{(e^x+2\operatorname{Cos.}\lambda+e^{-x})^2} x^3\,dx = \lambda\frac{\pi^2-\lambda^2}{2\operatorname{Sin.}\lambda}$ V. T. 124. N°. 7.

16) $\int \frac{e^x - e^{-x}}{(e^x + e^{-x} + 1)^2} x^{2a+1} dx = \frac{2a+1}{\sqrt{3}} (-1)^{a+1} (2\pi)^{2a+1} B''\left(\frac{1}{3}\right)$ V. T. 124. N°. 11.

17) $\int \frac{e^x - e^{-x}}{(e^x + e^{-x} - 1)^2} x^{2a+1} dx = \frac{2a+1}{\sqrt{3}} (-1)^{a+1} (2\pi)^{2a+1} B''\left(\frac{1}{6}\right)$ V. T. 124. N°. 12.

F. Algébr. rat. fract. à dén. mon. Exp. mon. en num. TABLE 126. Lim. 0 et ∞.

1) $\int \frac{e^{-x}}{x} dx = \infty$ Cisa de Grésy, Mém. Turin. 1821. 209. I. 26.

2) $\int \frac{e^{-px}}{x} dx = -A - lo - lp = \infty$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 20. — Cisa de Grésy, ib. 1821. 209. II. 34.

3) $\int \frac{e^{-x^2}}{x^2} dx = -\sqrt{\pi}$ Kramp, Réfr. 3. 72. — Oettinger, Cr. 35. 13.

4) $\int \frac{e^{-x^2}}{x^4} dx = \frac{2}{3}\sqrt{\pi}$

5) $\int \frac{e^{-x^2}}{x^6} dx = -\frac{4}{3.5}\sqrt{\pi}$

6) $\int \frac{e^{-x^2}}{x^8} dx = \frac{8}{3.5.7}\sqrt{\pi}$

7) $\int \frac{e^{-x^2}}{x^{2a}} dx = \frac{(-1)^a 2^{a-1}\sqrt{\pi}}{1^{a/2}}$

(4–7) Kramp, Réfr. 3. 72.

8) $\int \frac{e^x}{x^p} dx = \frac{\pi}{\Gamma(p)} \,Cosec.\, p\pi$ Cauchy, P. 28. 147. P. 1. § 2.

9) $= \Gamma(1-p)$ Svanberg, Transf. 3.

10) $\int \frac{e^{-px}}{x^{a+1}} dx = p^a \Gamma(-a)$

11) $\int \frac{e^{-x}}{x^{a+1}} dx = \Gamma(-a)$

(10–11), valeurs extraordinaires; Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826, p. 38.

12) $\int \frac{e^{-x^a}}{x^{b+1}} dx = -\frac{1^{-b/a/1}}{b}$ Oettinger, Cr. 35. 13.

13) $\int \frac{e^{-x^a}}{x^{ab+1}} dx = \infty$

14) $\int \frac{e^{-x^{-a}}}{x^{b+1}} dx = -\frac{a}{b} \cdot \frac{2a}{a+b} \cdot \frac{3a}{2a+b} \cdots$ } Oettinger, Cr. 35. 13.

15) $\int e^{xi} \frac{dx}{x^p} = \frac{q}{1-p} (-1)^{\frac{1-p}{2}}$; où $q = 0,906402$; Laplace, P. 15. 229.

16) $\int e^{-q^2x^2 - \frac{1}{x^2}} \frac{dx}{x^2} = \frac{1}{2} e^{-2q} \sqrt{\pi}$ Bonnet, L. 14. 249.

17) $\int e^{-\left(px^2 + \frac{q}{x^2}\right)i} \frac{dx}{x^{a+1}} = \left(\frac{p}{q}\right)^{\frac{a-1}{2}} e^{-2i\sqrt{pq}} \frac{1-i}{2} \sqrt{\frac{\pi}{2p}} \sum_0^\infty \frac{(a-2n-1)^{2n/2}}{4^{n/4}} \frac{1}{(4i\sqrt{pq})^n}$ Cauchy, P. 19. 511.

F. Alg. rat. fract. à dén. x^a pour a spécial. Expon. polynôme en numér. — TABLE 127. — Lim. 0 et ∞.

1) $\int \frac{1 - e^{px}}{x} e^{-x} dx = -l(1-p)$, $p^2 < 1$; Dienger, Cr. 46. 119.

2) $\int \frac{e^{-px} - 1}{x} e^{-x} dx = l \frac{1}{1+p}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 36.

3) $\int \frac{e^{-x} - e^{-px}}{x} dx = lp$ Lejeune-Dirichlet, Cr. 15. 258. — Liouville, L. 4. 317. — Grunert, Gr. 2. 266. — Arndt, Gr. 10. 253. — Schlömilch, Stud. I. § 6.

4) $\int \frac{e^{-px} - e^{-qx}}{x} dx = l \frac{q}{p}$ Cauchy, Cours. Leç. 33. — Id., Exerc. 1827. p. 112. — Id., C. R. 16. 422. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 20. — Cisa de Grésy, ib. 1821. 209. II. 34. — Pioch, Mém. Cour. Bruxelles. T. 15. P. 2. — Grunert, Gr. 2. 266. — Schlömilch, Stud. I. § 6.

5) $\int \frac{e^{-px} - e^{-qx}}{x} e^{-rxi} dx = l \frac{q + ri}{p + ri}$ Meyer, Int. Déf. 109.

6) $\int e^{-ax} (e^{-x} - 1)^b \frac{dx}{x} = -\Delta^b l a$ Laplace, Prob. I. 41.

7) $\int \frac{e^{-(q+r)x} - e^{-qx} - e^{-rx} + 1}{x} e^{-px} dx = l \frac{(p+q)(p+r)}{p(p+q+r)}$ V. T. 167. N°. 7.

8) $\int \frac{(e^{-px} - e^{-qx})(e^{-rx} - e^{-sx})}{x} e^{-x} dx = l \frac{(p+s+1)(q+r+1)}{(p+r+1)(q+s+1)}$ V. T. 167. N°. 8.

9) $\int \frac{e^{-x} - e^{-ax}}{x} e^{-px} x^{q-1} dx = \frac{(p+a)^{1-q} - (p+1)^{1-q}}{1-q} \Gamma(q)$ Cauchy, C. R. 16. 422.

10) $\int \frac{(e^{-qx}-1)^2}{x^2} e^{-x}\, dx = (2q+1)\, l(2q+1) - 2(q+1)\, l(q+1)$ V. T. 168. N°. 4.

11) $\int \frac{(e^{-qx}-1)^2}{x^2} e^{-px}\, dx = (p+2q)\, l(p+2q) - 2(p+q)\, l(p+q) + p\, l\, p$ V. T. 168. N°. 5.

12) $\int e^{-px}(e^{-x}-1)^a \frac{dx}{x^2} = \Delta^a . p\, l\, p$ Meyer, Int. Déf. 181.

13) $\int \frac{(b-c)e^{-ax} + (c-a)e^{-bx} + (a-b)e^{-cx}}{x^2} dx = (b-a)\, a\, l\, a + (c-a)\, b\, l\, b + (a-b)\, c\, l\, c$ V. T. 168. N°. 6.

14) $\int e^{-px}(e^{-x}-1)^b \frac{dx}{x^{q+1}} = \frac{\pi}{\Gamma(q+1)}\, Cosec.\{(q+1)\pi\}\, \Delta^b p^q$ Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826. p. 58; pour $q < b$, mais valeur extraordinaire pour $q > b$.

15) $\int \qquad = \frac{(-1)^{q+1}}{1^{q|1}} \Delta^b . p^q\, l\, p$, pour q entier, $< b$; Cauchy, Exerc. 1826. p. 58.

16) $\int \left\{\left(\frac{1}{x}+\frac{1}{2}\right)e^{-x} - \frac{1}{x}e^{-\frac{1}{2}x}\right\}\frac{dx}{x} = \frac{1}{2}(l\,2-1)$

17) $\int \left\{e^{-x} + \frac{e^{-x}}{x} - \frac{1}{x}\right\}\frac{dx}{x} = 1$

18) $\int \left\{p\, e^{-x} + \frac{e^{-px}}{x} - q\, e^{-x} - \frac{e^{-qx}}{x}\right\}\frac{dx}{x} = p\, l\, p - p - q\, l\, q + q$

(16–18) Stern, Gött. Stud. 1847.

19) $\int \left\{(p-\tfrac{1}{2})e^{-x} + \frac{x+2}{2x}\left(e^{-px} - e^{-\frac{1}{2}x}\right)\right\}\frac{dx}{x} = (p-\tfrac{1}{2})(l\,p-1)$ Meyer, Int. Déf. 121.

20) $\int \left\{1 - \frac{x+2}{2x}(1-e^{-x})\right\} e^{-qx}\frac{dx}{x} = -1 + \left(q+\frac{1}{2}\right) l\,\frac{q+1}{q}$ Cauchy, C.R. 16. 422.

21) $\int \left(\frac{e^{-x}-e^{-2x}}{x} - \frac{e^{-2x}}{x^2}\right) dx = 1 - l\,2$. Meyer, Int. Déf. 123.

22) $\int \frac{(1-e^{-x})^2}{x^2} e^{-2x}\, dx = 2\, l\, \frac{32}{27}$ V. T. 168. N°. 2.

23) $\int (e^{-x}-1)^2 \frac{e^{-qx}}{x^2}\, dx = (q+2)\, l(q+2) - 2(q+1)\, l(q+1) + q\, l\, q$ V. T. 168. N°. 3.

24) $\int \frac{(1-e^{-px})(1-e^{-qx})(1-e^{-rx})}{x^2} e^{-x} dx = (p+q+1)l(p+q+1) + (p+r+1)l(p+r+1) + (q+r+1)l(q+r+1) - (p+1)l(p+1) - (q+1)l(q+1) - (r+1)l(r+1) - (p+q+r)l(p+q+r)$ V. T. 168. N°. 8.

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25) $\int (1-e^{-px})^a \frac{e^{-x}}{x^2} dx = \sum_1^\infty \binom{a}{2n} (2np+1)\, l(2np+1) - \sum_1^\infty \binom{a}{2n-1} \{(2n-1)p+1\}\, l\{(2n-1)p+1\}$ V. T. 168. N°. 9.

26) $\int (1-e^{-px})^a \frac{e^{-qx}}{x^2} dx = \sum_0^\infty (-1)^n \binom{a}{n} (q+np)\, l(q+np)$ V. T. 168. N°. 10.

27) $\int \left\{ \frac{e^{-ax}}{(a-b)(a-c)(a-d)} + \frac{e^{-bx}}{(b-a)(b-c)(b-d)} + \frac{e^{-cx}}{(c-a)(c-b)(c-d)} + \frac{e^{-dx}}{(d-a)(d-b)(d-c)} \right\} \frac{dx}{x^3} =$
$= \frac{a^2\, l a}{(a-b)(a-c)(a-d)} + \frac{b^2\, l b}{(b-a)(b-c)(b-d)} + \frac{c^2\, l c}{(c-a)(c-b)(c-d)} + \frac{d^2\, l d}{(d-a)(d-b)(d-c)}$ V. T. 168. N°. 11.

28) $\int (1-e^{-px})^a \frac{e^{-x}}{x^3} dx = \frac{1}{2} \sum_1^\infty (-1)^{n-1} \binom{a}{n} (pn+1)^2\, l(pn+1)$ V. T. 168. N°. 13.

29) $\int (1-e^{-px})^a \frac{e^{-qx}}{x^3} dx = \frac{1}{2} \sum_1^\infty (-1)^{n-1} \binom{a}{n} (q+np)^2\, l(q+np)$ V. T. 168. N°. 14.

30) $\int \frac{(1-e^{-px})^a (1-e^{-qx})}{x^3} e^{-x} dx = \frac{1}{2} \sum_0^\infty (-1)^n \binom{a}{n} (q+pn+1)^2\, l(q+pn+1) +$
$+ \frac{1}{2} \sum_1^\infty (-1)^{n-1} \binom{a}{n} (pn+1)^2\, l(pn+1)$ V. T. 168. N°. 15.

31) $\int \left\{ \frac{q e^{-x}}{x} - \frac{1-e^{-qx}}{x^2} \right\} dx = q\, l q - q$

32) $\int \left\{ \frac{q^2 e^{-x}}{2x} - \frac{q}{x^2} + \frac{1-e^{-qx}}{x^3} \right\} dx = \frac{1}{2} q^2\, l q - \frac{3}{4} q^2$

33) $\int \left\{ \frac{q^3 e^{-x}}{6x} - \frac{q^2}{2x^2} + \frac{q}{x^3} - \frac{1-e^{-qx}}{x^4} \right\} dx = \frac{1}{6} q^3\, l q - \frac{11}{36} q^3$

(31–33) Sohnke, Samml.

F. Alg. rat. fract. à dén. x^a pour a général. Expon. polynôme en numér. **TABLE 128.** Lim. 0 et ∞.

1) $\int \frac{e^{-qx} - e^{-rx}}{x^{p+1}} dx = \frac{\Gamma(1-p)}{p} (r^p - q^p)$, $p < 1$;

2) $\int \frac{e^{-ax^c} - e^{-bx^c}}{x^c} dx = \frac{\Gamma\left(\frac{1}{c}\right)}{c-1} \left\{ b^{\frac{c-1}{c}} - a^{\frac{c-1}{c}} \right\}$, $b > a > 0$;

(1–2) Lindmann, Stockh. Handl. 1850. IV.

3) $\int e^{-bx} (e^{-x}-1)^c \frac{dx}{x^{q+1}} = - \frac{\pi}{Sin.\, q\pi\, \Gamma(q+1)} \Delta^c b^q$, $a < c$;

4) $\int \quad = - \frac{(-1)^{q+c}}{\Gamma(q+1)} \Delta^c b^q\, l b$, pour q entier;

(3–4) Cauchy, P. 28. 147. P. 1. § 2. — Laplace, Prob. 41. — Id., Mém. Acad. 1781. 29.

5) $\int \left(\frac{e^{-qx}-1}{x}\right)^c e^{-px}\,dx = \frac{(-1)^c}{1^{c-1/1}}\frac{\Delta^c}{\Delta p}.p^{c-1}\,lp$, après la différentiation mettez $\Delta p = q$; V. T. 168. N°. 17.

6) $\int \frac{(e^{-rx}-1)^c}{x^{q+1}} e^{-prx}\,dx = \frac{(-1)^c r^q}{\Gamma(q+1)} \Delta^c . p^q\, lp$ V. T. 168. N°. 18.

7) $\int \frac{e^{-bx}(e^{-x}-1)^c-(-x)^c}{x^{q+1}}\,dx = -\frac{\pi}{\Gamma(q+1)} Cosec.\, q\pi\, \Delta^c\, b^q$, $q < c$;

8) $\int \frac{e^{-bx}(e^{-x}-1)^{c-1}-(-x)^{c-1}\left(1-\frac{2b+c-1}{2}x\right)}{x^{q+1}}\,dx = -\frac{\pi}{\Gamma(q+1)} Cosec.\, q\pi\, \Delta^{c-1} b^q$, $c<q<c+1$;

9) $\int \frac{e^{-bx}(e^{-x}-1)^{c-2}-(-x)^{c-2}\left(1-\frac{2b+c-2}{2}x+\frac{6b(b+c-2)+(c-2)(3c-7)}{12}x^2\right)}{x^{q+1}}\,dx =$
$= -\frac{\pi}{\Gamma(q+1)} Cosec.\, q\pi\, \Delta^{c-2}\, b^q$, $c<q<c+1$;

(7–9) Cauchy, P. 18. 147. P. 3. §. 2.

F. Algébr. rat. fract. à dén. $x \pm q$. Expon. monôme. TABLE 129. Lim. 0 et ∞.

1) $\int \frac{e^{-px}}{x+1}\,dx = -e^p\, li.(e^{-p})$ Schlömilch, Beitr. III. 5. — Id., Gr. 5. 204.

2) $\int \frac{e^{-x}}{x+q}\,dx = -e^q\, li.(e^{-q})$ Winckler, Cr. 45. 102. — Schlömilch, Stud. I. 18. — Id., Gr. 5. 204.

3) $\int \frac{e^{-px}}{x+q}\,dx = -e^{pq}\, li.\,(e^{-pq})$ Schlömilch, Stud. I. 18.

4) $= -e^{pq}\, Ei.\,(-pq)$ Arndt, Gr. 10. 247.

5) $\int \frac{e^{pxi}}{x+q}\,dx = \pi e^{-pq} + i e^{-pq}\, li.\,(e^{pq})$ Meyer, Int. Déf. 264.

6) $\int \frac{e^{-px}}{x+q} x^a\,dx = (-1)^{a+1} q^a e^{pq} Ei.\,(-pq) + \frac{1}{p^a}\sum_1^a 1^{a-n/1}(-pq)^{n-1}$ Bierens de Haan, Verh. K. Akad. v. W. Dl. II. blad 19.

7) $\int \frac{e^{-px}}{1-x}\,dx = e^{-p}\, li.\,(e^p)$ Schlömilch, Beitr. III. 5. — Id., Gr. 5. 204.

8) $\int \frac{e^{-x}}{q-x}\,dx = e^{-q}\, li.\,(e^q)$ Schlömilch, Gr. 5. 204.

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9) $\int \frac{e^{-px}}{q-x}\,dx = e^{-pq}\,li.(e^{pq})$ Schlömilch, Stud. II. 20.

10) $= e^{-pq}\left\{\frac{1}{2}\,l\,\alpha^2 + Ei.(pq)\right\}$, où α indéterminé; Arndt, Gr. 10. 247.

11) $\int \frac{e^{pxi}}{q+xi}\,dx = i\,e^{pq}\,li.(e^{-pq})$ Meyer, Int. Déf. 264.

12) $\int \frac{e^{-px}}{x-q}\,x^a\,dx = -\,q^a\,e^{-pq}\,Ei.(-pq) + \frac{1}{p^a}\sum_1^a 1^{a-n/1}\,(-pq)^{n-1}$ Bierens de Haan, Verh. v. K. Ak. v. Wet. Dl. II. blad 19.

13) $\int \frac{e^{-x}}{lp-x}\,dx = \frac{1}{p}\,li.(p)$ Schlömilch, Gr. 5. 204.

F. Algébr. rat. fract. à dén. $x^2 \pm q^2$. Expon. monôme. TABLE 130. Lim. 0 et ∞.

1) $\int \frac{x\,e^{-px}}{1+x^2}\,dx = \sum_0^\infty (-1)^n \frac{1^{2n+1/1}}{p^{2n+2}}$

2) $\int \frac{e^{-px}}{1+x^2}\,dx = \sum_0^\infty (-1)^n \frac{1^{2n/1}}{p^{2n+1}}$

(1, 2) Bidone, Mém. Turin. 1812. 231. Art. 2. 33.

3) $= Sin.q.\,Ci.(q) + Cos.q\left\{\frac{1}{2}\pi - Si.(q)\right\}$ Arndt, Gr. 10. 225. — Schlömilch, Cr. 33. 325.

4) $\int \frac{e^{-px}}{q^2+x^2}\,dx = \frac{1}{q}\left\{Ci.(pq).Sin.pq - Si.(pq).Cos.pq + \frac{1}{2}\pi\,Cos.pq\right\}$, $0 < q < \infty$; Schlömilch, Cr. 33. 325. — Id., Stud. II. 21. — Arndt, Gr. 10. 225.

5) $\int \frac{x\,e^{-px}}{q^2+x^2}\,dx = -\,Ci.(pq).Cos.pq - Si.(pq).Sin.pq + \frac{1}{2}\pi\,Sin.pq$, $0 < q < \infty$; Arndt, Gr. 10. 225. — Schlömilch, Stud. II. 21. — Id., Cr. 33. 325, la trouvait fautivement négative.

6) $\int \frac{e^{pxi}}{q^2+x^2}\,dx = \frac{\pi}{2q}\,e^{-pq} - \frac{1}{2qi}\left\{e^{-pq}\,li.(e^{pq}) - e^{pq}\,li.(e^{-pq})\right\}$

7) $\int \frac{x\,e^{pxi}}{q^2+x^2}\,dx = \frac{\pi i}{2}\,e^{-pq} - \frac{1}{2}\left\{e^{-pq}\,li.(e^{pq}) + e^{pq}\,li.(e^{-pq})\right\}$

(6, 7) Meyer, Int. Déf. 267.

8) $\int \frac{e^{-px}}{x^2+q^2}\,x^{2a}\,dx = (-1)^a q^{2a-1}\left\{Ci.(pq).Sin.pq - Si.(pq).Cos.pq + \frac{1}{2}\pi\,Cos.pq\right\} + \frac{1}{p^{2a-1}}\sum_1^a 1^{2a-2n/1}(-p^2q^2)^{n-1}$

9) $\int \frac{e^{-px}}{x^2+q^2}\,x^{2a+1}\,dx = (-1)^{a+1} q^{2a}\left\{Ci.(pq).Cos.pq + Si.(pq).Sin.pq - \frac{1}{2}\pi\,Sin.pq\right\} + \frac{1}{p^{2a}}\sum_1^a 1^{2a-2n+1/1}(-p^2q^2)^{n-1}$

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10) $\int \frac{e^{-px}}{q^2 - x^2} dx = \frac{1}{2q} \{e^{-pq} li.(e^{pq}) - e^{pq} li.(e^{-pq})\}$ Schlömilch, Cr. 33. 325. — Id., Stud. II. 20.

11) $= \frac{1}{2q} \left\{- e^{pq} Ei.(-pq) - e^{-pq} \left(\frac{1}{2} l\alpha^2 - Ei.(pq)\right)\right\}$, où α indéterminé; Arndt, Gr. 10. 247.

12) $\int \frac{x e^{-px}}{q^2 - x^2} dx = \frac{1}{2} \{e^{-pq} li.(e^{pq}) + e^{pq} li.(e^{-pq})\}$ Schlömilch, Cr. 33. 325. — Id., Stud. II. 20.

13) $= \frac{1}{2} \left\{e^{pq} Ei.(-pq) - e^{-pq} \left(\frac{1}{2} l\alpha^2 - Ei.(pq)\right)\right\}$, où α indéterminé; Arndt, Gr. 10. 247.

14) $\int \frac{e^{-px}}{x^2 - q^2} x^{2a} dx = \frac{1}{2} q^{2a-1} \{e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq)\} + \frac{1}{p^{2a-1}} \sum_1^a 1^{2a-2n|1} (p^2 q^2)^{n-1}$

15) $\int \frac{e^{-px}}{x^2 - q^2} x^{2a+1} dx = -\frac{1}{2} q^{2a} \{e^{pq} Ei.(-pq) + e^{-pq} Ei.(pq)\} + \frac{1}{p^{2a}} \sum_1^a 1^{2a-2n+1|1} (p^2 q^2)^{n-1}$

Bierens de Haan, Verh. v. K. Ak. v. Wet. Dl. II. blad 19.

F. Algébr. rat. fract. à dén. $(x^a \pm q^a)^b$. Expon. monôme.

TABLE 131.

Lim. 0 et ∞.

1) $\int \frac{e^{-x}}{(q+x)^2} dx = \frac{1}{q} + e^q li.(e^{-q})$ V. T. 43. N°. 18.

2) $\int \frac{e^{-x}}{(q-x)^2} dx = -\frac{1}{q} + e^{-q} li.(e^q)$ V. T. 43. N°. 19.

3) $\int \frac{e^{-ax} x^{p-1}}{(1+x)^b} dx = \frac{\Gamma(p)}{a^p} \chi(p,b,a)$ Kummer, Cr. 17. 228. — Boncompagni, Cr. 25. 74.

4) $\int \frac{e^{-ax^c} x^{cp-1}}{(1+x)^b} dx = \frac{\Gamma(p)}{c a^p} \chi(p,b,a)$ Bonoompagni, Cr. 25. 74. Elle ne vaut que pour $c = 1$.

5) $\int \frac{e^{-px} x^a}{(x+q)^{c+1}} dx = \frac{(-1)^{a+c+1}}{1^{c/1}} \frac{d^{a+c}}{dp^a . dq^c} . \{e^{pq} li.(e^{-pq})\}$ Schlömilch, Stud. I. 18.

6) $\int \frac{e^{-px}}{(x+q)^a} dx = (-1)^a \frac{p^{a-1}}{1^{a-1/1}} e^{pq} Ei.(-pq) + \frac{1}{1^{a-1/1} q^{a-1}} \sum_1^{a-1} 1^{a-n-1/1} (-pq)^{n-1}$

7) $\int \frac{e^{-px}}{(x-q)^a} dx = (-1)^a \frac{p^{a-1}}{1^{a-1/1}} e^{-pq} Ei.(pq) + \frac{(-1)^{a-1}}{1^{a-1/1} q^{a-1}} \sum_1^{a-1} 1^{a-n-1/1} (pq)^{n-1}$

Bierens de Haan, Verh. v. K. Ak. v. Wet. Dl. II, blad 19.

8) $\int \frac{e^{xi}}{(q+xi)^p} dx = \frac{\pi e^{-q}}{\Gamma(p)}$ Poisson, P. 19. 404. N°. 68.

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F. Algébr. rat. fract. à dén. $(x^a \pm q^a)^b$. Expon. monôme. **TABLE 131 suite.** Lim. 0 et ∞.

$$9) \int \frac{e^{-px}}{(x^2+q^2)^2}\,dx = \frac{1}{2q^3}\Big[Ci.(pq).\,Sin.\,pq - Si.(pq).\,Cos.\,pq + \frac{1}{2}\pi\,Cos.\,pq - $$
$$- pq\Big\{Ci.(pq).\,Cos.\,pq + Si.(pq).\,Sin.\,pq - \frac{1}{2}\pi\,Sin.\,pq\Big\}\Big]$$

$$10) \int \frac{x\,e^{-px}}{(x^2+q^2)^2}\,dx = \frac{1}{2q^2}\Big[1 + pq\Big\{Ci.(pq).\,Sin.\,pq - Si.(pq).\,Cos.\,pq + \frac{1}{2}\pi\,Cos.\,pq\Big\}\Big]$$

$$11) \int \frac{e^{-px}}{(x^2-q^2)^2}\,dx = \frac{1}{4q^3}\Big[(pq-1)\,e^{pq}\,Ei.(-pq) + (1+pq)\,e^{-pq}\,Ei.(pq)\Big]$$

$$12) \int \frac{x\,e^{-px}}{(x^2-q^2)^2}\,dx = \frac{1}{4q^2}\Big[1 + pq\,\{e^{-pq}\,Ei.(pq) - e^{pq}\,Ei.(-pq)\}\Big]$$

(9) à (12): Bierens de Haan, Verh. v. K. Ak. v. Wet. Dl. II. blad 19.

F. Algébr. rat. fract. à autre dén. Expon. monôme. **TABLE 132.** Lim. 0 et ∞.

$$1) \int \frac{e^{-px}}{x^4-q^4}\,dx = \frac{1}{4q^3}\Big[e^{pq}\,Ei.(-pq) - e^{-pq}\,Ei.(pq) - 2Ci.(pq).\,Sin.\,pq + 2\,Si.(pq).\,Cos.\,pq - \pi\,Cos.\,pq\Big]$$

$$2) \int \frac{x\,e^{-px}}{x^4-q^4}\,dx = \frac{1}{4q^2}\Big[-e^{pq}\,Ei.(-pq) - e^{-pq}\,Ei.(pq) + 2Ci.(pq).\,Cos.\,pq + 2\,Si.(pq).\,Sin.\,pq - \pi\,Sin.\,pq\Big]$$

$$3) \int \frac{x^2\,e^{-px}}{x^4-q^4}\,dx = \frac{1}{4q}\Big[e^{pq}\,Ei.(-pq) - e^{-pq}\,Ei.(pq) + 2\,Ci.(pq).\,Sin.\,pq - 2\,Si.(pq).\,Cos.\,pq + \pi\,Cos.\,pq\Big]$$

$$4) \int \frac{x^3\,e^{-px}}{x^4-q^4}\,dx = \frac{1}{4}\Big[-e^{pq}\,Ei.(-pq) - e^{-pq}\,Ei.(pq) - 2Ci.(pq).\,Cos.\,pq - 2\,Si.(pq).\,Sin.\,pq + \pi\,Sin.\,pq\Big]$$

$$5) \int \frac{x^{4a}\,e^{-px}}{x^4-q^4}\,dx = \frac{1}{4}q^{4a-3}\Big[e^{pq}\,Ei.(-pq) - e^{-pq}\,Ei.(pq) - 2Ci.(pq).\,Sin.\,pq + 2\,Si.(pq).\,Cos.\,pq - \pi\,Cos.\,pq\Big]$$
$$+ \frac{1}{p^{4a-3}} \sum_1^a 1^{4a-4n/1}\,(p^4\,q^4)^{n-1}$$

$$6) \int \frac{x^{4a+1}\,e^{-px}}{x^4-q^4}\,dx = \frac{1}{4}q^{4a-2}\Big[-e^{pq}\,Ei.(-pq) - e^{-pq}\,Ei.(pq) + 2Ci.(pq).\,Cos.\,pq + 2\,Si.(pq).\,Sin.\,pq - \pi\,Sin.\,pq\Big]$$
$$+ \frac{1}{p^{4a-2}} \sum_1^a 1^{4a-4n+1/1}\,(p^4\,q^4)^{n-1}$$

$$7) \int \frac{x^{4a+2}\,e^{-px}}{x^4-q^4}\,dx = \frac{1}{4}q^{4a-1}\Big[e^{pq}\,Ei.(-pq) - e^{-pq}\,Ei.(pq) + 2Ci.(pq).\,Sin.\,pq - 2Si.(pq).\,Cos.\,pq + \pi\,Cos.\,pq\Big]$$
$$+ \frac{1}{p^{4a-1}} \sum_1^a 1^{4a-4n+2/1}\,(p^4\,q^4)^{n-1}$$

Sur les formules (1) à (7) voyez: Bierens de Haan, Verh. v. K. Akad. v. Wet. Dl. II. blad 19. Page 191.

F. Algébr. rat. fract. à autre dén. Expon. monôme. — **TABLE 152 suite.** — Lim. 0 et ∞.

$$8)\int\frac{x^{4a+3}e^{-px}}{x^4-q^4}dx=\frac{1}{4}q^{4a}\Big[-e^{pq}Ei.(-pq)-e^{-pq}Ei.(pq)-2Ci.(pq).Cos.pq-2Si.(pq).Sin.pq+\pi Sin.pq)\Big]+\frac{1}{p^{4a}}\sum_1^a 1^{4a-4n+3/1}(p^4q^4)^{n-1}$$

Bierens de Haan, Verh. v. K. Akad. v. Wet. Dl. II. blad 19.

$$9)\int\frac{e^{-px}}{q^3-q^2x+qx^2-x^3}dx=\frac{1}{2q^2}\Big[Ci.(pq)\{Sin.pq+Cos.pq\}+\{Si.(pq)-\tfrac{1}{2}\pi\}(Sin.pq-Cos.pq)-e^{pq}Ei.(-pq)\Big]$$

$$10)\int\frac{x\,e^{-px}}{q^3-q^2x+qx^2-x^3}dx=\frac{1}{2q}\Big[Ci.(pq)\{Cos.pq-Sin.pq\}+\{Si.(pq)-\tfrac{1}{2}\pi\}(Sin.pq+Cos.pq)-e^{-pq}Ei.(pq)\Big]$$

$$11)\int\frac{x^2e^{-px}}{q^3-q^2x+qx^2-x^3}dx=\frac{1}{2}\Big[Ci.(pq)(Cos.pq-Sin.pq)+\{Si.(pq)-\tfrac{1}{2}\pi\}(Sin.pq+Cos.pq)+e^{-pq}Ei.(pq)\Big]$$

(9–11) V. T. 129. N°. 9. & T. 131. N°. 6, 7.

$$12)\int\frac{e^{-px}}{q^3+q^2x+qx^2+x^3}dx=\frac{1}{2q^2}\Big[Ci.(pq)(Sin.pq-Cos.pq)-\{Si.(pq)-\tfrac{1}{2}\pi\}(Sin.pq+Cos.pq)+e^{-pq}Ei.(pq)\Big]$$

$$13)\int\frac{x\,e^{-px}}{q^3+q^2x+qx^2+x^3}dx=\frac{1}{2q}\Big[Ci.(pq)(Sin.pq+Cos.pq)+\{Si.(pq)-\tfrac{1}{2}\pi\}(Sin.pq-Cos.pq)-e^{-pq}Ei.(pq)\Big]$$

$$14)\int\frac{x^2e^{-px}}{q^3+q^2x+qx^2+x^3}dx=\frac{1}{2}\Big[-Ci.(pq)(Sin.pq+Cos.pq)+\{Si.(pq)-\tfrac{1}{2}\pi\}(Cos.pq-Sin.pq)+e^{pq}Ei.(-pq)\Big]$$

(12–14) V. T. 129. N°. 8. & T. 131. N°. 6, 7.

F. Alg. rat. fract. à dén. prod. de polyn. Expon. monôme. — **TABLE 133.** — Lim. 0 et ∞.

$$1)\int\left(e^{-x}-\frac{1}{1+x}\right)\frac{dx}{x}=-A$$ Schlömilch, Gr. 9. 5. — Id., Stud. I. 4. — Arndt, Gr. 10. 225. — Id., Gr. 10. 233.

$$2)\int\left(e^{-qx}-\frac{1}{1+x}\right)\frac{dx}{x}=-A-l\,q$$ Cauchy, P. 28. 147, P. 1. § 6.

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3) $\int\left(e^{-x}-\frac{1}{(1+x)^p}\right)\frac{dx}{x} = Z'(p)$ Lejeune-Dirichlet, Cr. 15. 258. — Grunert, Gr. 2. 266. — Schlömilch, Stud. I. 4.

4) $\int\left(\frac{e^{-x}-1}{x}+\frac{1}{1+x}\right)\frac{dx}{x} = A-1$ Arndt, Gr. 10. 233.

5) $\int\left(e^{-x}-\frac{1}{1+x^2}\right)\frac{dx}{x} = -A$ Arndt, Gr. 10. 225.

6) $\int\left\{\frac{e^{-xi}}{\left(1-\frac{xi}{q}\right)^q}+\frac{e^{xi}}{\left(1+\frac{xi}{q}\right)^q}\right\}dx = \frac{\pi}{\Gamma(q)}\left(\frac{e}{q}\right)^q$ Legendre, Exerc. 3. 40.

F. Algébr. rat. fract. à dén. x. Exp. binôme $e^{ax}\pm 1$ en dén. } à un terme. TABLE 134. Lim. 0 et ∞.

1) $\int\frac{1-e^{-x}}{e^x+1}\frac{dx}{x} = l\frac{\pi}{2}$ V. T. 171. N°. 1.

2) $\int\frac{1-e^{(1-q)x}}{e^x+1}\frac{dx}{x} = -Z'\left(\frac{q+1}{2}\right)+Z'\left(\frac{q}{2}\right)-A$ V. T. 171. N°. 2.

3) $\int\frac{e^{-qx}-e^{(q-1)x}}{e^{-x}+1}\frac{dx}{x} = l\,Cot.\frac{q\pi}{2p}$ V. T. 175. N°. 2.

4) $\int\frac{e^{-qx}-e^{-px}}{e^{-x}+1}\frac{dx}{x} = l\frac{\Gamma\left(\frac{q}{2}\right)\Gamma\left(\frac{p+1}{2}\right)}{\Gamma\left(\frac{p}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}$ V. T. 171. N°. 4.

5) $\int\frac{e^{-px}-e^{(p-q)x}}{1+e^{-qx}}\frac{dx}{x} = l\,Cot.\frac{p\pi}{2q}$ V. T. 175. N°. 15.

6) $\int\frac{1-e^{-px}}{1+e^{-x}}\frac{e^{-(q+1)x}}{x}dx = l\frac{\Gamma\left(\frac{p}{2}+1\right)\Gamma\left(\frac{p+q+1}{2}\right)}{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{p+q}{2}+1\right)}$ V. T. 171. N°. 3.

7) $\int\frac{e^{qx}+e^{-qx}-2}{e^x-1}\frac{dx}{x} = l(q\pi\,Cosec.\,q\pi)$ V. T. 175. N°. 4.

8) $\int\frac{(e^{qx}-e^{-qx})^2}{e^x+1}\frac{dx}{x} = -l(q\pi\,Cot.\,q\pi)$ V. T. 175. N°. 3.

9) $\int \frac{(e^{qx} - e^{-qx})^2}{e^{2x} - 1} \frac{dx}{x} = l\,(q\pi\, Cosec.\, q\pi)$ V. T. 175. N°. 7.

10) $\int \frac{(e^{qx} - e^{-qx})^2}{1 - e^{2px}} \frac{dx}{x} = l\left(p\, Sin. \frac{q\pi}{p}\right) - l\,q\pi$ V. T. 172. N°. 8.

11) $\int \frac{(e^{px} - e^{-px})^2}{1 - e^{-x}} \frac{e^{qx}}{x} dx = l \frac{\Gamma(q - 2p)\,\Gamma(q + 2p)}{\{\Gamma(q)\}^2}$ Malmsten, Cr. 35. 55.

12) $\int \frac{1 - e^{-qx}}{1 - e^{-x}} \frac{1 - e^{-px}}{x} e^{-x} dx = l \frac{\Gamma(p + q + 1)}{\Gamma(p + 1)\,\Gamma(q + 1)}$ V. T. 171. N°. 17.

13) $\int \frac{1 - e^{-qx}}{e^{x} - 1} \frac{1 - e^{-px}}{x} dx = l \frac{p + q}{pq} - l\,\mathrm{B}(p, q)$ V. T. 171. N°. 14.

14) $\int \frac{e^{-ax} - e^{-bx}}{1 + e^{-x}} \frac{1 + e^{-(2c+1)x}}{x} dx = -\, l \left[\left(\frac{c+1}{c}\right)^{\frac{b-1}{2}} \frac{\left(\frac{a+1}{2}\right)^{c/1} \left(\frac{b}{2}\right)^{c+1/1}}{\left(\frac{a}{2}\right)^{c+1/1} \left(\frac{b+1}{2}\right)^{c/1}} \right]$ V. T. 171. N°. 5.

15) $\int \frac{1 - e^{-qx}}{1 - e^{-x}} \frac{1 - e^{-px}}{x} e^{-rx} dx = l \frac{\Gamma(r)\,\Gamma(p + q + r)}{\Gamma(p + r)\,\Gamma(q + r)}$ V. T. 171. N°. 18.

16) $\int \frac{e^{-px} + e^{-qx}}{1 + e^{-rx}} \frac{1 + e^{(p+q-r)x}}{x} dx = l\left(Tang. \frac{q\pi}{2r} . Cot. \frac{p\pi}{2r}\right)$ V. T. 175. N°. 16.

17) $\int \frac{(1 - e^{-px})(1 - e^{-qx})(1 - e^{-rx})}{1 - e^{-x}} \frac{e^{-sx}\,dx}{x} = l \frac{\Gamma(p+q+s)\Gamma(p+r+s)\Gamma(q+r+s)\Gamma(s)}{\Gamma(p+s)\Gamma(q+s)\Gamma(r+s)\Gamma(p+q+r+s)}$ V. T. 171. N°. 21.

18) $\int \frac{1 - e^{(1-q)x}}{1 - e^{-x}} \frac{1 - e^{(\frac{1}{2}-q)x}}{e^{\frac{1}{2}x}} \frac{dx}{x} = l\,2^{2q-2}$ V. T. 177. N°. 15.

19) $\int \frac{1 - e^{-x}}{1 + e^{-x}} \frac{1}{e^{x} + e^{-x}} \frac{dx}{x} = \frac{1}{2} l\,2$ V. T. 175. N°. 18.

20) $\int \frac{1 - e^{-x}}{1 + e^{-x}} \frac{e^{-2x}}{e^{x} + e^{-x}} \frac{dx}{x} = l \frac{\pi}{2\sqrt{2}}$ V. T. 175. N°. 17.

F. Alg. rat. fract. à dén. mon. Exp. bin. $e^{ax} \pm 1$ en dén. } à plus. termes. TABLE 135. Lim. 0 et ∞.

1) $\int \left\{ \frac{1}{1 - e^{-x}} - \frac{1}{x} \right\} e^{-x} dx = \mathrm{A}$ V. T. 171. N°. 7.

2) $\int \left\{ \frac{e^{-x}}{x} - \frac{e^{-qx}}{e^{x} - 1} \right\} dx = Z'(1 + q)$ Schlömilch, Stud. I. 10. — Schaar, Mém. Cour. Brux. T. 22.

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3) $\int \left\{ \frac{e^{-qx}}{1-e^{-x}} - \frac{e^{-px}}{x} \right\} dx = lp - Z'(q)$ V. T. 171. N°. 9.

4) $\int \left\{ \frac{b\,e^{-bqx}}{1-e^{-x}} - \frac{b\,e^{-bx}}{x} \right\} dx = -\sum_1^b Z'\left(q + \frac{b-n}{b}\right)$ V. T. 171. N°. 10.

5) $\int \left\{ \frac{1}{2} - \frac{1}{1+e^{-\frac{1}{2}x}} \right\} \frac{e^{-x}}{x} dx = \frac{1}{2} l \frac{\pi}{4}$

6) $\int \left\{ \frac{1}{e^x+1} - \frac{1}{2} e^{-2x} \right\} \frac{dx}{x} = \frac{1}{2} l\, \pi$

(5, 6) Stern, Gött. Stud. 1847.

7) $\int \left\{ a - \frac{1-e^{-ax}}{1-e^{-x}} \right\} \frac{e^{-x}}{x} dx = l\,(1^{a/1})$ Liouville, L. 4. 317.

8) $\int \left\{ \frac{e^{-px} - e^{-(p+q)x}}{e^x - 1} - q\, e^{-x} \right\} \frac{dx}{x} = l \frac{\Gamma(p+1)}{\Gamma(p+q+1)}$ V. T. 171. N°. 12.

9) $\int \left\{ 1 - e^{-x} - \frac{(1-e^{-qx})(1-e^{-px})}{1-e^{-x}} \right\} \frac{dx}{x} = l\, \mathrm{B}(p, q)$ V. T. 175. N°. 1.

10) $\int \left\{ p - 1 - \frac{1-e^{(1-p)x}}{1-e^{-x}} \right\} \frac{e^{-x}}{x} dx = l\, \Gamma(p)$ Malmsten, Cr. 35. 55.

11) $\int \left\{ \frac{b}{x} - \frac{e^{(1-q)x}}{1-e^{-\frac{x}{b}}} \right\} e^{-x} dx = \sum_1^b Z'\left(q + \frac{b-n}{b}\right)$ V. T. 177. N°. 22.

12) $\int \left\{ \frac{1}{1-e^{-2x}} - \frac{2-e^{-x}}{2x} + \frac{1-e^{-x}}{2} \right\} \frac{e^{-x}}{x} dx = 0$

13) $\int \left\{ \frac{1}{1-e^{-x}} - \frac{1}{x} - \frac{1}{2} \right\} e^{-\frac{1}{2}x} \frac{dx}{x} = \frac{1}{2}(1 - l\,2)$

14) $\int \left\{ \frac{1}{1-e^{-2x}} - \frac{1}{2x} - \frac{1}{2} \right\} e^{-x} \frac{dx}{x} = \frac{1}{2}(1 - l\,2)$

15) $\int \left\{ \left[(p-1) - \frac{1}{1-e^{-x}} \right] e^{-x} + \left(\frac{1}{2} + \frac{1}{x} \right) e^{-px} \right\} \frac{dx}{x} = \left(p - \frac{1}{2}\right) lp - p + \frac{1}{2} l\, 2\pi$

16) $\int \left\{ \left(\frac{1}{x} + \frac{1}{2} \right) e^{-\frac{1}{2}x} - \left(\frac{1}{2} + \frac{1}{1-e^{-x}} \right) e^{-x} \right\} \frac{dx}{x} = \frac{1}{2} l\, 2\pi - \frac{1}{2}$

(12–16) Meyer, Int. Déf. 337, 338.

17) $\int \left\{ p\, e^{-x} - \frac{e^{-px}}{x} - \frac{1}{2} e^{-px} - \frac{1}{e^x - 1} \right\} \frac{dx}{x} = \left(p + \frac{1}{2}\right) lp - p + \frac{1}{2} l\, 2\pi$ Stern, Gött. Stud. 1847.

18) $\int \left\{\frac{e^{-x}}{x} - \frac{e^{-ax} - e^{-bx} - e^{(b-a)x}}{1-e^{-x}} e^{-px}\right\} dx = l \frac{\Gamma(b+p)\Gamma(a-b+p)}{\Gamma(a+p)}$ Malmsten, Cr. 35. 55.

19) $\int \left\{\frac{1}{e^x-1} - \frac{1}{e^{2x}-1} - \frac{e^{-\frac{1}{2}x}}{x} - \frac{e^{-x}}{2x}\right\} \frac{dx}{x} = 0$

20) $\int \left\{\frac{1}{2} e^{-x} + \frac{1}{x} e^{-x} - \frac{1}{e^x-1}\right\} \frac{dx}{x} = \frac{1}{2} l\, 2\pi - 1$

21) $\int \left\{\frac{e^{-x}}{x} - \frac{1}{2} \frac{e^{-x}+1}{e^x-1}\right\} \frac{dx}{x} = \frac{1}{2} l\, 2\pi - 1$

22) $\int \left\{\frac{e^{-x}}{x} - x e^{-x} - \frac{1}{2} \frac{e^{-x}+1}{e^x-1}\right\} \frac{dx}{x} = \frac{1}{2} l\, 2\pi$

23) $\int \left\{\frac{1}{x} - \frac{1}{2} e^{-x} - \frac{1}{e^x-1}\right\} \frac{dx}{x} = \frac{1}{2} l\, 2\pi$

24) $\int \left\{ap - \frac{1}{2}(a-1) - \frac{a}{1-e^{-x}} - \frac{e^{-(a-1)x}}{1-e^{\frac{x}{a}}}\right\} \frac{e^{-x}}{x} dx = l \left\{\Gamma(p+1)\Gamma\left(p-\frac{1}{a}+1\right)\Gamma\left(p-\frac{2}{a}+1\right) \dots \Gamma\left(p-\frac{a+1}{a}\right)\right\}$

25) $\int \left\{\frac{a-1}{2} + \frac{a-1}{1-e^{-x}} + \frac{e^{(1-a)x}}{1-e^{\frac{x}{a}}} + \frac{e^{-apx}}{1-e^{-x}}\right\} \frac{e^{-x}}{x} dx = l \frac{\Gamma(pa+1)}{\left\{\Gamma(p+1)\Gamma\left(p-\frac{1}{a}+1\right)\Gamma\left(p-\frac{2}{a}+1\right) \dots \Gamma\left(p-\frac{a+1}{a}\right)\right\}}$

26) $= \frac{1}{2}(a-1)\, l\, 2\pi - \left(ap + \frac{1}{2}\right) la$

27) $\int \left\{\frac{e^{(1-p)x}}{1-e^x} - \frac{e^{(1-p)qx}}{1-e^{qx}} - \frac{e^x}{1-e^x} + \frac{e^{qx}}{1-e^{qx}}\right\} \frac{dx}{x} = q\, lp$

Stern, Gött. Stud. 1847. (19–27)

28) $\int \left\{\frac{1}{e^x-1} - \frac{pe^{-px}}{1-e^{-px}} + \left(pq + \frac{p+1}{2}\right) e^{-px} + (1-pq)e^{-x}\right\} \frac{dx}{x} = \frac{p-1}{2} l 2\pi + \left(\frac{1}{2} - pq\right) lp$ V. T. 175. N°. 19.

29) $\int \left\{\frac{e^{-qx}}{1-e^{-x}} - \frac{e^{-pqx}}{1-e^{-px}} - \frac{p-1}{1-e^{-px}} e^{-px} - \frac{p-1}{2} e^{-px}\right\} \frac{dx}{x} = \frac{p-1}{2} l 2\pi + \left(\frac{1}{2} - pq\right) lp$ V. T. 175. N°. 20.

30) $\int \left\{q - \frac{1}{2} \frac{e^{-rx} - e^{-px}}{x} + \frac{p\, e^{-pqx}}{1-e^{-px}} - \frac{r\, e^{-rqx}}{1-e^{-rx}}\right\} \frac{dx}{x} = (p-r)\left\{\frac{1}{2} - q + \frac{1}{2} l\pi - l\Gamma(q) + \frac{1}{2} l2\right\}$ V. T. 176. N°. 3.

31) $\int \left\{\frac{e^{-qx}}{1-e^{-x}} - \frac{e^{-pqx} + (p-1)e^{\frac{1}{2}px}}{1-e^{-px}}\right\} \frac{dx}{x} = \frac{1}{2}(p-1)\, l\, 2 + \left(\frac{1}{2} - pq\right) lp$ V. T. 177. N°. 24.

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1) $\int \frac{1}{e^x - e^{-x}} \frac{dx}{x} = \frac{1}{2} l\, 2$ V. T. 172. N°. 2.

2) $\int \frac{e^x - e^{-x}}{e^{2x} + e^{-2x}} \frac{dx}{x} = l\, Tang. \frac{3\pi}{8}$ V. T. 172. N°. 4.

3) $\int \frac{(e^{qx} - e^{-qx})^2}{e^x - e^{-x}} \frac{dx}{x} = l\, Cos.\, q\pi$ V. T. 175. N°. 6.

4) $\int \frac{1 - e^{2(q-p)x}}{e^{qx} + e^{(q-2p)x}} \frac{dx}{x} = l\, Cot. \frac{q\pi}{2p}$ V. T. 172. N°. 9.

5) $\int \frac{e^{qx} - e^{-qx}}{e^x - e^{-x}} \frac{e^{-x}}{x} dx = -l\,(q\pi\, Cosec.\, q\pi)$ V. T. 175. N°. 7.

6) $\int \frac{1 - e^{-qx}}{e^x - e^{-x}} \frac{1 - e^{-(q+1)x}}{x} dx = q\, l\, 2,\ q > 1;$ V. T. 172. N°. 3.

7) $\int \frac{(1 - e^{-x})^2}{e^x + e^{-x}} \frac{dx}{x} = l \frac{4}{\pi}$ V. T. 172. N°. 1.

8) $\int \frac{\{1 - e^{(q-p)x}\}^2}{e^{qx} - e^{(q-2p)x}} \frac{dx}{x} = l\, Cosec. \frac{q\pi}{2p}$ V. T. 172. N°. 10.

9) $\int \frac{e^{qx} - e^{-qx}}{e^{px} + e^{-px}} \frac{dx}{x} = l\, Tang. \left(\frac{p+q}{4p}\pi\right)$ V. T. 172. N°. 6.

10) $\int \frac{e^{qx} + e^{-qx}}{e^{px} - e^{-px}} \frac{dx}{x} = l\, Sec. \frac{q\pi}{2p}$ V. T. 172. N°. 7.

11) $\int \frac{e^{qx} - e^{-qx}}{e^{\pi x} + e^{-\pi x}} \frac{dx}{x} = l\, Tang. \left(\frac{\pi}{4} + \frac{q}{4}\right)$

12) $\int \frac{e^{qx} + e^{-qx} - 2}{e^{\pi x} - e^{-\pi x}} \frac{x}{} = -l\, Cos. \frac{1}{2} q$

(11, 12) Legendre, Exerc. 5. 45.

13) $\int \frac{e^{(p-q)x} + e^{(q-p)x} - 2}{e^{px} - e^{-px}} \frac{dx}{x} = l\, Cosec. \frac{q\pi}{2p}$ V. T. 175. N°. 10.

14) $\int \frac{e^{qx} + e^{-qx} - 2}{e^{px} - e^{-px}} \frac{dx}{x} = l\, Sec. \frac{q\pi}{2p}$ V. T. 175. N°. 9.

15) $\int \frac{e^{qx} + e^{-qx} - e^{rx} - e^{-rx}}{e^{px} - e^{-px}} \frac{dx}{x} = l \left(Cos. \frac{r\pi}{2p} . Sec. \frac{q\pi}{2p}\right)$ V. T. 175. N°. 14.

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16) $\int \frac{e^{(p-q)x} - e^{(p-r)x} - e^{(r-p)x} + e^{(q-p)x}}{e^{px} - e^{-px}} \frac{dx}{x} = l\left(Sin.\frac{r\pi}{2p} . Cosec.\frac{q\pi}{2p}\right)$ V. T 175. N°. 13.

17) $\int \frac{e^{qx} - e^{-qx}}{e^{\pi x} - e^{-\pi x}} \frac{dx}{x^p} = \Gamma(1-p) \sum_0^\infty \left[\frac{1}{\{(2n+1)\pi - q\}^{1-p}} - \frac{1}{\{(2n+1)\pi + q\}^{1-p}}\right]$ Malmsten, Cr. 38. 1.

18) $\int \frac{e^{qx} + e^{-qx}}{e^{\pi x} + e^{-\pi x}} dx = \Gamma(1-p) \sum_0^\infty (-1)^n \left[\frac{1}{\{(2n+1)\pi - q\}^{1-p}} - \frac{1}{\{(2n+1)\pi + q\}^{1-p}}\right]$

F. Alg. rat. fract. à dén. mon. Expon. trinôme en dén. TABLE 137. Lim. 0 et ∞.

1) $\int \frac{1}{e^x + 1 + e^{-x}} \frac{dx}{x} = \frac{1}{2} l 3$ V. T. 174. N°. 2.

2) $\int \frac{e^x}{e^{2x} - 1 + e^{-2x}} \frac{dx}{x} = l \frac{2}{\sqrt{3}}$ V. T. 174. N°. 3.

3) $\int \frac{1}{e^x + e^{-x} + 2Cos.\frac{a\pi}{b}} \frac{dx}{x} = \frac{1 + Cos.\frac{a\pi}{b}}{Sin.\frac{a\pi}{b}} \left\{ Tang.\frac{a\pi}{2b} l2b + 2\sum_1^{b-1} (-1)^{n-1} Sin.\frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+n+1}{2b}\right)}{\Gamma\left(\frac{n+1}{2b}\right)} \right\}$, pour $a+b$ impair;

4) $= \frac{1 + Cos.\frac{a\pi}{b}}{Sin.\frac{a\pi}{b}} \left\{ Tang.\frac{a\pi}{2b} lb + 2\sum_1^{\frac{b-1}{2}} (-1)^{n-1} Sin.\frac{na\pi}{b} l \frac{\Gamma\left(\frac{b-n+1}{b}\right)}{\Gamma\left(\frac{n+1}{b}\right)} \right\}$, pour $a+b$ pair; V. T. 174. N°. 5, 6.

5) $\int \frac{1}{e^x + e^{-x} + 2\,Cos.\lambda} \frac{dx}{x^{1-q}} = Cosec.\lambda\, \Gamma(q) \sum_1^\infty (-1)^{n-1} \frac{Sin.\,n\lambda}{n^q}$ V. T. 174. N°. 4.

6) $\int \frac{e^x + e^{-x}}{e^{2x} + e^{-2x} + 2\,Cos.2\lambda} \frac{dx}{x^{1-q}} = Sec.^2 \lambda\, \Gamma(q) \sum_1^\infty (-1)^n \frac{Cos.\{(2n+1)\lambda\}}{(2n+1)^q}$ V. T. 174. N°. 13.

7) $\int \frac{1 - e^{(p-q)x}}{e^x + e^{-x} + 2Cos.\frac{a\pi}{b}} \frac{e^{-px}}{x} dx = Cosec.\frac{a\pi}{b} \sum_1^{b-1} (-1)^{n-1} Sin.\frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+q+n}{2b}\right)\Gamma\left(\frac{p+n}{2b}\right)}{\Gamma\left(\frac{b+p+n}{2b}\right)\Gamma\left(\frac{q+n}{2b}\right)}$, pour $a+b$ impair;

8) $= Cosec.\frac{a\pi}{b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} Sin.\frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+q-n}{b}\right)\Gamma\left(\frac{p+n}{b}\right)}{\Gamma\left(\frac{b+p-n}{b}\right)\Gamma\left(\frac{q+n}{b}\right)}$, pour $a+b$ pair; V. T. 174. N°. 7, 8.

9) $\displaystyle\int\frac{(1-e^{-x})^2}{e^x+e^{-x}+2Cos.\frac{a\pi}{b}}\frac{dx}{x}=Cosec.\frac{a\pi}{b}\sum_1^{b-1}(-1)^{n-1}Sin.\frac{na\pi}{b}\,l\frac{\Gamma\left\{\left(\frac{b+n+1}{2b}\right)\right\}^2\Gamma\left(\frac{n+2}{2b}\right)\Gamma\left(\frac{n}{2b}\right)}{\left\{\Gamma\left(\frac{n+1}{2b}\right)\right\}^2\Gamma\left(\frac{n+b}{2b}\right)\Gamma\left(\frac{n+b+2}{2b}\right)}$, pour $a+b$ impair;

10) $\displaystyle=Cosec.\frac{a\pi}{b}\sum_1^{\frac{b-1}{2}}(-)^{n-1}Sin.\frac{na\pi}{b}\,l.\frac{\left\{\Gamma\left(\frac{b-n+1}{b}\right)\right\}^2\Gamma\left(\frac{n+2}{b}\right)\Gamma\left(\frac{n}{b}\right)}{\left\{\Gamma\left(\frac{n+1}{b}\right)\right\}^2\Gamma\left(\frac{b-n}{b}\right)\Gamma\left(\frac{b-n+2}{b}\right)}$, pour $a+b$ pair;

V. T. 174. N°. 9, 10.

11) $\displaystyle\int\left\{e^{-x}Tang.\frac{a\pi}{2b}-\frac{2e^{-px}Sin.\frac{a\pi}{b}}{e^x+e^{-x}+2Cos.\frac{a\pi}{2b}}\right\}\frac{dx}{x}=Tang.\frac{a\pi}{2b}l2b+2\sum_1^{b-1}(-1)^{n-1}Sin.\frac{na\pi}{b}\,l\frac{\Gamma\left(\frac{b+p+n}{2b}\right)}{\Gamma\left(\frac{p+n}{2b}\right)}$, pour $a+b$ pair;

12) $\displaystyle=Tang.\frac{a\pi}{2b}lb+2\sum_1^{\frac{b-1}{2}}(-1)^{n-1}Sin.\frac{na\pi}{b}\,l\frac{\Gamma\left(\frac{b+p-n}{b}\right)}{\Gamma\left(\frac{p-n}{b}\right)}$, pour $a+b$ impair.

V. T. 174. N°. 11, 12.

13) $\displaystyle\int\left\{q-\frac{1}{2}+\frac{(1-e^{-x})(1-qx)-xe^{-x}}{e^{-2x}+1-2e^{-x}}e^{(1-q)x}\right\}\frac{e^{-x}}{x}dx=q-\frac{1}{2}-l\Gamma\left(\frac{1}{2}\right)+l\Gamma(q)-\frac{1}{2}l2$ V. T. 174. N°. 1.

1) $\displaystyle\int\frac{1}{e^{\pi x}+e^{-\pi x}}\frac{dx}{q^2+x^2}=\frac{1}{4q}\left\{Z'\left(\frac{q}{2}+\frac{3}{4}\right)-Z'\left(\frac{q}{2}+\frac{1}{4}\right)\right\}$

2) $\displaystyle\int\frac{1}{e^{\pi x}+e^{-\pi x}}\frac{dx}{1+x^2}=1-\frac{1}{4}\pi$

3) $\displaystyle\int\frac{1}{e^{\pi x}+e^{-\pi x}}\frac{dx}{1+4x^2}=\frac{1}{4}l2$

Legendre, Exerc. 5. 50.

4) $\displaystyle\int\frac{e^{px}-e^{-px}}{e^{\pi x}-e^{-\pi x}}\frac{dx}{1+x^2}=-\frac{1}{2}p\,Cos.p+\frac{1}{2}Sin.p\,l\{2(1+Cos.p)\},\ p<\pi;$ Legendre, Exerc. 5. 46. — Schlömilch, Beitr. II. § 9.

5) $\displaystyle\int\frac{e^{(\pi-\lambda)x}-e^{(\lambda-\pi)x}}{e^{\pi x}-e^{-\pi x}}\frac{dx}{q^2+x^2}=\frac{1}{q}\sum_1^{\infty}\frac{Sin.\,n\lambda}{q+n},\ \lambda^2<\pi^2;$ Schlömilch, Cr. 42. 125. il trouve fautivement $x\,dx$ au lieu de dx.

6) $\displaystyle\int\frac{e^{px}+e^{-px}}{e^{\pi x}-e^{-\pi x}}\frac{x}{1+x^2}dx=\frac{1}{2}(p\,Sin.p-1)+\frac{1}{2}Cos.p\,l\{2(1+Cos.p)\}$ Legendre, Exerc. 5. 46.

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7) $\int \frac{e^{px}+e^{-px}}{e^{\pi x}-e^{-\pi x}} \frac{x}{1+x^2} dx = \frac{1}{2} Cos. p\, l(2+2\, Cos. p) + \frac{1}{4} p\, Sin. p - \frac{1}{2}$, valeur fautive; (V. N°. 6.) Poisson, Mém. Inst. 1811. 163. N°. 27.

8) $\int \frac{e^{(\pi-\lambda)x}+e^{(\lambda-\pi)x}}{e^{\pi x}-e^{-\pi x}} \frac{x}{q^2+x^2} dx = \frac{1}{2q} + \sum_1^\infty \frac{Cos. n\lambda}{q+n}$, $\lambda^2 \leqq \pi^2$; Schlömilch, Cr. 42. 125.

9) $\int \frac{x}{e^{2\pi x}-1} \frac{dx}{q^2+x^2} = -\frac{1}{4} q + \frac{1}{2} l\, q - \frac{1}{2} Z'(q)$ Legendre, Exerc. 5. 49.

10) $\int \frac{x}{e^{2\pi x}-1} \frac{dx}{1+x^2} = \frac{1}{2} A - \frac{1}{4}$ Poisson, P. 18. 295. N°. 25. — Id., Mém Inst. 1811. 163. N°. 29. — Legendre, Exerc. 5. 49.

11) $\int \frac{1}{e^{2\pi q x}-1} \frac{x}{1+x^2} dx = \frac{1}{2} l\, q + \frac{1}{4q} - \frac{1}{2} Z'(1+q)$ Schaar, Mém. Cour. Brux. T. 22.

12) $\int \frac{x}{e^{\pi x}-e^{-\pi x}} \frac{dx}{1+x^2} = l\, 2 - \frac{1}{2}$ V. T. 387. N°. 3.

13) $\int \frac{e^{px}-e^{-px}}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}} \frac{dx}{1+x^2} = \frac{1}{2}\pi\, Sin. p - \frac{1}{2} Cos. p\, l \frac{1+Sin. p}{1-Sin. p}$, $p < \frac{1}{2}\pi$;

14) $\int \frac{1}{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}} \frac{dx}{1+x^2} = \frac{1}{2} l\, 2$

(13, 14) Schlömilch, Beitr. II. 9. — Id., Stud. II. 19.

15) $\int \frac{e^{px}+e^{-px}}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}} \frac{x}{1+x^2} dx = -1 + \frac{1}{2}\pi\, Cos. p + \frac{1}{2} Sin. p\, l \frac{1+Sin. p}{1-Sin. p}$, $\frac{1}{2}\pi \geqq p \geqq 0$;

16) $\int \frac{1}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}} \frac{x}{1+x^2} dx = \frac{1}{4}\pi - \frac{1}{2}$

(15, 16) Schlömilch, Beitr. II. 7.

17) $\int \frac{1}{e^{\frac{1}{4}\pi x}+e^{-\frac{1}{4}\pi x}} \frac{dx}{1+x^2} = \frac{1}{2\sqrt{2}} \left(\pi - l \frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ Schlömilch, Beitr. II. 9. — Id., Stud. II. 19.

18) $\int \frac{1}{e^{\frac{1}{4}\pi x}-e^{-\frac{1}{4}\pi x}} \frac{x}{1+x^2} dx = \frac{1}{4}\pi\sqrt{2} - 1 + \frac{1}{2\sqrt{2}} l \frac{\sqrt{2}+1}{\sqrt{2}-1}$ Schlömilch, Beitr. II. 7.

19) $\int \frac{1}{e^{2\pi x}-1} \frac{x}{(q^2+x^2)^2} dx = -\frac{1}{8q^3} - \frac{1}{4q^2} + \frac{1}{4q} \frac{dZ'(q)}{dq}$ Cisa de Grésy, Mém. Turin. 1811. 209. II. 62.

20) $= \frac{1}{4q^4} \sum_0^\infty \frac{B_{2n+1}}{q^{2n}}$ Cisa de Grésy, Mém. Turin. 1811. 209. II. 61. — Plana, Mém. Turin. 1820.

21) $\int \frac{1}{e^{2\pi x}-1} \frac{x}{q^2-x^2} dx = \frac{1}{4q^4} \sum_0^\infty (-1)^n \frac{B_{2n+1}}{n+1} q^{-2n}$

22) $\int \frac{1}{e^{2\pi x}-1} \frac{x}{(q^2-x^2)^2} dx = \frac{1}{4q^4} \sum_0^\infty (-1)^n \frac{B_{2n+1}}{q^{2n}}$

(21, 22) Plana, Mém. Turin. 1820.

1) $\int e^{-x}\,dx\sqrt{x} = \frac{1}{2}\sqrt{\pi}$ Euler, Calc. Int. 4. S. 5. 211. — Plana, Cr. 17. 1.

2) $\int e^{-px}dx\sqrt{x} = \frac{1}{2p}\sqrt{\frac{\pi}{p}}$ Dienger, Cr. 46. 119.

3) $\int e^{-x}dx\sqrt[q]{x^b} = \frac{q\ .\ 2q\ .\ 3q\ \ldots}{b+q\,.\,b+2q\,.\,b+3q\ldots}$ Oettinger, Cr. 35. 13.

4) $\int e^{-bx}x^{a-\frac{1}{2}}\,dx = \frac{1^{a/2}}{(2b)^a}\sqrt{\frac{\pi}{b}}$ Schlömilch, Stud. I. 12.

5) $\int e^{-4x\sqrt{b}}(x+x^2)^{q-\frac{1}{2}}\,dx = \frac{\Gamma(q-\frac{1}{2})}{2^{2q+1}\sqrt{\pi}}\frac{e^{2\sqrt{b}}}{b^q}\left\{\Gamma(q)\psi(1-q,b)+\Gamma(-q)b^q\psi(1+q,b)\right\}$ Kummer, Cr. 17. 228.

6) $\int e^{-a^2(x+\frac{1}{x})}dx\sqrt{x} = \frac{1}{a}\left(1+\frac{1}{2a^2}\right)e^{-2a^2}\sqrt{\pi}$ Cauchy, P. 28. 147. P. I. § 4.

7) $\int e^{-\frac{1+x^2}{2qx}}\,dx\sqrt{x} = \frac{1+q}{\sqrt[q]{e}}\sqrt{2q\pi}$ Legendre, Exerc. 3. 51.

8) $\int e^{-\left(px+\frac{q}{x}\right)}x^{c-\frac{1}{2}}\,dx = \left(\frac{q}{p}\right)^{\frac{c}{2}}\sqrt{\frac{\pi}{p}}\,e^{-2\sqrt{pq}}\sum_0^\infty\frac{(c-n)^{2n/1}}{2^{n/2}(2\sqrt{pq})^n}$ Schlömilch, Cr. 33. 268. — Id., Stud. I. 17.

9) $\int e^{-x^a}x^{(b+\frac{1}{2})a-1}\,dx = \frac{1^{b/2}}{2^b a}\sqrt{\pi}$ Kramp, Réfr. 3. 67.

10) $\int e^{-\frac{1+x^2}{2qx}}x^{\frac{2b+1}{2}}\,dx = \frac{\sqrt{2q\pi}}{\sqrt[q]{e}}\sum_0^\infty\frac{(b-n+1)^{2n/1}}{2^n\,1^{n/1}}(-q)^n$ Legendre, Exerc. 3. 52.

11) $\int\frac{dx\sqrt{x}}{e^x+e^{-x}} = \frac{1}{2}\sqrt{\pi}\sum_0^\infty(-1)^n\frac{1}{\sqrt{(2n+1)^3}}$ V. T. 187, N°. 18.

12) $\int\frac{e^x-e^{-x}}{(e^x+e^{-x})^2}\,dx\sqrt{x} = \frac{1}{2}\sqrt{\pi}\sum_0^\infty(-1)^n\frac{1}{\sqrt{(2n+1)}}$ V. T. 140. N°. 19.

13) $\int\frac{e^x-e^{-x}}{(e^x+e^{-x}+1)^2}\,dx\sqrt{x} = \frac{\sqrt{\pi}}{2\,Sin.\frac{1}{3}\pi}\sum_1^\infty(-1)^{n-1}\frac{Sin.\frac{n\pi}{3}}{\sqrt{n}}$ V. T. 140. N°. 20.

1) $\int\frac{e^{-x}}{\sqrt{x}}\,dx = \sqrt{\pi}$ Euler, Calc. Int. 4. S. 5. 211. — Cauchy, Cours. Leç. 33. — Bidone, Mém. Turin. 1812. 231. Art. 1, 20. — Binet, P. 27. 123. — Plana, Cr. 17. 1. — Grunert, Gr. 2. 266.

2) $\int \frac{e^{-qx}}{\sqrt{x}} dx = \sqrt{\frac{\pi}{q}}$ Bidone, Mém. Turin. 1812. 231. Tableau. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 34. — Dienger, Cr. 46. 119.

3) $\int \frac{e^{pxi}}{\sqrt{x}} dx = e^{\frac{1}{4}i\pi}\sqrt{\frac{\pi}{p}}$ Schlömilch, Stud. I. 13.

4) $\int e^{-qx} \frac{x^a}{\sqrt{x}} dx = \frac{1^{a/2}}{(2q)^a}\sqrt{\frac{\pi}{q}}$ Raabe, Int. 165.

5) $\int e^{-\frac{1+x^2}{2qx}} \frac{dx}{\sqrt{x}} = \frac{\sqrt{2q\pi}}{\sqrt[q]{e}}$ Legendre, Exerc. 3. 50. — Bidone, Mém. Turin. 1812. 231. Art. 2. 34. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 38.

6) $\int e^{-q^2\left(x+\frac{1}{x}\right)} \frac{dx}{\sqrt{x}} = \frac{1}{q} e^{-2q^2}\sqrt{\pi}$ Cauchy, P. 28. 147. P. 1. 4.

7) $\int e^{-\frac{1+x^2}{2qx}} \frac{dx}{x\sqrt{x}} = \frac{2q}{\sqrt[q]{e}}\sqrt{\frac{\pi}{2q}}$

8) $\int e^{-\frac{1+x^2}{2qx}} \frac{dx}{x^2\sqrt{x}} = \frac{1+q}{\sqrt[q]{e}}\sqrt{2q\pi}$

(7–8) Legendre, Exerc 3. 53.

9) $\int e^{-\left(px+\frac{q}{x}\right)} \frac{dx}{\sqrt{x}} = e^{-2\sqrt{pq}}\sqrt{\frac{\pi}{p}}$ Cauchy, Sav. Etr. 1827. 124. Note. 16.

10) $\int e^{-x}\sqrt[q]{x^b}\frac{dx}{x} = \frac{2q \cdot 3q \cdot 4q \ldots}{b \cdot b+q \cdot b+2q \ldots}$ Oettinger, Cr. 35. 13.

11) $\int e^{-\frac{1+x^2}{2qx}} \frac{dx}{x^{\frac{2b+1}{2}}} = \frac{\sqrt{2q\pi}}{\sqrt[q]{e}} \sum_0^\infty \frac{b^{2n/1}}{2^{n/2}} q^n$ Legendre, Exerc. 3. 52.

12) $\int e^{-\left(px+\frac{q}{x}\right)} \frac{dx}{x^{c+1}} = \left(\frac{p}{q}\right)^{\frac{c}{2}} e^{-2\sqrt{pq}}\sqrt{\frac{\pi}{p}} \sum_0^\infty \frac{(c-n)^{2n/1}}{2^{n/2}(2\sqrt{pq})^n}$ Schlömilch, Cr. 33. 268. — Id., Stud. I. 17.

13) $\int \frac{e^{-qx}-1}{\sqrt{x}} e^{-x} dx = \left\{1 - \frac{1}{\sqrt{(1+q)}}\right\}\sqrt{\pi}$ V. T. 178. N°. 2.

14) $\int \frac{e^{-px}-e^{-qx}}{\sqrt{x}} e^{-x} dx = \left\{\frac{1}{\sqrt{(1+q)}} - \frac{1}{\sqrt{(1+p)}}\right\}\sqrt{\pi}$ V. T. 178. N°. 3.

15) $\int \frac{e^{xp\sqrt{}} + e^{-p\sqrt{x}}}{\sqrt{x}} e^{-x} dx = 2e^{\frac{1}{4}p^2}\sqrt{\pi}$ V. T. 37. N°. 14.

16) $\int \frac{e^{-px}-e^{-qx}}{x^{2-\frac{1}{a}}} dx = \frac{a\Gamma\left(\frac{1}{a}\right)}{a-1}\left(q^{\frac{a-1}{a}} - p^{\frac{a-1}{a}}\right), q > p > 0;$ Lindmann, Stock. Handl. 1850. IV.

17) $\int \frac{Sin.p \sqrt{\{\sqrt{(p^2+x^2)}+x\}} - Cos.p \sqrt{\{\sqrt{(p^2+x^2)}-x\}}}{\sqrt{(p^2+x^2)}} e^{-x} dx = 0$

18) $\int \frac{Sin.p \sqrt{\{\sqrt{(p^2+x^2)}-x\}} + Cos.p \sqrt{\{\sqrt{(p^2+x^2)}+x\}}}{\sqrt{(p^2+x^2)}} e^{-x} dx = 0$

Cauchy, Sav. Etr. 1827. 599. P. I. § 6.

19) $\int \frac{1}{e^x + e^{-x}} \frac{dx}{\sqrt{x}} = \sqrt{\pi} \sum_0^\infty \frac{(-1)^n}{\sqrt{(2n+1)}}$

20) $\int \frac{1}{e^x + e^{-x} + 1} \frac{dx}{\sqrt{x}} = \frac{\sqrt{\pi}}{Sin.\frac{1}{3}\pi} \sum_1^\infty (-1)^{n-1} \frac{Sin.\frac{1}{3}n\pi}{\sqrt{n}}$

Malmsten, Cr. 38. 1., dans 19) il a faut. $\sum_1^\infty$;

21) $\int \frac{Cos.\lambda - e^{-x} - e^{-ax} Cos.\{(a+1)\lambda\} + e^{-(a+1)x} Cos.a\lambda}{e^x + e^{-x} - 2\,Cos.\lambda} \frac{dx}{\sqrt{x}} = \sqrt{\pi} \sum_1^a \frac{Cos.n\lambda}{\sqrt{n}}$ V. T. 178. N°. 6.

22) $\int \frac{Sin.\lambda - e^{-ax} Sin.\{(a+1)\lambda\} + e^{-(a+1)x} Sin.a\lambda}{e^x + e^{-x} - 2\,Cos.\lambda} \frac{dx}{\sqrt{x}} = \sqrt{\pi} \sum_1^a \frac{Sin.n\lambda}{\sqrt{n}}$ V. T. 178. N°. 7.

F. Algébr. rat. ent. Expon. sous forme irrat. — TABLE 141. — Lim. 0 et ∞.

1) $\int e^{-x} x\,dx \sqrt{(1-e^{-2x})} = \frac{1}{4}\pi\left(\frac{1}{2} + l\,2\right)$ V. T. 162. N°. 1.

2) $\int e^{-2x} x\,dx \sqrt{(1-e^{-2x})} = \frac{1}{3}\left(\frac{4}{3} - l\,2\right)$ V. T. 162. N°. 2.

3) $\int e^{-x}\, x\,dx\,(1-e^{-2x})^{\frac{2a-1}{2}} = \frac{1^{a/2}\pi}{2^{a+2}\,1^{a/1}} \{A + Z'(a+1) + 2\,l\,2\}$ V. T. 162. N°. 3.

4) $\int \frac{x}{\sqrt{(e^x-1)}} dx = 2\pi\, l\,2$ V. T. 181. N°. 1.

5) $\int \frac{x^2}{\sqrt{(e^{2x}-1)}} dx = \frac{1}{2}\pi\left\{(l2)^2 + \frac{1}{12}\pi^2\right\}$ V. T. 164. N°. 1.

6) $\int \frac{x}{\sqrt{(e^{2x}-1)}} dx = \frac{1}{2}\pi\, l\,2$ V. T. 163. N°. 2.

7) $\int \frac{x\,e^{-x}}{\sqrt{(e^{2x}-1)}} dx = 1 - l\,2$ V. T. 163. N°. 3.

8) $\int \frac{x\,e^{-2x}}{\sqrt{(e^{2x}-1)}} dx = \frac{1}{8}\pi(2\,l\,2 - 1)$ V. T. 163. N°. 4.

9) $\int \frac{x e^{-3x}}{\sqrt{(e^{2x}-1)}}\,dx = \frac{1}{9}(5-6\,l\,2)$ V. T. 163. N°. 5.

10) $\int \frac{x e^{-4x}}{\sqrt{(e^{2x}-1)}}\,dx = \frac{3}{16}\pi\left(l\,2-\frac{7}{12}\right)$ V. T. 163. N°. 6.

11) $\int \frac{x e^{-5x}}{\sqrt{(e^{2x}-1)}}\,dx = \frac{8}{15}\left(\frac{47}{60}-l\,2\right)$ V. T. 163. N°. 7.

12) $\int \frac{x e^{-x}}{\sqrt{(e^{2x}-e^{-2x})}}\,dx = \frac{1}{8}\pi\,l\,2$ V. T. 163. N°. 12.

13) $\int \frac{x e^{-3x}}{\sqrt{(e^{2x}-e^{-2x})}}\,dx = \frac{1}{4}(1-l\,2)$ V. T. 163. N°. 13.

14) $\int \frac{e^{2x} x^2}{\sqrt{(e^{2x}-1)^3}}\,dx = \pi\,l\,2$ V. T. 141. N°. 4.

15) $\int \frac{e^{2x} x^3}{\sqrt{(e^{2x}-1)^3}}\,dx = \frac{3\pi}{2}\left\{(l\,2)^2+\frac{1}{12}\pi^2\right\}$ V. T. 141. N°. 5.

16) $\int \frac{e^{-x} x}{\sqrt[3]{(1-e^{-2x})}}\,dx = \frac{\pi^2}{54}+\frac{5\pi\,l\,3}{18\sqrt{3}}$ V. T. 163. N°. 8.

17) $\int \frac{e^{-2x} x}{\sqrt[3]{(1-e^{-2x})}}\,dx = -\frac{\pi^3}{54}+\frac{5\pi\,l\,3}{18\sqrt{3}}$ V. T. 163. N°. 9.

18) $\int \frac{x}{\sqrt[3]{(e^{3x}-1)}}\,dx = \frac{\pi}{3\sqrt{3}}\left(l\,3+\frac{\pi}{3\sqrt{3}}\right)$ V. T. 163. N°. 10.

19) $\int \frac{x}{\sqrt[3]{(e^{3x}-1)^2}}\,dx = \frac{\pi}{3\sqrt{3}}\left(l\,3-\frac{\pi}{3\sqrt{3}}\right)$ V. T. 163. N°. 11.

20) $\int \frac{e^{-ax} x^h}{\sqrt[b]{(1-e^{-bx})^{b-c}}}\,dx = 1^{h/1}\sum_0^\infty \frac{(b-c)^{n/b}}{b^{n/b}}\frac{1}{(a+bn)^{h+1}}$ V. T. 164. N°. 2.

21) $\int \frac{x}{e^x+p^2-1}\frac{e^x}{\sqrt{(e^x-1)}}\,dx = \frac{2\pi}{p}\,l\,(1+p)$ V. T. 181. N°. 3.

22) $\int \frac{x}{p^2 e^x+(q^2-p^2)}\frac{e^x}{\sqrt{(e^x-1)}}\,dx = \frac{\pi}{pq}\,l\,\frac{p}{p+q}$ V. T. 181. N°. 10.

23) $\int \frac{x}{p^2 e^x-(p^2+q^2)}\frac{e^x}{\sqrt{(e^x-1)}}\,dx = -\frac{\pi}{pq}\,\mathit{Arctang.}\frac{q}{p}$ V. T. 181. N°. 11.

24) $\int \frac{\{a\sqrt{(e^{-x}-1)}-bi\}^{-p}+\{a\sqrt{(e^{-x}-1)}+bi\}^{-b}}{(e^{-x}-1)^{\frac{3-p}{2}}}\,x e^{-x}dx = \frac{4}{b}\frac{\pi}{p-1}\{a^{-p}-(a+b)^{-p}\}$ V.T.184. N° 18.

1) $\int e^{ix}(ix)^{p-1}\,dx = 2\,Sin.\,p\pi\,\Gamma(p)$

2) $\int e^{ix}(-ix)^{p-1}\,dx = 0$

3) $\int e^{ix}(r+ix)^{q-1}\,dx = \frac{\pi e^{-r}}{\Gamma(1-q)}$

4) $\int e^{ix}(r-ix)^{q-1}\,dx = 0$

5) $\int e^{ix}(ix)^{p-1}(-ix)^{q-1}\,dx = 2\,Sin.\,p\pi\,\Gamma(p+q-1)$

$, 0 < p < 1, 1 \geqq q > \ldots, r > 0;$ Cayley, L. 12. 231.

6) $\int e^{-x^2} x^2\,dx = \frac{1}{2}\sqrt{\pi}$

7) $\int e^{-px^2} x^2\,dx = \frac{1}{2p}\sqrt{\frac{\pi}{p}}$

Ohm, Ausw. 20.

8) $\int e^{-x^2} x^{2a}\,dx = \frac{1^{a/2}}{2^a}\sqrt{\pi}$ Fourier, Chal. 370. — Poisson, Chal. 75.

9) $\int e^{-x^2} x^{2a+1}\,dx = 0$ Poisson, Chal. 75.

10) $\int e^{-x^2+2px} x\,dx = p\,e^{p^2}\sqrt{\pi}$

11) $\int e^{-x^2+2px} x^2\,dx = \frac{1+2p^2}{2} e^{p^2}\sqrt{\pi}$

12) $\int e^{-px^2+2qx} x^{a+1}\,dx = \frac{\sqrt{\pi}}{2^a p\sqrt{p}}\frac{d^a}{dq^a}.q\,e^{\frac{q^2}{p}}$

13) $\int e^{-px^2+2qx} x\,dx = \frac{q}{p}\sqrt{\frac{\pi}{p}}\,e^{\frac{q^2}{p}}$

Dienger, Cr. 46. 119.

14) $\int e^{-px^2-qx} x^a\,dx = (-1)^a\left(\frac{q}{2p}\right)^a e^{\frac{q^2}{4p}}\sqrt{\frac{\pi}{p}}\sum_0^\infty \frac{a^{2n/-1}}{1^{n/1}}\left(\frac{p}{q^2}\right)^n$

15) $\int e^{(px^2+qx)i} x^a\,dx = (-1)^a(1+i)\left(\frac{q}{2p}\right)^a e^{-\frac{q^2 i}{4p}}\sqrt{\frac{\pi}{2p}}\sum_0^\infty \frac{a^{2n/-1}}{1^{n/1}}\left(\frac{pi}{q^2}\right)^n$

16) $\int e^{-(px^2+qx)i} x^a\,dx = (-1)^a(1-i)\left(\frac{q}{2p}\right)^a e^{\frac{q^2 i}{4p}}\sqrt{\frac{\pi}{2p}}\sum_0^\infty \frac{a^{2n/-1}}{1^{n/1}}\left(\frac{p}{q^2 i}\right)^n$

Cauchy, P. 19. 511.

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17) $\int e^{-e^x} x e^x dx = -A$ V. T. 273. N°. 1.

18) $\int e^{-qe^x} x e^x dx = -\frac{1}{q}(A + lq)$ V. T. 273. N°. 2.

19) $\int e^{-e^{2x}} x e^x dx = -\frac{1}{4}(A + l4)\sqrt{\pi}$ V. T. 273. N°. 3.

20) $\int e^{-qe^{2x}} x e^x dx = -\frac{1}{4}(A + l4q)\sqrt{\frac{\pi}{q}}$ V. T. 273. N°. 4.

F. Algébr. rat. ent. x. Expon. binôme en dén. **TABLE 143.** Lim. — ∞ et ∞.

1) $\int \frac{e^{px} x}{e^x + q} dx = \frac{\pi q^{p-1}}{Sin. p\pi}(lq - \pi\, Cot. p\pi), p < 1$; V. T. 180. N°. 1.

2) $\int \frac{e^{px} x}{e^x - 1} dx = (\pi\, Cosec. p\pi)^2, 0 < p < 1$; V. T. 183. N°. 1.

3) $\int \frac{x}{e^x - 1} \frac{dx}{e^{(p-1)x}} = (\pi\, Cosec. p\pi)^2$ V. T. 183. N°. 2.

4) $\int \frac{1 - e^{-x}}{1 - e^{2ax}} e^{-(a-1)x} x\, dx = \left(\frac{\pi}{2a} Tang. \frac{\pi}{2a}\right)^2, a > 2$; V. T. 180. N°. 14.

5) $\int \frac{1 - e^x}{1 - e^{2ax}} e^{-(a-2)x} x\, dx = \left(\frac{\pi}{2a} Tang. \frac{\pi}{2a}\right)^2, a > 2$; V. T. 180. N°. 15.

6) $\int \frac{1 - e^{2x}}{1 - e^{2bx}} e^{ax} x\, dx = \left(\frac{\pi}{2b}\right)^2 \frac{Sin. \left\{\frac{a+1}{b}\pi\right\} . Sin. \frac{\pi}{b}}{Sin.^2 \frac{a\pi}{2b} . Sin.^2 \left\{\frac{a+2}{2b}\pi\right\}}$ V. T. 180. N°. 16.

7) $\int \frac{x}{e^x + e^{-x}} dx = 0$ V. T. 180. N°. 2.

8) $\int \frac{x}{qe^{-x} + e^x} dx = \frac{\pi}{2q} l\frac{1}{q}$ V. T. 180. N°. 8.

9) $\int \frac{x}{p^2 e^{-x} + q^2 e^x} dx = \frac{\pi}{2pq} l\frac{q}{p}$ V. T. 180. N°. 10.

10) $\int \frac{x}{e^x - e^{-x}} dx = \frac{1}{4}\pi^2$ V. T. 180. N°. 11.

11) $\int \frac{e^{-px} x}{e^x - e^{-x}} dx = \left(\frac{1}{2}\pi \, Cosec. \left\{\frac{p+1}{2}\pi\right\}\right)^2, \ p^2 < 1;$ V. T. 180. N°. 12.

12) $\int \frac{1 - e^{-\frac{2x}{b}}}{e^x - e^{-x}} x \, dx = -\left(\frac{1}{2}\pi \, Tang. \frac{\pi}{b}\right)^2, \ b > 2;$ V. T. 185. N°. 7.

13) $\int \frac{1 - e^{\frac{2x}{b}}}{e^x - e^{-x}} x \, dx = -\left(\frac{1}{2}\pi \, Tang. \frac{\pi}{b}\right)^2, \ b > 2;$ V. T. 185. N°. 8.

14) $\int \frac{1 - e^{px}}{e^x - e^{-x}} x \, dx = -\left(\frac{1}{2}\pi \, Tang. \frac{1}{2} p \pi\right)^2, \ p < 1;$ V. T. 183. N°. 4.

15) $\int \frac{1 - e^{\frac{2x}{b}}}{e^x - e^{-x}} e^{\left(\frac{a}{b}-1\right)x} x \, dx = \frac{1}{4}\pi^2 \frac{Sin. \left\{\frac{a+1}{b}\pi\right\} . Sin. \frac{\pi}{b}}{Sin.^2 \frac{a\pi}{b} . Sin.^2 \left\{\frac{a+2}{b}\pi\right\}}$ V. T. 185. N°. 9.

1) $\int \frac{x e^x}{(q + e^x)^{p+1}} dx = \frac{1}{p q^p} \{l q - A - Z'(p)\}$ V. T. 182. N°. 4.

2) $\int \frac{x e^x}{(q + e^x)^2} dx = \frac{1}{q} l q$ V. T. 182. N°. 1.

3) $\int \frac{x e^x}{(q + e^x)^{b+\frac{1}{2}}} dx = \frac{2}{2b-1} q^{\frac{1}{2}-b} \left\{l(q-1) + 2 l 2 - \sum_1^{b-2} \frac{1}{n} - 2 \sum_{b-1}^{2b-3} \frac{1}{n}\right\}$ V. T. 185. N°. 10.

4) $\int \frac{x e^x}{(a^2 + b^2 e^{2x})^p} dx = \frac{\Gamma(p-\frac{1}{2})\sqrt{\pi}}{4 a^{2p-1} b \Gamma(p)} \left\{2 l \frac{a}{2p} - A - Z'(p-\frac{1}{2})\right\}$ V. T. 182. N°. 6.

5) $\int \left(\frac{1}{e^x + e^{-x}}\right)^{2a+1} x \, dx = 0$ V. T. 183. N°. 7.

6) $\int \frac{x}{(q^2 e^{-x} + e^x)^{2a+1}} dx = \frac{1^{a/2}}{2^{a/2}} \frac{\pi l q}{2 q^{2a+1}}$ V. T. 183. N°. 8.

7) $\int \frac{x}{(q^2 e^{-x} + e^x)^{2a}} dx = \frac{1^{a/1}}{2^{a/1}} \frac{l q}{2 q^{2a}}$ V. T. 183. N°. 9.

8) $\int \frac{x}{(q^2 e^x + e^{-x})^p} dx = \frac{l q}{2 q^p} \frac{\{\Gamma(\frac{1}{2}p)\}^2}{\Gamma(p)}$ V. T. 183. N°. 10.

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9) $\displaystyle\int\frac{x\,e^{-x}}{(a+e^x)^{b+1}}\,dx = \frac{1}{b\,a^b}\left\{-l\,a+\sum_1^a\frac{1}{n}\right\}$ V. T. 180. N°. 7.

10) $\displaystyle\int\frac{x}{e^x+q}\,\frac{dx}{e^{-x}+1} = \frac{(l\,q)^2}{2\,(q+1)}$ V. T. 183. N°. 12.

11) $\displaystyle\int\frac{x}{q\,e^{-x}+1}\,\frac{dx}{e^x-1} = \frac{\pi^2+(l\,q)^2}{2\,(q+1)}$ V. T. 183. N°. 14.

12) $\displaystyle\int\frac{e^{(p-1)x}}{e^x+q}\,\frac{x}{e^x+1}\,dx = \frac{\pi}{q-1}\,\frac{q^p\,l\,q\,Sin.\,p\,\pi-(1-q^p)\,\pi\,Cos.\,p\,\pi}{Sin.^2\,p\,\pi}$, $p^2<1$; V. T. 183. N°. 13.

13) $\displaystyle\int\frac{e^{px}}{q\,e^{-x}+1}\,\frac{x}{e^x-1}\,dx = \frac{\pi}{1+q}\,\frac{\pi+q^p\,(Sin.\,p\,\pi\,l\,q-\pi\,Cos.\,p\,\pi)}{Sin.^2\,p\,\pi}$, $p^2<1$; V. T. 183. N°. 15.

F. Algébr. rat. ent. x^a.
Expon. polynôme en dén. **TABLE 145.** Lim. $-\infty$ et ∞.

1) $\displaystyle\int\left(\frac{x}{e^{\frac{1}{2}x}-e^{-\frac{1}{2}x}}\right)^2dx = \frac{2}{3}\pi^2$ V. T. 182. N°. 3.

2) $\displaystyle\int\frac{e^x-e^{-x}}{(e^x+e^{-x})^2}\,x^2\,dx = 0$ V. T. 143. N°. 7.

3) $\displaystyle\int\frac{e^x+e^{-x}}{(e^x-e^{-x})^2}\,x^2\,dx = 0$ V. T. 143. N°. 10.

4) $\displaystyle\int\frac{p^2\,e^x-q^2\,e^{-x}}{(p^2\,e^x+q^2\,e^{-x})^2}\,x^2\,dx = \frac{\pi}{p\,q}\,l\,\frac{p}{q}$ V. T. 143. N°. 9.

5) $\displaystyle\int\frac{p+(1-p)\,e^{-x}}{(1-e^{-x})^2}\,e^{-px}\,x^2\,dx = 2\,\pi^2\,Cosec.^2\,p\,\pi$ V. T. 143. N°. 8.

6) $\displaystyle\int\frac{e^x-q^2\,e^{-x}}{(q^2\,e^{-x}+e^x)^{2a}}\,x^2\,dx = \frac{1^{a-1/2}}{2^{a-1/2}}\,\frac{4\,l\,q}{q^{2a-1}}\,\frac{\pi}{2\,a+1}$ V. T. 144. N°. 6.

7) $\displaystyle\int\frac{e^x-q^2\,e^{-x}}{(q^2\,e^{-x}+e^x)^{2a+1}}\,x^2\,dx = \frac{1^{a/1}}{2^{a/2}}\,\frac{l\,q}{2\,a\,q^{2a}}$ V. T. 144 N°. 7.

8) $\displaystyle\int\frac{q^2\,e^x-e^{-x}}{(q^2\,e^x+e^{-x})^{p+1}}\,x^2\,dx = \frac{l\,q}{q^p}\,\frac{\{\Gamma(\frac{1}{2}p)\}^2}{\Gamma\,(p+1)}$ V. T. 144. N°. 8.

9) $\displaystyle\int\frac{x^2}{e^x-1}\,\frac{dx}{1+q\,e^{-x}} = \frac{\pi^2+(l\,q)^2}{6\,(1+q)}\,l\,q$ V. T. 184. N°. 1.

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10) $\int \frac{x - lq}{e^x - 1} \frac{x}{1 - q e^{-x}} dx = \frac{4\pi^2 + (lq)^2}{6(q-1)} lq$ V. T. 184. N°. 8.

11) $\int \frac{x - lq}{e^x - 1} \frac{x e^{px}}{1 - q e^{-x}} dx = \frac{\pi^2}{q - 1} \frac{(q^p + 1) lq - 2\pi (q^p - 1) Cot. p\pi}{Sin.^2 p\pi}, p^2 < 1;$ V. T. 184. N°. 7.

12) $\int \frac{x^3}{e^x - 1} \frac{dx}{1 + q e^{-x}} = \frac{\{\pi^2 + (lq)^2\}^2}{24(1+q)}$ V. T. 184. N°. 2.

13) $\int \frac{x^4}{e^x - 1} \frac{dx}{1 + q e^{-x}} = \frac{(\pi^2 + (lq)^2\}^2}{360} \frac{7\pi^2 + 3(lq)^2}{1 + q} lq$ V. T. 184. N°. 3.

14) $\int \frac{x^5}{e^x - 1} \frac{dx}{1 + q e^{-x}} = \frac{\{\pi + (lq)^2\}^2}{720} \frac{\{3\pi^2 + (lq)^2\}^2}{1 + q}$ V. T. 184. N°. 4.

15) $\int \frac{e^x - q e^{-x}}{(e^x + q)^2} \frac{x^2}{(1 + e^{-x})^2} dx = \frac{1}{q+1} (lq)^2$ V. T. 144. N°. 10.

16) $\int \frac{e^x + q e^{-x}}{(q e^{-x} + 1)^2} \frac{x^2}{(1 - e^x)^2} dx = \frac{1}{q+1} \{\pi^2 + (lq)^2\}$ V. T. 144. N°. 11.

17) $\int \frac{x^{2a+1}}{e^x + e^{-x}} dx = 0$ V. T. 180. N°. 3.

18) $\int \frac{x^{2a}}{e^x + e^{-x}} dx = 2 . 1^{2a/1} \sum_0^\infty (-1)^n \frac{1}{(2n+1)^{2a+1}}$ V. T. 180. N°. 4.

19) $\int \frac{x^{2a}}{e^{\frac{1}{4}bx} + e^{-\frac{1}{4}bx}} dx = \frac{(-1)^{a+1}}{b} (4\pi)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{2n-1}{4b}\right)$ V. T. 185. N°. 12.

20) $\int \frac{e^{qx} x^a}{e^x + e^{-x}} dx = \frac{1}{2} \pi (-1)^a \frac{d^a}{dq^a} . Sec. \frac{1}{2} q\pi$ V. T. 180. N°. 6.

21) $\int \frac{e^{bx} + e^{-bx}}{e^x + e^{-x}} x^{2a} dx = \frac{(-1)^{a+1}}{b} 2 \left(\frac{2\pi}{b}\right)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{2n-1}{4b}\right) Cos. \left\{\frac{2n-1}{2} a\pi\right\}$ V. T. 184. N°. 5.

22) $\int \frac{e^{bx} - e^{-bx}}{e^x - e^{-x}} x^{2a} dx = \frac{(-1)^{a+1}}{b} 2 \left(\frac{2\pi}{b}\right)^{2a+1} \sum_1^b (-1)^{n-1} B'' \left(\frac{n}{2b}\right) Sin. n a \pi$ V. T. 184. N°. 6.

23) $\int \frac{x^2}{e^x + 2 Cos. \lambda + e^{-x}} dx = \lambda Cosec. \lambda \frac{\pi^2 - \lambda^2}{3}$ V. T. 184. N°. 9.

24) $\int \frac{x^4}{e^x + 2 Cos. \lambda + e^{-x}} dx = 2\lambda \frac{\pi^2 - \lambda^2}{5} \frac{7\pi^2 - 3\lambda^2}{Sin. \lambda}$ V. T. 184. N°. 10.

25) $\int \frac{x^{2a}}{e^x + e^{-x} + 1} dx = \frac{(-1)^{a+1}}{\sqrt{3}} 2 (2\pi)^{2a+1} B'' \left(\frac{1}{3}\right)$ V. T. 184. N°. 11.

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26) $\int \frac{x^{2a}}{e^x + e^{-x} - 1} dx = \frac{(-1)^{a+1}}{\sqrt{3}} 2 (2\pi)^{2a+1} B''\left(\frac{1}{6}\right)$ V. T. 184. N°. 12.

27) $\int \frac{x^{2a}}{e^x + e^{-x} - 2 \, Cos. \, 2 p \pi} dx = (-1)^{a+1} (2\pi)^{a+1} \frac{B''(p)}{Sin. \, 2 p \pi}, \; 0 < p < 1$; Raabe, Cr. 42. 348.

F. Algébr. rat. fract. Expon. } Dén. à fact. x^a. TABLE 146. Lim. — ∞ et ∞.

1) $\int e^{-x^2 - \frac{p}{4x^2}} \frac{dx}{x^2} = \frac{2}{p} e^{-p} \sqrt{\pi}$ Meyer, Int. Déf. 152.

2) $\int e^{-px^2 - \frac{q}{x^2}} \frac{dx}{x^2} = e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{q}}$

3) $\int e^{-px^2 - \frac{q}{x^2}} \frac{dx}{x^{2a}} = \left(\frac{p}{q}\right)^{\frac{a}{2}} e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{p}} \sum_0^\infty \frac{(a-n)^{2n/1}}{1^{n/1}} \left(\frac{1}{4pq}\right)^n$

4) $\int e^{\left(px^2 + \frac{q}{x^2}\right)i} \frac{dx}{x^{2a}} = \left(\frac{p}{q}\right)^{\frac{a}{2}} e^{2i\sqrt{pq}} (1+i) \sqrt{\frac{\pi}{2p}} \sum_0^\infty \frac{a^{2n/-1}}{1^{n/1}} \left(\frac{i}{4pq}\right)^n$

5) $\int e^{-\left(px^2 + \frac{q}{x^2}\right)i} \frac{dx}{x^{2a}} = \left(\frac{p}{q}\right)^{\frac{a}{2}} e^{-2i\sqrt{pq}} (1-i) \sqrt{\frac{\pi}{2p}} \sum_0^\infty \frac{a^{2n/-1}}{1^{n/1}} \left(\frac{1}{4pqi}\right)^n$

} Cauchy, P. 19. 151.

6) $\int \frac{e^{-pxi}}{q^2 + x^2} \frac{dx}{x^r} = \frac{\pi e^{-pq} e^{\frac{1}{2} r \pi i}}{q^{r+1}}$

7) $\int \frac{e^{-pxi}}{q^2 + x^2} \frac{dx}{x^{2a}} = (-1)^a \frac{\pi}{q^{2a+1}} e^{-pq}$

8) $\int \frac{e^{-pxi}}{q^2 + x^2} \frac{dx}{x^{2a-1}} = (-1)^{a-1} \frac{\pi i}{q^{2a}} e^{-pq}$

} Meyer, Int. Déf. 274.

9) $\int \frac{e^{-pxi}}{1 + x^2} \frac{dx}{(xi)^{1-q}} = (-1)^{q-1} \pi e^p$

10) $\int \frac{e^{-pxi}}{1 - x^2} \frac{dx}{(xi)^{1-q}} = -\frac{1}{2} \pi \, Cos. \left(\frac{1}{2} q \pi - p\right)$

} $, 0 < q < 1$; Meyer, Int. Déf. 156.

11) $\int \frac{e^{(p-1)x} - e^{(q-1)x}}{e^x + e^{-x}} \frac{dx}{x} = \pi \, l \left(Tang. \frac{1}{4} p \pi . Cot. \frac{1}{4} q \pi\right)$ V. T. 180. N°. 7.

12) $\int \frac{e^{(p-1)x} - e^{(q-1)x}}{e^x - e^{-x}} \frac{dx}{x} = \pi \, l \left(Sin. \frac{1}{2} p \pi . Cosec. \frac{1}{2} q \pi\right)$ V. T. 180. N°. 18.

13) $\int \frac{e^{-px} - e^{-qx}}{1 + e^{-2qx}} \frac{dx}{x} = l\, Cot. \frac{p\pi}{4q}$ V. T. 183, N°. 17.

14) $\int \frac{e^{-px} - e^{-qx}}{1 + e^{-rx}} \frac{dx}{x} = l \left(Tang. \frac{q\pi}{2r} . Cot. \frac{p\pi}{2r} \right)$ V. T. 183. N°. 18.

F. Algébr. rat. fract. Expon. } Dén. sans fact. x^a. TABLE 147. Lim. — ∞ et ∞.

1) $\int \frac{e^{xi}}{q + xi} dx = 2\pi e^{-q}$ Poisson, P. 19. 404. N°. 68. — Liouville, Cr. 13. 219.

2) $\int \frac{(-xi)^p}{q + xi} e^{xi} dx = 2\pi q^p e^{-q}$ } Cayley, L. 12. 231.

3) $\int \frac{(xi)^p}{q + xi} e^{-xi} dx = 0$ }

4) $\int \frac{e^{-pxi}}{(q + xi)^q} dx = 0$ Ohm, Ausw. 23.

5) $\int \frac{e^{xi}}{(q + xi)^p} dx = \frac{2\pi e^{-q}}{\Gamma(p)}$ Poisson, P. 19. 404. N°. 73. — Laplace, Prob. 33. — Liouville, Cr. 13. 219. — Lobatschewsky, Mém. Kasan. 1835. 211.

6) $\int \frac{e^{pxi}}{(q + xi)^r} dx = \frac{2\pi}{\Gamma(r)} p^{r-1} e^{-pq}$ Cauchy, Lim. Imag. 101. — Id., Exerc. 1827. p. 141.

7) $\int \frac{e^{pxi}}{(q - xi)^r} dx = 0$ Cauchy, Lim. Imag. 105. — Id., Exerc. 1827. p. 141.

8) $\int \frac{e^{pxi}}{1 + x^2} dx = \pi e^{-p}$ } Cauchy, Cours. Leç. 39.

9) $\int \frac{e^{pxi}}{1 + x^2} (-xi)^{q-1} dx = \pi e^{-p}, q < 1;$ }

10) $\int \frac{e^{-pxi}}{q^2 + x^2} dx = \frac{\pi}{q} e^{-pq}$ Lejeune-Dirichlet, Cr. 4. 94. — Schlömilch, Stud. II. 17.

11) $\int \frac{e^{pxi}}{q^2 + x^2} dx = \frac{\pi}{q} e^{-pq}$ Minding, Taf. 6.

12) $\int \frac{e^{(p-r)xi}}{q^2 + x^2} dx = \frac{\pi}{q} e^{(p-r)q}$, pour $0 < r < p$; } Ohm, Ausw. 23.

13) $= \frac{\pi}{q} e^{(r-p)q}$, pour $p < r < \infty$; }

14) $\int (-x i)^p e^{xi} \frac{dx}{q^2+x^2} = \pi q^{p-1} e^{-q}$ Cayley, L. 12. 231.

15) $\int (x i)^{p+1} e^{-rxi} \frac{dx}{q^2+x^2} = \pi q^p e^{-rq}$ Cauchy, P. 19. 511.

16) $\int \frac{e^{-pxi}}{(s+xi)^r} \frac{dx}{q^2+x^2} = \frac{\pi}{q} \frac{e^{-pq}}{(q+s)^r}$

17) $\int \frac{e^{-pxi}}{(a+xi)^r (b+xi)^s \ldots\ldots} \cdot \frac{dx}{q^2+x^2} = \frac{\pi e^{-pq}}{q} \frac{1}{(a+q)^r (b+q)^s \ldots\ldots}$

, où a, b,... peuvent être aussi des fractions;

} Lejeune-Dirichlet, Cr. 4. 94. — Schlömilch, Stud. II. 27.

18) $\int \frac{e^{-pxi}}{(a+xi)^s (b+xi)^t \ldots} \frac{dx}{q^2+r^2x^2} = \frac{\pi}{q} e^{\frac{-pq}{r}} (q+ar)^{-s} (q+br)^{-t} \ldots$ Ohm, Ausw. 93.

, où a, b ... peuvent être aussi des fractions;

19) $\int (x i)^{r+1} e^{-pxi} \frac{dx}{q^2-x^2} = \pi q^r \, Cos. \left\{\frac{r+2}{2}\pi - pq\right\}$ Cauchy, P. 19. 511.

F. Algébr. irrat. Expon. TABLE 148. Lim. — ∞ et ∞.

1) $\int e^{-px} dx \sqrt{x} = \frac{1}{2p} \sqrt{\frac{\pi}{p}}$ Ohm, Ausw. 20.

2) $\int e^{xi} \frac{dx}{\sqrt{(q+xi)}} = 2 e^{-q} \sqrt{\pi}$

3) $\int e^{q+xi} \frac{dx}{\sqrt{(q+xi)}} = 2\sqrt{\pi}$

4) $\int e^{q+xi+\frac{pi}{4(q+xi)}} \frac{dx}{\sqrt{(q+xi)}} = (e^{\sqrt{pi}} + e^{-\sqrt{pi}}) \sqrt{\pi}$

5) $\int e^{q+xi+\frac{pi}{4(q+xi)}} \frac{dx}{\sqrt{(q+xi)}} = \frac{4}{2-\sqrt{pi}} e^{-\sqrt{pi}} \sqrt{\pi}$

} Cauchy, P. 19. 511.

F. Algébr. Expon. TABLE 149. Lim. diverses 0 et p.

1) $\int_0^{\frac{1}{\infty}} \frac{k}{k^2+(b-x)^2} x^{p-1} e^{-qx} dx = \frac{\pi}{2\Gamma(p)} b^{p-1} e^{-bq}$, où il faut mettre $k=0$ après l'intégration; Cauchy, P. 28. 147. P. 1. 3.

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2) $\int_{-1}^{0} \frac{e^{-x}}{x} dx = -\infty$ V. T. 45. N°. 7.

3) $\int_{-\infty}^{0} \frac{e^{-x}}{x} dx = -\infty$ V. T. 45. N°. 9.

4) $\int_{0}^{\frac{a}{c}} \frac{e^{(bc-1)x} - e^{-(bc+1)x}}{x} dx = \frac{1}{2} l \left(\frac{1+bc}{1-bc}\right)^2 + Ei.\left(-\frac{a}{c}+ab\right) - Ei.\left(-\frac{a}{c}-ab\right)$ Arndt, Gr. 11. 70.

5) $\int_{0}^{l2} (e^x - 1)^{a-1} x e^x dx = \frac{1}{a}\left\{l2 + (-1)^a \sum_1^\infty \frac{1}{a+n+1}\right\}$ V. T. 151. N°. 11.

6) $\int_{0}^{l2} \frac{x}{1-e^{-x}} dx = \frac{1}{12}\pi^2$ V. T. 160. N°. 1.

7) $\int_{0}^{l2} \frac{e^x x^2}{(e^x-1)^2} dx = \frac{1}{6}\pi^2 - 2(l2)^2$ V. T. 149. N°. 6.

8) $\int_{0}^{l2} \frac{e^x - e^{-x}}{(e^{\frac{1}{2}x} - e^{-\frac{1}{2}x})^4} x^2 dx = \frac{\pi}{4} l2 - 2(l2)^2$ V. T. 149. N°. 9

9) $\int_{0}^{l2} \frac{x}{e^x + e^{-x} - 2} dx = \frac{1}{8}\pi l2$ V. T. 160. N°. 2.

10) $\int_{0}^{l(1+p)} (1+x) e^x dx = (1+p) l(1+p)$ V. T. 42. N°. 4.

11) $\int_{0}^{l\frac{1+p}{1-p}} \frac{x}{(p^2-1)(e^x+e^{-x}) - 2(p^2+1)} dx = -\frac{1}{p} Arcsin.p,\ p^2 < 1;$ V. T. 186. N°. 2.

12) $\int_{0}^{l\frac{1+p}{1-p}} \frac{e^x - 1}{(p^2-1)(e^{2x}+1)+2(p^2+1)e^x} \cdot \frac{x e^x}{\sqrt{\{(p^2-1)(e^{2x}+1)+2(p^2+1)e^x\}}} dx = \frac{\pi}{2p} Arcsin.p, p \leqq 1;$ V. T. 160. N°. 17.

13) $\int_{0}^{l\frac{1+p}{1-p}} \frac{1-e^x}{(p^2-q^2)(1+e^{2x}) + 2(p^2+q^2)e^x} \cdot \frac{x e^x dx}{\sqrt{\{(p^2-1)(e^{2x}+1)+2(p^2+1)e^x\}}}$

$= \frac{\pi}{2pq\sqrt{(1-q^2)}} l \frac{pq + \{1-\sqrt{(1-q^2)}\}\{1-\sqrt{(1-p^2)}\}}{pq - \{1-\sqrt{(1-q^2)}\}\{1-\sqrt{(1-p^2)}\}}$ V. T. 166. N°. 17.

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14) $\int_0^{2l\,Cot.\frac{1}{2}\lambda} \frac{x}{\sqrt{\{2(1+Cos.^2\lambda)e^x - Sin.^2\lambda(1+e^{2x})\}}} \frac{e^x}{1-e^x} dx = \frac{1}{4}\pi \frac{\pi - 2\lambda}{Cos.\lambda}$ V. T. 166. N°. 7.

15) $\int_0^q \frac{e^{-px}}{x-q} dx = -\infty$ Arndt, Gr. 10. 247.

16) $\int_0^p (e^{-kqx} - e^{-krx}) \frac{dx}{x} = l\frac{r}{q}$, pour $k = \infty$; Schlömilch, Gr. 11. 63.

1) $\int_1^\infty e^{-\frac{x}{q}} \frac{dx}{\sqrt{(x-1)}} = \frac{\sqrt{q\pi}}{\sqrt[q]{e}}$ Legendre, Exerc. 3. 50.

2) $\int_{-\infty}^{-1} \frac{e^{-x}}{x} dx = -\infty$ V. T. 45. N°. 8.

3) $\int_{-p}^\infty \frac{e^{-x}}{x} dx = -\frac{1}{2} l\,\alpha^2 - li.(p)$, où α est indéterminé; Arndt, Gr. 10. 247.

4) $\int_a^\infty \frac{e^{-x}}{x} dx = -A - l\,a - \sum_1^\infty (-1)^n \frac{a^n}{n\,1^{n/1}}$ Clausius, Cr. 34. 123.

5) $\qquad = -Ei.(-a)$ Beez, Gr. 19. 419.

6) $\int_q^\infty \frac{e^{-x}}{x^2} dx = Ei.(-q) + \frac{1}{q} e^{-q}$ V. T. 150. N°. 8.

7) $\int_p^\infty \frac{e^{-x}}{x^a} dx = \frac{(-1)^a}{1^{a/1}} \left\{A - \sum_1^{a-1} \frac{1}{n}\right\} + \sum_1^{a-1} \frac{1}{1^{n-1/1}} \frac{(-1)^n}{(a-n)p^{a-n}} + \frac{(-1)^{a-1}}{1^{a-1/1}} \left\{lp + \sum_1^\infty \frac{(-p)^n}{a^{n/1}}\right\}$ Arndt, Gr. 10. 233.

8) $\int_p^\infty \frac{e^{\frac{1}{2}qx}}{e^{qx} - e^{-qx}} \frac{dx}{x} = \frac{1}{2pq} + \frac{1}{\pi} \sum_1^\infty (-1)^n \frac{1}{2n} Arctang. \frac{2n\pi}{pq} + \frac{1}{2\pi} \sum_0^\infty \frac{(-1)^n}{2n+1} l \left\{1 + \left(\frac{2n+1}{pq}\pi\right)^2\right\}$

9) $\int_p^\infty \frac{1}{e^{\frac{1}{2}qx} - e^{-\frac{1}{2}qx}} \frac{dx}{x} = \frac{1}{pq} + \frac{2}{\pi} \sum_1^\infty \frac{(-1)^n}{2n} Arctang. \frac{2n\pi}{pq}$

10) $\int_p^\infty \frac{1}{e^{\frac{1}{2}qx} + e^{-\frac{1}{2}qx}} \frac{dx}{x} = \frac{1}{\pi} \sum_0^\infty \frac{(-1)^n}{2n+1} l \left\{1 + \left(\frac{2n+1}{pq}\pi\right)^2\right\}$

(8–10: Raabe, Int. 419.)

11) $\int_q^\infty \frac{x}{\sqrt{(e^x - q^2)}} \frac{e^x}{e^x - q^2 + 1} dx = 2\pi l(1+q)$ V. T. 181. N°. 2.

12) $\int_q^\infty \frac{e^{-px}}{x-q} dx = \infty$ Arndt, Gr. 10. 247.

13) $\int_{lp}^\infty \frac{e^{-x}}{x^2} dx = li(p) - \frac{1}{p\, lp}$ V. T. 150. N°. 14.

14) $\int_{-lp}^\infty \frac{e^{-x}}{x} dx = -li(p)$ V. T. 45. N°. 5.

15) $\int_{-\frac{1}{2}Cot.^2\lambda}^\infty \frac{e^{-a^2 x}}{\sqrt{(Cos.^2 \lambda + 2x\, Sin.^2 \lambda)}} dx = \frac{1}{2a\, Sin.\lambda} e^{\frac{1}{2}a^2 Cot.^2 \lambda} \sqrt{2\pi}$ Vieille, Exerc. p. 165.

1) $\int x^{2a}\, lx\, dx = \frac{1}{(2a+1)^2}$ Arndt, Gr. 6. 187.

2) $\int x^{p-1} \left(l\frac{1}{x}\right)^a dx = \frac{1^{a/1}}{p^{a+1}}$ Euler, Calc. Int. 4. S. 3 § 7. — Legendre, Mém. Inst. 1809. 416. N°. 41. — Id., Exerc. 2. 40. — Id., Exerc. 4. 147.

3) $\int x^p (lx)^a dx = (-1)^a \frac{1^{a/1}}{(1+p)^{a+1}}$ Euler, N. C. Petr. 14. 129. — Id., Calc. Int. 4. S. 5. N°. 13. — Oettinger, Cr. 35. 13.

4) $\int x^{q+ri-1} \left(l\frac{1}{x}\right)^{p-1} dx = \frac{\Gamma(p)}{(q+ri)^p}$ V. T. 113. N°. 17.

5) $\int x^{p-1} dx \sqrt{\left(l\frac{1}{x}\right)} = \frac{1}{2p}\sqrt{\frac{\pi}{p}}$ V. T. 139. N°. 2.

6) $\int (1-x)^{q-1} x^{p-1}\, lx\, dx = \frac{Z'(p) - Z'(p+q)}{\Gamma(p+q)} \Gamma(p)\Gamma(q)$ Raabe, Int. 228. — Féaux, Funct. Transc. p. 36.

6') $= -\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \sum_0^q \frac{1}{n+p-1}$, pour q entier; Feaux, Funct. Transc. p. 36.

7) $\int (1-x^q)^a x^{p-1} (lx)^b dx = (-1)^{a+b} 1^{b/1} \Delta^a \frac{1}{q p^{b+1}}$

8) $= (-1)^b 1^{b/1} \sum_0^\infty \binom{a}{n} \frac{(-1)^n}{(p+nq)^{b+1}}$

Oettinger, Cr. 38. 162. (7, 8)

9) $\int (x-1)^c x^{b-1} \left(l\frac{1}{x}\right)^{q-1} dx = \Gamma(q)\,\Delta^c . b^{-q}$ Legendre, Exerc. 4. 147.

10) $\int \left\{\left(l\frac{1}{x}\right)^{q-1} - x^{p-1}(1-x)^{q-1}\right\} dx = \frac{\Gamma(p+q)-\Gamma(p)}{q\,\Gamma(p+q)}\,\Gamma(1+q)$ V. T. 113. N°. 9.

11) $\int l(1+x)\,x^{a-1}\,dx = \frac{1}{a}\left\{l2 + (-1)^a \sum_0^\infty \frac{1}{a+1+n}\right\}$ V. T. 3. N°. 2.

12) $\int l(1-x)\,x\,dx = -\frac{3}{4}$ Euler, N. C. Petr. 14. 129.

13) $\int l(1+x^2)\,x^{p-1}\,dx = \frac{1}{2p}\left\{2\,l2 + Z'\left(\frac{p+1}{4}\right) - Z'\left(\frac{p+3}{4}\right)\right\}$ V. T. 3. N°. 13.

14) $\int (p\,x^{p-1} - q\,x^{q-1})\,l(1-x^2)\,dx = Z'\left(\frac{1}{2}q+1\right) - Z'\left(\frac{1}{2}p+1\right)$ V. T. 3. N°. 14.

15) $\int l(q+lx)\,x^{p-1}\,dx = \frac{1}{p}\left\{e^{-pq}\,Ei.(pq) + lq\right\}$ V. T. 169. N°. 1.

16) $\int l(q-lx)\,x^{p-1}\,dx = \frac{1}{p}\left\{e^{pq}\,Ei.(-pq) + lq\right\}$ V. T. 169. N°. 2.

F. Alg. rat. fract.. Dén. môn. ou bin. Logar. en num. lx. TABLE 152. Lim. 0 et 1.

1) $\int lx \frac{dx}{x^{a-1}} = -\frac{1}{a^2}$ Euler, Mém. Petersb. 1814.

2) $\int lx \frac{dx}{1+x} = -\frac{1}{2}\sum_1^\infty \frac{1}{n^2}$ Euler, Calc. Int. 4. S. 5. § 47. — Id., N. C. Petr. 19. 66.

3) $\int \qquad = -\frac{1}{12}\pi^2$ Euler, Calc. Int. 4. S. 5. § 78. — Kausler, Mém. Petersb. T. 3. — Plana, Mém. Turin. 1818. 7. IV. 21.

4) $\int lx \frac{x}{1+x}\,dx = -1 + \frac{1}{12}\pi^2$ V. T. 42. N°. 1, et T. 152. N°. 3. — Kausler, Mém. Petersb. T. 3. p. 414. trouve faut. $\frac{1}{12}\pi^2$.

5) $\int lx \frac{x^2}{1+x}\,dx = \frac{3}{4} - \frac{1}{12}\pi^2$ V. T. 42. N°. 1, et T. 152. N°. 4. — Kausler, Mém. Petersb. T. 3. p. 114. trouve faut. $-\frac{1}{12}\pi^2$.

6) $\int lx \frac{dx}{1-x} = -\sum_1^\infty \frac{1}{n^2}$ Euler, Calc. Int. 4. S. 5. § 47. — Id., N. C. Petr. 19. 66.

7) $\qquad = -\frac{1}{6}\pi^2$ Euler, Calc. Int. 4. S. 3. § 78. — Plana, Mém. Turin. 1818. 7. IV. 21. — Schaeffer, Cr. 30. 277.

8) $\int l\,x \frac{x}{1-x}\,dx = 1 - \frac{1}{6}\pi^2$ V. T. 42. N°. 1 et T. 152. N°. 7.

9) $\int l\,x \frac{1+x}{1-x}\,dx = 1 - 2\sum_1^\infty \frac{1}{n^2}$ Euler, N. C. P. 14. 129.

10) $\int l\,x \frac{x^{p-1}}{1-x}\,dx = -\sum_0^\infty \frac{1}{(p+n)^2}$ Binet, P. 27. 123.

11) $\int l\,x \frac{dx}{1+x^2} = -\sum_0^\infty (-1)^n \frac{1}{(2n+1)^2}$ Bidone, Mém. Turin. 1812. 231. Art. 13. N°. 37.

12) $\int l\,x \frac{x}{1+x^2}\,dx = -\frac{1}{48}\pi^2$ Euler, Calc. Int. 4. S. 5. 49. — Id., N. C. P. 19. 66.

13) $\int l\,x \frac{dx}{1-x^2} = -\frac{1}{8}\pi^2$ Euler, Calc. Int. 4. S. 3. 78. — Id., N. C. P. 19. 30. — Poisson, Mém. Inst. 1811. 163. N°. 27. (fautiv. $-\frac{1}{8}\pi$) — Plana, Mém. Turin. 1818. 7. IV. 21.

14) $\int l\,x \frac{x}{1-x^2}\,dx = -\frac{1}{24}\pi^2$ Euler, Calc. Int. 4. S. 3. 78. — Plana, Mém. Turin. 1818. 7. IV. 21.

15) $\int l\,x \frac{1-x}{1+x^3}\,dx = -\frac{2}{27}\pi^3$ Euler, Calc. Int. 4. S. 3. 80. — Id., N. C. P. 19. 30.

16) $\int l\,x \frac{x}{1-x^4}\,dx = -\frac{1}{32}\pi^2$ V. T. 152. N°. 12, 14.

17) $\int l\,x \frac{x^3}{1-x^4}\,dx = -\frac{1}{96}\pi^2$ V. T. 152. N°. 12, 14.

18) $\int l\,x \frac{x^{a-1}}{1-x^{2a}}\,dx = -\frac{\pi^2}{8a^2}$ Euler, Calc. Int. 4. S. 3. 77. — Id., N. C. P. 19. 30.

19) $\int l\,x \frac{1-x^2}{1+x^{2p}} x^{p-2}\,dx = -\frac{1}{4}\left(\frac{\pi}{p}\right)^2 Sin. \frac{\pi}{2p} . Sec.^2 \frac{\pi}{2p}$

20) $\int l\,x \frac{1+x^2}{1-x^{2p}} x^{p-2}\,dx = -\frac{1}{4}\left(\frac{\pi}{p}\right)^2 Sec.^2 \frac{\pi}{2p}$

} Euler, Calc. Int. 4. S. 3. 80. — Id., N. C. P. 19. 30.

21) $\int l\left(\frac{1}{x}\right) \frac{x^{a-1}+x^{b-a-1}}{1-x^b}\,dx = \left(\frac{\pi}{b}\right)^2 Cosec.^2 \frac{a\pi}{b}$ Legendre, Exerc. 2. 44. — Id., Mém. Inst. 1809. 416. N°. 45.

1) $\int lx \frac{1+2x}{1+x+x^2} dx = -\frac{1}{9}\pi^2$

2) $\int lx \frac{1-2x}{1-x+x^2} dx = -\frac{1}{18}\pi^2$

Euler, Calc. Int. 4. S. 3. 105. — Id., ib. S. 5. 50. — Id., N. C. P. 19. 30.

3) $\int lx \frac{dx}{1-x+x^2} = -\frac{4}{27}\pi^2$ Euler, Calc. Int. 4. S. 3. 80. — Id., N. C. P. 19. 30. — Legendre, Mém. Inst. 1809. 416. N°. 51.

4) $\int lx \frac{x}{1-x+x^2} dx = -\frac{5}{108}\pi^2$ V. T. 153. N°. 2, 3.

5) $\int lx \frac{1-x^2}{1+(e^{2p}+e^{-2p})x^2+x^4} dx = \frac{p}{2}\frac{\pi}{e^p-e^{-p}}$ V. T. 125. N°. 5.

6) $\int lx \frac{Cos.\lambda - x}{1-2x\,Cos.\lambda+x^2} dx = \frac{1}{6}\pi^2 - \frac{1}{2}\pi\lambda + \frac{1}{4}\lambda^2$ Euler, N. C. P. 19. 66. — Id., Calc. Int. 4. S. 5. 46.

7) $\int lx \frac{x}{1-x^2+x^4} dx = -\frac{1}{27}\pi^2$ Euler, Calc. Int. 4. S. 3. 80. — Id., N. C. P. 19. 30.

8) $\int lx \frac{1-x^2}{1+2px^2+x^4} dx = \frac{\pi}{2\sqrt{\{2(p-1)\}}} l \frac{\sqrt{(p-1)}-\sqrt{(p+1)}+\sqrt{2}}{\sqrt{(p-1)}+\sqrt{(p+1)}-\sqrt{2}}$ V. T. 125. N°. 4.

9) $\int lx \frac{1-x^2}{1+2x^2\,Cos.2\lambda+x^4} dx = -\frac{1}{4}\pi\lambda\,Cosec.\lambda$ V. T. 125. N°. 6.

10) $\int lx \frac{1-x^2}{1+x^2}\frac{dx}{x} = -\infty$

11) $\int lx \frac{1+x^2}{1-x^2}\frac{dx}{x} = -\infty$

Euler, N. C. P. 19. 30. — Id., Calc. Int. 4. S. 3. 81.

12) $\int lx \frac{x^{p-q}+x^{p+q}}{1+x^{2p}}\frac{dx}{x} = \frac{\pi^2}{4p^2} Sin.\frac{q\pi}{2p}.\,Sec.^2\frac{q\pi}{2p}$

13) $\int lx \frac{x^{p-q}-x^{p+q}}{1-x^{2p}}\frac{dx}{x} = -\frac{\pi^2}{4p^2} Sec.^2\frac{q\pi}{2p}$

Euler, Calc. Int. 4. S. 3. 74. — Id., N. C. P. 19. 30.

14) $\int lx \frac{dx}{(1+x)^2} = \frac{1}{2} l \frac{1}{2}$ V. T. 122. N°. 1.

15) $\int lx \frac{x}{(1+x^2)^2} dx = \frac{1}{4} l \frac{1}{2}$ V. T. 335. N°. 1.

16) $\int lx \frac{x^2}{1-x^2}\frac{dx}{1+x^4} = -\frac{\pi^2}{16(2+\sqrt{2})}$ Euler, Calc. Int. 4. S. 3. 80. — Id., N. C. P. 19. 30.

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17) $\int l\,x \frac{(p+q)(x^{p-q}-x^{q-p})+(p-q)(x^{p+q}-x^{-(p+q)})}{(x^p+x^{-p})^2}\frac{dx}{x} = \frac{\pi}{2p}\,Sec.\frac{q\pi}{2p}\,,\; p>q;$ V. T. 5. N°. 18.

18) $\int l\,x \frac{(p+q)(x^{p-q}-x^{q-p})+(q-p)(x^{p+q}-x^{-(p+q)})}{(x^p-x^{-p})^2}\frac{dx}{x} = -\frac{\pi}{2p}\,Tang.\frac{q\pi}{2p}\,,\; p>q;$ V. T. 5. N°. 17.

19) $\int l\,x \frac{x^p-x^{-p}}{(x^p+x^{-p})^2}\frac{dx}{x} = \frac{\pi}{4p^2}$ V. T. 5. N°. 23.

20) $\int l\,x \frac{1-x^2}{1+x^2}\frac{dx}{x\left(x+\frac{1}{x}\right)^{2p}} = \frac{\{\Gamma(p)\}^2}{8p\,\Gamma(2p)}$ V. T. 5. N°. 24.

21) $\int \left\{\frac{1+p\,l\,x}{1-x}+\frac{x\,l\,x}{(1-x)^2}\right\} x^{p-1}\,dx = -1$ Arndt, Gr. 10. 253.

F. Alg. rat. fract. à dén. binôme. Logar. en num. $(lx)^2$ et $(lx)^3$. TABLE 154. Lim. 0 et 1.

1) $\int (l\,x)^2 \frac{dx}{1+x^2} = \frac{1}{16}\pi^3$

2) $\int (l\,x)^2 \frac{1+x^2}{1+x^4}\,dx = \frac{3\sqrt{2}}{64}\pi^3$

Euler, Calc. Int. 4. S. 3. 84. — Id., ib. S. 5. 49. — Id., N. C. P. 19. 30.

3) $\int (l\,x)^2 \frac{1-x^4}{1-x^6}\,dx = \frac{1}{27}\pi^3\sqrt{3}$

4) $\int (l\,x)^2 \frac{1-x^2}{1-x^6}\,x\,dx = \frac{1}{243}\pi^3\sqrt{3}$

Legendre, Mém. Inst. 1809. 416. N°. 50. — Id., Exerc. 2. 49.

5) $\int (l\,x)^2 \frac{x^{p-q-1}+x^{p+q-1}}{1+x^{2p}}\,dx = \frac{\pi^3}{8p^3}\left(2\,Sec.^3\frac{q\pi}{2p} - Sec.\frac{q\pi}{2p}\right)$

6) $\int (l\,x)^2 \frac{x^{p-q-1}-x^{p+q-1}}{1-x^{2p}}\,dx = \frac{\pi^3}{4p^3}\,Sin.\frac{q\pi}{2p}.\,Sec.^3\frac{q\pi}{2p}$

Euler, Calc. Int. 4. S. 3. 82. — Id., N. C. P. 19. 30.

7) $\int (l\,x)^2 \frac{x^{q-1}-x^{p-q-1}}{1-x^p}\,dx = 2\left(\frac{\pi}{p}\,Cosec.\frac{q\pi}{p}\right)^3.\,Cos.\frac{q\pi}{p}$

8) $\int (l\,x)^3 \frac{x^{q-1}+x^{p-q-1}}{1-x^p}\,dx = \left(\frac{\pi}{p}\,Cosec.\frac{q\pi}{p}\right)^4\left(2+4\,Cos.^2\frac{q\pi}{p}\right)$

Legendre. Exerc. 2. 44. — Id., Mém. Inst. 1809. 416. N°. 45.

9) $\int (l\,x)^3 \frac{dx}{1+x} = -\frac{21}{4}\sum_1^\infty \frac{1}{n^4}$ Euler, Calc. Int. 4. S. 5. 47. — Id., N. C. P. 19. 66.

F. Alg. rat. fract. à dén. binôme. Logar. en num. $(lx)^2$ et $(lx)^3$. TABLE 154 suite. Lim. 0 et 1.

10) $\int (lx)^3 \frac{dx}{1+x} = -\frac{7}{120}\pi^4$

11) $\int (lx)^3 \frac{dx}{1-x} = -\frac{1}{15}\pi^4$

Euler, Calc. Int. 4. S. 3. 96. — Id., N. C. P. 19. 30.

12) $= -6 \sum_1^\infty \frac{1}{n^4}$ Euler, Calc. Int. 4. S. 5. 47. — Id., N. C. P. 19. 66.

13) $\int (lx)^3 \frac{x}{1+x^2} dx = -\frac{7}{1920}\pi^4$ Euler, Calc. Int. 4. S. 5. 49. — Id., N. C. P. 19. 66.

14) $\int (lx)^3 \frac{dx}{1-x^2} = -\frac{1}{16}\pi^4$ Euler, Calc. Int. 4. S. 3. 95. — Id., N. C. P. 19. 30.

15) $\int (lx)^3 \frac{x}{1-x^2} dx = -\frac{1}{240}\pi^4$ V. T. 154. N°. 10, 12.

16) $\int (lx)^3 \frac{x}{1-x^4} dx = -\frac{1}{256}\pi^4$ V. T. 154. N°. 13, 15.

17) $\int (lx)^3 \frac{x^3}{1-x^4} dx = -\frac{1}{3840}\pi^4$ V. T. 154. N°. 13, 15.

F. Alg. rat. fract. à dén. binôme. Log. en num. $(lx)^4, (lx)^5, (lx)^6, (lx)^7$. TABLE 155. Lim. 0 et 1.

1) $\int (lx)^4 \frac{dx}{1+x^2} = \frac{5}{64}\pi^5$ Euler, Calc. Int. 4. S. 3. 94. — Id., N. C. P. 19. 30.

2) $\int (lx)^5 \frac{dx}{1+x} = -\frac{31}{252}\pi^6$

3) $\int (lx)^5 \frac{dx}{1-x} = -\frac{8}{63}\pi^6$

Euler, Calc. Int. 4. S. 3. 97. — Id., N. C. P. 19. 30.

4) $\int (lx)^5 \frac{dx}{1-x^2} = -\frac{1}{8}\pi^6$ Euler, Calc. Int. 4. S. 3. 95. — Id., N. C. P. 19. 30.

5) $\int (lx)^5 \frac{x}{1-x^2} dx = -\frac{1}{504}\pi^6$ V. T. 155. N°. 2, 3.

6) $\int (lx)^5 \frac{dx}{1+x} = -\frac{465}{4}\sum_1^\infty \frac{1}{n^6}$

7) $\int (lx)^5 \frac{dx}{1-x} = -120\sum_1^\infty \frac{1}{n^6}$

Euler, Calc. Int. 4. S. 5. 47. — Id., N. C. P. 19. 66

F. Alg. rat. fract. à dén. binôme. Log. en num. $(lx)^4, (lx)^5, (lx)^6, (lx)^7$.	TABLE 155 suite.	Lim. 0 et 1.

8) $\int (lx)^6 \frac{dx}{1+x^2} = \frac{61}{256}\pi^7$ Euler, Calc. Int. 4. S. 3. 94. — Id., N. C. P. 19. 30

9) $\int (lx)^7 \frac{dx}{1+x} = -\frac{127}{1680}\pi^8$ Euler, Calc. Int. 4. S. 5. 47.

10) $\int (lx)^7 \frac{dx}{1-x^2} = -\frac{17}{32}\pi^8$ Euler, N. C. P. 19. 30.

F. Alg. rat. fract. à dén. trinôme. Log. en num. $(lx)^a$ pour a spécial.	TABLE 156.	Lim. 0 et 1.

1) $\int (lx)^2 \frac{dx}{1+x+x^2} = \frac{8}{243}\pi^3 \sqrt{3}$

2) $\int (lx)^2 \frac{dx}{1-x+x^2} = \frac{10}{243}\pi^3 \sqrt{3}$

(1, 2) Euler, Calc. Int. 4. S. 3. 105. — Id., N. C. P. 19. 30. — Legendre, Exerc. 2. 49. — Id., Mém. Inst. 1809. 416. N°. 50.

3) $\int (lx)^2 \frac{dx}{1+2x\,Cos.\lambda+x^2} = \frac{1}{2}\lambda\, Cosec.\lambda \frac{\pi^2-\lambda^2}{3}$ Legendre, Exerc. 4. 105.

4) $\int (lx)^2 \frac{dx}{1-2x\,Cos.\lambda+x^2} = 2\lambda\, Cosec.\lambda \left(\frac{1}{6}\pi^2 - \frac{1}{4}\pi\lambda + \frac{1}{12}\lambda^2\right)$ Euler, N. C. P. 19. 66.

5) $\int (lx)^4 \frac{dx}{1+2x\,Cos.\lambda+x^2} = \frac{1}{5}\lambda\, Cosec.\lambda\, (\pi^2-\lambda^2)(7\pi^2-3\lambda^2)$ Legendre, Exerc. 4. 105.

F. Alg. rat. fract. à dén. binôme $x \pm b$. Log. en num. $(lx)^a$ pour a général.	TABLE 157.	Lim. 0 et 1.

1) $\int (lx)^{2a} \frac{dx}{1 \pm x} = 0$ (fautif) Euler, Calc. Int. 4. S. 5. 47.

2) $\int (lx)^{2a} \frac{dx}{1+x} = \frac{2^{2a}-1}{2^{2a}} 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}}$

3) $\int (lx)^{2a} \frac{dx}{1-x} = 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}}$

(2, 3) Raabe, Cr. 42. 348.

4) $\int (lx)^{2a-1} \frac{dx}{1+x} = 1^{2a-1/1} \left(\frac{1}{2^{2a-1}} - 1\right) \sum_1^\infty \frac{1}{n^{2a}}$ Euler, Calc. Int. 4. S. 5. 47.

5) $= -\frac{2^{2a-1}-1}{2a}\pi^{2a} B_{2a-1}$ Arndt, Gr. 6. 434.

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6) $\int (l\,x)^{2a-1} \frac{dx}{1-x} = -\frac{2^{2a-2}}{a} \pi^{2a} B_{2a-1}$ Arndt, Gr. 6. 434.

7) $= -1^{2a-1/1} \sum_1^\infty \frac{1}{n^{2a}}$ Euler, Calc. Int. 4. S. 5. 47.

8) $\int \left(l\frac{1}{x}\right)^{a-1} \frac{dx}{1+x} = 1^{a/1} \sum_0^\infty \frac{(-1)^n}{(1+n)^a}$ Arndt, Gr. 6. 434.

9) $\int \left(l\frac{1}{x}\right)^{a-1} \frac{dx}{1-x} = 1^{a/1} \sum_0^\infty \frac{1}{(1+n)^a}$ Euler, N. C. P. 14. 129. — Arndt, Gr. 6. 434.

10) $\int \left(l\frac{1}{x}\right)^{p+1} \frac{dx}{1-qx} = \frac{p+1}{q} \Gamma(p+1) \sum_0^\infty \frac{q^n}{n^{p+2}}$ V. T. 117. N°. 11.

11) $\int \left(l\frac{1}{x}\right)^{b-1} \frac{x^a}{1+x} dx = 1^{b/1} \sum_0^\infty \frac{(-1)^n}{(a+n+1)^b}$, $b > 1$; Arndt, Gr. 6. 434.

12) $\int \left(l\frac{1}{x}\right)^{b-1} \frac{x^a}{1-x} dx = 1^{b/1} \sum_0^\infty \frac{1}{(a+n+1)^b}$ Binet, P. 27. 123. — Arndt, Gr. 6. 434.

13) $\int \left(l\frac{1}{x}\right)^q x^{b-1} (x-1)^c \left(b + \frac{cx}{x-1}\right) dx = \Gamma(q) \Delta^c . b^{-q}$ V. T. 118. N°. 18.

14) $\int \left(l\frac{1}{x}\right)^{a-1} \frac{1-x^b}{1-x} dx = 1^{a/1} \sum_1^b \frac{1}{n^a}$ Euler, N. C. Petr. 14. 129.

F. Alg. rat. fract. à autre dén. binôme. Log. en num. $(lx)^a$ pour a général. TABLE 158. Lim. 0 et 1.

1) $\int (l\,x)^a \frac{dx}{1+x^2} = (-1)^a 1^{a/1} \sum_1^\infty \frac{(-1)^n}{(2n+1)^{a+1}}$ Bidone, Mém. Turin. 1812. 231. Art. 3. 37.

2) $\int (l\,x)^{2a} \frac{dx}{1+x^2} = \frac{(-1)^{a+1}}{2} (2\pi)^{2a+1} B''\left(\frac{1}{4}\right)$

3) $\int (l\,x)^{2a} \frac{x^b + x^{-b}}{1+x^2} dx = \frac{(-1)^{a+1}}{b} \left(\frac{2\pi}{b}\right)^{2a+1} \sum_1^b (-1)^{n+1} B''\left(\frac{2n-1}{4b}\right) Cos.\left(\frac{2n-1}{2} b\pi\right)$

4) $\int (l\,x)^{2a} \frac{dx}{1-x^2} = \frac{2^{2a+1}-1}{2^{2a+1}} 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}}$

(2–4) Raabe, Cr. 42. 348.

5) $\int \left(l\frac{1}{x}\right)^{2a-} \frac{dx}{1-x^2} = \frac{2^{2a}-1}{4a} \pi^{2a} B_{2a-1}$ V. T. 120. N°. 18.

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6) $\int (l\,x)^{2a-1} \frac{x}{1-x^2}\,dx = (-1)^a \frac{\pi^{2a}}{4a} B_{2a-1}$ Plana, Mém. Turin. 1820.

7) $\int (l\,x)^{p-1} \frac{dx}{1-x^2} = \frac{\Gamma(p)}{(-1)^{p-1}} \sum_0^\infty \frac{1}{(2n+1)^p}$ V. T. 336. N°. 17.

8) $\int (l\,x)^{p-1} \frac{x^q}{1-x^2}\,dx = \frac{\Gamma(p)}{(-1)^{p-1}} \sum_0^\infty \frac{1}{(q+1+2n)^p}$ V. T. 336. N°. 18.

9) $\int (l\,x)^{2a} \frac{x^{-b}-x^b}{1-x^2}\,dx = \frac{(-1)^{a+1}}{b}\left(\frac{2\pi}{b}\right)^{2a+1} \sum_1^b (-1)^{n-1} B''\left(\frac{n}{2b}\right) Sin.\, n b \pi$ Raabe, Cr. 42. 348.

10) $\int \left(l\frac{1}{x}\right)^r \frac{x^{p-1}}{1-x^q}\,dx = 1^{r/1} \sum_0^\infty \frac{1}{(p+nq)^{r+1}}$ Oettinger, Cr. 38. 162.

11) $\int (l\,x)^{2a} \frac{x^{b-1}-x^{2c-b-1}}{1-x^{2c}}\,dx = \frac{(-1)^{a+1}}{c}(2\pi)^{2a+1} \sum_1^{c-1} B''\left(\frac{n}{2c}\right) Sin. \frac{n b \pi}{c}$

12) $\int (l\,x)^{2a} \frac{x^{-1}+x^{2c-b-1}}{1+x^{2c}}\,dx = \frac{(-1)^{a+1}}{c}(2\pi)^{2a+1} \sum_1^{c} B''\left(\frac{2n-1}{4c}\right) Sin.\left(\frac{2n-1}{2c} b\pi\right)$

(11, 12) Raabe, Cr. 42. 348.

13) $\int \frac{x^p - x^{-p}}{1-x^2} (l\,x)^{2a}\,dx = \left(\frac{\pi}{2}\right)^{2a+1} \frac{d^{2a}}{d p^{2a}}. Tang.\, 2 p\pi, p < 1;$ V. T. 121. N°. 15.

14) $\int \left(l\frac{1}{x}\right)^{2a-1} \frac{1+x}{1-x}\frac{dx}{x} = \frac{2^{2a-1}}{a} \pi^{2a} B_{2a-1}$ V. T. 118. N°. 15.

15) $\int (lx)^{2a} \frac{dx}{x^{1+q}+x^{1-q}} = \frac{1}{2}(-1)^{a+1}\left(\frac{2\pi}{q}\right)^{2a+1} B''\left(\frac{1}{4}\right)$ V. T. 120. N°. 15.

16) $\int (l\,x)^{2a-1} \frac{dx}{x^{1+q}-x^{1-q}} = \frac{1}{4}(-1)^a \left(\frac{2\pi}{q}\right)^{2a} B'\left(\frac{1}{2}\right)$ V. T. 120. N°. 19.

F. Alg. rat. fract. à dén. trinôme. Log. en num. $(l x)^a$ pour a général. TABLE 159. Lim. 0 et 1.

1) $\int (l\,x)^{2a} \frac{dx}{1+x+x^2} = \frac{(-1)^{a+1}}{\sqrt{3}}(2\pi)^{2a+1} B''\left(\frac{1}{3}\right)$

2) $\int (l\,x)^{2a} \frac{dx}{1-x+x^2} = \frac{(-1)^{a+1}}{\sqrt{3}}(2\pi)^{2a+1} B''\left(\frac{1}{6}\right)$

(1, 2) Raabe, Cr. 42. 348.

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3) $\displaystyle\int (l\,x)^{2a} \frac{d\,x}{1+x^2-2\,x\,Cos.\lambda} = Cosec.\lambda \left[1^{2a/1} \left\{ \lambda \sum_1^\infty \frac{1}{n^{4a}} - \frac{\lambda^3}{1.2.3} \sum_1^\infty \frac{1}{n^{4a-2}} + \ldots \right\} \right.$

$\displaystyle\left. + \frac{(-1)^a}{2a+1} \lambda^{2a-1} \left\{ 2a(2a+1) \sum_1^\infty \frac{1}{n^2} - \frac{2a+1}{2} \lambda\pi + \frac{\lambda^2}{2} \right\} \right]$

4) $\displaystyle\int (l\,x)^{2a+1} \frac{Cos.\lambda + x}{1+x^2-2x\,Cos.\lambda} dx = 1^{2a+1/1} \left\{ \sum_1^\infty \frac{1}{n^{4a+2}} - \frac{\lambda^2}{1.2} \sum_1^\infty \frac{1}{n^{4a}} + \frac{\lambda^4}{1.2.3.4} \sum_1^\infty \frac{1}{n^{4a-2}} - \ldots \right\}$

$\displaystyle + \frac{(-1)^a}{2a+2} \lambda^{2a} \left\{ (2a+1)(2a+2) \sum_1^\infty \frac{1}{n^2} - \frac{2a+2}{2} \lambda\pi + \frac{1}{2}\lambda^2 \right\}$

(3 et 4) Euler, Calc. Int. 4. S. 4. 46.

5) $\displaystyle\int (l\,x)^{2b} \frac{d\,x}{1+x^2-2\,x\,Cos.2p\pi} = \frac{(-1)^{b+1}}{2} (2\pi)^{2b+1} Cosec.2p\pi . B''(p), 0 < p < 1;$ Raabe, Cr. 42. 348.

6) $\displaystyle\int (l\,x)^{2b-1} \frac{Cos.2p\pi - x}{1+x^2-2\,x\,Cos.2p\pi} d\,x = \frac{(-1)^a}{2} (2\pi)^{2b} B'(p) - \frac{(2\pi)^{2b}}{4b} B_{2b-1}$ V. T. 125. N°. 9.

7) $\displaystyle\int \left(l\frac{1}{x} \right)^{r-1} \frac{p\,Cos.\lambda - p^2\,x}{1-2\,p\,x\,Cos.\lambda + p^2 x^2} x^{q-1}\, d\,x = \Gamma(r) \sum_1^\infty \frac{p^n\,Cos.\,n\lambda}{(q+n-1)^r}$ Kummer, Cr. 17. 210.

1) $\displaystyle\int l(1+x) \frac{d\,x}{x} = \frac{1}{12}\pi^2$ Ohm, Ausw. 16.

2) $\displaystyle\int l(1+x) \frac{d\,x}{1+x^2} = \frac{1}{8}\pi\,l2$ Bertrand, L. 8. 110. — Serret, L. 9. 436. — Grunert, Gr. 4. 113. — Id., Gr. 6. 448. — Hill, Cr. 3. 102.

3) $\displaystyle\int l(1+x) \frac{(p-1)\,x^{p-1} - p\,x^{-p}}{x} d\,x = 2\,l2 - \pi\,Cosec.p\pi, p < 1$ V. T. 5. N°. 1.

4) $\displaystyle\int l(1+p\,x) \frac{1-x^2}{1+x^2} d\,x = \frac{1}{2} \frac{(1+p)^2}{1+p^2} l(1+p) - \frac{1}{2} \frac{p}{1+p^2} l2 - \frac{\pi}{4} \frac{p^2}{1+p^2}$ V. T. 6. N°. 1.

5) $\displaystyle\int l(1-x) \frac{d\,x}{x} = -\frac{1}{6}\pi^2$ Euler, N. C. Petr. 14. 129. — Schaeffer, Cr. 30. 277.

6) $\displaystyle\int l(1+x^2) \frac{d\,x}{x} = \frac{1}{24}\pi^2$ V. T. 152. N°. 12.

7) $\displaystyle\int l(1+x^2) \frac{d\,x}{1+x^2} = \frac{1}{2}\pi\,l2 - \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 157. N. 21.

8) $\int l(1+x^2)\frac{dx}{(1+px)^2} = \frac{1}{p}\left\{\frac{p}{1+p^2}\, l\frac{1+p}{\sqrt{2}} - \frac{1}{1+p}\, l\,2 + \frac{1}{4}p^2\pi\right\}$ V. T. 6. N°. 1.

9) $\int l(1+x^2)\frac{dx}{x(1+x^2)} = \frac{1}{2}\left\{(l\,2)^2 - \frac{1}{12}\pi^2\right\}$ V. T. 160. N°. 17.

10) $\int l(1-x^2)\frac{dx}{x} = -\frac{1}{12}\pi^2$ Ohm, Ausw. 16.

11) $\int l(1-x^4)\frac{dx}{x} = -\frac{1}{24}\pi^2$ V. T. 152. N°. 17.

12) $\int l(1+x+x^2)\frac{dx}{x} = \frac{1}{9}\pi^2$ V. T. 153. N°. 1.

13) $\int l(1-x+x^2)\frac{dx}{x} = \frac{1}{18}\pi^2$ V. T. 153. N°. 2.

14) $\int l(1-2x\,Cos.\lambda+x^2)\frac{dx}{x} = \frac{1}{3}\pi^2 - \pi\lambda + \frac{1}{2}\lambda^2$ V. T. 153. N°. 6.

15) $\int l\frac{1+x}{1-x}\,\frac{dx}{x} = \frac{1}{4}\pi^2$ Ohm, Ausw. 16.

16) $\int l\frac{1-x^2\,Cot\,hp.^2\lambda}{1+x^2\,Cot\,hp.^2\lambda}\,\frac{dx}{1-(1-x^2)\,Cos\,hp.^2\lambda} = \frac{2\lambda\, l\, Sin\,hp.\lambda}{Sin\,hp.\lambda.\,Cos\,hp.\lambda}$ V. T. 343 N°. 12.

17) $\int l\frac{1+p\sqrt{(1-x^2)}}{1-p\sqrt{(1-x^2)}}\,\frac{dx}{1+x^2} = \pi\,Arcsin.\,p\,, p \leqq 1$; Raabe, Int. 421.

18) $\int l\frac{1+Cos.\mu\sqrt{(1-x^2)}}{1-Cos.\mu\sqrt{(1-x^2)}}\,\frac{dx}{Tang.^2\lambda+x^2} = 2\pi Cot.\lambda.\, l\left[Cos.\left\{\frac{1}{2}(\lambda-\mu)\right\}.Cosec.\left\{\frac{1}{2}(\lambda+\mu)\right\}\right]$ V. T. 343. N°. 20.

19) $\int l\frac{1+Sin.\lambda\sqrt{(1-x^2)}}{1-Sin.\lambda\sqrt{(1-x^2)}}\,\frac{dx}{1-x^2} = \pi\lambda$ V. T. 340. N°. 10.

20) $\int l\frac{x+\sqrt{(1-x^2)}}{x-\sqrt{(1-x^2)}}\,\frac{x}{1-x^2}\,dx = \frac{1}{4}\pi^2$ V. T. 340. N°. 14.

1) $\int (lx)^2 . l(1+x)\frac{dx}{x} = \frac{7}{360}\pi^4$ V. T. 154. N°. 10.

2) $\int (lx)^2 . l(1-x)\frac{dx}{x} = -\frac{1}{45}\pi^4$ V. T. 154. N°. 11.

3) $\int (lx)^2 . l(1+x^2)\frac{dx}{x} = \frac{7}{2880}\pi^4$ V. T. 154. N°. 13.

4) $\int (lx)^2 . l(1-x^2)\frac{dx}{x} = -\frac{1}{360}\pi^4$ V. T. 154. N°. 15.

5) $\int (lx)^2 . l(1-x^4)\frac{dx}{x} = -\frac{1}{2880}\pi^4$ V. T. 154. N°. 17.

6) $\int (lx)^4 . l(1+x)\frac{dx}{x} = \frac{31}{1260}\pi^6$ V. T. 155. N°. 2.

7) $\int (lx)^4 . l(1-x)\frac{dx}{x} = -\frac{8}{315}\pi^6$ V. T. 155. N°. 3.

8) $\int (lx)^4 . l(1-x^2)\frac{dx}{x} = -\frac{1}{2560}\pi^6$ V. T. 155. N°. 5.

9) $\int (lx)^{2a} . l(1+x)\frac{dx}{x} = \frac{2^{2a+1}-1}{(2a+1)(2a+2)}\pi^{2a+2}\,B_{2a+1}$ V. T. 157. N°. 5.

10) $\int (lx)^{2a} . l(1-x)\frac{dx}{x} = \frac{-2^{2a}}{(a+1)(2a+1)}\pi^{2a+2}\,B_{2a+1}$ V. T. 157. N°. 6.

11) $\int (lx)^{2a} . l(1-x^2)\frac{dx}{x} = \frac{-1}{(2a+1)(2a+2)}\pi^{2a+2}\,B_{2a+1}$ V. T. 158. N°. 6.

12) $\int (lx)^{2a} . l(1-2x\,Cos.2p\pi+x^2)\frac{dx}{x} = \frac{(2\pi)^{2a+2}}{2a+1}\left\{(-1)^{a+1}B'(p)-\frac{1}{2^{a+2}}B_{2a+1}\right\}$, $p<1$; Raabe, Cr. 42. 348.

13) $\int \left(l\frac{1}{x}\right)^{r-1} . l(1-2px\,Cos.\lambda+p^2x^2)\frac{dx}{x} = \frac{2\Gamma(r+1)}{r}\sum_1^\infty \frac{p^n\,Cos.n\lambda}{(n+1)^{r+1}}$ V. T. 159. N°. 7.

14) $\int \left(l\frac{1}{x}\right)^{a-1} . l(1+x)\frac{dx}{x} = \frac{1^{a+1/1}}{a}\sum_0^\infty \frac{(-1)^n}{(1+n)^{a+1}}$ V. T. 157. N°. 8.

15) $\int \left(l\frac{1}{x}\right)^{a-1} . l(1-x)\frac{dx}{x} = -\frac{1^{a+1/1}}{a}\sum_0^\infty \frac{1}{(1+n)^{a+1}}$ V. T. 157. N°. 9.

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16) $\int\left(l\frac{1}{x}\right)^p . l(1-qx)\frac{dx}{x} = -\Gamma(p+1)\sum_0^\infty \frac{q^n}{n^{q+1}}$ V. T. 157. N°. 10.

17) $\int\left(l\frac{1}{x}\right)^{a-1} l(1-x^2)\frac{dx}{x} = \frac{1^{a+1/1}}{2^{a+1}}\sum_0^\infty \frac{1}{(1+n)^{a+1}}$ V. T. 157. N°. 8, 9.

18) $\int l(1+x^2)\frac{2x^2 lx-(1+x^2)}{(1+x^2)^2}\frac{dx}{x} = -\frac{1}{2}l2$ V. T. 153. N°. 15.

19) $\int l(1-x^2)\frac{(1-3x^4)lx+(1+x^4)}{(1+x^4)^2}dx = \frac{-\pi^2}{8(2+\sqrt{2})}$ V. T. 153. N°. 16.

F. Algébr. irrat. ent. Logar. en numér. TABLE 162. Lim. 0 et 1.

1) $\int lx\,dx\sqrt{(1-x^2)} = -\frac{1}{4}\pi\left(\frac{1}{2}+l2\right)$

2) $\int x\,lx\,dx\sqrt{(1-x^2)} = -\frac{1}{3}\left(\frac{3}{4}-l2\right)$

Euler, Calc. Int. 4. S. 3. 152, 154. — Id., Act. Petr. 1777. II. 3.

3) $\int lx\,dx\sqrt{(1-x^2)^{2a-1}} = -\frac{1^{a/2}}{2^{a+1}\,1^{a/1}}\frac{\pi}{2}(A+Z'(a+1)+2l2)$ Lindmann, Stockh. Handl. 1850. III.

4) $\int x^{p-1}\left(l\frac{1}{x}\right)^{a-\frac{1}{2}}dx = \frac{1^{a/2}}{(2p)^a}\sqrt{\frac{\pi}{p}}$ V. T. 114. N°. 8.

5) $\int l(1+p^2x^2)dx\sqrt{(1-x^2)} = \frac{1}{2}\pi\left\{l\frac{1+\sqrt{(1+p^2)}}{2}+\frac{\sqrt{(1+p^2)}-1}{p^2}-\frac{1}{2}\right\}$ V. T. 335. N°. 6.

6) $\int l(1+p^2x^2-p^2)dx\sqrt{(1-x^2)} = \frac{1}{2}\pi\left\{l\frac{1+\sqrt{(1-p^2)}}{2}-\frac{1}{2}\frac{1-\sqrt{(1-p^2)}}{1+\sqrt{(1-p^2)}}\right\}$ V. T. 335. N°. 5.

F. Algébr. irrat. fract. Logar. en num. lx. TABLE 163. Lim. 0 et 1.

1) $\int lx\frac{dx}{\sqrt{(1-x^2)}} = -1-\sum_1^\infty \frac{1^{n/2}}{2^{n/2}}\frac{1}{(2n+1)^2}$ Arndt, Gr. 6. 187.

2) $= -\frac{1}{2}\pi l2$ Euler, Calc. Int. 4. S. 3. 117. — Id., N. C. Petr. 14. 129. — Id., Act. Petr. 1777. II. 3. — Id., Mém. Pétersb. T. 6. p. 30. — Legendre, Exerc. 2. 43. — Id., Mém. Inst. 1809. 416. N°. 44. — Kausler, Mém. Pétersb. T. 3. — Oettinger, Cr. 38. 162. — Arndt, Gr. 6. 187.

3) $\int lx\frac{x}{\sqrt{(1-x^2)}}dx = l2-1$ Euler, Calc. Int. 4. S. 3. 144, 164. — Id, Act. Petr. 1777. II. 3. — Kausler, Mém. Pétersb. T. 3. — Oettinger, Cr. 38. 162.

4) $\int lx \frac{x^2}{\sqrt{(1-x^2)}} dx = -\frac{\pi}{4}\left(l2 - \frac{1}{2}\right)$

5) $\int lx \frac{x^3}{\sqrt{(1-x^2)}} dx = -\frac{2}{3}\left(\frac{5}{6} - l2\right)$

6) $\int lx \frac{x^4}{\sqrt{(1-x^2)}} dx = -\frac{3}{16}\pi\left(l2 - \frac{7}{12}\right)$

7) $\int lx \frac{x^5}{\sqrt{(1-x^2)}} dx = -\frac{8}{15}\left(\frac{47}{60} - l2\right)$

Euler, Calc. Int. 4. S. 3. 147, sqq. — Id., Act. Petr. 1777. II. 3. — Kausler, Mém. Pétersb. T. 3.

8) $\int lx \frac{dx}{\sqrt[3]{(1-x^2)}} = -\frac{\pi^2}{54} - \frac{5\pi}{18\sqrt{3}} l3$

9) $\int lx \frac{x}{\sqrt[3]{(1-x^2)}} dx = \frac{\pi^2}{54} - \frac{5\pi}{18\sqrt{3}} l3$

Euler, Mém. Pétersb. T. 6. p. 30.

10) $\int lx \frac{dx}{\sqrt[3]{(1-x^3)}} = -\frac{\pi}{3\sqrt{3}}\left(l3 + \frac{\pi}{3\sqrt{3}}\right)$

11) $\int lx \frac{x}{\sqrt[3]{(1-x^3)^2}} dx = -\frac{\pi}{3\sqrt{3}}\left(l3 - \frac{\pi}{3\sqrt{3}}\right)$

12) $\int lx \frac{x}{\sqrt{(1-x^4)}} dx = -\frac{1}{8}\pi l2$

13) $\int lx \frac{x^3}{\sqrt{(1-x^4)}} dx = \frac{1}{4}(l2 - 1)$

Euler, Calc. Int. 4. S. 3. 157, sqq. — Id., Act. Petr. 1777. II. 3. (où les formules 10 et 11 sont fautives).

14) $\int lx \frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}} = -\frac{1}{2} lp \,.\, F'(p) - \frac{1}{4}\pi F'\left\{\sqrt{(1-p^2)}\right\}$ V. T. 347. N°. 4.

15) $\int lx \frac{1}{(1-p)^2 - 4px^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{\pi}{2(1-p^2)} l\frac{1+p}{2}, \; p^2 < 1;$ V. T. 346. N°. 4.

16) $\int lx \frac{1+q-2qx^2}{(1+q)^2 - 4qx^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{4}\pi l \frac{1+q}{4}$ V. T. 346. N°. 5.

17) $\int lx \frac{1-q+2qx^2}{(1-q)^2 + 4qx^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{4}\pi l \frac{1-q}{4}, \; q^2 < 1;$ V. T. 346. N°. 1.

18) $\int l\frac{1}{x} \frac{x^{-\frac{1}{2}} + x^{-\frac{1}{2}}}{1-x} dx = 2\pi^2$ Poisson, Mém. Inst. 1811. 163. N°. 54.

19) $\int l\frac{1}{x} \frac{x^{a-1}}{\sqrt[b]{(1-x^b)^{b-c}}} dx = \sum_0^\infty \frac{(b-c)^{n/b}}{b^{n/b}} \frac{1}{(a+bn)^2}$ Legendre, Mém. Inst. 1809. 416. N°. 24.

F. Algébr. irrat. fract.
Log. en num. $(lx)^a$

TABLE 164.

Lim. 0 et 1.

1) $\int \left(l\frac{1}{x}\right)^2 \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2}\pi \left\{(l\,2)^2 + \frac{1}{12}\pi^2\right\}$ Legendre, Exerc. 2. 43. — Id., Mém. Inst. 1809. 416. N°. 44.

2) $\int \left(l\frac{1}{x}\right)^h \frac{x^{a-1}}{\sqrt[b]{(1-x^b)^{b-c}}}\,dx = 1^{h/1}\sum_0^\infty \frac{(b-c)^{n/b}}{b^{n/b}}\frac{1}{(nb+a)^{h+1}}$ Legendre, Mém. Inst. 1809. 416. N°. 24.

3) $\int \left(l\frac{1}{x}\right)^{2a-1} \frac{dx}{(1-x)\sqrt{x}} = \frac{2^{2a}-1}{4a}(2\pi)^{2a}\mathrm{B}_{2a-1}$ V. T. 120. N°. 20.

4) $\int (lx)^{2a}\frac{x^{\frac{1}{2}b-1}}{1+x^b}\,dx = \frac{(-1)^{a+1}}{2b}(4\pi)^{2a+1}\sum_1^b(-1)^{n-1}\mathrm{B}''\left(\frac{2n-1}{4b}\right)$

5) $\int (lx)^{2a}\frac{1+x^{b-c}}{1+x^b}x^{\frac{1}{2}c-1}\,dx = \frac{(-1)^{a+1}}{b}(2\pi)^{2a+1}\sum_1^b \mathrm{B}''\left(\frac{2n-1}{4b}\right)Sin.\left\{\frac{2n-1}{2b}c\pi\right\}$

6) $\int (lx)^{2a}\frac{1-x^{b-c}}{1-x^b}x^{\frac{1}{2}c-1}\,dx = \frac{(-1)^{a+1}}{b}(2\pi)^{2a+1}\sum_1^b \mathrm{B}''\left(\frac{n}{2b}\right)Sin.\frac{nc\pi}{b}$

(4–6) Raabe, Cr. 42. 348.

F. Algébr. irrat. fract.
Log. en num. de fonct. ent.

TABLE 165.

Lim. 0 et 1.

1) $\int l(1+x)\frac{dx}{\sqrt{(1-x^2)}} = \frac{\pi}{2}l\,2 - 2\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 11.

2) $\int l(1-x)\frac{dx}{\sqrt{(1-x^2)}} = \frac{\pi}{2}l\,2 + 2\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 12.

3) $\int l(1+px)\frac{dx}{x\sqrt{(1-x^2)}} = \frac{1}{8}\{\pi^2 - 4(Arccos.p)^2\}$, $p^2 < 1$; V. T. 339. N°. 28.

4) $\int l(r+px)\frac{x}{1-qx^2}\frac{dx}{\sqrt{(1-x^2)}} = \frac{\pi}{\sqrt{q(1-q)}}\,l\frac{p\sqrt{q}-\{1-\sqrt{(1-q)}\}\{r+\sqrt{(r^2-p^2)}\}}{p\sqrt{q}+\{1-\sqrt{(1-q)}\}\{r+\sqrt{(r^2-p^2)}\}}$, $q<1, p<r$; Lobatschewsky, Mém. Kasan. 1835. 1.

5) $\int l(1+x^2)\frac{dx\sqrt{(1-x^4)}}{x} = \frac{2\pi\sqrt{2\pi}}{\{\Gamma(\frac{1}{4})\}^2} - \frac{\{\Gamma(\frac{1}{4})\}^2}{2\sqrt{2\pi}}$ V. T. 12. N°. 9.

6) $\int l(1+qx^2)\frac{dx}{\sqrt{(1-x^2)}} = \pi\,l\frac{1+\sqrt{(1+q^2)}}{2}$ V. T. 334. N°. 8.

7) $\int l(1+x^2\,Tang.^2\lambda)\frac{dx}{\sqrt{(1-x^2)}} = \pi\,l\left(Cos.^2\frac{1}{2}\lambda.Sec.\lambda\right)$ V. T. 334. N°. 9.

8) $\int l(1+q^2x^2)\frac{x^2}{\sqrt{(1-x^2)}}dx = \frac{1}{2}\pi\left\{l\frac{1+\sqrt{(1+q^2)}}{2} + \frac{1-\sqrt{(1-q^2)}}{q^2} + \frac{1}{2}\right\}$, $q<1$; V. T. 335. N°. 7.

9) $\int l(1+px^2)\frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}}=\frac{1}{2}l\frac{2+2p}{\sqrt{p}}.F'(p)-\frac{1}{8}\pi F'\{\sqrt{(1-p^2)}\}, p^2<1;$ V. T. 348. N°. 10.

10) $\int l(1+x^2 Cot.^2\lambda)\frac{dx}{\sqrt{(1-x^2)(1-p^2x^2)}}=\pi F\{\sqrt{(1-p^2)},\lambda\}-2F'(p)Y\{\sqrt{(1-p^2)},\lambda\}-$
$-2F'(p)lSin.\lambda-\frac{1}{2}\pi F'\{\sqrt{(1-p^2)}\}-F'(p)lp-\{E'(p)-F'(p)\}[F\{\sqrt{(1-p^2)},\lambda\}]^2$, $p^2<1;$ V. T. 348. N°. 14.

11) $\int l(1-x^2)\frac{dx}{\sqrt{(1-x^2)}}=\pi l2$ V. T. 163. N°. 2.

12) $\int l(1-x^2)\frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}}=l\frac{\sqrt{(1-p^2)}}{p}.F'(p)-\frac{1}{2}\pi F'\{\sqrt{(1-p^2)}\}, p<1;$ V. T. 347. N°. 11.

13) $\int l(1-p^2x^2 Sin.^2\lambda)\frac{dx}{\sqrt{(1-x^2)(1-p^2x^2)}}=E'(p)F(p,\lambda)-2F'(p)Y(p,\lambda), p<1;$ V. T. 348. N°. 17.

14) $\int l(1-x^2 Sin.\lambda)\frac{dx}{\sqrt{(1-x^2)}}=2\pi l Cos.\frac{1}{2}\lambda$ V. T. 334. N°. 13.

15) $\int l(1-x^2 Sinhp.^2\lambda)\frac{dx}{\sqrt{(1-x^2)}}=2\pi l Coshp.\frac{1}{2}\lambda$ V. T. 334. N°. 10.

16) $\int l(1-x^2 Coshp.^2\lambda)\frac{dx}{\sqrt{(1-x^2)}}=\pi l\frac{1+Sinhp.\lambda}{2}$ V. T. 334. N°. 12.

17) $\int l(1-p^2x^2)\frac{x^2}{\sqrt{(1-x^2)}}dx=\frac{1}{2}\pi\left\{l\frac{1+\sqrt{(1-p^2)}}{2}-\frac{1}{2}\frac{1-\sqrt{(1-p^2)}}{1+\sqrt{(1-p^2)}}\right\}, p^2<1;$ V. T. 335. N°. 5.

18) $\int l(1-p^2x^2)\frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}}=\frac{1}{2}l(1-p^2).F'(p), p<1;$ V. T. 348. N°. 12.

19) $\int l(1-px^2)\frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}}=\frac{1}{2}l\frac{2-2p}{\sqrt{p}}F'(p)-\frac{1}{8}\pi F'\{\sqrt{(1-p^2)}\}, p<1;$ V. T. 348. N°. 11.

20) $\int l(1-p^2x^2)\frac{dx}{(1-p^2x^2)\sqrt{(1-p^2x^2)(1-x^2)}}=\frac{1}{1-p^2}\left[(p^2-2)F'(p)+\left\{2+\frac{1}{2}l(1-p^2)\right\}E'(p)\right], p^2<1;$ V. T. 348. N°. 18.

21) $\int l(1-p^2x^2)\frac{dx}{1-p^2x^2}\sqrt{\frac{1-x^2}{1-p^2x^2}}=\frac{1}{p^2}\left[\left(2-p^2+\frac{1}{2}l(1-p^2)\right\}F'(p)+\left\{2+\frac{1}{2}l(1-p^2)\right\}E'(p)\right], p^2<1;$ V. T. 348. N°. 20.

22) $\int l(Sec.^2\lambda-x^2 Tang.^2\lambda)\frac{dx}{\sqrt{(1-x^2)}}=\pi l\left(Cos.^2\frac{1}{2}\lambda . Sec.\lambda\right)$ V. T. 334. N°. 9.

23) $\int l(1-p^2x^2)\frac{x^2}{1-p^2x^2}\frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}} = \frac{1}{p^2(1-p^2)}$ $\left[\left\{2+\frac{1}{2}l(1-p^2)\right\}E'(p)-\left\{2-p^2+\frac{1}{2}(1-p^2)l(1-p^2)\right\}F'(p)\right]$, $p^2<1$; V. T. 348. N°. 19.

24) $\int l(1-p^2x^4)\frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}} = \frac{1}{2}l\frac{4(1-p^2)}{p}.F'(p)-\frac{1}{4}\pi F'\{\sqrt{(1-p^2)}\}$, $p^2<1$; V. T. 348. N°. 16.

25) $\int l(1+p^2+2px)\frac{dx}{\sqrt{(1-x^2)}} = \sum_0^\infty \frac{1}{1+2n}\frac{2^{n/2}}{1^{n/2}}\left(\frac{2p}{1+p^2}\right)^{2n+1}$, $p\leqq 1$; V. T. 334. N°. 19.

26) $\int l(1+p^2-p^2x^2)\frac{x^2}{\sqrt{(1-x^2)}}dx = \frac{1}{2}\pi\left\{l\frac{1+\sqrt{(1+p^2)}}{2}+\frac{\sqrt{(1+p^2)}-1}{p^2}-\frac{1}{2}\right\}$ V. T. 335. N°. 6.

27) $\int l\{1-(Cos.^2\lambda+p^2Sin.^2\lambda)x^2\}\frac{dx}{\sqrt{(1-x^2)(1-p^2x^2)}} = \pi F\{\sqrt{(1-p^2)},\lambda\}-2F'(p)Y\{\sqrt{(1-p^2)},\lambda\}+$ $+F'(p)l\frac{\sqrt{(1-p^2)}}{p}-\frac{1}{2}\pi F'\{\sqrt{(1-p^2)}\}-\{E'(p)-F'(p)\}\{F(p,\lambda)\}^2$, $p<1$; V. T. 348. N°. 15.

28) $\int l\{1-x^2+x^2\sqrt{(1-p^2)}\}\frac{dx}{\sqrt{(1-x^2)(1-p^2x^2)}} = \frac{1}{2}l\frac{2\sqrt{(1-p^2)^3}}{1+\sqrt{(1-p^2)}}.F'(p)$, $p^2<1$; V. T. 348. N°. 13.

F. Alg. irrat. fract. Log. en num. de fonct. fract. TABLE 166. Lim. 0 et 1.

1) $\int l\frac{1+x^2\,Sin.\lambda}{1-x^2\,Sin.\lambda}\frac{dx}{\sqrt{(1-x^2)}} = \pi\, l\frac{1+Sin.\frac{1}{2}\lambda}{Cos.\frac{1}{2}\lambda}$ V. T. 334. N°. 22.

2) $\int l\frac{1-x^2\,Sin.^2\lambda}{1-x^2\,Sin.^2\mu}\frac{dx}{\sqrt{(1-x^2)}} = 2\pi l\left(Cos.\frac{1}{2}\lambda.\,Sec.\frac{1}{2}\mu\right)$ V. T. 334. N°. 23.

3) $\int l\frac{Sin.^2\lambda-x^2\,Cos.^2\mu}{1-x^2\,Sin.^2\mu}\frac{dx}{\sqrt{(1-x^2)}} = \pi\, l\, Cot.\frac{1}{2}\mu.\,Tang.\left(\frac{1}{2}Arcsin.\frac{Sin.\mu}{Sin.\lambda}\right)$ V. T. 334. N°. 24.

4) $\int l\frac{Cos.^2\lambda+x^2\,Sin.^2\lambda}{Cos.^2\mu+x^2\,Sin.^2\mu}\frac{dx}{\sqrt{1-x^2}} = 2\pi l\left(Cos.\frac{1}{2}\lambda.\,Sec.\frac{1}{2}\mu\right)$ V. T. 334. N°. 23.

5) $\int l\frac{1+x}{1-x}\frac{dx}{x\sqrt{(1-x^2)}} = \frac{1}{2}\pi^2$ V. T. 340. N°. 2.

6) $\int l\frac{1+qx}{1-qx}\frac{dx}{x\sqrt{(1-x^2)}} = \pi\, Arcsin.\, q$ V. T 340. N°. 3.

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7) $\int l\frac{1+x\,Sin.\lambda}{1-x\,Sin.\lambda}\frac{dx}{x\sqrt{(1-x^2)}}=\pi\lambda$ Lobatschewsky, Mém. Kasan. 1835. 1.

8) $\int\left\{l\frac{1+x}{1-x}-2x\right\}\frac{dx}{x^3\sqrt{(1-x^2)}}=\frac{1}{4}\pi^2$ V. T. 340. N°. 6.

9) $\int l\frac{1+\sqrt{(1-x^2)(Sin.^2\lambda-x^2\,Sin.^2\mu)}}{1-\sqrt{(1-x^2)(Sin.^2\lambda-x^2\,Sin.^2\mu)}}\frac{dx}{\sqrt{(1-x^2)}}=\pi l\frac{1}{2}\left\{Cos.^2\frac{1}{2}\lambda+\sqrt{\left(Cos.^2\frac{1}{2}\lambda+Sin.^2\frac{1}{2}\mu.Cos.^2\frac{1}{2}\mu\right)}\right\}$ V. T. 348. N°. 2.

10) $\int l\frac{1-x\,Cos\,h\,p.\lambda.\,Cos\,h\,p.\mu\sqrt{(1-x^2\,Cos\,h\,p.^2\lambda.\,Tang\,h\,p.^2\mu)}}{1+x\,Cos\,h\,p.\lambda.\,Cos\,h\,p.\mu\sqrt{(1-x^2\,Cos\,h\,p.^2\lambda.\,Tang\,h\,p.^2\lambda)}}\frac{dx}{\sqrt{(1-x^2)}}=$

$=\pi\,l\frac{4\,Sin\,h\,p.\lambda}{\{Sin\,h\,p.\lambda+\sqrt{(1-Cos\,h\,p.^2\lambda.\,Cos\,h\,p.^2\mu)}\}(1+Sin\,h\,p.\lambda)}$ V. T. 348. N°. 3.

11) $\int l\frac{1+q\sqrt{(1-p^2x^2)}}{1-q\sqrt{(1-p^2x^2)}}\frac{dx}{\sqrt{(1-p^2x^2)(1-x^2)}}=\pi\,\mathrm{F}\{\sqrt{(1-p^2)},Arcsin.q\}$ V. T. 348. N°. 22.

12) $\int l\frac{1+x\,Cos.\mu}{1-x\,Cos.\mu}\frac{1}{1+x\,Cos.\lambda}\frac{dx}{\sqrt{(1-x^2)}}=\frac{2\pi}{Sin.\lambda}l\frac{Cos.\{\frac{1}{4}(\pi-2\lambda)\}}{Cos.\{\frac{1}{2}(\lambda-\mu)\}}$ V. T. 343. N°. 13.

13) $\int l\frac{1+x\,Cos.\mu}{1-x\,Cos.\mu}\frac{1}{1-x^2\,Cos.\lambda}\frac{dx}{\sqrt{(1-x^2)}}=\frac{\pi}{Sin.\lambda}l\frac{1+Sin.\lambda}{Sin.\lambda+Sin.\mu}$ V. T. 343. N°. 19.

14) $\int l\frac{1+qx\,Cos.\lambda}{1-qx\,Cos.\lambda}\frac{1}{1-x^2Cos.^2\lambda}\frac{dx}{\sqrt{(1-x^2)}}=\frac{\pi}{Sin.\lambda}l\frac{1+Sin.\lambda}{Sin.\lambda+\sqrt{(1-q^2Cos.^2\lambda)}}$ V. T. 343. N°. 17.

15) $\int l\frac{1+x\,Cos.\lambda}{1-x\,Cos.\lambda}\frac{x}{1-x^2\,Cos.^2\lambda}\frac{dx}{\sqrt{(1-x^2)}}=2\pi\,Cosec.\,2\lambda\,.\,l\,Sin.\lambda$ V. T. 343. N°. 14.

16) $\int l\frac{1+x\,Cos.\mu}{1-x\,Cos.\mu}\frac{x}{1-x^2\,Cos.^2\lambda}\frac{dx}{\sqrt{(1-x^2)}}=\frac{2\pi}{Sin.2\lambda}l\frac{Sin.\{\frac{1}{2}(\mu+\lambda)\}}{Cos.\{\frac{1}{2}(\mu-\lambda)\}}$ V. T. 343. N°. 10.

17) $\int l\frac{1+px}{1-px}\frac{x}{1-qx^2}\frac{dx}{\sqrt{(1-x^2)}}=\frac{\pi}{\sqrt{q(1-q)}}l\frac{p\sqrt{q}+\{1-\sqrt{(1-q)}\}\{1-\sqrt{(1-p^2)}\}}{p\sqrt{q}-\{1-\sqrt{(1-q)}\}\{1-\sqrt{(1-p^2)}\}}$ Lobatschewsky, Mém. Kasan. 1835. 1.

18) $\int l\frac{1+x\,Cos\,h\,p.\lambda}{1-x\,Cos\,h\,p.\lambda}\frac{x}{1-x^2\,Cos\,h\,p.^2\lambda}\frac{dx}{\sqrt{(1-x^2)}}=-2\pi\frac{l\,Sin\,h\,p.\lambda}{Sin\,h\,p.\lambda\,.\,Cos\,h\,p.\lambda}$ V. T. 343. N°. 18.

19) $\int l\frac{1+x}{1-x}\frac{x}{1-Cos.^2\lambda.\,Cos.^2\mu-x^2\,Sin.^2\mu}\frac{dx}{\sqrt{(x^2-Cos.^2\lambda)}}=$

$=\frac{\pi}{2\,Sin.\lambda.\,Sin.\mu}l\frac{Sin.\mu+\sqrt{(1-Cos^2\lambda.\,Cos.^2\mu)}}{Sin.\mu(1+Sin.\lambda)}$ V. T. 347. N°. 14.

20) $\int l\frac{1+x\,Cos\,h\,p.\,\mu}{1-x\,Cos\,h\,p.\,\mu}\frac{x}{1-x^2\,Cos.^2\lambda}\frac{dx}{\sqrt{(1-x^2)}} =$

$= \frac{2\pi}{Sin.2\lambda}\,l\left[Cot\,h\,p.\left(\frac{1}{2}\,Arccos\,h\,p.\frac{Cos\,h\,p.\,\mu}{Cos.\,\lambda}\right)Tang\,h\,p.\left\{\frac{1}{2}\,Arccot\,h\,p.\frac{Tang.\,\lambda}{Tang\,h\,p.\,\mu}\right\}\right]$ V. T. 343. N°. 21.

21) $\int lx\frac{x\,dx}{\sqrt{(1-x^2)}}\left\{\frac{1}{1-x^2}\,l\frac{1-x\,Sin.\,\lambda}{1+x\,Sin.\,\lambda}+\frac{2\,Sin^2.\lambda}{(1-x^2Sin^2.\lambda)^2}\right\} = \pi\lambda$ V. T. 166. N°. 7.

F. Alg. rat. ent. Log. en dén. lx. TABLE 167. Lim. 0 et 1.

1) $\int\frac{x^n}{lx}dx = \infty$ Euler, Calc. Int. 4. S. 5. 21. — Legendre, Exerc. 3. 57.

2) $\int\frac{1-x}{lx}dx = -l\,2$ Euler, Calc. Int. 4 S. 5. 5. — Id., N. C. Petr. 19. 66. — Legendre, Exerc. 5. 3.

3) $\int\frac{1-x^p}{lx}dx = -l\,(p+1)$ Euler, Calc. Int. 4. S. 5, 5. — Id., N. C. Petr. 19. 66. — Poisson, P. 18. 295. N°. 25. — Legendre, Exerc. 3. 57. — Id., ib. 5. 3. — Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 36. — Plana, Mém. Turin. 1818. 7. Art. 14. Add. — Cisa de Grésy, Mém. Turin. 1821. 209. I. 29. — Arndt, Gr. 10. 253.

4) $\int\frac{x^p-x^q}{lx}dx = l\frac{p+1}{q+1}$ Euler, Act. Petr. 1777. 2. 29. — Id., Calc. Int. 4. S. 5. 5, 22. — Id., N. C. P. 19. 66. — Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 36.

5) $\int\frac{x^q-1}{lx}x^p\,dx = l\frac{p+q+1}{q+1}$ Legendre, Exerc. 3. 57. — Cisa de Grésy, Mém. Turin. 1821. 209. I. 29.

6) $\int\frac{x^{p-1}-x^{q-1}}{lx}x^{ri}\,dx = l\frac{p+ri}{q+ri}$ Meyer, Int. Déf. 109.

7) $\int\frac{(x^q-1)(x^r-1)}{lx}x^{p-1}\,dx = l\frac{p+q+r}{p+q} - l\frac{p+r}{p}$ Binet, P. 27. 123.

8) $\int\frac{(x^p-x^q)(x^r-x^s)}{lx}dx = l\frac{(p+r+1)(q+s+1)}{(p+s+1)(q+r+1)}$ Euler, N. C. Petr. 20. 59.

9) $\int x^{a-1}(x-1)^b\frac{dx}{lx} = \Delta^b.\,la$ V. T. 127. N°. 6.

10) $\int(x-1)^n\frac{dx}{lx} = \sum_0^\infty\binom{a}{n}l\,(a-n+1)$ Euler, Calc. Int. 4. S. 5. 29. — Id., N. C. Petr. 19. 66. — Stern, Gött. Stud. 1847.

1) $\int\left(\frac{1-x}{lx}\right)^2 dx = l\frac{27}{16}$

2) $\int\left(\frac{1-x}{lx}\right)^2 x\,dx = 2\,l\,\frac{32}{27}$

Euler, N. C. Petr. 19. 66.

3) $\int\frac{x^{q-1}(x-1)^2}{(l\,x)^2}dx = (q+2)\,l(q+2)-2(q+1)\,l(q+1)+q\,l\,q$ Euler, N. C. P. 20. 59.

4) $\int\left(\frac{x^q-1}{l\,x}\right)^2 dx = (2q+1)\,l(2q+1)-2(q+1)\,l(q+1)$

5) $\int\left(\frac{x^q-1}{l\,x}\right)^2 x^p dx = (2q+p+1)\,l(2q+p+1)-2(q+p+1)\,l(q+p+1)+(p+1)\,l(p+1)$

Legendre, Exerc. 3. 58.

6) $\int\frac{(q-r)\,x^{p-1}+(r-p)\,x^{q-1}+(p-q)\,x^{r-1}}{(l\,x)^2}dx =$
$= (q-r)\,p\,l\,p+(r-p)\,q\,l\,q+(p-q)\,r\,l\,r$ Euler, Calc. Int. 4. S. 5. 23. — Id., N. C. Petr. 19. 66. — Id., N. C. Petr. 20. 59.

7) $\int\left\{1-\left(\frac{1}{2}-\frac{1}{l\,x}\right)(1-x)\right\}x^{q-1}\frac{dx}{l\,x} = 1+\left(q+\frac{1}{2}\right)l\frac{q}{q+1}$ V. T. 124. N°. 20.

8) $\int(1-x^p)(1-x^q)(1-x^r)\frac{dx}{(l\,x)^2} = (p+q+1)\,l(p+q+1)+(p+r+1)\,l(p+r+1)$
$+(q+r+1)\,l(q+r+1)-(p+1)\,l(p+1)-(q+1)\,l(q+1)$
$-(r+1)\,l(r+1)-(p+q+r)\,l(p+q+r)$

9) $\int(1-x^p)^a\frac{dx}{(lx)^2} = \sum_1^\infty\binom{a}{2n}(2np+1)\,l(2np+1)-\sum_0^\infty\binom{a}{2n-1}\{(2n-1)p+1\}\,l\{(2n-1)p+1\}$

10) $\int(1-x^p)^a\frac{x^q\,dx}{(l\,x)^2} = \sum_0^\infty(-1)^n\binom{a}{n}(q+np+1)\,l(q+np+1)$

Stern, Gött. Stud. 1847.

11) $\int\left\{\frac{x^p}{(p-q)(p-r)(p-s)}+\frac{x^q}{(q-p)(q-r)(q-s)}+\frac{x^r}{(r-p)(r-q)(r-s)}+\frac{x^s}{(s-p)(s-q)(s-r)}\right\}\frac{dx}{x(lx)^3} =$
$= \frac{p^2\,lp}{(p-q)(p-r)(p-s)}+\frac{q^2 lq}{(q-p)(q-r)(q-s)}+\frac{r^2\,lr}{(r-p)(r-q)(r-s)}+\frac{s^2\,ls}{(s-p)(s-q)(s-r)}$ Euler, N. C. Petr. 20. 59.

12) $\int\left\{\frac{1-x}{lx}+\frac{x}{(lx)^2}\right\}dx = l\,2-1$ V. T. 127. N°. 21.

13) $\int(1-x^p)^a\frac{dx}{(l\,x)^3} = \frac{1}{2}\sum_1^\infty(-1)^n\binom{a}{n}(p\,n+1)^2\,l(p\,n+1)$ Stern, Gött. Stud. 1847.

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14) $\int (1-x^p)^a \frac{x^q}{(l\,x)^3}\,dx = \frac{1}{2}\sum_0^\infty (-1)^n \binom{a}{n} (pn+q+1)^2\, l(pn+q+1)$

15) $\int (1-x^p)^a (1-x^q) \frac{dx}{(l\,x)^3} = -\frac{1}{2}\sum_0^\infty (-1)^n \binom{a}{n} (q+pn+1)^2\, l\,(q+pn+1)$
$+\frac{1}{2}\sum_1^\infty (-1)^n \binom{a}{n} (pn+1)\, l(pn+1)$

Stern, Gött. Stud. 1847.

16) $\int \frac{x^{a-1}}{(l\,x)^q}\,dx = \frac{1^{q/1}}{q^{a+1}}$ (fautif) Oettinger, Cr. 35. 13.

17) $\int \left(\frac{x^q-1}{l\,x}\right)^a x^{p-1}\,dx = \frac{1}{1^{a-1/1}} \frac{\Delta^a}{\Delta p} . p^{a-1} lp$, en posant $\Delta p = q$; Legendre, Exerc. 3. 58.

18) $\int \left(\frac{x^q-1}{l\,x}\right)^a x^{pq-1}\,dx = \frac{1}{1^{a-1/1}}\Delta^a . \{(pq)^{a-1}\, l\,p\,q\}$

19) $\int \frac{x^{p-1}}{(l\,x)^{b+1}}(x-1)^a\,dx = \frac{1}{1^{b/1}}\Delta^a . p^b\, lp$

20) $\int \frac{x^{pr-1}}{(l\,x)^{b+1}}(x^r-1)^a\,dx = \frac{r^{-b}}{1^{b/1}}\Delta^a . p^b\, lp$

Cauchy, P. 28. 147. P. 1. § 2.

21) $\int \frac{x^{q-1}-x^{r-1}}{(l\,x)^{p+1}}\,dx = (-1)^{p+1}\Gamma(1-p)\frac{q^p-r^p}{p}$, $p < 1$; V. T. 128. N°. 1.

22) $\int \frac{x^{b-1}}{\left(l\frac{1}{x}\right)^{q+1}}(x-1)^c\,dx = -\frac{\pi}{\Gamma(q+1)} Cosec.\,p\,\pi\,\Delta^c . b^q$

23) $= \frac{(-1)^{q+1}}{1^{q/1}}\Delta^c . b^q\, l\,b$, pour q entier;

, $q < b$; V. T. 128. N°. 3, 4.

24) $\int \frac{dx}{lx}\left\{(p-q)+\frac{x^{q-1}-x^{p-1}}{lx}\right\} = p-q+q\,l\,q-p\,l\,p$ V. T. 127. N°. 18.

F. Alg. rat. ent. Log. en dén. de forme $a \pm (lx)^b$. TABLE 169. Lim. 0 et 1.

1) $\int \frac{x^{p-1}}{q+l\,x}\,dx = -e^{-pq}\,Ei.\,(p\,q)$ V. T. 129. N°. 9.

2) $\int \frac{x^{p-1}}{q-l\,x}\,dx = e^{pq}\,Ei.\,(-p\,q)$ V. T. 129. N°. 4.

3) $\int \frac{x}{q^2+(l\,x)^2}\,d\,x = \sum_1^\infty (-1)^n \frac{1^{2n/1}}{q^{2n+1}}$ V. T. 130. N°. 2.

4) $\int \frac{x\,l\,x}{q^2+(l\,x)^2}\,d\,x = -\sum_1^\infty (-1)^n \frac{1^{2n+1/1}}{q^{2n-1}}$ V. T. 130. N°. 1.

5) $\int \frac{x^{q-1}}{1+(l\,x)^2}\,d\,x = \sum_0^\infty (-1)^n \frac{1^{2n/1}}{q^{2n+1}}$ V. T. 130. N°. 2.

6) $\int \frac{x^{q-1}\,l\,x}{1+(l\,x)^2}\,d\,x = -\sum_0^\infty (-1)^n \frac{1^{2n+1/1}}{q^{2n-2}}$ V. T. 130. N°. 1.

7) $\int \frac{x^{p-1}}{q^2+(l\,x)^2}\,d\,x = \frac{1}{q}\left\{Ci.(p\,q).Sin.p\,q - Si.(p\,q).Cos.pq + \frac{1}{2}\pi\,Cos.p\,q\right\}$ V. T. 130. N°. 4.

8) $\int \frac{x^{p-1}\,lx}{q^2+(l\,x)^2}\,d\,x = Ci.(p\,q).Cos.p\,q + Si.(p\,q).Sin.p\,q - \frac{1}{2}\pi\,Sin.p\,q$ V. T. 130. N°. 5.

9) $\int \frac{x^{p-1}}{q^2-(l\,x)^2}\,d\,x = \frac{1}{2\,q}\left\{e^{-pq}\,Ei.(p\,q) - e^{pq}\,Ei.(-p\,q)\right\}$ V. T. 130. N°. 10.

10) $\int \frac{x^{p-1}\,lx}{q^2-(l\,x)^2}\,d\,x = -\frac{1}{2}\left\{e^{-pq}\,Ei.(p\,q) + e^{pq}\,Ei.(-p\,q)\right\}$ V. T. 130. N°. 12.

11) $\int \frac{x^{p-1}}{q^4-(lx)^4}\,dx = \frac{1}{4\,q^3}\left\{e^{-pq}\,Ei.(pq) - e^{pq}\,Ei.(-pq) + 2\,Ci.(pq).Sin.pq - 2\,Si.(pq).Cos.pq + \pi\,Cos.pq\right\}$ V. T. 132. N°. 1.

12) $\int \frac{x^{p-1}\,lx}{q^4-(lx)^4}\,dx = \frac{1}{4\,q^2}\left\{-e^{-pq}\,Ei.(pq) - e^{pq}\,Ei.(-pq) + 2\,Ci.(pq).Cos.pq + 2\,Si.(pq).Sin.pq - \pi\,Sin.pq\right\}$ V. T. 132. N°. 2.

13) $\int \frac{x^{p-1}(lx)^2}{q^4-(lx)^4}\,dx = \frac{1}{4q}\left\{e^{-pq}\,Ei.(pq) - e^{pq}\,Ei.(-pq) - 2\,Ci.(pq).Sin.pq + 2\,Si.(pq).Cos.pq - \pi\,Cos.pq\right\}$ V. T. 132. N°. 3.

14) $\int \frac{x^{p-1}(lx)^3}{q^4-(lx)^4}\,dx = \frac{1}{4}\left\{-e^{-pq}\,Ei.(pq) - e^{pq}\,Ei.(pq) - 2\,Ci.(pq).Cos.pq - 2\,Si.(pq).Sin.pq + \pi\,Sin.pq\right\}$ V. T. 132. N°. 4.

15) $\int \frac{x^{q-1}}{(p+l\ \ )^2}\,d\,x = -\frac{1}{p}\left\{1 + p\,q\,e^{-pq}\,Ei.(p\,q)\right\}$ V. T. 169. N°. 1.

16) $\int \frac{x^{q-1}}{(p-l\,x)^2}\,d\,x = \frac{1}{p}\left\{1 - p\,q\,e^{pq}\,Ei.(-pq)\right\}$ V. T. 169. N°. 2.

17) $\int \frac{x^{p-1}}{\{q^2+(l.x)^2\}^2}\,d\,x = \frac{1}{2q^3}\left\{Ci.(pq).Sin.p\,q - Si.(pq).Cos.p\,q + \frac{1}{2}\pi\,Cos.p\,q\right\} - \frac{p}{2\,q^2}\left\{Ci.(pq).Cos.p\,q + Si.(p\,q).Sin.p\,q - \frac{1}{2}\pi\,Sin.p\,q\right\}$ V. T. 169. N°. 7.

1) $\int \frac{x^p - x}{x} \frac{dx}{lx} = lq$ Binet, P. 27. 123.

2) $\int \frac{x^p - x^q}{x} \frac{dx}{lx} = l\frac{p}{q}$ Cauchy, Cours. Leç. 33.

3) $\int \left\{1 + \frac{1}{lx} + \frac{1}{x\,lx}\right\} \frac{dx}{lx} = -1$ V. T. 127. N°. 17.

4) $\int \left\{\frac{x^q - 1}{x(lx)^2} - \frac{q}{lx}\right\} dx = q\,lq - q$

5) $\int \left\{\frac{x^q - 1}{x(lx)^3} - \frac{q}{x(lx)^2} - \frac{q^2}{2\,lx}\right\} dx = \frac{1}{2}q^2\,lq - \frac{3}{4}q^2$

6) $\int \left\{\frac{x^q - 1}{x(lx)^4} - \frac{q}{x(lx)^3} - \frac{q^2}{2x(lx)^2} - \frac{q^3}{6\,lx}\right\} dx = \frac{1}{6}q^3\,lq - \frac{11}{36}q^3$

(4–6) Sohnke, Samml.

7) $\int \left\{x - \left(\frac{1}{1 - lx}\right)^q\right\} \frac{dx}{x\,lx} = -Z'(q)$ V. T. 112. N°. 8.

8) $\int \frac{l(1 - x^a)}{1 + (lx)^2} \frac{dx}{x} = \pi \left\{ l\Gamma\left(\frac{a}{2\pi} + 1\right) + \frac{1}{2}la + \frac{a}{2\pi}\left(l\frac{a}{2\pi} - 1\right)\right\}$ V. T. 378. N°. 4.

9) $\int \left\{x - \frac{1}{1 - lx}\right\} \frac{dx}{x\,lx} = A$ V. T. 133. N°. 1.

10) $\int \left\{x - \frac{1}{1 + (lx)^2}\right\} \frac{dx}{x\,lx} = A$ V. T. 133. N°. 5.

11) $\int \left\{x^q - \frac{1}{(1 - lx)}\right\} \frac{dx}{x\,lx} = lq + A$ V. T. 133. N°. 2.

12) $\int \left\{x - \frac{1}{(1 - lx)^p}\right\} \frac{dx}{x\,lx} = -Z'(p)$ V. T. 133. N°. 3.

13) $\int \left\{\frac{x - 1}{lx} - \frac{1}{1 - lx}\right\} \frac{dx}{x\,lx} = -1 + A$ V. T. 133. N°. 4.

14) $\int \frac{x\,lx + 1 - x}{x(lx)^2}\, l(1 + x)\,dx = l\frac{2}{\pi}$ V. T. 171. N°. 1.

15) $\int l(1 + x)^2 \frac{2q\,lx(x^q + x^{-q}) - (x^q - x^{-q})}{(lx)^2} \frac{x^q - x^{-q}}{x} dx = 2l\frac{\mathrm{Tang.}\,q\pi}{q\pi}$ V. T. 175. N°. 3.

F. Alg. rat. fract. à dén. mônome.
Log. en dén.

TABLE 170 suite.

Lim. 0 et 1.

16) $\int l(1-x)^2 \frac{2\,q\,l\,x\,(x^q + x^{-q}) - (x^q - x^{-q})}{(l\,x)^2} \frac{x^q - x^{-q}}{x} d\,x = 2\,l \frac{Sin.\,2\,q\,\pi}{2\,q\,\pi}$ V. T. 175. N°. 4.

17) $\int l(1-x^2)^2 \frac{2\,q\,l\,x\,(x^q + x^{-q}) - (x^q - x^{-q})}{(l\,x)^2} \frac{x^q - x^{-q}}{x} d\,x = 4\,l \frac{Sin.\,q\,\pi}{q\,\pi}$ V. T. 175. N°. 7.

18) $\int \frac{l\,x}{1-x^a} \frac{\{1+(lx)^2\}ax^a - (1-x^2)2lx.l(1-x^2)}{\{1+(l\,x)^2\}^2} \frac{dx}{x} = \pi\left\{-l\Gamma\left(\frac{a}{2\pi}\right) + 2la + \frac{a}{2\pi}\left(l\frac{a}{2\pi} - 1\right)\right\}$ V.T.170. N°. 8.

F. Alg. rat. fract. à dén. $1 \pm x$.
Log. en dén. $(l\,x)^a$.

TABLE 171.

Lim. 0 et 1.

1) $\int \frac{1-x}{1+x} \frac{d\,x}{l\,x} = l\frac{2}{\pi}$ Euler, N. C. Petr. 20. 59. — Legendre, Exerc. 5. 8.

2) $\int \frac{1-x^{q-1}}{1+x} \frac{d\,x}{l\,x} = Z'\left(\frac{q}{2}\right) - Z'\left(\frac{q+1}{2}\right) - Z'\left(\frac{1}{2}\right)$ Legendre, Exerc. 5. 8.

3) $\int \frac{1-x^p}{1+x} \frac{x^q}{l\,x} d\,x = l \frac{\Gamma\left(\frac{1}{2}p+1\right)\Gamma\left(\frac{p+q+1}{2}\right)}{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{p+q}{2}+1\right)}$ Stern, Gött. Stud. 1847.

4) $\int \frac{x^{p-1} - x^{q-1}}{1+x} \frac{d\,x}{l\,x} = l \frac{\Gamma\left(\frac{q}{2}\right)\Gamma\left(\frac{p+1}{2}\right)}{\Gamma\left(\frac{p}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}$

5) $\int \frac{x^{p-1} - x^{q-1}}{1+x} \frac{1+x^{2a+1}}{l\,x} d\,x = l\left\{\left(\frac{a}{a+1}\right)^{\frac{a-1}{2}} \frac{\left(\frac{p}{2}\right)^{a+1/1} \left(\frac{q+1}{2}\right)^{a/1}}{\left(\frac{p+1}{2}\right)^{a/1} \left(\frac{q}{2}\right)^{a+1/1}}\right\}$

(4, 5) Kummer, Cr. 17. 210.

6) $\int \left\{\frac{1}{1+x} - \frac{x}{2}\right\} \frac{d\,x}{l\,x} = -\frac{1}{2} l\,\pi$ V. T. 135. N°. 6.

7) $\int \left(\frac{1}{l\,x} + \frac{1}{1-x}\right) d\,x = A$ Legendre, Exerc. 5. 12. — Cauchy, P. 28. 147. P. 1. § 6.

8) $\int \left(\frac{1}{l\,x} + \frac{x^{q-1}}{1-x}\right) d\,x = -Z'(q)$ Cauchy, P. 28. 147. P. 1. § 6. — Gausz, 1812. — Lejeune-Dirichlet, Cr. 15. 258. — Schaar, Mém. Cour. Brux. T. 22.

9) $\int \left(\frac{x^{p-1}}{l\,x} + \frac{x^{q-1}}{1-x}\right) d\,x = l\,p - Z'(q)$ Legendre, Exerc. 5. 12. — Arndt, Gr. 10. 250.

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10) $\int\left(\frac{b\,x^{bq-1}}{1-x}+\frac{b\,x^{b-1}}{l\,x}\right)dx = -\sum_1^b Z'\left(q+\frac{b-n}{b}\right)$ Arndt, Gr. 10. 253.

11) $\int\left(\frac{1-x^{q-1}}{1-x}-q+1\right)\frac{dx}{lx} = l\,\Gamma(q)$ Kummer, Cr. 35. 1.

12) $\int\left(\frac{x^p-x^{p+q}}{1-x}-q\right)\frac{dx}{lx} = l\frac{\Gamma(p+q+1)}{\Gamma(p+1)}$ Plana, Mém. Turin. 1818. 7. Art 4. Add.

13) $\int\left\{\frac{1}{lx}+\frac{x^a-x^b-x^{a-b}}{1-x}x^{p-1}\right\}dx = l\frac{\Gamma(a+p)}{\Gamma(b+p)\Gamma(a+b+p)}$ V. T. 135. N°. 18.

14) $\int\frac{1-x^q}{1-x}\,\frac{1-x^p}{lx}\,dx = l\,\mathrm{B}(p,q)-l\frac{p+q}{pq}$ Binet, P. 27. 123.

15) $\int\frac{1-x^a}{1-x}\,\frac{1-x^b}{lx}\,dx = l\,\frac{1^{a/1}}{(b+1)^{a/1}}$, pour a entier, b fraction;

16) $= -\,l\,\frac{1^{a+b/1}}{1^{a/1}\,1^{b/1}}$, pour a et b entiers;

(15–16) Cisa de Grésy, Mém. Turin. 1821. 209. I. 30.

17) $= -\,l\,\frac{\Gamma(a+b+1)}{\Gamma(a+1)\,\Gamma(b+1)}$, pour a et b fractions; Legendre, Exerc. 4. 111. — Cisa de Grésy, Mém. Turin. 1821. 209. I. 30.

18) $\int\frac{1-x^q}{1-x}\,\frac{1-x^p}{lx}\,x^{r-1}\,dx = l\,\frac{\Gamma(q+r)\Gamma(p+r)}{\Gamma(r)\Gamma(p+q+r)}$ Legendre, Exerc. 4. 111.

19) $\int\frac{x^p-x^q}{1-x}\,\frac{x^r-x^s}{lx}\,dx = l\,\frac{(1+p+s)(1+q+r)}{(1+p+r)(1+q+s)}$ Schlömilch, Gr. 4. 167.

20) $\int\frac{(1-x^p)(1-x^q)(1-x^r)}{1-x}\,\frac{dx}{lx} = l\,\frac{\Gamma(p+1)\Gamma(q+1)\Gamma(r+1)\Gamma(p+q+r+1)}{\Gamma(p+q+1)\Gamma(p+r+1)\Gamma(q+r+1)}$

21) $\int\frac{(1-x^p)(1-x^q)(1-x^r)}{1-x}\,x^{s-1}\,\frac{dx}{lx} = l\,\frac{\Gamma(p+s)\Gamma(q+s)\Gamma(r+s)\Gamma(p+q+r+s)}{\Gamma(p+q+s)\Gamma(p+r+s)\Gamma(q+r+s)\Gamma(s)}$

22) $\int\frac{(1-x^p)(1-x^q)(1-x^r)(1-x^s)}{1-x}\,\frac{dx}{lx} =$

$$= l\frac{\Gamma(p+1)\Gamma(q+1)\Gamma(r+1)\Gamma(s+1)\Gamma(p+q+r+1)\Gamma(p+q+s+1)\Gamma(p+r+s+1)\Gamma(q+r+s+1)}{\Gamma(p+q+1)\Gamma(p+r+1)\Gamma(p+s+1)\Gamma(q+r+1)\Gamma(q+s+1)\Gamma(r+s+1)\Gamma(p+q+r+s+1)}$$

(20–22) Stern, Gött. Stud. 1847.

23) $\int\left(\frac{1}{lx}+\frac{1}{1-x}-\frac{1}{2}\right)\frac{dx}{lx} = -1+\frac{1}{2}\,l\,2\pi$ V. T. 135. N°. 20.

24) $\int\left(\frac{1}{lx}+\frac{1}{2}\frac{1+x}{1-x}\right)\frac{dx}{lx}=\frac{1}{2}l\,2\pi-1$ V. T. 135. N°. 21.

25) $\int\left(\frac{1}{lx}-lx+\frac{1}{2}\frac{1+x}{1-x}\right)\frac{dx}{lx}=\frac{1}{2}l\,2\pi$ V. T. 135. N°. 22.

26) $\int\left(\frac{1}{lx}+\frac{1}{2}x+\frac{1}{1-x}\right)\frac{dx}{lx}=\frac{1}{2}l\,2\pi$ V. T. 135. N°. 23.

27) $\int\left(p+\frac{x^{p-1}}{lx}-\frac{1}{2}x^{p-1}-\frac{1}{1-x}\right)\frac{dx}{lx}=-\left(p+\frac{1}{2}\right)lp+p-\frac{1}{2}l\,2\pi$ V. T. 135. N°. 17.

F. Alg. rat. fract. à dén. $1 \pm x^a$. Log. en dén. $(lx)^b$. **TABLE 172.** Lim. 0 et 1.

1) $\int\frac{(1-x)^2}{1+x^2}\frac{dx}{lx}=l\frac{\pi}{4}$ Legendre, Exerc. 5. 3.

2) $\int\frac{1}{1-x^2}\frac{dx}{lx}=-\frac{1}{2}l\,2$ Euler, Calc. Int. 4. S. 3. 115.

3) $\int\frac{1-x^q}{1-x^2}\frac{1-x^{q+1}}{lx}dx=-q\,l2,\ q>-1;$ Legendre, Exerc. 4. 113.

4) $\int\frac{1-x^2}{1+x^4}\frac{dx}{lx}=l\,Cot.\frac{3\pi}{8}$ Euler, Calc. Int. 4. S. 3. 111.

5) $\int\left\{\frac{1}{1-x^2}+\frac{1}{2\,lx}-\frac{1}{2}\right\}\frac{dx}{lx}=\frac{1}{2}(l2-1)$ V. T. 135. N°. 14.

6) $\int\frac{x^{p+q-1}-x^{p-q-1}}{1+x^{2p}}\frac{dx}{lx}=l\,Tang.\left\{\frac{p+q}{4p}\pi\right\}$

7) $\int\frac{x^{p+q-1}+x^{p-q-1}}{1-x^{2p}}\frac{dx}{lx}=l\,Cos.\frac{q\pi}{4p}$

(6, 7) Euler, N. C. Petr. 19. 30. — Id., Calc. Int. 4. S. 5. 30.

8) $\int\frac{(1-x^q)^2}{1-x^p}x^{p-q-1}\frac{dx}{lx}=l\left(\frac{p}{q\pi}Sin.\frac{q\pi}{p}\right),\ p>q;$ Euler, N. Act. Petr. I. P. 2. 29.

9) $\int\frac{1-x^{2p-2q}}{1+x^{2p}}\frac{x^{q-1}}{lx}dx=l\,Tang.\frac{q\pi}{2p}$

10) $\int\frac{(1-x^{p-q})^2}{1-x^{2p}}\frac{x^{q-1}}{lx}dx=l\,Sin.\frac{q\pi}{2p}$

(9, 10) Euler, N. A. Petr. 7. 64.

1) $\int \frac{lx}{4\pi^2 q^2 + (lx)^2} \frac{dx}{1-x^2} = -\frac{1}{2} lq - \frac{1}{4q} + \frac{1}{2} Z'(1+q)$ V. T. 138. N°. 11.

2) $\int \frac{1}{q^2 + (lx)^2} \frac{dx}{1-x} = \frac{1}{2}\left\{-\frac{q}{4\pi} + l\frac{q}{2\pi} - Z'\left(\frac{q}{2\pi}\right)\right\}$ V. T. 138. N°. 9.

3) $\int \frac{lx}{4\pi^2 + (lx)^2} \frac{dx}{1-x} = \frac{1}{4} - \frac{1}{2} A$ V. T. 138. N°. 10.

4) $\int \frac{lx}{q^2 - (lx)^2} \frac{dx}{1-x} = \frac{4\pi^4}{q^4} \overset{\infty}{\underset{0}{\Sigma}} (-1)^{n-1} \frac{B_{2n+1}}{(n+1) q^{2n}} \pi^{2n}$ V. T. 138. N°. 22.

5) $\int \frac{lx}{\{q^2 + (lx)^2\}^2} \frac{dx}{1-x} = -\frac{\pi^2}{q^4} \overset{\infty}{\underset{0}{\Sigma}} \frac{B_{2n+1}}{q^{2n}} (2\pi)^{2n}$ V. T. 138. N°. 21.

6) $\int \frac{lx}{\{q^2 - (lx)^2\}^2} \frac{dx}{1-x} = \frac{\pi^2}{q^4} \overset{\infty}{\underset{0}{\Sigma}} (-1)^{n-1} \frac{B_{2n+1}}{q^{2n}} (2\pi)^{2n}$ V. T. 138. N°. 23.

7) $\int \frac{1}{\pi^2 + (lx)^2} \frac{dx}{1+x^2} = \frac{4-\pi}{4\pi}$ V. T. 138. N°. 2.

8) $\int \frac{1}{\pi^2 + 4(lx)^2} \frac{dx}{1+x^2} = \frac{1}{4} \pi l 2$ V. T. 138. N°. 3.

9) $\int \frac{1}{q^2 + (lx)^2} \frac{dx}{1+x^2} = \frac{1}{4q}\left\{Z'\left(\frac{2q+3}{4\pi}\right) - Z'\left(\frac{2q+1}{4\pi}\right)\right\}$ V. T. 138. N°. 1.

10) $\int \frac{lx}{\pi^2 + 4(lx)^2} \frac{dx}{1+x^2} = \frac{2-\pi}{16}$ V. T. 138. N°. 16.

11) $\int \frac{1}{\pi^2 + (lx)^2} \frac{lx}{1-x^2} dx = \frac{1}{2\pi} - l 2$ V. T. 138. N°. 12.

12) $\int \frac{1}{q^2 + (lx)^2} \frac{x}{1-x^2} dx = -\frac{1}{2}\left\{\frac{\pi}{2q} + l\frac{\pi}{q} + Z'\left(\frac{q}{\pi}\right)\right\}$ V. T. 138. N°. 9.

13) $\int \frac{lx}{\pi^2 + (lx)^2} \frac{x}{1-x^2} dx = \frac{1}{4} - \frac{1}{2} A$ V. T. 138. N°. 10.

14) $\int \frac{lx}{q^2\pi^2 + (lx)^2} \frac{x}{1-x^2} dx = \frac{1}{4q^4} \overset{\infty}{\underset{0}{\Sigma}} (-1)^n \frac{B_{2n+1}}{(n+1) q^{2n}}$ V. T. 138. N°. 22.

15) $\int \frac{lx}{\{q^2 + (lx)^2\}^2} \frac{x}{1-x^2} dx = -\frac{\pi^2}{4q^4} \overset{\infty}{\underset{0}{\Sigma}} \frac{B_{2n+1}}{q^{2n}} \pi^{2n}$ V. T. 138. N°. 21.

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16) $\displaystyle\int \frac{lx}{\{q^2-(lx)^2\}^2}\frac{x}{1-x^2}dx = \frac{\pi^2}{4q^4}\sum_0^\infty (-1)^{n+1}\frac{B_{2n+1}}{q^{2n}}\pi^{2n}$ V. T. 138. N°. 23.

17) $\displaystyle\int \frac{1}{\frac{1}{16}\pi^2+(lx)^2}\frac{dx}{1-x^2} = \frac{1}{\pi\sqrt{2}}\left\{\pi + l\frac{\sqrt{2}-1}{\sqrt{2}+1}\right\}$ V. T. 138. N°. 18.

18) $\displaystyle\int \frac{lx}{\frac{1}{16}\pi^2+(lx)^2}\frac{dx}{1-x^2} = -\frac{1}{4}\sqrt{\pi+1} + \frac{1}{2\sqrt{2}}l\frac{\sqrt{2}-1}{\sqrt{2}+1}$ V. T. 138. N°. 19.

F. Alg. rat. fract. à dén. trinôme.
Log. en dén. TABLE 174. Lim. 0 et 1.

1) $\displaystyle\int \left\{q-\frac{1}{2}+\frac{(1-x)(1+qlx)+xlx}{(1-x)^2}x^{q-1}\right\}\frac{dx}{lx} = \frac{1}{2}-q+l\Gamma\left(\frac{1}{2}\right)-l\Gamma(q)+\frac{1}{2}l2$ Arndt, Gr. 10. 455.

2) $\displaystyle\int \frac{x}{1+x^2+x^4}\frac{dx}{lx} = -\frac{1}{2}l3$ Euler, Calc. Int. T. 4. S. 3. 112.

3) $\displaystyle\int \frac{x}{1-x^2+x^4}\frac{dx}{lx} = l\left(\frac{1}{2}\sqrt{3}\right)$ Euler, Calc. Int. T. 4. S 3. 116.

4) $\displaystyle\int \frac{1}{1+x^2+2x\,Cos.\lambda}\frac{dx}{\left(l\frac{1}{x}\right)^{1-q}} = Cosec.\lambda.\Gamma(q)\sum_1^\infty(-1)^{n-1}\frac{Sin.n\lambda}{n^q}$

5) $\displaystyle\int \frac{1}{1+x^2+2xCos.\frac{a\pi}{b}}\frac{dx}{l\frac{1}{x}} = \frac{1+Cos.\frac{a\pi}{b}}{Sin.\frac{a\pi}{b}}\left\{Tang.\frac{a\pi}{2b}l2b+2\sum_1^{b-1}(-1)^{n-1}Sin.\frac{na\pi}{b}.l\frac{\Gamma\left(\frac{b+n+1}{2b}\right)}{\Gamma\left(\frac{n+1}{2b}\right)}\right\}$, pour $a+b$ impair;

6) $\displaystyle = \frac{1+Cos.\frac{a\pi}{b}}{Sin.\frac{a\pi}{b}}\left\{Tang.\frac{a\pi}{2b}lb+2\sum_1^{\frac{b-1}{2}}(-1)^{n-1}Sin.\frac{na\pi}{b}.l\frac{\Gamma\left(\frac{b-n+1}{b}\right)}{\Gamma\left(\frac{n+1}{b}\right)}\right\}$, pour $a+b$ pair;

7) $\displaystyle\int \frac{1-x^{p-q}}{1+x^2+2xCos.\frac{a\pi}{b}}\frac{x^q}{l\frac{1}{x}}dx = Cosec.\frac{a\pi}{b}\sum_1^{b-1}(-1)^{n-1}Sin.\frac{na\pi}{b}.l\frac{\Gamma\left(\frac{p+b+n}{2b}\right)\Gamma\left(\frac{q+n}{2b}\right)}{\Gamma\left(\frac{q+b+n}{2b}\right)\Gamma\left(\frac{p+n}{2b}\right)}$, pour $a+b$ impair;

Les formules (4) à (7) sont déduites par Malmsten, Cr. 38. 1. Partout $a < b$.

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8) $\int \frac{1-x^{b-q}}{1+x^2+2x\,Cos.\frac{a\pi}{b}} \frac{x^q}{l\frac{1}{x}} dx = Cosec.\frac{a\pi}{b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} Sin.\frac{na\pi}{b} . l \frac{\Gamma\left(\frac{p+b-n}{b}\right)\Gamma\left(\frac{q+n}{b}\right)}{\Gamma\left(\frac{q+b-n}{b}\right)\Gamma\left(\frac{p+n}{b}\right)}$, pour $a+b$ pair;

9) $\int \frac{(1-x)^2}{1+x^2+2x\,Cos.\frac{a\pi}{b}} \frac{dx}{l\frac{1}{x}} = Cosec.\frac{a\pi}{b} \sum_1^{b-1} (-1)^{n-1} Sin.\frac{na\pi}{b} . l \frac{\left\{\Gamma\left(\frac{b+n+1}{2b}\right)\right\}^2 \Gamma\left(\frac{n+2}{2b}\right)\Gamma\left(\frac{n}{2b}\right)}{\left\{\Gamma\left(\frac{n+1}{2b}\right)\right\}^2 \Gamma\left(\frac{b+n}{2b}\right)\Gamma\left(\frac{b+n+2}{2b}\right)}$, pour $a+b$ impair;

10) $= Cosec.\frac{a\pi}{b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} Sin.\frac{na\pi}{b} . l \frac{\left\{\Gamma\left(\frac{b-n+1}{b}\right)\right\}^2 \Gamma\left(\frac{n+2}{b}\right)\Gamma\left(\frac{n}{b}\right)}{\left\{\Gamma\left(\frac{n+1}{b}\right)\right\}^2 \Gamma\left(\frac{b-n}{b}\right)\Gamma\left(\frac{b-n+2}{b}\right)}$, pour $a+b$ pair;

11) $\int \left\{ Tang.\frac{a\pi}{2b} - \frac{2x^q Sin.\frac{a\pi}{b}}{1+x^2+2x\,Cos.\frac{a\pi}{b}} \right\} \frac{dx}{l\frac{1}{x}} = Tang.\frac{a\pi}{2b} . l2b + 2 \sum_1^{b-1} (-1)^{n-1} Sin.\frac{na\pi}{b} . l \frac{\Gamma\left(\frac{q+b+n}{2b}\right)}{\Gamma\left(\frac{q+n}{2b}\right)}$, pour $a+b$ impair;

12) $= Tang.\frac{a\pi}{2b} . lb + 2 \sum_1^{\frac{b-1}{2}} (-1)^{n-1} Sin.\frac{na\pi}{b} . l \frac{\Gamma\left(\frac{q+b-n}{b}\right)}{\Gamma\left(\frac{q+n}{b}\right)}$, pour $a+b$ pair;

13) $\int \frac{1+x^2}{1+x^4+2x^2 Cos.\lambda} \frac{dx}{(lx)^{1-q}} = Sec.\frac{1}{2}\lambda\,\Gamma(q) \sum_1^{\infty} (-1)^n \frac{Cos.\{(n+\frac{1}{2})\lambda\}}{(2n+1)^q}$

Les formules (8) à (13) sont déduites par Malmsten, Cr. 38. 1. Partout $a < b$.

1) $\int \frac{(1-x^q)(1-x^p)-(1-x)^2}{1-x} \frac{dx}{xlx} = lB(p,q)$ Binet, P. 27. 123. — Id., C. R. 9. 39.

2) $\int \frac{x^{q-1}-x^{-q}}{1+x} \frac{dx}{lx} = l\,Tang.\frac{1}{2}q\pi$ Kummer, Cr. 17. 210.

3) $\int \frac{(x^q-x^{-q})^2}{1+x} \frac{dx}{lx} = l(q\pi\,Cot.q\pi)$ Binet, P. 27. 123.

4) $\int \frac{x^q+x^{-q}-2}{1-x} \frac{dx}{lx} = l\frac{Sin.q\pi}{q\pi}$ Legendre, Exerc. 5. 3. — Binet, P. 27. 123.

5) $\int \frac{x^q - x^{-q}}{1+x^2} \frac{dx}{lx} = l\,Tang.\left(\frac{q+1}{4}\pi\right)$ V. T. 136. N°. 11.

6) $\int \frac{(x^q - x^{-q})^2}{1-x^2} \frac{dx}{lx} = l\,Cos\,q\pi$ Binet, P. 27. 123.

7) $\int \frac{(x^q - x^{-q})^2}{1-x^2} \frac{x}{lx} dx = l\frac{Sin.\,q\pi}{q\pi}$ Binet, P. 27. 123.

8) $\int \frac{x^q + x^{-q} - 2}{1-x^2} \frac{dx}{lx} = l\,Cos.\frac{1}{2}q\pi$ V. T. 136 N°. 12.

9) $\int \frac{x^{p-q} - x^{2p} + x^{p+q}}{1-x^{2p}} \frac{dx}{x\,lx} = l\,Cos.\frac{q\pi}{2p}$ Euler, N. Act. Petr. 1777 P. 2. 29. — Legendre, Exerc. 4. 112.

10) $\int \frac{x^q - 2x^p + x^{2p-q}}{1-x^{2p}} \frac{dx}{x\,lx} = l\,Sin.\frac{q\pi}{2p}$ Euler, N. Act. Petr. 1777 P. 2. 29. — Legendre, Exerc. 4. 112.

11) $\int \frac{x^{p+q} - x^{p-q}}{1+x^{2p}} \frac{dx}{x\,lx} = l\,Tang.\left(\frac{p+q}{4p}\pi\right)$ Euler, Calc. Int. 4. S. 3. 110.

12) $\int \left\{\frac{x^{p-1}}{1-x} - \frac{x^{pq-1}}{1-x^q} - \frac{1}{x(1-x)} + \frac{1}{x(1-x^2)}\right\} \frac{dx}{lx} = q\,l\,p$ V. T. 135. N°. 17.

13) $\int \frac{x^q - x^r - x^{2p-r} + x^{2p-q}}{1-x^{2p}} \frac{dx}{x\,lx} = l\left(Sin.\frac{q\pi}{2p}.\,Cosec.\frac{r\pi}{2p}\right)$ Euler, N. Act. Petr. 1777. P. 2. 29. — Id., N. C. P. 7. 64.

14) $\int \frac{x^{p-q} - x^{p-r} - x^{p+r} + x^{p+q}}{1-x^{2p}} \frac{dx}{x\,lx} = l\left(Cos.\frac{q\pi}{2p}.\,Sec.\frac{r\pi}{2p}\right)$ Euler, N. Act. Petr. 1777. P. 2. 29. — Id., N. C. P. 7. 64.

15) $\int \frac{x^p - x^{r-p}}{1+x^r} \frac{dx}{x\,lx} = l\,Tang.\frac{p\pi}{2r}$ Euler, N. A. Petr. 1777. P. 2. 29.

16) $\int \frac{x^p - x^q}{1+x^r} \frac{1+x^{r-p-q}}{x} \frac{dx}{lx} = l\left\{Tang.\frac{p\pi}{2r}.\,Cot.\frac{q\pi}{2r}\right\}$ Euler, N. A. Petr. 7. 64.

17) $\int \frac{1-x}{1+x} \frac{x^2}{1+x^2} \frac{dx}{lx} = l\frac{2\sqrt{2}}{\pi}$ Legendre, Exerc. 5. 3.

18) $\int \frac{1-x}{1+x} \frac{1}{1+x^2} \frac{dx}{lx} = -\frac{1}{2}l2$ V. T. 171. N°. 1. et T. 175. N°. 17.

19) $\int \left\{\frac{1}{1-x} - \frac{p\,x^{p-1}}{1-x^p} + \left(pq - \frac{p+1}{2}\right)x^{p-1} + (1-pq)\right\} \frac{dx}{lx} = \frac{1-p}{2}l2\pi + \left(pq - \frac{1}{2}\right)lp$ Arndt, Gr. 10. 455.

20) $\int \left\{\frac{x^{q-1}}{1-x} - \frac{x^{pq-1}}{1-x^p} - \frac{p-1}{1-x^p}x^{p-1} - \frac{1}{2}(p-1)x^{p-1}\right\} \frac{dx}{lx} = \frac{1-p}{2}l2\pi + \left(pq - \frac{1}{2}\right)lp$ Arndt, Gr. 10. 455.

1) $\int\left(\frac{1}{1-x^2}+\frac{2-x}{2\,lx}-\frac{1-x}{x}\right)\frac{dx}{lx}=0$ V. T. 135. N°. 12.

2) $\int\left\{p-1-\frac{1}{1-x}+\left(\frac{1}{2}-\frac{1}{lx}\right)x^{p-1}\right\}\frac{dx}{lx}=\left(\frac{1}{2}-p\right)lp+p-\frac{1}{2}l\,2\pi$ V. T. 135. N°. 15.

3) $\int\left\{\left(q-\frac{1}{2}\right)\frac{x^{p-1}-x^{r-1}}{lx}+\frac{p\,x^{pq-1}}{1-x^p}-\frac{r\,x^{r-1}}{1-x^r}\right\}\frac{dx}{lx}=(p-r)\left\{\frac{1}{2}-q+l\Gamma\left(\frac{1}{2}\right)-l\Gamma(q)+\frac{1}{2}l2\right\}$ Arndt, Gr. 10. 455.

4) $\int\frac{x^{-q}+x^q}{1-x^2}\frac{dx}{(lx)^p}=\frac{\Gamma(1-p)}{\pi^{p-1}}\sum_0^\infty(-1)^{p+n}\left\{\frac{1}{(2n+1-q)^{1-p}}-\frac{1}{(2n+1+q)^{1-p}}\right\}$ V T. 136. N°. 18.

5) $\int\frac{x^{-q}-x^q}{1-x^2}\frac{dx}{(lx)^p}=(-1)^p\frac{\Gamma(1-p)}{\pi^{p-1}}\sum_0^\infty\left\{\frac{1}{(2n-1-q)^{1-p}}-\frac{1}{(2n-1+q)^{1-p}}\right\}$ V. T. 136. N°. 17.

6) $\int\frac{x^{-p}-x^p}{1-x^2}\frac{dx}{\frac{1}{4}\pi^2+(lx)^2}=Sin.\frac{1}{2}p\pi-\pi\,Cos.\frac{1}{2}p\pi.l\frac{1+Sin.\frac{1}{2}p\pi}{1-Sin.\frac{1}{2}p\pi},\ p<1;$ V. T. 138. N°. 13.

7) $\int\frac{x^{-p}+x^p}{1-x^2}\frac{lx}{\frac{1}{4}\pi^2+(lx)^2}dx=1-\frac{1}{2}\pi\,Cos.\frac{1}{2}p\pi+\frac{1}{2}Sin.\frac{1}{2}p\pi.l\frac{1-Sin.\frac{1}{2}p\pi}{1+Sin.\frac{1}{2}p\pi},p<1;$ V. T. 138. N°. 15.

8) $\int\frac{x^{-p}-x^p}{1-x^2}\frac{dx}{\pi^2+(lx^2)}=\frac{1}{2\pi}\left[-p\pi Cos.p\pi+Sin.p\pi.l\{2(1+Cos.p\pi)\}\right],p<1;$ V. T. 138. N°. 4.

9) $\int\frac{x^{-p}+x^p}{1-x^2}\frac{lx}{\pi^2+(lx)^2}dx=\frac{1}{2}\left[1-p\pi\,Sin.p\pi-Cos.p\pi\,l\{2(1+Cos.p\pi)\}\right]$ V. T. 138. N°. 6.

10) $\int\frac{x^{p-1}-x^{1-p}}{1-x^2}\frac{dx}{q^2+(lx)^2}=\frac{\pi}{q}\sum_1^\infty\frac{Sin.np\pi}{q+n\pi},\ p^2<1;$ V. T. 138. N°. 5.

11) $\int\frac{x^{p-1}+x^{1-p}}{1-x^2}\frac{lx}{q^2+(lx)^2}dx=-\frac{\pi}{2q}-\sum_1^\infty\frac{Cos.np\pi}{q+n\pi},\ p^2<1;$ V. T. 138. N°. 8.

12) $\int\frac{lx}{x(1-x^{2q})}\frac{x^{2q}}{\pi^2+(lx)^2}dx=-\frac{1}{2}lq-\frac{1}{4q}+\frac{1}{2}Z'(1+q)$ V. T. 138. N°. 11.

13) $\int\frac{lx}{x(1-x^q)}\frac{x^q}{4\pi^2+(lx)^2}dx=-\frac{1}{2}lq-\frac{1}{4q}+\frac{1}{2}Z'(1+q)$ V. T. 138. N°. 11.

1) $\int \frac{1}{(1+x)\sqrt{x}} \frac{dx}{\pi^2+(lx)^2} = \frac{1}{2\pi} l2$ V. T. 138. N°. 3.

2) $\int \frac{1}{(1+x)\sqrt{x}} \frac{dx}{4\pi^2+(lx)^2} = \frac{4-\pi}{8\pi}$ V. T. 138. N°. 2.

3) $\int \frac{1}{(1+x)\sqrt{x}} \frac{dx}{q^2+(lx)^2} = \frac{1}{4q}\left\{Z'\left(\frac{q+3\pi}{4\pi}\right) - Z'\left(\frac{q+\pi}{4\pi}\right)\right\}$ V. T. 138. N°. 1.

4) $\int \frac{1}{(1+x)\sqrt{x}} \frac{dx}{\pi^2+4(lx)^2} = \frac{1}{4\pi\sqrt{2}}\left\{\pi - l\frac{\sqrt{2}+1}{\sqrt{2}-1}\right\}$ V. T. 138. N°. 18.

5) $\int \frac{lx}{(1-x)\sqrt{x}} \frac{dx}{\pi^2+(lx)^2} = \frac{1}{2} - \frac{1}{4}\pi$ V. T. 138. N°. 16.

6) $\int \frac{lx}{(1-x)\sqrt{x}} \frac{dx}{\pi^2+4(lx)^2} = -\frac{\pi}{2\sqrt{2}} + 1 + \frac{1}{2\sqrt{2}} l\frac{\sqrt{2}-1}{\sqrt{2}+1}$ V. T. 138. N°. 19.

7) $\int \frac{1}{(1+\sqrt{x})\sqrt{x^3}} \frac{dx}{\pi^2+(lx)^2} = \frac{1}{2\pi\sqrt{2}}\left\{\pi - l\frac{\sqrt{2}+1}{\sqrt{2}-1}\right\}$ V. T. 138. N°. 18.

8) $\int \frac{lx}{(1-\sqrt{x})\sqrt{x^3}} \frac{dx}{\pi^2+(lx)^2} = \frac{-\pi}{2\sqrt{2}} + 1 + \frac{1}{2\sqrt{2}} l\frac{\sqrt{2}-1}{\sqrt{2}+1}$ V. T. 138. N°. 19.

9) $\int \frac{x^{-p}-x^p}{(1-x)\sqrt{x}} \frac{dx}{\pi^2+(lx)^2} = \frac{1}{2} Sin.p\pi + \frac{1}{2}\pi Cos.p\pi . l\frac{1-Sin.p\pi}{1+Sin.p\pi}, p < \frac{1}{2};$ V.T. 138. N°. 13.

10) $\int \frac{x^{-p}+x^p}{(1-x)\sqrt{x}} \frac{lx}{\pi^2+(lx)^2} dx = 1 - \frac{1}{2}\pi Cos.p\pi + \frac{1}{2} Sin.p\pi . l\frac{1-Sin.p\pi}{1+Sin.p\pi}, p < \frac{1}{2};$ V. T. 138. N°. 15.

11) $\int \frac{x^{-p}-x^p}{(1-x)\sqrt{x}} \frac{dx}{4\pi^2+(lx)^2} = \frac{1}{4\pi}\left[2p\pi Cos.2p\pi + Sin.2p\pi . l2\{(1+Cos.2p\pi)\}\right]$ V. T. 138. N°. 4.

12) $\int \frac{x^{-p}+x^p}{(1-x)\sqrt{x}} \frac{lx}{4\pi^2+(lx)^2} dx = \frac{1}{2}\left[1 - 2p\pi Sin.2p\pi - Cos.2p\pi . l\{2(1+Cos.2p\pi)\}\right]$ V. T. 138. N°. 6.

13) $\int \frac{x^{\frac{p-1}{2}} - x^{\frac{1-p}{2}}}{(1-x)\sqrt{x}} \frac{dx}{q^2+(lx)^2} = \frac{2\pi}{q}\sum_1^\infty \frac{Sin.2np\pi}{q+2n\pi}, p < 1;$ V. T. 138. N°. 5.

14) $\int \frac{x^{\frac{p-1}{2}} + x^{\frac{1-p}{2}}}{(1-x)\sqrt{x}} \frac{lx}{q^2+(lx)^2} dx = -\frac{\pi}{q} - 2\pi\sum_1^\infty \frac{Cos.2np\pi}{q+2n\pi}, p < 1;$ V. T. 138. N°. 8.

15) $\int \frac{1-x^{q-1}}{1-x} \frac{1-x^{q-1}}{\sqrt{x}} \frac{dx}{lx} = -l2^{2q-2}$ Legendre, Exerc. 4. 113. — Cisa de Grésy, Mém. Turin. 1821. 209. I. 31.

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16) $\int\left(\frac{1}{1-x}+\frac{1}{lx}-\frac{1}{2}\right)\frac{dx}{lx\sqrt{x}}=\frac{1}{2}(l2-1)$ V. T. 135. N°. 13.

17) $\int\left\{\frac{1}{lx}-\frac{1}{2}-\frac{\sqrt{x}}{lx}\right\}\frac{dx}{lx}=\frac{1}{2}(l2-1)$ V. T. 127. N°. 16.

18) $\int\left\{\left(\frac{1}{lx}-\frac{1}{2}\right)\sqrt{x}+\left(\frac{1}{2}+\frac{1}{1-x}\right)x\right\}\frac{dx}{xlx}=\frac{1}{2}l2\pi-\frac{1}{2}$ V. T. 135. N°. 16.

19) $\int\left\{\frac{1}{x}-\frac{1}{1+\sqrt{x}}\right\}\frac{dx}{lx}=\frac{1}{2}l\frac{4}{\pi}$ V. T. 135. N°. 5.

20) $\int\left\{\frac{1}{1-x}-\frac{2}{1-x^2}+\frac{\sqrt{\frac{1}{x}}}{lx}-\frac{1}{2lx}\right\}\frac{dx}{lx}=0$ V. T. 135. N°. 19.

21) $\int\left\{\frac{a-1}{2}+\frac{a-1}{1-x}+\frac{x^{a-1}}{1-\sqrt[a]{\frac{1}{x}}}+\frac{x^{ap}}{1-x}\right\}\frac{dx}{lx}=\left(ap+\frac{1}{2}\right)la-\frac{1}{2}(a-1)l2\pi$ V. T. 135. N°. 26.

22) $\int\left(\frac{b}{lx}+\frac{x^{q-1}}{1-\sqrt[b]{x}}\right)dx=-\sum_1^b Z'\left(q+\frac{b-n}{b}\right)$ Arndt, Gr. 10. 253.

23) $\int\left\{\left(p-\frac{1}{2}\right)x+\left(\frac{1}{2}-\frac{1}{lx}\right)\left(x^{p-1}-\sqrt{\frac{1}{x}}\right)\right\}\frac{dx}{lx}=\left(\frac{1}{2}-p\right)(lp-1)$ V. T. 127. N°. 19.

24) $\int\left\{\frac{x^{q-1}}{1-x}-\frac{x^{pq-1}+(p-1)x^{\frac{1}{2}p-1}}{1-x^p}\right\}\frac{dx}{lx}=\frac{1}{2}(1-p)l2+\left(pq-\frac{1}{2}\right)lp$ Arndt, Gr. 10. 455.

25) $\int l(1-x)\frac{q\,lx\,\{x^{\frac{1}{2}q}+x^{-\frac{1}{2}q}\}-(x^{\frac{1}{2}q}-x^{-\frac{1}{2}q})}{x(lx)^2}(x^{\frac{1}{2}q}-x^{-\frac{1}{2}q})dx=l\frac{Sin.q\pi}{q\pi}$ V. T. 175. N°. 4.

F. Algébr. rat. Log. en dén. sous forme irrat. — TABLE 178. — Lim. 0 et 1.

1) $\int\frac{x^{p-1}}{\sqrt{l\frac{1}{x}}}dx=\sqrt{\frac{\pi}{p}}$ Bonnet, L. 17. 265.

2) $\int\frac{x^q-1}{\sqrt{lx}}dx=\left\{\frac{1}{\sqrt{(1+q)}}-1\right\}\sqrt{-\pi}$

3) $\int\frac{x^p-x^q}{\sqrt{lx}}dx=\left\{\frac{1}{\sqrt{(1+p)}}-\frac{1}{\sqrt{(1+q)}}\right\}\sqrt{-\pi}$

(2, 3) Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 36.

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4) $\int \frac{1}{1+x^2} \frac{dx}{\sqrt{l\frac{1}{x}}} = \sqrt{\pi} \sum_1^\infty \frac{(-1)^n}{\sqrt{(2n+1)}}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 37.

5) $\int \frac{1}{1+x+x^2} \frac{dx}{\sqrt{l\frac{1}{x}}} = \frac{\sqrt{\pi}}{Sin.\frac{1}{3}\pi} \sum_1^\infty (-1)^{n-1} \frac{Sin.\frac{1}{3}n\pi}{\sqrt{n}}$ V. T. 140. N°. 20.

6) $\int \frac{Cos.\lambda - x - x^a Cos.\{(a+1)\lambda\} + x^{a+1} Cos.a\lambda}{1-2x\,Cos.\lambda + x^2} \frac{dx}{\sqrt{lx}} = \Gamma\left(\frac{1}{2}\right) \sum_1^a \frac{Cos.n\lambda}{\sqrt{n}}$

7) $\int \frac{Sin.\lambda - x^a Sin.\{(a+1)\lambda\} + x^{a+1} Sin.a\lambda}{1-2x\,Cos.\lambda + x^2} \frac{dx}{\sqrt{lx}} = \Gamma\left(\frac{1}{2}\right) \sum_1^a \frac{Sin.n\lambda}{\sqrt{n}}$

(6, 7) Bonnet, L. 17. 265.

8) $\int \frac{x^{p-1} - x^{q-1}}{\left(l\frac{1}{x}\right)^{2-\frac{1}{c}}} dx = \frac{c\,\Gamma\left(\frac{1}{c}\right)}{c-1} \left(q^{\frac{c-1}{c}} - p^{\frac{c-1}{c}}\right)$, $q > \quad > 0$; V. T. 140. N°. 16.

1) $\int l(1+x) \frac{dx}{x^{2-p}} = \frac{\pi\,Cosec.p\pi}{1-p}$, $0 < p < 1$; V. T. 18. N°. 5.

2) $\int l(1+x) \frac{dx}{x^{1+p}} = \frac{\pi}{p} Cosec.p\pi$, $0 < p < 1$; V. T. 22. N°. 1.

3) $\int l(1+qx) \frac{dx}{x^{1-p}} = -\frac{\pi}{(p+1)q^{p-1}} Cosec.p\pi$, $p < 1$; V. T. 18. N°. 7.

4) $\int l(1-x) \frac{dx}{x^{2-p}} = \frac{\pi}{p-1} Cot.p\pi$, $0 < p < 1$; V. T. 18. N°. 8.

5) $\int l(1+x^2) \frac{dx}{x^{2-p}} = \frac{\pi}{1-p} Sec.\frac{1}{2}p\pi$, $0 < p < 1$; V. T. 19. N°. 8.

6) $\int l(1-x^2) \frac{dx}{x^{3-p}} = \frac{\pi}{p-2} Cot.\frac{1}{2}p\pi$, $0 < p < 1$; V. T. 19. N°. 9.

7) $\int l(1+x^3) \frac{dx}{x^3} = \frac{\pi}{3}\sqrt{3}$ V. T. 19. N°. 10.

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8) $\int l(1+x^4)\frac{dx}{x^4} = \frac{1}{3}\pi\sqrt{2}$ V. T. 19. N°. 13.

9) $\int l(1+x^6)\frac{dx}{x^2} = 2\pi$ V. T. 19. N°. 17.

10) $\int l(1+x^6)\frac{dx}{x^4} = \frac{1}{3}\pi$ V. T. 19. N°. 16.

11) $\int l(1+x^6)\frac{dx}{x^6} = \frac{2}{5}\pi$ V. T. 19. N°. 15.

12) $\int l\frac{(x+1)(x+q^2)}{(x+q)^2}\frac{dx}{x} = (lq)^2$

13) $\int l\frac{1+x}{\sqrt{(x^2+2x\,Cos.\lambda+1)}}\frac{dx}{x} = \frac{1}{2}\lambda^2,\ 0<\lambda<\pi;$

(12, 13) $, q>1;$ Minding, Tafeln. II.

14) $\int l(1-x^2)\frac{(p-1)x^p+(p+1)x^{-p}}{x^2}dx = -\pi\,Tang.\tfrac{1}{2}p\pi,\ 1>p>0;$ V. T. 22. N°. 7.

15) $\int l\frac{1+2x+x^2}{1+2x\,Cos.\lambda+x^2}\frac{dx}{x^{1-p}} = 2\pi\frac{1-Cos.p\lambda}{p\,Sin.p\pi}$

16) $\int l\frac{1+2x+x^2}{1+2x\,Cos.\lambda+x^2}\frac{x^p+x^{-p}}{x}dx = \frac{4\pi}{p\,Sin.p\pi}(1-Cos.p\lambda)$

(15, 16) $, p<1;$ Legendre, Exerc. 4. 104.

17) $\int l\frac{(x+1)(x+q^2)}{(x+q)^2}\frac{dx}{x^{1-p}} = \frac{\pi}{p}\frac{(q^p-1)^2}{Sin.p\pi},\ p^2<1,\ q>1;$

18) $\int l\frac{x+1}{\sqrt{(x^2+2x\,Cos.\lambda+1)}}\frac{dx}{x^{1-p}} = \frac{\pi}{p\,Sin.p\pi}(1-Cos.p\lambda),\ 0<\lambda<\pi,\ p^2<1;$

(17, 18) Minding, Tafeln. II.

19) $\int lx.l\frac{a^2+2bx+x^2}{a^2-2bx+x^2}\frac{dx}{x} = 2\pi\,la.\,Arcsin.\frac{b}{a},\ a \geqq b;$ Schlömilch, Gr. 4. 316.

20) $\int lx.l\frac{1+b^2x^2}{1+a^2x^2}\frac{dx}{x^2} = \pi(b-a)+\pi\,l\frac{a^a}{b^b}$ V. T. 45. N°. 1.

21) $\int l\left(1+\frac{x^2}{p^2}\right).l\left(1+\frac{q^2}{x^2}\right)\frac{dx}{x^2} = 2\pi\frac{p+q}{pq}l\frac{p+q}{p}-\frac{2\pi}{p}$ Schlömilch, Gr. 4. 71.

22) $\int l(1-x).\{p\,lx-1\}\frac{dx}{x^{p+1}} = (\pi\,Cosec.\,p\pi)^2,\ p<1;$ V. T. 183. N°. 2.

23) $\int l(1+x^2)\frac{\{(p-2)x^p-(q-2)x^q\}lx+x^q-x^p}{(lx)^2}\frac{dx}{x^3} = 2\pi l\left\{Tang.\frac{1}{4}q\pi.Cot.\frac{1}{4}p\pi\right\},\ p<1,\ q<1;$ V.T. 180. N°. 7.

24) $\int l(1-x^2)\frac{\{(p-2)x^p-(q-2)x^q\}lx+x^q-x^p}{(lx)^2}\frac{dx}{x^3}=2\pi l\left\{Sin.\frac{1}{2}q\pi.Cosec.\frac{1}{2}p\pi\right\}, p<1, q<1;$ V. T. 180. N°. 13.

25) $\int l(1-x)\{(p-1)lx+1\}\frac{dx}{x^{2-p}} = -(\pi\, Cosec.\, p\,\pi)^2,\ 0<p<1;$ V. T. 183. N°. 1.

TABLE 180.

F. Alg. rat. fract. à dén. binôme. Log. $(l\,x)^a$. Lim. 0 et ∞.

1) $\int\frac{x^{p-1}}{x+q}lx\,dx = \pi q^{p-1}\,Cosec.\,p\pi\,(lq-\pi\,Cot.\,p\,\pi),\ 0<p<1,\ q>1;$ Minding, Taf. II.

2) $\int\frac{lx}{1+x^2}dx = 0$ Hill, Cr. 3. 101. — Bidone, Mém. Turin. 1812. 231. Art. 3. 37.

3) $\int\frac{(lx)^{2a+1}}{1+x^2}dx = 0$

4) $\int\frac{(lx)^{2a}}{1+x^2}dx = 2.1^{2a/1}\sum_0^\infty(-1)^n\frac{1}{(2n+1)^{2a+1}}$

(3, 4) Bidone, Mém. Turin. 1812. 231. Art. 3. 37.

5) $= (-1)^{a+1}(2\pi)^{2a+1}\,B'\left(\frac{1}{4}\right)$ Raabe, Cr. 42. 348.

6) $\int\frac{x^q}{1+x^2}(lx)^a\,dx = \frac{1}{2}\pi\frac{d^a}{dq^a}.\,Sec.\frac{1}{2}q\,\pi$ Moigno, Calc. Int. 136.

7) $\int\frac{x^{p-1}-x^{q-1}}{1+x^2}\frac{dx}{lx} = \pi\,l\left(Tang.\frac{1}{4}p\,\pi\,.\,Cot.\,\frac{1}{4}q\,\pi\right)$ Cauchy, Lim. Imag. 128.

8) $\int\frac{lx}{q^2+x^2}dx = \frac{\pi}{2q}lq$ Schlömilch, Gr. 4. 316. — Arndt, Gr. 11. 70.

9) $\int\frac{lpx}{q^2+x^2}dx = \frac{\pi}{2q}lpq$ Arndt, Gr. 11. 70.

10) $\int\frac{lx}{p^2+q^2x^2}dx = \frac{\pi}{2pq}l\frac{p}{q}$ Lindmann, Stockh. Handl. 1850. II.

11) $\int\frac{lx}{1-x^2}dx = -\frac{1}{4}\pi^2$

12) $\int\frac{x^p\,lx}{1-x^2}dx = -\frac{1}{4}\left\{\pi\,Cosec.\left(\frac{p+1}{2}\pi\right)\right\}^2,\ p^2<1;$

(11, 12) Ohm, Ausw. 16.

13) $\int \frac{x^{p-1} - x^{q-1}}{1-x^2} \frac{dx}{lx} = \pi\, l \left(Sin. \frac{1}{2} p \pi . Cosec. \frac{1}{2} q \pi\right)$ Cauchy, Lim. Imag. 129.

14) $\int \frac{1-x}{1-x^{2a}} x^{a-2}\, l x\, dx = -\left(\frac{\pi}{2a} Tang. \frac{\pi}{2a}\right)^2,\ a > 1;$

15) $\int \frac{1-x^2}{1-x^{2a}} x^{a-3}\, l x\, dx = -\left(\frac{\pi}{2a} Tang. \frac{\pi}{a}\right)^2,\ a > 2;$

Schlömilch, Gr. 12. 208.

16) $\int \frac{1-x^2}{1-x^{2b}} x^{a-1}\, l x\, dx = -\left(\frac{\pi}{2b}\right)^2 \frac{Sin. \left\{\frac{a+1}{b}\pi\right\} . Sin. \frac{\pi}{b}}{Sin.^2 \frac{a\pi}{2b} . Sin.^2 \left\{\frac{a+2}{2b}\pi\right\}}$ Lindmann, Gr. 14. 94.

1) $\int l(1+x^2) \frac{dx}{1+x^2} = \pi\, l 2$

2) $\int l(q^2+x^2) \frac{dx}{1+x^2} = \pi\, l(1+q)$

Hill, Cr. 3. 101.

3) $\int l(1+p^2 x^2) \frac{dx}{1+x^2} = \pi\, l(1+p)$

4) $\int l\left(1+\frac{p^2}{x^2}\right) \frac{dx}{1+x^2} = \pi\, l(1+p)$

Schlömilch, Gr. 4. 71.

5) $\int l\left(\frac{x+1}{x-1}\right)^2 \frac{x}{1+x^2} dx = \frac{1}{2}\pi^2$ V. T. 340. N°. 14.

6) $\int l\left(1+\frac{1}{x^2}\right) \frac{dx}{1+p^2 x^2} = \frac{\pi}{p} l(1+p)$ Schlömilch, Gr. 4. 71.

7) $\int l(x^2+q^2) \frac{dx}{p^2+x^2} = \frac{\pi}{p} l(p+q)$ V. T. 265. N°. 12.

8) $\int l(x^2-q^2)^2 \frac{dx}{p^2+x^2} = \frac{\pi}{p} l(p^2+q^2)$ V. T. 265. N°. 13.

9) $\int l(x^4-q^4)^2 \frac{dx}{p^2+x^2} = \frac{\pi}{p} l(p^2+q^2)(p+q)^2$ V. T. 265. N°. 15.

10) $\int l\left(1+\frac{q^2}{x^2}\right)\frac{dx}{p^2+x^2}=\frac{\pi}{p}l\frac{p+q}{p}$

11) $\int l\left(1+\frac{q^2}{x^2}\right)\frac{dx}{p^2-x^2}=\frac{\pi}{p}\,Arctang.\frac{q}{p}$

Cauchy, Lim. Imag. Add. 28, 30.

12) $\int l\frac{x^2+Sec.^2\lambda}{1+x^2}\frac{dx}{1+x^2}=\pi\, l\left(Cos.^2\frac{1}{2}\lambda . Sec.\lambda\right)$ V. T. 334. N°. 9.

13) $\int l\frac{Cos.^2\lambda+x^2}{1+x^2}\frac{dx}{1+x^2}=2\pi\, l\, Cos.\frac{1}{2}\lambda$ V. T. 334. N°. 13.

14) $\int l\frac{x^2+Cos.^2\lambda}{x^2+Cos.^2\mu}\frac{dx}{1+x^2}=2\pi\, l\left(Cos.\frac{1}{2}\lambda . Sec.\frac{1}{2}\mu\right)$ V. T. 334. N°. 23.

15) $\int l\frac{1+Sin.2\lambda+x^2}{1-Sin.2\lambda+x^2}\frac{dx}{1+x^2}=\pi\, l\frac{1+Sin.\lambda}{Cos.\lambda}$ V. T. 334. N°. 22.

16) $\int l\frac{Cos.^2\mu-Cos.^2\lambda+x^2 Sin.^2\lambda}{Cos.^2\mu+x^2}\frac{dx}{1+x^2}=\pi l\left\{Cot.\frac{1}{2}\mu . Tang.\left(\frac{1}{2}Arcsin.\left(\frac{Sin.\mu}{Sin.\lambda}\right)\right)\right\}$ V. T. 334. N°. 24.

17) $\int l(1-e^{-2\pi x})\frac{dx}{p^2+x^2}=-\frac{\pi}{2p}l\left[\left(\frac{e}{p}\right)^p\Gamma(p)\sqrt{\frac{p}{2\pi}}\right]$ Liouville, L. 17. 448.

F. Alg. rat. fract. à dén. $(a\pm x^b)^c$. Log. TABLE 182. Lim. 0 et ∞ .

1) $\int lx\frac{dx}{(q+x)^2}=\frac{1}{q}lq\,,\; q<1$; Schlömilch, Beitr. III. § 10.

2) $\int (lx)^2\frac{x}{(1+x^2)^2}dx=0$ V. T. 153. N°. 15 et T. 187. N°. 5.

3) $\int (lx)^2\frac{dx}{(1-x)^2}=\frac{2}{3}\pi^2$ Minding, Taf. II.

4) $\int lx\frac{dx}{(q+x)^{p+1}}=\frac{1}{p\,q^p}\{lq-A-Z'(p)\}$ Schlömilch, Beitr. III. § 9.

5) $\int (lx)^2\frac{dx}{(1+x^2)^q}=\frac{\Gamma(q-\frac{1}{2})}{2\,\Gamma(q)}\sqrt{\pi}\left\{A+2l2+Z'\left(\frac{2q-1}{2}\right)\right\}$ V. T. 333. N°. 7.

6) $\int lx\frac{dx}{(a^2+b^2x^2)^p}=\frac{\Gamma(p-\frac{1}{2})}{a^{2p-1}\,b\,\Gamma(p)}\frac{\sqrt{\pi}}{4}\left\{2l\frac{a}{2p}-A-Z'\left(p-\frac{1}{2}\right)\right\}$ Lindmann, Gr. 16. 94.

7) $\int lx\frac{dx}{(a+x)^{b+2}}=\frac{1}{(b+1)a^{b+1}}\left\{la-\sum_1^a\frac{1}{n}\right\}$ Schlömilch, Beitr. III. § 10.

8) $\int l(1+x)\frac{x}{(q^2+x^2)^2}dx = \frac{1}{2(1+q^2)}\left(lq+\frac{\pi}{2q}\right)$ V. T. 24. N°. 3.

9) $\int l(1-x)^2\frac{x}{(q^2+x^2)^2}dx = \frac{1}{1+q^2}\left(lq-\frac{\pi}{2q}\right)$ V. T. 24. N°. 4.

10) $\int l(1+x)\frac{q^2-x^2}{(q^2+x^2)^2}dx = \frac{1}{1+q^2}\left(lq-\frac{1}{2}q\pi\right)$ V. T. 24. N°. 1.

11) $\int l(1-x)^2\frac{q^2-x^2}{(q^2+x^2)^2}dx = \frac{-2}{1+q^2}\left(lq+\frac{1}{2}q\pi\right)$ V. T. 24. N°. 2.

12) $\int l(1+x^2)\frac{dx}{(1+x^2)^2} = \frac{\pi}{2}\left(l2-\frac{1}{2}\right)$ V. T. 163. N°. 4.

13) $\int l(q+x)\frac{dx}{(1-x)^2} = \frac{2qlq}{1+q}$ V. T. 24. N°. 12.

14) $\int l(1-x)^2\frac{dx}{(q+x)^2} = \frac{2}{1+q}lq$ V. T. 24. N°. 12.

15) $\int l(q^2+x^2)\frac{dx}{(1+x)^2} = \frac{2q^2}{1+q^2}lq-\frac{q\pi}{1+q^2}, q<1;$ V. T. 24. N°. 1.

16) $\int l(q^2+x^2)\frac{dx}{(1-x)^2} = \frac{-2q^2}{1+q^2}lq+\frac{q\pi}{1+q^2}$ V. T. 24. N°. 2.

17) $\int l(q^2+x^2)\frac{x}{(1-x^2)^2}dx = \frac{2q^2lq}{1+q^2}$ V. T. 24. N°. 12.

18) $\int l(q^2+x^2)\frac{p^2-x^2}{(p^2+x^2)^2}dx = -\frac{\pi}{p+q}$ V. T. 24. N°. 8.

19) $\int l(q^2+x^2)\frac{p^2+x^2}{(p^2-x^2)^2}dx = -\frac{\pi q}{p^2+q^2}$ V. T. 24. N°. 11.

20) $\int l(1-x^2)^2\frac{x}{(q^2+x^2)^2}dx = \frac{2}{1+q^2}lq$ V. T. 24. N°. 12.

21) $\int l(p^2-x^2)^2\frac{q^2-x^2}{(q^2+x^2)^2}dx = -\frac{2q\pi}{p^2+q^2}$ V. T. 24. N°. 11.

22) $\int l\left(\frac{1+x}{1-x}\right)^2\frac{x}{(q^2+x^2)^2}dx = \frac{1}{1+q^2}\frac{\pi}{2q}$ V. T. 24. N°. 13.

1) $\int \frac{lx}{x}\frac{x^p}{x-1}\,dx = (\pi\,Cosec.\,p\,\pi)^2$, $0 < p < 1$; Minding, Tafeln. II.

2) $\int \frac{lx}{x-1}\frac{dx}{x^p} = (\pi\,Cosec.\,p\,\pi)^2$ Svanberg, Transf. § 5.

3) $\int l\left(\frac{1+x}{1-x}\right)^2 \frac{dx}{x(1+x^2)} = \frac{1}{2}\pi^2$ Schlömilch, Gr. 4. 316.

4) $\int lx\,\frac{1-x^{-p}}{1-x^2}\,dx = \left(\frac{1}{2}\pi\,Tang.\,\frac{1}{2}p\,\pi\right)^2$, $p < 1$; Schlömilch, Gr. 12. 208.

5) $\int lx\,\frac{(p+q)(x^{q-p}-x^{p-q})+(p-q)(x^{p+q}-x^{-p-q})}{(x^p-x^{-p})^2}\frac{dx}{x} = \frac{\pi}{p}\,Tang.\,\frac{q\pi}{2p}$ V. T. 22. N°. 14.

6) $\int lx\,\frac{(p-q)(x^{p+q}-x^{-p-q})+(p+q)(x^{p-q}-x^{q-p})}{(x^p+x^{-p})^2}\frac{dx}{x} = \frac{\pi}{p}\,Sec.\,\frac{q\pi}{2p}$ V. T. 22. N°. 15.

7) $\int lx\left(\frac{x}{1+x^2}\right)^{2a+1}\frac{dx}{x} = 0$

8) $\int lx\left(\frac{x}{q^2+x^2}\right)^{2a+1}\frac{dx}{x} = \frac{1^{a/2}}{2^{a/2}}\frac{\pi\,l\,q}{2\,q^{2a+1}}$

9) $\int lx\left(\frac{x}{q^2+x^2}\right)^{2a}\frac{dx}{x} = \frac{1^{a-1/1}}{a^{a/1}}\frac{lq}{2\,q^{2a}}$

10) $\int lx\left(\frac{x}{q^2+x^2}\right)^{p}dx = \frac{p}{2\,q^p}\frac{\{\Gamma(\frac{1}{2}p)\}^2}{\Gamma(p)}$

11) $\int l\frac{x}{q}\left(\frac{x}{q^2+x^2}\right)^{p}\frac{dx}{x} = 0$

(7–11) Schlömilch, Gr. 4. 316.

12) $\int \frac{lx}{x+q}\frac{dx}{x+1} = \frac{1}{2}\frac{(lq)^2}{q-1}$

13) $\int \frac{lx}{x+q}\frac{x^p}{x+1}\,dx = \frac{\pi}{q-1}\,\frac{q^p\,l\,q\,.\,Sin.\,p\,\pi+(1-q^p)\,\pi\,Cos.\,p\,\pi)}{Sin.^2\,p\,\pi}$

14) $\int \frac{lx}{x+q}\frac{dx}{x-1} = \frac{1}{2(1+q)}\{\pi^2+(l\,q^2)\}$

15) $\int \frac{lx}{x+q}\frac{x^p}{x-1}\,dx = \frac{\pi}{1+q}\,\frac{\pi+q^p(Sin.\,p\,\pi\,l\,q-\pi\,Cos.\,p\,\pi)}{Sin.^2\,p\,\pi}$

(12–15) $p^2 < 1$, $q > 1$; Minding, Taf. II.

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16) $\int lx \frac{p+x^2}{p^2+x^2}\frac{dx}{1+x^2} = \frac{1}{2}\frac{\pi}{1+p} lp$ V. T. 346. N°. 7.

17) $\int \frac{x^{p-1}-x^{q-1}}{1+x^{2q}}\frac{dx}{lx} = l\, Tang. \frac{p-q}{4q}$

18) $\int \frac{x^{p-1}-x^{q-1}}{1+x^r}\frac{dx}{lx} = l\left(Tang. \frac{p\pi}{2r}. Cot. \frac{q\pi}{2r}\right)$

(17, 18) Euler, N. A. Petr. 7. 64.

F. Alg. rat. fract. à autre dén. Log. d'autre forme. — TABLE 184. — Lim. 0 et ∞.

1) $\int \frac{(lx)^2}{x-1}\frac{dx}{x+q} = \frac{1}{3(1+q)} lq \{\pi^2+(lq)^2\}$

2) $\int \frac{(lx)^3}{x-1}\frac{dx}{x+q} = \frac{1}{4(1+q)} \{\pi^2+(lq)^2\}^2$

3) $\int \frac{(lx)^4}{x-1}\frac{dx}{x+q} = \frac{1}{15(1+q)} \{\pi^2+(lq)^2\}^2 \{7\pi^2+2(lq)^2\} lq$

4) $\int \frac{(lx)^5}{x-1}\frac{dx}{x+q} = \frac{1}{6(1+q)} \{\pi^2+(lq)^2\}^2 \{3\pi^2+(lq)^2\}^2$

(1–4) , $q>1$; Dedekind, Eul. Int. S. 23. — Minding, Taf. II; il a fautivement dans les dénominateurs 6, 24, 360, 720, au lieu de 3, 4, 15, 6;

5) $\int \frac{(lx)^{2a}}{1+x^2}(x^{-b}+x^b)\,dx = \frac{(-1)^{a+1}}{b} 2(2q\pi)^{2a+1} \sum_1^b (-1)^{n-1} B''\left(\frac{2n-1}{4b}\right) Cos.\left(\frac{2n-1}{2}a\pi\right)$

6) $\int \frac{(lx)^{2a}}{1-x^2}(x^{-b}-x^b)\,dx = \frac{(-1)^{a+1}}{b} 2(2q\pi)^{2a+1} \sum_1^b (-1)^{n-1} B''\left(\frac{n}{2b}\right) Sin.\, n a\pi$

(5, 6) Raabe, Cr. 42. 348.

7) $\int \frac{lx.\, l\frac{x}{q}}{x-q}\frac{x^p}{x-1}\,dx = \frac{\pi^2}{q-1}\frac{(q^p+1)lq - 2\pi(q^p-1)\,Cot.\, p\pi}{Sin.^2\, p\pi}$

8) $\int \frac{lx.\, l\frac{x}{q}}{x-q}\frac{dx}{x-1} = \frac{4\pi^2+(lq)^2}{6(q-1)} lq$

(7, 8) , $p^2<1$, $q>1$; Minding, Taf. II.

9) $\int (lx)^2 \frac{dx}{1+2x\,Cos.\lambda+x^2} = \lambda\, Cosec.\, \lambda \frac{\pi^2-\lambda^2}{3}$ Legendre, Exerc. 4. 105.

10) $\int (lx)^4 \frac{dx}{1+2x\,Cos.\lambda+x^2} = \frac{2}{5}\lambda\, Cosec.\, \lambda (\pi^2-\lambda^2)(7\pi^2-3\lambda^2)$ Legendre, Exerc. 4. 105.

$$11) \int (lx)^{2a} \frac{dx}{1+x+x^2} = \frac{(-1)^{a+1}}{\sqrt{3}} 2(2\pi)^{2a+1} B''\left(\frac{1}{3}\right)$$

$$12) \int (lx)^{2a} \frac{dx}{1-x+x^2} = \frac{(-1)^{a+1}}{\sqrt{3}} 2(2\pi)^{2a+1} B''\left(\frac{1}{6}\right)$$

$$13) \int (lx)^{2a} \frac{dx}{1-2x\,Cos.2p\pi+x^2} = (-1)^{a+1}(2\pi)^{2a+1} Cosec.2p\pi B''(p)$$

(11–13) $, p < 1;$ Raabe, Cr. 42. 348.

$$14) \int l(1+x^2) \frac{dx}{x(1+x^2)} = \frac{1}{12}\pi^2$$ V. T. 152. N°. 14.

$$15) \int l(1+x) \frac{xlx-x-q}{(x+q)^2} \frac{dx}{x} = \frac{1}{q-1} lq$$ V. T. 183. N°. 12.

$$16) \int l(1-x) \frac{xlx-x-q}{(x+q)^2} \frac{dx}{x} = \frac{1}{2(1-q)} \{\pi^2 + (lq)^2\}$$ V. T. 183. N°. 14.

$$17) \int l(1-x) \frac{x}{1-x^2} dx = \infty$$ V. T. 181. N°. 5.

$$18) \int l\left(1+\frac{b^2}{x^2}\right) \frac{(a-xi)^{-p}+(a+xi)^{-p}}{2} dx = \frac{\pi}{p-1}\left\{\left(\frac{1}{a}\right)^p - \left(\frac{1}{a+b}\right)^p\right\}$$ Cauchy, Lim. Imag. Add. 29.

$$19) \int \left\{\frac{p-1}{(1+x)^2} - \frac{(1+x)^{-1}-(1+x)^{-p}}{x}\right\} \frac{dx}{l(1+x)} = l\Gamma(p)$$ Féaux, Fonct. Transc.

F. Alg. irrat. fract. Log. **TABLE 185.** Lim. 0 et ∞.

$$1) \int lx \frac{dx}{x\sqrt{x}} = 2\pi$$ V. T. 28. N°. 1.

$$2) \int lx \frac{1-x}{(1+x)^2} \frac{dx}{\sqrt{x}} = -\frac{1}{2}\pi$$ V. T. 28. N°. 1.

$$3) \int lx \frac{1+x}{(1-x)^2} \frac{dx}{\sqrt{x}} = 0$$ V. T. 28. N°. 2.

$$4) \int lx \frac{dx}{\sqrt{(1+x^2)\{1+(1-p^2)x^2\}}} = -\frac{1}{2} F'(p)\, l(1-p^2),\ p^2 < 1;$$ V. T. 347. N°. 13.

$$5) \int lx \frac{dx}{\sqrt{(1+x^2)(x^2+1-p^2)}} = \frac{1}{2} F'(p)\, l(1-p^2),\ p^2 < 1;$$ V. T. 347. N°. 13.

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6) $\int l x \frac{1-x^{\frac{2}{a}}}{1-x^2} dx = \left(\frac{1}{2}\pi \, Tang. \frac{\pi}{a}\right)^2 , \; a > 2;$

7) $\int l x \frac{1-x^{-\frac{2}{a}}}{1-x^2} dx = \left(\frac{1}{2}\pi \, Tang. \frac{\pi}{a}\right)^2 , \; a > 2;$

8) $\int l x \frac{1-x^{\frac{2}{a}}}{1-x^2} x^{\frac{b}{a}-1} dx = -\left(\frac{\pi}{2}\right)^2 \frac{Sin. \left\{\frac{b+1}{a}\pi\right\} . Sin. \frac{\pi}{a}}{Sin.^2 \frac{b\pi}{2a} . Sin.^2 \left\{\frac{b+2}{2a}\pi\right\}}$

(6–8) Lindmann, Gr. 16. 94.

9) $\int l x \frac{dx}{(q+x)^{b+\frac{3}{2}}} = \frac{2}{(2b+1) q^{b+\frac{1}{2}}} \left\{ lq + 2l2 - \sum_1^{b-1} \frac{1}{n} - 2 \sum_b^{2b-1} \frac{1}{n} \right\}$ Schlömilch, Beitr. III. § 10.

10) $\int l x \frac{dx}{(1-x^2)^{\frac{1}{2}+a}} = -\frac{1^{a|2}}{2^{a+1} \, 1^{a|1}} \frac{\pi}{2} \left\{ 1 + 2l2 + \sum_1^{a} \frac{1}{n} \right\}$ V. T. 331. N°. 10.

11) $\int (lx)^{2a} \frac{x^{\frac{1}{2}b-1}}{1+x^b} dx = \frac{(-1)^{a+1}}{b} (4\pi)^{2a+1} \sum_1^{b} (-1)^{n-1} B'' \left(\frac{2n-1}{4b}\right)$ Raabe, Cr. 42. 348.

12) $\int l(1-x)^2 \frac{dx}{x\sqrt{x}} = 0$ V. T. 28. N°. 2.

13) $\int l(1+x) \frac{dx}{x^{p+\frac{1}{2}}} = \frac{2}{2p-1} \pi \, Sec. \, p\pi , \; p^2 < \frac{1}{4};$ V. T. 28. N°. 5.

14) $\int l \frac{1 - Cot h p^2 . \lambda + x^2}{1 + Cot h p^2 . \lambda + x^2} \frac{2}{1 + (1 - Cos h p^2 . \lambda) x^2} \frac{dx}{\sqrt{(1+x^2)}} = \frac{2 \lambda \, l \, Sin \, h p . \lambda}{Sin h p . \lambda . Cos h p . \lambda}$ V. T. 343. N°. 12.

F. Alg. Log. de fonct. irrat. TABLE 186. Lim. 0 et ∞.

1) $\int l \frac{\sqrt{(1+x^2)} + \sqrt{(1-p^2)}}{\sqrt{(1+x^2)} - \sqrt{(1-p^2)}} \frac{dx}{\sqrt{(1+x^2)}} = \pi \, Arccos. \, p$, $p^2 \leqq 1;$

2) $\int l \frac{\sqrt{(1+x^2)} + p}{\sqrt{(1+x^2)} - p} \frac{dx}{\sqrt{(1+x^2)}} = \pi \, Arcsin. \, p$

(1–2) Raabe, Int. 421.

3) $\int l(1+\sqrt{x})^2 \frac{dx}{(q+x)^2} = \frac{1}{1+q} \left(l q + \frac{\pi}{\sqrt{q}} \right)$ V. T. 24. N°. 3.

4) $\int l(1-\sqrt{x})^2 \frac{dx}{(q+x)^2} = \frac{1}{1+q} \left\{ l q - \frac{\pi}{\sqrt{q}} \right\}$ V. T. 24. N°. 4.

F. Alg. Log. de fonct. irrat.	TABLE 186 suite.	Lim. 0 et ∞ .

5) $\int l \frac{x}{\sqrt[3]{(1+x^3)}} \frac{dx}{1+x^3} = -\frac{\pi l 3}{3\sqrt{3}} - \frac{\pi^2}{27}$

6) $\int l \frac{x}{\sqrt[3]{(1+x^3)}} \frac{x}{1+x^3} dx = -\frac{\pi l 3}{3\sqrt{3}} + \frac{\pi^2}{27}$

7) $\int l \frac{x}{\sqrt[3]{(1+x^3)}} \frac{dx}{1-x+x^2} = -\frac{2\pi l 3}{3\sqrt{3}}$

8) $\int l \frac{x}{\sqrt[3]{(1+x^3)}} \frac{1-x}{1+x^3} dx = -\frac{2}{27}\pi^2$

(5–8) Euler, Calc. Int. 4. S. 3. 161. — Id., Act. Petr. 1777. II. 3.

F. Alg. Log.	TABLE 187.	Lim. 1 et ∞ .

1) $\int (lx)^p \frac{dx}{x^2} = \Gamma(1+p)$ V. T. 42. N°. 2.

2) $\int lx \frac{dx}{1+x^2} = \sum_0^\infty \frac{1}{(2n+1)^2}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 37.

3) $\int lx \frac{dx}{1-x^2} = -\frac{1}{8}\pi^2$ Ohm, Ausw. 16.

4) $\int lx \frac{x}{(1+x^2)^2} dx = \frac{1}{4} l \frac{1}{2}$ V. T. 335. N°. 1.

5) $\int lx \frac{dx}{x^2\sqrt{(x^2-1)}} = 1 - l2$ V. T. 103. N°. 3.

6) $\int (lx)^a \frac{dx}{1+x^2} = 1^{a/1} \sum_0^\infty (-1)^n \frac{1}{(2n+1)^{a+1}}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N . 37.

7) $\int l(1+x^2) \frac{dx}{1+x^2} = \frac{\pi}{2} l2 + \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 7.

8) $\int l(1+x^2) \frac{dx}{x(1+x^2)} = \frac{1}{2}\left\{\frac{1}{6}\pi^2 - (l2)^2\right\}$ V. T. 160. N°. 17 et T. 184. N°. 15.

9) $\int \frac{1}{lp - lx} \frac{dx}{x^2} = \frac{1}{p} li.p$ V. T. 43. N°. 5.

10) $\int \frac{1}{q + lx} \frac{dx}{x^2} = -e^q Ei.(-q)$ V. T. 129. N°. 9.

11) $\int \frac{1}{q - lx} \frac{dx}{x^2} = e^{-q} Ei.(q)$ V. T. 129. N°. 4.

12) $\int \frac{1}{q^2 + (lx)^2} \frac{dx}{x^2} = Ci.(q).Sin\,q - Si.(q).Cos.\,q + \frac{1}{2}\pi\,Cos.\,q$ V. T. 130. N°. 4.

13) $\int \frac{lx}{q^2 + (lx)^2} \frac{dx}{x^2} = Ci.(q).Cos.\,q + Si.(q).Sin.\,q - \frac{1}{2}\pi\,Sin.\,q$ V. T. 130. N°. 5.

14) $\int \frac{1}{q^2 - (lx)^2} \frac{dx}{x^2} = \frac{1}{2q}\{e^{-q}Ei.(q) - e^{q}Ei.(-q)\}$ V. T. 130. N°. 10.

15) $\int \frac{lx}{q^2 - (lx)^2} \frac{dx}{x^2} = -\frac{1}{2q}\{e^{-q}Ei.(q) + e^{q}Ei.(-q)\}$ V. T. 130. N°. 12.

16) $\int \frac{(lx)^b}{1 + (lx)^a} \frac{dx}{x^2} = \frac{b}{a}\,Cosec.\,\frac{b\pi}{a}$ V. T. 43. N°. 17.

17) $\int \frac{dx\sqrt{lx}}{1 + x^2} = \frac{1}{2}\sqrt{\pi}\sum_0^\infty(-1)^n \frac{1}{\sqrt{(2n+1)}}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 37.

18) $\int \frac{dx}{x^2\sqrt{lx}} = \sqrt{\pi}$ Euler, Calc. Int. 4. S. 5. 211.

19) $\int \frac{1}{1 + x^2} \frac{dx}{\sqrt{lx}} = \sqrt{\pi}\sum_0^\infty(-1)^n \frac{1}{\sqrt{(2n+1)}}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 37.

F. Alg. Log. TABLE 188. Lim. diverses 0 et p.

1) $\int_{-1}^{0} l(1-x)\frac{dx}{x} = \frac{1}{12}\pi^2$ Schaeffer, Cr. 30. 277.

2) $\int_{-1}^{0} l(x-1)^2 \frac{dx}{1+x^2} = -\frac{1}{4}\pi\, l2$ Hill, Cr. 3. 101.

3) $\int_0^{\sqrt{\frac{1}{2}}} lx \frac{dx}{\sqrt{(1-x^2)}} = -\frac{1}{4}\pi\, l2 - \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 271. N°. 4.

4) $\int_0^{\sqrt{\frac{1}{2}}} l(1-x^2)^2 \frac{dx}{\sqrt{(1-x^2)}} = \pi\, l2 + 2\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 271. N°. 6.

5) $\int_0^{\frac{1}{2}} l(1-x)\frac{dx}{x} = \frac{1}{2}(l2)^2 - \frac{1}{12}\pi^2$

6) $\int_0^{2} l(1-x)\frac{dx}{x} = -\frac{1}{4}\pi^2 + \pi i\, l2$

(5, 6) Schaeffer, Cr. 30. 277.

7) $\int_0^{\frac{1}{e}} \frac{dx\sqrt[q]{x}}{x\sqrt{\{-(1+lx)\}}} = \frac{\sqrt{q\pi}}{\sqrt[q]{e}}$ V. T. 150. N°. 1.

8) $\int_0^{\frac{-1+\sqrt{5}}{2}} l(1-x)\frac{dx}{x} = -\frac{1}{10}\pi^2 + \frac{1}{5}\left(l\frac{1+\sqrt{5}}{2}\right)^2 + \frac{2}{5}\left(l\frac{-1+\sqrt{5}}{2}\right)^2 l\frac{3-\sqrt{5}}{2}$

9) $\int_0^{\frac{3-\sqrt{5}}{2}} l(1-x)\frac{dx}{x} = -\frac{1}{15}\pi^2 - \frac{1}{5}\left(l\frac{1+\sqrt{5}}{2}\right)^2 + \frac{3}{5}l\frac{1+\sqrt{5}}{2}l\frac{3-\sqrt{5}}{2}$

10) $\int_0^{\frac{1-\sqrt{5}}{2}} l(1-x)\frac{dx}{x} = \frac{1}{15}\pi^2 - \frac{3}{10}\left(l\frac{1+\sqrt{5}}{2}\right)^2 + \frac{2}{5}l\frac{-1+\sqrt{5}}{2}l\frac{3-\sqrt{5}}{2} + l\frac{-1+\sqrt{5}}{2}l\frac{1+\sqrt{5}}{2}$

11) $\int_0^{\frac{1+\sqrt{5}}{2}} l(1-x)\frac{dx}{x} = -\frac{7}{30}\pi^2 + \frac{3}{10}\left(l\frac{1+\sqrt{5}}{2}\right)^2 - \frac{2}{5}l\frac{-1+\sqrt{5}}{2}l\frac{3-\sqrt{5}}{2} + \pi i l\frac{1+\sqrt{5}}{2}$

12) $\int_0^{\frac{-1-\sqrt{5}}{2}} l(1-x)\frac{dx}{x} = \frac{1}{10}\pi^2 + \frac{4}{5}\left(l\frac{1+\sqrt{5}}{2}\right)^2 - \frac{2}{5}l\frac{-1+\sqrt{5}}{2}l\frac{3-\sqrt{5}}{2} + l\frac{-1+\sqrt{5}}{2}l\frac{1+\sqrt{5}}{2}$

13) $\int_0^{\frac{3+\sqrt{5}}{2}} l(1-x)\frac{dx}{x} = -\frac{4}{15}\pi^2 + \frac{1}{2}\left(l\frac{3+\sqrt{5}}{2}\right)^2 + \frac{1}{5}\left(l\frac{1+\sqrt{5}}{2}\right)^2 - \frac{3}{5}l\frac{-1+\sqrt{5}}{2}l\frac{3-\sqrt{5}}{2} + \pi i l\frac{3+\sqrt{5}}{2}$

(8–13: Schaeffer, Cr. 30. 277.)

14) $\int_{\frac{1}{q}}^{0} l(q+x)^2 \frac{dx}{1+x^2} = Arctang.\frac{1}{q}.\,l(1+q^2)$ Hill, Cr. 3. 101.

15) $\int_0^{q} l(1+qx)\frac{dx}{1+x^2} = l(1+q^2).\,Arctang.\,q$ Bertrand, L. 8. 110. — Grunert, Gr. 4. 113.

16) $\int_0^{e^{-q}} \frac{x^{a-1}}{q+lx}dx = -\infty$ V. T. 150. N°. 12.

17) $\int_0^{\frac{1}{e}} l\left(2l\frac{1}{x}-1\right)\frac{x^{2a-1}}{lx}dx = -\frac{1}{2}\{Ei.(-a)\}^2$ V. T. 383. N°. 3.

1) $\int_{e^{-q}}^{1} \frac{x^{a-1}}{q+lx}\,dx = \infty$ V. T. 149. N°. 15.

2) $\int_{1}^{\frac{1}{e}} \frac{lx}{(1-lx)^2}\frac{dx}{x} = \frac{1}{2}e-1$ V. T. 112. N°. 5.

3) $\int_{1}^{e^{-\frac{a}{c}}} \frac{x^{bc}-x^{-bc}}{lx}\,dx = \frac{1}{2}l\left(\frac{1+bc}{1-bc}\right)^2 + Ei.\left(-\frac{a}{c}+ab\right) - Ei.\left(-\frac{a}{c}-ab\right)$ V. T. 149. N°. 4.

4) $\int_{1}^{e} \frac{dx}{x^2\,lx} = \infty$ V. T. 112. N°. 4.

5) $\int_{-1}^{+1} l(px-q)\frac{x}{1-rx^2}\frac{dx}{\sqrt{(1-x^2)}} = \frac{\pi}{\sqrt{r(1-r)}}\,l\frac{p\sqrt{r}-\{1-\sqrt{(1-r)}\}\{q+\sqrt{(q^2-p^2)}\}}{p\sqrt{r}+\{1-\sqrt{(1-r)}\}\{q+\sqrt{(q^2-p^2)}\}}$

6) $\int_{-1}^{+1} l\left(\frac{1-x^a}{1-x}\right)\frac{x}{1-x^2 Sin.^2\lambda}\frac{dx}{\sqrt{(1-x^2)}} = 2\pi Cosec.\lambda \sum_{1}^{\frac{1}{2}(a-1)} l\frac{1-2g Tang.\frac{1}{2}\lambda + h Tang.^2\frac{1}{2}\lambda}{1+2g Tang.\frac{1}{2}\lambda + h Tang.^2\frac{1}{2}\lambda}$

, où $g = Cos.\left(\frac{2n+1}{a}\pi\right) + Cos.\left(\frac{1}{4}\pi + \frac{2n+1}{2a}\pi\right)\sqrt{\left\{2\,Sin.\left(\frac{2n+1}{a}\pi\right)\right\}}$,

$h = 1 + 2\,Sin.\left(\frac{2n+1}{a}\pi\right) + 2\,Sin.\left(\frac{1}{4}\pi + \frac{2n+1}{2a}\pi\right)\sqrt{\left\{2\,Sin.\left(\frac{2n+1}{a}\pi\right)\right\}}$;

7) $\int_{-1}^{+1} l\left(\frac{1-x^a}{1-x}\right)\frac{x}{\sqrt{(1-x^2)}}\,dx = \pi - 2\pi \sum_{1}^{\frac{1}{2}(a-1)} \left[Cos.\left(\frac{1}{4}\pi - \frac{2n+1}{a}\pi\right)\sqrt{\left\{2\,Sin.\left(\frac{2n+1}{a}\pi\right)\right\}}\right]$

(5–7) Lobatschewsky, Mém. Kasan. 1835. 1.

8) $\int_{-\infty}^{\infty} l\left(1+\frac{pi}{x}\right)\frac{dx}{q+xi} = 2\pi\,l\frac{p+q}{q}$

9) $\int_{-\infty}^{\infty} l\left(1+\frac{pi}{x}\right)\frac{dx}{q-xi} = 0$

(8–9) Cauchy, Lim. Imag. Add. N°. 27.

10) $\int_{p}^{\infty} l(1+x)\frac{dx}{x^2} = \frac{1}{p}l(1+p) - l\frac{1+p}{p}$ Schlömilch, Gr. 4. 71.

11) $\int_{-q}^{q} l(p-x)\,dx\sqrt{(q^2-x^2)} = \frac{1}{2}q\pi\left[lp - l\frac{pq-\sqrt{(p^2-q^2)}}{q^2} + \frac{\{pq-\sqrt{(p^2-q^2)}\}^2}{2q^2}\right]$ Plana, Mém. Turin. 1820. 389.

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12) $\int_{-q}^{q} l(1-px)dx\sqrt{(q^2-x^2)} = \frac{1}{2}q^2\pi l\left\{2\frac{1-\sqrt{(1-p^2q^2)}}{p^2q^2}\right\} + \frac{\pi}{4p^2}\left\{1-\sqrt{(1-p^2q^2)}\right\}^2$ Plana, Mém. Turin. 1820. 389.

13) $\int_{-q}^{q} l(x-r)\frac{x}{q-px^2}\frac{dx}{\sqrt{(q^2-x^2)}} = \frac{\pi q}{\sqrt{p(1-p)}} l\frac{q\sqrt{p}-\{1-\sqrt{(1-p)}\}\{r+\sqrt{(r^2-q^2)}\}}{q\sqrt{p}+\{1-\sqrt{(1-p)}\}\{r+\sqrt{(r^2-q^2)}\}}$ Lobatschewsky. Mém. Kasan. 1835. 1.

14) $\int_{p}^{q}\frac{(r+lx)^s}{x}dx = \frac{1}{s+1}\{(r+lq)^{s+1}-(r+lp)^{s+1}\}$

15) $\int_{p}^{q}(lx+2a\pi i)^s\frac{dx}{x} = \frac{1}{1+s}\{(lq+2a\pi i)^{s+1}-(lp+2a\pi i)^{s+1}\}$

16) $\int_{p}^{q}\frac{1}{lx+r}\frac{dx}{x} = l((r+lq))-l((r+lp))$

17) $\int_{p}^{q}\frac{1}{lx+2a\pi i}\frac{dx}{x} = l((lq+2a\pi i))-l((lp+2a\pi i))$

(14–17) Ohm, Ausw. 5, 6.

18) $\int_{a}^{b}(lx)^2\frac{dx}{\sqrt{(b^2-x^2)(x^2-a^2)}} = \frac{1}{b}l\,a\,b.\,\mathrm{F}'\left\{\sqrt{\frac{a^2-b^2}{a^2}}\right\}$ Roberts, L. 14. 238.

F. Alg. Log. de Log. | TABLE 190. | Lim. 0 et 1.

1) $\int ll\frac{1}{x}\,x^{a-1}\,dx = -\frac{1}{a}(\mathrm{A}+la)$ Malmsten, Cr. 38. 1.

2) $\int ll\frac{1}{x}.\left(l\frac{1}{x}\right)^{p-1}x^{a-1}\,dx = -\frac{\Gamma(p)}{a^p}\{\mathrm{Z}'(p)-la\}$ V. T. 377. N°. 2.

3) $\int ll\frac{1}{x}\,x^{a-1}\frac{dx}{\sqrt{l\frac{1}{x}}} = -\sqrt{\frac{\pi}{a}}\{\mathrm{A}+2l2+la\}$ V. T. 273. N°. 4.

4) $\int ll\frac{1}{x}\frac{1}{1+x^2}\frac{dx}{\sqrt{l\frac{1}{x}}} = \sqrt{\pi}\sum_{0}^{\infty}(-1)^{n+1}\frac{l(2n+1)+2l2+\mathrm{A}}{\sqrt{(l2n+1)}}$ V. T. 381. N°. 4.

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5) $\int ll\frac{1}{x}\,\frac{x^{-\frac{b}{c}}+x^{\frac{b}{c}}}{1+x^2}\,dx=\frac{1}{2}\pi Sec.\frac{b\pi}{2c}(l\pi-A)-\pi\sum_0^\infty\left\{\frac{l\left\{(2n+1)\pi-\frac{b\pi}{c}\right\}}{(2n+1)\pi-\frac{b\pi}{c}}-\frac{l\left\{(2n+1)\pi+\frac{b\pi}{c}\right\}}{(2n+1)\pi+\frac{b\pi}{c}}\right\}$ V. T. 274. N°. 1.

6) $\int ll\frac{1}{x}\,\frac{x^{-\frac{b}{c}}-x^{\frac{b}{c}}}{1-x^2}\,dx=\frac{1}{2}\pi Tang.\frac{b\pi}{2c}(l\pi+A)-\pi\sum_0^\infty\left\{\frac{l\left\{(2n+1)\pi-\frac{b\pi}{c}\right\}}{(2n+1)\pi-\frac{b\pi}{c}}-\frac{l\left\{(2n+1)\pi+\frac{b\pi}{c}\right\}}{(2n+1)\pi+\frac{b\pi}{c}}\right\}$ V. T. 274. N°. 4.

7) $\int ll\frac{1}{x}\,\frac{dx}{(1+x)^2}=\frac{1}{2}\left\{Z'\left(\frac{1}{2}\right)+l\,2\pi\right\}$ Malmsten, Cr. 38. 1.

8) $\int ll\frac{1}{x}\,\frac{1}{1+x+x^2}\,\frac{dx}{\sqrt{l\frac{1}{x}}}=\frac{\sqrt{\pi}}{Sin.\frac{1}{3}\pi}\sum_1^\infty(-1)^n Sin.\frac{1}{3}n\pi\,\frac{l\,n+2\,l\,2+A}{\sqrt{(2n+1)}}$ V. T. 381. N°. 15.

9) $\int ll\frac{1}{x}\,\frac{dx}{1+2x\,Cos.\lambda+x^2}=\frac{1}{2}\pi\,Cosec.\lambda\,l\,\frac{(2\pi)^{\frac{\lambda}{\pi}}\Gamma\left(\frac{1}{2}+\frac{\lambda}{2\pi}\right)}{\Gamma\left(\frac{1}{2}-\frac{\lambda}{2\pi}\right)}$ Malmsten, Cr. 38. 1.

10) $\int l\{a^2+(lx)^2\}\,\frac{dx}{1+x^2}=\pi\,l\,\frac{2\Gamma\left(\frac{2a+3\pi}{4\pi}\right)}{\Gamma\left(\frac{2a+\pi}{4\pi}\right)}-\frac{1}{2}\pi\,l\frac{\pi}{2}$ V. T. 275. N°. 17.

11) $\int l\{a^2+(lx)^2\}\,\frac{x^{-\frac{b}{c}}+x^{\frac{b}{c}}}{1+x^2}\,dx=\pi Sec.\frac{b\pi}{2c}l2c\pi+2\pi\sum_1^c(-1)^{n-1}Cos.\left\{\left(n-\frac{1}{2}\right)\frac{b\pi}{c}\right\}l\frac{\Gamma\left(\frac{2a+2\pi n-\pi}{4c\pi}+\frac{1}{2}\right)}{\Gamma\left(\frac{2a+2\pi n-\pi}{4c\pi}\right)}$, pour $b+c$ impair;

V. T. 275. N°. 12, 13.

12) $=\pi Sec.\frac{b\pi}{2c}lc\pi+2\pi\sum_1^{\frac{c-1}{2}}(-1)^{n-1}Cos.\left\{\left(n-\frac{1}{2}\right)\frac{b\pi}{c}\right\}l\frac{\Gamma\left(\frac{2a-2\pi n+\pi}{2c\pi}+1\right)}{\Gamma\left(\frac{2a+2\pi n-\pi}{2c\pi}\right)}$, pour $b+c$ pair;

13) $\int l\left\{\frac{1}{4}\pi^2c^2+(lx)^2\right\}\frac{x^{-\frac{b}{c}}+x^{\frac{b}{c}}}{1+x^2}\,dx=\pi\,Sec.\frac{b\pi}{2c}\,l\pi+$, pour $b+c$ impair;

$+\pi\sum_1^c(-1)^{n-1}Cos.\left\{\left(n-\frac{1}{2}\right)\frac{b\pi}{c}\right\}l\left\{\left(\frac{c+1}{2}-n\right)Cot.\left(\frac{\pi}{4}-\frac{2n-1}{4c}\pi\right)\right\}$ V. T. 275. N°. 16.

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14) $\int l\left\{\frac{\pi^2}{4}+(lx)^2\right\}\frac{dx}{1+x^2}=\frac{1}{2}\pi l\frac{\pi^2}{8}$ V. T. 275. N°. 1.

15) $\int l\{a^2+(lx)^2\}\frac{x^{-\frac{b}{c}}-x^{\frac{b}{c}}}{1-x^2}dx=\pi\, Tang.\frac{b\pi}{2c}l2c\pi+2\pi\sum_1^{c-1}(-1)^{n-1}Sin.\frac{nb\pi}{c}l\frac{\Gamma\left(\frac{a+\pi n}{2c\pi}+\frac{1}{2}\right)}{\Gamma\left(\frac{a+\pi n}{2c\pi}\right)}$, pour $b+c$ impair; V. T. 275. N°. 14, 15.

16) $=\pi\, Tang.\frac{b\pi}{2c}lc\pi+2\pi\sum_1^{\frac{c-1}{2}}(-1)^{n-1}Sin.\frac{nb\pi}{c}l\frac{\Gamma\left(\frac{a-\pi n}{c\pi}+1\right)}{\Gamma\left(\frac{a+\pi n}{c\pi}\right)}$, pour $b+c$ pair;

17) $\int l\left\{\frac{1}{4}\pi^2c^2+(lx)^2\right\}\frac{x^{-\frac{b}{c}}-x^{\frac{b}{c}}}{1-x^2}dx=-\pi\, Tang.\frac{b\pi}{2c}l\pi+$
$+\pi\sum_0^{c-1}(-1)^{n-1}Sin.\frac{bn\pi}{c}\; l\left\{\left(\frac{1}{2}c-n\right)Cot.\left(\frac{\pi}{4}-\frac{n\pi}{2c}\right)\right\}$, pour $b+c$ impair; V. T. 275. N°. 17.

18) $\int l\{a^2+(lx)^2\}\frac{dx}{(1+x)\sqrt{x}}=2\pi l\frac{2\Gamma\left(\frac{a+3\pi}{4\pi}\right)}{\Gamma\left(\frac{a+\pi}{4\pi}\right)}+\pi l\pi$ V. T. 275. N°. 18.

19) $\int l\{a^2+(lx)^2\}\frac{1+x^{\frac{2}{3}}}{1+x^{\frac{2}{3}}+x^{\frac{4}{3}}}\frac{dx}{\sqrt[3]{x^2}}=-\pi l\pi-2\pi\, Sin.\frac{\pi}{3}l\frac{6\Gamma\left(\frac{a+4\pi}{6\pi}\right)\Gamma\left(\frac{a+5\pi}{6\pi}\right)}{\Gamma\left(\frac{a+\pi}{6\pi}\right)\Gamma\left(\frac{a+2\pi}{6\pi}\right)}$ V. T. 275. N°. 19.

20) $\int\left\{(p-1)x+\frac{(1-lx)^{-1}-(1-lx)^{-p}}{l(1-lx)}\right\}\frac{dx}{xlx}=-l\Gamma(p)$ V. T. 378. N°. 10.

21) $\int\left\{\frac{x}{lx}+\frac{1}{(1-lx)^2l(1-lx)}\right\}\frac{dx}{x}=0$ V. T. 378. N°. 9.

22) $\int\left\{x-\frac{(1-lx)^{-(p+1)}}{l(1-lx)}\right\}\frac{dx}{xlx}=-lp$ V. T. 378. N°. 11.

F. Alg. Log. de Log. — **TABLE 191.** — Lim. 0 ou 1 et ∞.

1) $\int_0^\infty llx\frac{dx}{1+x^2}=\frac{1}{2}\pi l\left(\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}\sqrt{2\pi}\right)$ Malmsten, Cr. 38. 1.

2) $\int_0^\infty l\,l\,x \frac{dx}{1+x+x^2} = \frac{\pi}{\sqrt{3}} l \left(\frac{\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{3}\right)} \sqrt[3]{2\pi}\right)$

3) $\int_0^\infty l\,l\,x \frac{x^{a-1}-x^{-a-1}}{x^b-x^{-b}} dx = \frac{\pi}{2b} Tang. \frac{a\pi}{2b} l 2\pi + \frac{\pi}{b} \sum_1^{b-1} (-1)^{n-1} Sin. \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b+n}{2b}\right)}{\Gamma\left(\frac{n}{2b}\right)}$, pour $a+b$ impair;

4) $= \frac{\pi}{2b} Tang. \frac{a\pi}{2b} l\pi + \frac{\pi}{b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} Sin. \frac{na\pi}{b} l \frac{\Gamma\left(\frac{b-n}{b}\right)}{\Gamma\left(\frac{n}{b}\right)}$, pour $a+b$ pair;

5) $\int_0^\infty l\,l\,x \frac{x^{a-2}}{1+x^2+x^4+\dots+x^{2a-2}} dx = \frac{\pi}{2a} Tang. \frac{\pi}{2a} l 2\pi + \frac{\pi}{a} \sum_1^{a-1} (-1)^{n-1} Sin. \frac{n\pi}{a} l \frac{\Gamma\left(\frac{a+n}{2a}\right)}{\Gamma\left(\frac{n}{2a}\right)}$, pour a pair;

6) $= \frac{\pi}{2a} Tang. \frac{\pi}{2a} l\pi + \frac{\pi}{a} \sum_1^{\frac{a-1}{2}} (-1)^{n-1} Sin. \frac{n\pi}{a} l \frac{\Gamma\left(\frac{a-n}{a}\right)}{\Gamma\left(\frac{n}{a}\right)}$, pour a impair;

7) $\int_1^\infty l\,l\,x \frac{dx}{1-x+x^2} = \frac{2\pi}{\sqrt{3}} \left\{\frac{5}{6} l 2\pi - l\Gamma\left(\frac{1}{6}\right)\right\}$

8) $\int_1^\infty l\,l\,x \frac{x^{a-1}+x^{-a-1}}{x^b+x^{-b}} dx = \frac{\pi}{2b} Sec. \frac{a\pi}{2b} l 2\pi + \frac{\pi}{b} \sum_1^{b} (-1)^{n-1} Cos. \frac{(n-\frac{1}{2})a\pi}{b} l \frac{\Gamma\left(\frac{b+n-\frac{1}{2}}{2b}\right)}{\Gamma\left(\frac{n-\frac{1}{2}}{2b}\right)}$, pour $a+b$ impair;

9) $= \frac{\pi}{2b} Sec. \frac{a\pi}{2b} l\pi + \frac{\pi}{b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} Cos. \frac{(n-\frac{1}{2})a\pi}{b} l \frac{\Gamma\left(\frac{b-n+\frac{1}{2}}{b}\right)}{\Gamma\left(\frac{n-\frac{1}{2}}{b}\right)}$, pour $a+b$ pair;

Les integrales 2 à 9 sont déduites par Malmsten, Cr. 38. 1, où il y a plusieurs fautes.

1) $\int x \, Sin.\, a x \, dx = \frac{1}{a^2}(Sin.\, a - a\, Cos.\, a)$ Kummer, Cr. 35. 1.

2) $\int x \, Cos.\, a x \, dx = \frac{1}{a^2}(a\, Sin.\, a + Cos.\, a - 1)$ Dienger, Cr. 46. 119.

3) $\int x^2 \, Cos.\, a x \, dx = \frac{1}{a^3}\{(a^2-2)\, Sin.\, a + 2a\, Cos.\, a\}$ V. T. 192. N°. 1.

4) $\int x^{q-1}\, Sin.\, 2p\pi x \, dx = \frac{1-q}{(lq)^2 + 4p^2\pi^2} \frac{2p\pi}{q}$ Kummer, Cr. 35. 1.

5) $\int \frac{Sin.\, p x}{x} dx = Si.(p) = \sum_1^\infty \frac{1}{2n-1} \frac{p^{2n-1}}{1^{2n-1/1}}$ Arndt, Gr. 10. 225. — Schlömilch, Cr. 33. 316.

6) $\int (1-x^2)^{\frac{2a-1}{2}} Cos.\, p x \, dx = \frac{1^{a/2}}{2^{a+2}\, 1^{a/1}} \left\{1 + \sum_1^\infty (-1)^n \frac{(\frac{1}{2}p)^{2n}}{1^{n/1}(a+1)^{n/1}}\right\}$ Bessel, Abh. Berlin. 1824. I.

7) $\int (1-x^2)^{a-\frac{3}{2}} Cos.\, 2 p x \, dx = \frac{\Gamma(a-\frac{1}{2})}{2.1^{a/1}} \sqrt{\pi} \sum_0^\infty (-1)^n \frac{p^{2n}}{1^{n/1}\, a^{n/1}}$

8) $\int (1-x^2)^{a-b-1} x^{2b-1} Cos.\, p x \, dx = \frac{1^{b/1}\, 1^{a-b/1}}{2.1^{a/1}} \sum_0^\infty (-1)^n \frac{b^{n/1}}{2^{2n/1}\, a^{n/1}} p^{2n}$ Schlömilch, Stud. I. 24.

9) $\int x^{b-1}(1-x)^{a-b-1} Cos.(\sqrt{p x})\, dx = \frac{1^{b/1}\, 1^{a-b/1}}{1^{a/1}} \sum_0^\infty \frac{b^{n/1}}{2^{2n/1}\, a^{n/1}} p^n$

(7–9: Schlömilch, Stud. I. 24.)

10) $\int Sin.\left\{\frac{1}{2}p\left(x + \frac{1}{x}\right)\right\} . Sin.\left\{\frac{1}{2}p\left(x - \frac{1}{x}\right)\right\} \frac{dx}{1-x^2} = -\frac{1}{2}\pi\, Sin.\, p$ Cauchy, Lim. Imag. Add. 17.

11) $\int \frac{Cos.\, q x - Cos.\left(\frac{q}{x}\right)}{x - \frac{1}{x}} \frac{dx}{x} = -\frac{1}{2}\pi\, Sin.\, q$

12) $\int \left\{\frac{Cos.\, q x}{x - \frac{1}{x}} - b \frac{Cos.\left(\frac{q}{x^b}\right)}{x^b - x^{-b}}\right\} \frac{dx}{x} = \frac{1}{2}\pi(Cos.\, q\, .\, l b - Sin.\, q)$

(11–12: Cauchy, P. 19. 511.)

13) $\int \left\{\frac{Sin.\, \lambda}{1 + 2x\, Cos.\, \lambda + x^2} - \frac{\lambda}{(1+x)^2}\right\} dx = 0$ Malmsten, Cr. 38. 1.

1) $\int x\, Sin.\, x\, dx = 0$

2) $\int x\, Cos.\, x\, dx = -1$

Boncompagni, Cr. 25. 74.

3) $\int x^2\, Sin.\, qx\, dx = -\frac{2}{q^3}$ Cisa de Grésy, Mém. Turin. 1821. 209. II. 50. — Oettinger, Cr. 38. 216.

4) $\int x^4\, Sin.\, qx\, dx = \frac{24}{q^4}$

5) $\int x\, Cos.\, qx\, dx = -\frac{1}{q^2}$

6) $\int x^3\, Cos.\, qx\, dx = \frac{6}{q^4}$

7) $\int x^5\, Cos.\, qx\, dx = -\frac{120}{q^6}$

8) $\int x^{2a}\, Sin.\, qx\, dx = (-1)^a \frac{1^{2a/1}}{q^{2a+1}}$

9) $\int x^{2a}\, Cos.\, qx\, dx = 0$

10) $\int x^{2a-1}\, Sin.\, qx\, dx = 0$

11) $\int x^{2a-1}\, Cos.\, qx\, dx = (-1)^a \frac{1^{2a-1/1}}{q^{2a}}$

Oettinger, Cr. 38. 216.

12) $\int x^{p-1}\, Sin.\, x\, dx = \Gamma(p)\, Sin. \frac{1}{2} p\pi$

13) $\int x^{p-1}\, Cos.\, x\, dx = \Gamma(p)\, Cos. \frac{1}{2} p\pi$

, $p^2 < 1$;

Cauchy, Sav. Etr. 1827. 124. Note 3. — Plana, Mém. Bruxelles. 1837. — Boncompagni, Cr. 25. 74.

14) $\int x^{p-1}\, Sin.\, qx\, dx = \frac{\Gamma(p)}{q^p}\, Sin. \frac{1}{2} p\pi$

15) $\int x^{p-1}\, Cos.\, qx\, dx = \frac{\Gamma(p)}{q^p}\, Cos. \frac{1}{2} p\pi$

Legendre, Exerc. 3. 55. — Plana, Mém. Bruxelles. 1837. — Oettinger, Cr. 38. 216. — Schlömilch, Stud. I. 13. (pour $1 > q > 0$ et $q^2 < 1$ resp.). — Raabe, Int. 416. (pour tout p et q).

16) $\int x^{p-2}\, Sin.\, q x\, d x = \frac{1}{(1-p)\, q^{p-1}}\, \Gamma(p)\, Cos.\, \frac{1}{2} p \pi$ Plana, Mém. Brux. 1837.

17) $\int x\, Sin.\, (x^2).\, Sin.\, p x\, d x = \frac{1}{8} p \sqrt{2\pi}.\left\{Cos.\left(\frac{1}{4}p^2\right) + Sin.\left(\frac{1}{4}p^2\right)\right\}$

18) $\int x\, Cos.\, (x^2)\, Sin.\, p x\, d x = -\frac{1}{8} p \sqrt{2\pi}.\left\{Cos.\left(\frac{1}{4}p^2\right) - Sin.\left(\frac{1}{4}p^2\right)\right\}$

(17, 18) Cauchy, Lim. Imag. § 192. — Id., Sav. Etr. 1827. 124. Note 2.

19) $\int x^{p-1}\, Sin.\, (q x^r)\, d x = \frac{\Gamma\left(\frac{p}{r}\right)}{r \sqrt[r]{q^p}}\, Sin.\, \frac{p\pi}{2r}$

20) $\int x^{p-1}\, Cos.\, (q x^r)\, d x = \frac{\Gamma\left(\frac{p}{r}\right)}{r \sqrt[r]{q^p}}\, Cos.\, \frac{p\pi}{2r}$

(19, 20) Raabe, Int. 416.

21) $\int \frac{x\, Sin.\, x}{Cos.\, \lambda - Cos.\, x}\, d x = 2\pi\, l(1 - Cos.\, \lambda + i\, Sin.\, \lambda)$

22) $\int \frac{x\, Sin.\, x}{e^q + e^{-q} - 2\, Cos.\, x}\, d x = \pi\, l\, (1 - e^{-q})$

(21, 22) Poisson, P. 18. 295. N°. 37.

23) $\int \frac{x\, Sin.\, x}{p - Cos.\, x}\, d x = \pi\, l\{2\,(1+p)\} - i\, Arctang.\, \frac{\sqrt{(1-p^2)}}{1+p}$ Plana, Mém. Turin. 1820.

F. Alg. rat. fract. à dén. x.
Circ. Dir. en num. d'un fact. mon.

TABLE 194.

Lim. 0 et ∞.

1) $\int \frac{Sin.\, x}{x}\, d x = \frac{1}{2}\pi$ Mascheroni, Adn. 52. — Euler, Calc. Int. T. 4. S. 5. 139. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 2. — Fourier, Chal. 415. — Laplace, P. 15. 229. — Lobatschewsky, Mém. Kasan. 1835. 211. — Id., Cr. 24. 164. — Schlömilch, Gr. 1. 417. — Id., Cr. 36. 268. — Id., Beitr. III. § 4.

2) $\int \frac{Cos.\, x}{x}\, d x = A$ (fautive) Mascheroni, Adn. 45. — Boncompagni, Cr. 24. 75.

3) $= \infty$ Laplace, P. 15. 229.

4) $= -A - l\, 0$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 5.

5) $\int \frac{Sin.\, p x}{x}\, d x = \frac{1}{2}\pi\, ,\, p > 0;$

6) $= 0 \quad ,\, p = 0;$

7) $= -\frac{1}{2}\pi, p < 0;$

(5–7) Poisson, Chaleur. 102, 158. — Id., Mém. Acad. 1823. 571. N°. 12. — Cauchy, Cours. Leç. 33. — Id., Exerc. 1826. P. 95. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 53. — Pioch, Mém. Courr. Brux. T. 15. P. 2. — Libri, Cr. 7. 224. — Besge, L. 14. 31.

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8) $\int \frac{Sin. p x}{x} d x = \frac{1}{2} \pi$, p très-petit ; Cauchy, Sav. Etr. 1827. 599. S. 2.

Sur la formule (5) seule voyez encore: Legendre, Exerc. 3. 46. — Cauchy, Lim. Imag. Add. 16. — Id., Cours. Leç. 33. — Laplace, Probab. L. 1. 25. — Bidone, Mém. Turin. 1812. 231. Art. 2. 18. — Poisson, P. 16. 215. N°. 2. — Lobatto, Cr. 11. 171. — Raabe, Cr. 23. 105. — Oettinger, Cr. 38. 216. — Bonnet, L. 14. 249. — Schlömilch, Gr. 1. 417. — Id., Cr. 36. 268. — Id., Stud. I. 13. — Lindmann, Gr. 16. 94.

9) $\int \frac{Cos. p x}{x} d x = \infty$ Legendre, Exerc. 3. 46. — Cauchy, Cours. Leç. 33. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 53.

10) $= -\infty$ Lobatto, Cr. 11. 171. (fautive).

11) $= - A - l a - l 0$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 6.

12) $\int \frac{Tang. x}{x} d x = \frac{1}{2} \pi$ Schlömilch, Gr. 4. 316.

13) $\int \frac{Tang. p x}{x} d x = \frac{1}{2} \pi$ Legendre, Exerc. 5. 35. — Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 38. — Plana, Mém. Turin. 1818. 7. I. 13.

14) $\int \frac{Sin.^{2a+1} x}{x} d x = (-1)^a \binom{-\frac{1}{2}}{a} \frac{\pi}{2}$ Raabe, Cr. 23. 105. — Id., Cr. 25. 160.

15) $= \frac{1}{2} \pi \frac{1^{a/2}}{2^{a/2}}$ Raabe, Cr. 23. 105. — Schlömilch, Gr. 4. 316 (pour a fraction à dénominateur et numérateur impairs).

16) $\int \frac{Sin.^3 q x}{x} d x = \frac{1}{4} \pi$

17) $\int \frac{Sin.^5 q x}{x} d x = \frac{3}{16} \pi$

18) $\int \frac{Sin.^{2a} q x}{x} d x = \infty$

19) $\int \frac{Sin.^{2a+1} q x}{x} d x = \frac{\pi}{2^{2a+1}} \sum_0^a (-1)^n \binom{2a+1}{a+n+1}$

(16–19) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 10, 15.

20) $\int \frac{Sin. \{(p-q)x\}}{x} d x = \frac{1}{2} \pi$, $p > q$;

21) $= 0$, $p = q$;

22) $= - \frac{1}{2} \pi$, $p < q$;

(20–22) Pioch, Mém. Courr. Brux. T. 15. P. 2.

1) $\int \frac{Sin.\,x.\,Cos\,q\,x}{x}\,dx = \frac{1}{2}\pi,\ q^2 \leqq 1;$

2) $= 0,\ q^2 > 1;$

Fourier, Chal. 357. — Schlömilch, Stud. I. 21.

3) $\int \frac{Sin.\,q\,x.\,Cos.\,x}{x}\,dx = \frac{1}{2}\pi,\ q > 1;$

4) $= 0,\ q < 1;$

Serret, L. 8. 489.

5) $\int \frac{Sin.\,q\,x.\,Cos.\,p\,x}{x}\,dx = \frac{1}{2}\pi,\ q > p;$

6) $= 0,\ q < p;$

Legendre, Exerc. 8. 46. — Schlömilch, Cr. 36. 268. — Id., Stud. I. 21. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.

7) $= \frac{1}{4}\pi,\ q = p;$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.

Libri, Cr. 7. 224 et Arndt, Gr. 11. 70 trouvent les mêmes formules (5) à (7) pour les limites respectives $q > p > -q,\ -q > p > -\infty$ et $+q < p < \infty,\ q = \pm p$.

8) $\int \frac{Sin.\,q\,x.\,Sin.^2\,p\,x}{x}\,dx = 0,\ -\infty \leqq q < -2p;$

9) $= -\frac{1}{8}\pi,\ q = -2p;$

10) $= -\frac{1}{4}\pi,\ -2p < q < 0;$

11) $= 0,\ q = 0;$

12) $= \frac{1}{4}\pi,\ 0 < q < 2p;$

13) $= \frac{1}{8}\pi,\ q = 2p;$

14) $= 0,\ 2p < q \leqq \infty;$

Dienger, Gr. 12. 416.

15) $\int \frac{Sin.\,q\,x.\,Cos.^2\,p\,x}{x}\,dx = \frac{1}{2}\pi,\ p > 2q;$

16) $= \frac{3}{8}\pi,\ p = 2q;$

17) $= \frac{1}{4}\pi,\ p < 2q;$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.

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18) $\int \frac{Sin.^2 qx . Cos.^3 px}{x} dx = \frac{1}{16} l \frac{(2q+p)^3 (p-2q)^3 (2q+3p)(3p-2q)}{9p^8}$, $p > 2q$;

19) $= \frac{1}{16} l \frac{(2q+p)^3 (2q-p)^3 (2q+3p)(3p-2q)}{9p^8}$, $3p > 2q > p$;

20) $= \frac{1}{16} l \frac{(2q+p)^3 (2q-p)^3 (2q+3p)(2q-3p)}{9p^8}$, $3p < 2q$;

21) $\int \frac{Sin.^2 qx . Cos.^3 qx}{x} dx = \frac{1}{16} l\, 15$

22) $\int \frac{Sin.^2 x . Cos.^3 \frac{2}{3} x}{x} dx = \infty$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.

23) $\int \frac{Sin.^{2a+1} x . Cos.^{2b} x}{x} dx = \frac{\Gamma(a+\frac{1}{2})\Gamma(b+\frac{1}{2})}{2\Gamma(a+b+1)}$

24) $\int \frac{Sin.^{2a+1} x . Cos.^{2b-1} x}{x} dx = \frac{\Gamma(a+\frac{1}{2})\Gamma(b+\frac{1}{2})}{2\Gamma(a+b+1)}$

25) $\int \frac{Sin.^{2a+1} x . Cos.^{2b} x}{x} dx = \frac{\pi}{2} \frac{1^{a/2} 1^{b/2}}{2^{a+b/2}}$

26) $\int \frac{Sin.^{2a+1} x . Cos.^{2b-1} x}{x} dx = \frac{\pi}{2} \frac{1^{a/2} 1^{b/2}}{2^{a+b/2}}$

Schlömilch, Gr. 4. 316, où a et b peuvent aussi être des fractions à numérateur et à dénominateur impairs.

27) $\int \frac{Sin. px . Sin. qx . Sin. rx}{x} dx = 0$, $-\infty \leqq p < -(q+r)$;

28) $= -\frac{1}{8}\pi$, $p = -(q+r)$;

29) $= -\frac{1}{4}\pi$, $-(q+r) < p < q-r$;

30) $= -\frac{1}{8}\pi$, $p = q-r$;

31) $= 0$, $q-r < p < r-q$;

32) $= \frac{1}{8}\pi$, $p = r-q$;

33) $= \frac{1}{4}\pi$, $r-q < p < r+q$;

34) $= \frac{1}{8}\pi$, $p = r+q$;

35) $= 0$, $r+q < p \leqq \infty$;

$, p < q < r$; Dienger, Cr. 12. 210.

F. Alg. rat. fract. à dén. x. Circ. Dir. en num. polyn.	TABLE 196.	Lim. 0 et ∞.

1) $\int \frac{Cos. x - Cos. q x}{x} dx = \frac{1}{2} l q^2$ Raabe, Cr. 23. 105. — Arndt, Gr. 11. 70.

2) $\int \frac{Cos. q x - Cos. p x}{x} dx = \frac{1}{2} l \frac{p^2}{q^2}$ Poisson, P. 16. 215 N°. 2. — Bidone, Mém. Turin. 1812. 231. Art. 1. 6. — Cisa de Grésy, Mém. Turin. 1821. 209. II. § 54. — Raabe, Cr. 23. 105 (pour q et p aussi des fractions).

3) $\int \frac{Sin.^2 q x - Sin.^2 p x}{x} dx = \frac{1}{2} l \frac{q}{p}$

4) $\int \frac{Sin.^{2a} q x - Sin.^{2a} p x}{x} dx = \frac{1}{2^{2a+1}} l \frac{q}{p} \sum_1^a (-1)^{n-1} \binom{2a}{a+n}$

(3, 4) Bidone, Mém. Turin. 1812. 231. Tableau.

5) $\int \frac{Cos.^{2a} q x - Cos.^{2a} p x}{x} dx = \frac{1}{2} l \left(\frac{p}{q}\right)^2 \left\{1 - \frac{(a+1)^{a/1}}{4^{a/4}}\right\}$

6) $\int \frac{Cos.^{2a+1} q x - Cos.^{2a+1} p x}{x} dx = \frac{1}{2} l \left(\frac{p}{q}\right)^2$

(5, 6) Schlömilch, Gr. 5. 152.

7) $\int \frac{Cos. \lambda - Cos. b \lambda x}{x} Sin. a x \, dx = \frac{1}{2} \pi (Cos. \lambda - 1) \quad , \frac{a}{b} > \lambda > 0;$

8) $\qquad = \frac{1}{2} \pi \, Cos. \lambda \quad , \frac{a}{b} < \lambda < \infty;$

(7, 8) Arndt, Gr. 11. 70.

9) $\int \frac{3 - 4 Sin.^2 q x}{x} Sin.^2 q x \, dx = \frac{1}{2} l 2$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 10.

F. Alg. rat. fract. à dén. x^a pour a spécial. Circ. Dir. en num. monôme d'un fact.	TABLE 197.	Lim. 0 et ∞.

1) $\int \frac{Sin. q x}{x^2} dx = q(1 - A - l q - l 0) = \infty$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7.

2) $\int \frac{Cos. q x}{x^2} dx = \infty$

3) $\qquad = \infty - \frac{1}{2} q \pi$

(2, 3) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 8. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 53.

4) $\int \frac{Sin. q x}{x^3} dx = -\infty - \frac{1}{4} q \pi^2$

5) $\int \frac{Cos. q x}{x^3} dx = \infty^2 - \frac{3}{4} q^2 + \frac{1}{2} q^2 \{A + l q + l 0\}$

(4, 5) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7, 8.

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6) $\int \frac{Sin.^2 x}{x^2} dx = \frac{1}{2}\pi$ Lindmann, Gr. 16. 94.

7) $\int \frac{Sin.^2 qx}{x^2} dx = \frac{1}{2} q \pi$

8) $\int \frac{Sin.^3 qx}{x^2} dx = \frac{3}{4} q\, l\, 3$

9) $\int \frac{Sin.^4 qx}{x^2} dx = \frac{1}{4} q \pi$

10) $\int \frac{Sin.^5 qx}{x^2} dx = 5q \frac{3\,l\,3 - l\,5}{16}$

11) $\int \frac{Sin.^6 qx}{x^2} dx = \frac{3}{16} q \pi$

12) $\int \frac{Sin.^{10} qx}{x^2} dx = \frac{35}{256} q \pi$

(7–12) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 11, 14.

13) $\int \frac{Sin.^3 qx}{x^3} dx = \frac{3}{8} q^2 \pi$

14) $\int \frac{Sin.^4 qx}{x^3} dx = q^2\, l\, 2$

15) $\int \frac{Sin.^5 qx}{x^3} dx = \frac{5}{32} q^2 \pi$

16) $\int \frac{Sin.^6 qx}{x^3} dx = 3q^2 \frac{8\,l\,2 - 3\,l\,3}{16}$

(13–16) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 12, 15.

17) $\int \frac{Sin.^4 qx}{x^4} dx = \frac{1}{2} q^3 \pi$

18) $\int \frac{Sin.^5 qx}{x^4} dx = 5q^3 \frac{25\,l\,5 - 27\,l\,3}{96}$

(17–18) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 11, 14.

19) $\int \frac{Sin.^3 qx}{x^5} dx = \infty$

20) $\int \frac{Sin.^5 qx}{x^5} dx = \frac{125}{384} q^4 \pi$

21) $\int \frac{Sin.^6 qx}{x^5} dx = \frac{27\,l\,3 - 32\,l\,2}{16} q^4$

(19–21) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 18, 15, 12.

1) $\int \frac{Sin.\,px.\,Sin.\,qx}{x^2}\,dx = \frac{1}{2}p\pi \;,\; p \leqq q\;;$

2) $= \frac{1}{2}q\pi \;,\; p \geqq q\;;$

Ohm, Ausw. 18.

3) $\int \frac{Sin.^2\,qx.\,Cos.^2\,px}{x^2}\,dx = \frac{2q-p}{4}\pi \;,\; q > p\;;$

4) $= \frac{1}{4}q\pi \;,\; q = p\;;$

5) $= \frac{1}{4}q\pi \;,\; q < p\;;$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.

6) $\int \frac{Sin.^2\,px.\,Sin.\,qx}{x^3}\,dx = \frac{1}{2}p^2\pi \;,\; q \geqq 2p\;;$

7) $= \frac{1}{8}\pi(4pq - q^2) \;,\; q \leqq 2p\;;$

8) $\int \frac{Sin.px.Sin.qx.Sin.rx}{x^3}\,dx = \frac{1}{2}pq\pi \;,\; r \geqq p+q\;;$

9) $= \frac{1}{4}\pi(pq+pr+qr) - \frac{1}{8}\pi(p^2+q^2+r^2) \;,\; r < p+q\;;$

où $p<q<r$;

10) $\int \frac{Sin.^2\,px.\,Sin^2\,qx}{x^4}\,dx = \frac{1}{6}p^2\pi(3q-p) \;,\; p \leqq q\;;$

11) $\int \frac{Sin.^3\,px.\,Sin.\,qx}{x^4}\,dx = \frac{1}{2}p^3\pi \;,\; q > 3p\;;$

12) $= \frac{1}{48}\pi\{24p^3 - (3p-q)^3\} \;,\; p \leqq q \leqq 3p\;;$

13) $= \frac{1}{48}\pi\{24p^2q - (p+q)^3\} \;,\; q \leqq p\;;$

Ohm, Ausw. 18.

1) $\int \frac{1 - Cos.\,qx}{x^2}\,dx = \frac{1}{2}\pi q \;,\; q > 0\;;$

2) $= -\frac{1}{2}\pi q \;,\; q < 0\;;$

Poisson, Mém. Acad. 1816. 71. N°. 16. — Id., Chal. 100.

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3) $\int \frac{Cos.\, q\, x - Cos.\, p\, x}{x^2}\, dx = \frac{p-q}{2}\pi$ Poisson, P. 16. 215. N°. 7. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 9.

4) $\int \frac{Sin.\, x - x\, Cos.\, x}{x^2}\, dx = 1$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7.

5) $\int \frac{p\, Cos.\, q\, x - r\, x\, Sin.\, q\, x + s}{x^2}\, dx = (r - p\, q)\frac{\pi}{2}$ Cellérier, L. 8. 255.

6) $\int \frac{Sin.\, q\, x - q\, x\, Cos.\, q\, x}{x^3}\, dx = \frac{1}{4}\pi q^2 \quad , q > 0;$

7) $= -\frac{1}{4}\pi q^2, \quad q < 0;$

(6, 7) Poisson, Mém. Acad. 1816. 71. N°. 16.

8) $\int \frac{Sin.\, x - x\, Cos.\, x}{x^3}\, dx = \frac{1}{4}\pi$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 9.

9) $\int \frac{x^3 - Sin.^3\, x}{x^5}\, dx = \frac{13}{32}\pi$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 13.

F. Alg. rat. fract. à dén. x^a pour a général. Circ. Dir. en num. mon. d'un fact. $Sin.^b x$. TABLE 200. Lim. 0 et ∞.

1) $\int \frac{Sin.\, x}{x^{p+1}}\, dx = \frac{C}{1-p}\frac{1}{p}\, Sin.\, \frac{1}{2} p\, \pi$

2) $\int \frac{Sin.\, x}{x^p}\, dx = \frac{C}{1-p}\, Sin.\left\{(2\alpha + 1)(1-p)\frac{\pi}{2}\right\}$, où α entier arbitraire;

3) $= \frac{C}{1-p}\, Cos.\, \frac{1}{2} p\, \pi$

4) $= 1$, pour p très-petit;

(1–4) , où $C = 0{,}906402$; $0 < p < 1$; Laplace, P. 15. 229.

5) $= \frac{\pi}{2\,\Gamma(p)}\, Cosec.\, \frac{1}{2} p\, \pi$, $2 > p > 0$; Schlömilch, Cr. 33. 353. — Id., Beitr. III. § 4.

6) $\int \frac{Sin.\, x}{x^{a+1+p}}\, dx = \frac{(-1)^{a+2}}{p^{a-1/1}}\, Sin.\left\{\frac{a+p}{2}\pi\right\}\Gamma(1-p)$, $p < 1$; Plana, Mém. Brux. 1837.

7) $\int \frac{Sin.\, q\, x}{x^p}\, dx = \frac{q^{p-1}}{2\,\Gamma(p)}\, \pi\, Cosec.\, \frac{1}{2} p\, \pi$, $2 > p > 0$; Schlömilch, Gr. 6. 200. — Id., Stud. I. 13.

8) $= \frac{\Gamma(1-p)}{q^{1-p}}\, Cos.\, \frac{1}{2} p\, \pi$ Lobatto, Int. 74.

9) $\int \frac{Sin.\, q x}{x^p} dx = \infty, p \geq 2$; Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7.

10) $\int \frac{Sin.\, q x}{x^{a+1}} dx = q^a \frac{i^{-a} - i^a}{2i} \infty$ Cisa de Grésy, Mém. Turin. 1821. 209. II. 51.

11) $= -1^{-1-a/1} q^a\, Sin. \frac{1}{2} a\pi, a < 1$; Oettinger, Cr. 38. 216.

12) $= q^a \Gamma(-a)\, Sin. \frac{1}{2} a\pi$ (val. extraord.) Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826. p. 53.

13) $\int \frac{Sin.\,(x-a)}{x^p} dx = \Gamma(1-p)\, Cos. \left(\frac{1}{2} p\pi + a\right), 0 < p < 1$; Svanberg, L. 11. 197.

14) $\int \frac{Sin.^{2b} q x}{x^{2a}} dx = (-1)^a \frac{\pi}{2^{2b}} \frac{q^{2a-1}}{1^{2a-1/1}} \sum_1^b (-1)^n \binom{2b}{b+n} (2n)^{2a-1}$

15) $\int \frac{Sin.^{2b} q x}{x^{2a+1}} dx = (-1)^a \frac{1}{2^{2b-1}} \frac{q^{2a}}{1^{2a/1}} \sum_1^b (-1)^{n-1} \binom{2b}{b+n} (2n)^{2a}\, l\, 2n$

(14, 15): $, 1 < a < b$;

16) $\int \frac{Sin.^{2b+1} q x}{x^{2a}} dx = (-1)^{a+1} \frac{1}{2^{2b}} \frac{q^{2a-1}}{1^{2a-1/1}} \sum_1^b (-1)^{n-1} \binom{2b+1}{b+n+1} (2n+1)^{2a-1} l(2n+1)$

17) $\int \frac{Sin.^{2b+1} q x}{x^{2a+1}} dx = (-1)^a \frac{\pi}{2^{2b+1}} \frac{q^{2a}}{1^{2a/1}} \sum_1^{b+1} (-1)^{n-1} \binom{2b+1}{b+n} (2n-1)^{2a}$

(16, 17): $, 0 < a < b+1$;

(14–17): Bidone, Mem. Turin. 1812. 231. Art. 1. N°. 11, 12, 14. 15.

18) $\int \frac{Sin.^b x}{x^a} dx = \frac{(-1)^{\frac{a+b}{2}-1}}{2^{b-1} 1^{a-1/1}} \frac{\pi}{2} \sum_1^{\frac{b-1}{2}} (-1)^n \binom{b}{n} \binom{b-2}{n}^{a-1}$, a et b impairs;

19) $= \frac{(-1)^{\frac{a+b}{2}}}{2^{b-1} 1^{a-1/1}} \frac{\pi}{2} \sum_0^{\frac{b}{2}-1} (-1)^n \binom{b}{n} \binom{b-2}{n}^{a-1}$, a et b pairs;

(18, 19): $, a \leq b$; Lindmann, Gr. 17. 455.

20) $\int \frac{Sin.^b x}{x^{a+1}} dx = \frac{\pi}{2^b 1^{a/1}} Cosec. \left\{\frac{a+b}{2}\pi\right\} \left\{b^a - \binom{b}{1}(b-2)^a + \binom{b}{2}(b-4)^a + \ldots + \binom{b}{\frac{1}{2}b-1} 2^a\right\}$, b pair, a impair;

21) $= \frac{\pi}{2^b 1^{a/1}} Cosec. \left\{\frac{a+b}{2}\pi\right\} \left\{b^a - \binom{b}{1}(b-2)^a + \binom{b}{2}(b-4)^a + \ldots + \binom{b}{\frac{1}{2}b-1)} 1^a\right\}$, b impair, a pair;

22) $= \frac{(-1)^{\frac{a+b}{2}}}{2^b 1^{a/1}} \left\{b^a\, l\, b - \binom{b}{1}(b-2)^a l(b-2) + \ldots\right\}$, a et b pairs, ou a et b impairs;

Les formules (20) à (22) se trouvent Cauchy, P. 28. 147. P. 1. § 2.

1) $\int \frac{Cos.\,x}{x^p}\,dx = \frac{C}{1-p}\,Cos.\left\{(2\alpha+1)(1-p)\frac{\pi}{2}\right\}$, où α entier arbitraire;

2) $= \frac{C}{1-p}\,Sin.\,\frac{1}{2}\,p\,\pi$

3) $= \frac{1}{2}\,p\,\pi$, pour p très-petit;

(1–3): où $C=0{,}906402$; $0<p<1$; Laplace, P. 15. 229

4) $= \frac{\pi}{2\,\Gamma(p)}\,Sec.\,\frac{1}{2}\,p\,\pi,\ 1>p>0$; Schlömilch, Cr. 33. 353. — Id., Beitr. III. § 4.

5) $\int \frac{Cos.\,x}{x^{a+1+p}}\,dx = \frac{(-1)^{a+1}}{p^{a-1|1}}\,Cos.\left\{\frac{a+p}{2}\,\pi\right\}\Gamma(1-p),\ p<1$; Plana, Mém. Brux. 1837.

6) $\int \frac{Cos.\,q\,x}{x^p}\,dx = \frac{q^{p-1}}{2\,\Gamma(p)}\,\pi\,Sec.\,\frac{1}{2}\,p\,\pi,\ 1>p>0$; Schlömilch, Gr. 6. 200. — Id., Stud. I. 13.

7) $= \frac{\Gamma(1-p)}{q^{1-p}}\,Sin.\,\frac{1}{2}\,p\,\pi$ Lobatto, Int. 74.

8) $= \infty,\ p \geqq 1$; Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7.

9) $\int \frac{Cos.\,2\,q\,x}{x^{2a}}\,dx = (-1)^a\,\frac{(2\,q)^{2a-1}}{1^{2a-1|1}}\,\frac{\pi}{2}$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 11.

10) $\int \frac{Cos.\,q\,x}{x^{a+1}}\,dx = q^a\,\frac{i^{-a}+i^a}{2}\,\infty$ Cisa de Grésy, Mém. Turin. 1821. 209. II. N°. 51.

11) $= 1^{-a-1/1}\,q^a\,Cos.\,\frac{1}{2}\,a\,\pi,\ a<1$; Oettinger, Cr. 38. 216.

12) $= q^a\,\Gamma(-a)\,Cos.\,\frac{1}{2}\,a\,\pi$ (val. extraord.) Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826. p. 57.

13) $\int \frac{Cos.^a x}{x^b}\,dx = \frac{(-1)^{\frac{b+a}{2}-1}}{2^{a-1}\,1^{b-1|1}}\,\frac{\pi}{2}\,\sum_0^{\frac{a-1}{2}}(-1)^n\binom{a}{n}\left(\frac{a}{2}-n\right)^{b-1}$, a et b impairs;

14) $= \frac{(-1)^{\frac{b+a}{2}}}{2^{a-1}\,1^{b-1|1}}\,\frac{\pi}{2}\,\sum_0^{\frac{a}{2}-1}(-1)^n\binom{a}{n}\left(\frac{a}{2}-n\right)^{b-1}$, a et b pairs;

(13–14): $a \geqq b > 0$; Lindmann, Gr. 17. 455.

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1) $\int \frac{Sin.\, x.\, Sin.\, q x}{x^p} d x = \frac{\pi}{4 \Gamma(p)} Sec. \frac{1}{2} p \pi \left\{(1-q)^{p-1} - (1+q)^{p-1}\right\}, q < 1;$

2) $= \frac{\pi}{4 \Gamma(p)} Sec. \frac{1}{2} p \pi \left\{(q-1)^{p-1} - (1+q)^{p-1}\right\}, q > 1;$

3) $\int \frac{Sin.\, x.\, Cos.\, q x}{x^p} d x = \frac{\pi}{4 \Gamma(p)} Cosec. \frac{1}{2} p \pi \left\{(1-q)^{p-1} + (1+q)^{p-1}\right\}, q < 1;$

4) $= \frac{\pi}{4 \Gamma(p)} Cosec. \frac{1}{2} p \pi \left\{(q+1)^{p-1} - (q-1)^{p-1}\right\}, q > 1;$

Schlömilch, Stud. I. 22.

5) $\int \frac{Sin.\, a x}{x} \left(\frac{Sin.\, x}{x}\right)^a d x = \frac{1}{2} \pi$ Hoppe, Cr. 40. 142.

6) $\int \frac{Sin.\, b q x}{x} \left(\frac{Sin.\, x}{x}\right)^b d x = \frac{1}{2} \pi \left\{1 - \frac{1}{2^{b-1} 1^{b|1}} \sum_{0}^{\frac{a-bq}{2}} (-1)^n \frac{b^{n|-1}}{1^{n|1}} (b - b q - 2 n)^b\right\}$

7) $\int Cos.\, b q x \left(\frac{Sin.\, x}{x}\right)^b d x = \frac{\pi}{2^{b-1}} \sum_{0}^{\frac{a \pm bq}{2}} (-1)^n \frac{b^{n|-1}}{1^{b|1} 1^{n|1}} (b \pm b q - 2 n)^{b-1}$

Lobatschewsky, Cr. 24. 164. Dans 7) on peut prendre le double signe à volonté.

8) $\int Cos.\, a x \left(\frac{Sin.\, x}{x}\right)^b d x = 0, a \geqq b;$ Hoppe, Cr. 40. 142.

9) $\int Cos.\, q x \left(\frac{Sin.\, x}{x}\right)^b d x = \frac{\pi}{1^{b|1} 2^b} \sum_{0}^{\infty} (-1)^n \binom{b}{n} (q + b - 2 n)^{b-1}$ Laplace, Probab. L. 1. 42.

10) $\int Cos.\left(q x - \frac{1}{4} a \pi\right) \frac{Sin.\, x}{x^p} d x = \frac{\pi}{4 \Gamma(p)} \left\{\left(\frac{Sin.\, \frac{1}{4} a \pi}{Cos.\, \frac{1}{2} p \pi} + \frac{Cos.\, \frac{1}{4} a \pi}{Sin.\, \frac{1}{2} p \pi}\right) (1-q)^{p-1} + \right.$

$\left. + \left(\frac{Cos.\, \frac{1}{4} a \pi}{Sin.\, \frac{1}{2} p \pi} - \frac{Sin.\, \frac{1}{4} a \pi}{Cos.\, \frac{1}{2} p \pi}\right) (1+q)^{p-1}\right\}, q < 1;$

11) $= \frac{\pi}{4 \Gamma(p)} \left\{\left(\frac{Sin.\, \frac{1}{4} a \pi}{Cos.\, \frac{1}{2} p \pi} - \frac{Cos.\, \frac{1}{4} a \pi}{Sin.\, \frac{1}{2} p \pi}\right) (q-1)^{p-1} + \right.$

$\left. + \left(\frac{Cos.\, \frac{1}{4} a \pi}{Sin.\, \frac{1}{2} p \pi} - \frac{Sin.\, \frac{1}{4} a \pi}{Cos.\, \frac{1}{2} p \pi}\right) (1+q)^{p-1}\right\}, q > 1;$

Ces form. (10) et (11) se trouvent Meyer, Int. Déf. 448.

12) $\int Cos.\left(\frac{a+1}{2} \pi + p x\right) \frac{d x}{x^{q+1}} = 0$

13) $\int Cos.\left(\frac{a+1}{2} \pi - p x\right) \frac{d x}{x^{q+1}} = \frac{\pi p^q}{\Gamma(q+1)}$

, valeurs extraord.; Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826. p. 57.

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14) $\int \frac{Cos.\left\{\frac{r-1}{2}\pi+2(p+q)x\right\}+Cos.\left\{\frac{r+1}{2}\pi+2(p-q)x\right\}}{x^{r+1}}dx = 0 \quad ,p>q;$

15) $\quad = \frac{(q-p)^r}{\Gamma(r+1)}\pi, p<q;$

, val. extraord.; Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826. p. 57.

16) $\int \frac{Sin. 2qx. Sin^b x}{x^{b+1}} dx = (-1)^b \frac{\pi}{2^{b+1}}, 2q<b;$

17) $\quad = 0, \quad 2q>b$, et q entier;

18) $\int \frac{Cos. 2ax. Sin.^b x}{x^{b+1}} dx = 0 \quad a+b$ impair;

19) $\int \frac{Sin. 2cx. Sin.^b x}{x^{a+1}} dx = \frac{\pi}{2^{b+1} 1^{a/1}} Sec.\left\{\frac{a+b}{2}\pi\right\} \Delta^b.(2c-b)^a, 2c<b;$

20) $\quad = \frac{\pi}{2^{b+1} 1^{a/1}} Sec.\left\{\frac{a+b}{2}\pi\right\}\left\{\sum_0^\infty (-1)^n \binom{b}{n}(b+2c-2n)^a - \right.$
$\left. - \sum_0^\infty (-1)^n \binom{b}{n}(b-2c-2n)^a\right\}, 2c>b;$

(19–20) $, a<b+1, a+b$ pair;

21) $\quad = \frac{(-1)^{\frac{a+b-1}{2}}}{2^b 1^{a/1}} \Delta^b.\left\{(2c-b)^a l(2c-b)\right\} \quad , a+b$ impair;

22) $\int \frac{Cos. 2cx. Sin.^b x}{x^{a+1}} dx = (-1)^{\frac{b}{2}-1} \frac{\pi}{2^{b+1} 1^{a/1}} Cosec. \frac{1}{2} a\pi \Delta^b.(2c-b)^a, b$ pair;

23) $\quad = (-1)^{\frac{b-1}{2}} \frac{\pi}{2^{b+1} 1^{a/1}} Sec. \frac{1}{2} a\pi \Delta^b.(2c-b)^a \quad , b$ impair;

24) $\quad = -\frac{\pi}{2^{b+1} 1^{a/1}} Cosec\left\{\frac{a+b}{2}\pi\right\} \Delta^b.(2c-b)^a, \quad$ tout b;

(22–24) $, 2c>b;$

25) $\quad = -\frac{\pi}{2^{b+1} 1^{a/1}} Cosec.\left\{\frac{a+b}{2}\pi\right\}\left\{\sum_0^\infty (-1)^n \binom{b}{n}(b+2c-2n)^a - \right.$
$\left. - \sum_0^\infty (-1)^n \binom{b}{n}(b-2c-2n)^a\right\}, 2c<b;$

26) $\quad = 0 \quad , a+b$ impair $\quad , 2c>b;$

(16–26) Cauchy, P. 28. 147. P. I. § 2.

27) $\int \frac{\mathrm{Cos}\,2cx.\mathrm{Sin}.^b x}{x^{a+1}}\,dx = (-1)^{\frac{a+b+1}{2}} \frac{1}{2^b\,1^{a/1}} \Delta^b \{(2c-b)^a l(2c-b)\}, a+b$ pair, $2c > b$; Cauchy, P. 28. 147. P. I. § 2.

28) $= (-1)^{\frac{a+b-1}{2}} \frac{1}{2^b\,1^{a/1}} \sum_0^\infty (-1)^n \binom{b}{n} (b \pm 2c - 2n)^a, a+b$ impair; $2c < b$; Dans 28) on peut prendre le double signe à volonté.

29) $= (-1)^{\frac{a+b}{2}+1} \frac{1}{2^b\,1^{a/1}} \Delta^b \{(2c-b)^a l(2c-b)\}, a+b$ pair; $2c < b$;

30) $\int \frac{\mathrm{Sin}.\,bx.\,\mathrm{Cos}.\,x}{x} \left(\frac{\mathrm{Sin}.\,x}{x}\right)^{b-1} dx = \frac{1}{2}\pi$ Hoppe, Cr. 40. 142.

31) $\int \frac{\mathrm{Sin}.^b x}{x^{a+1}} \mathrm{Cos}.\left\{2cx + (a-b+1)\frac{\pi}{2}\right\} dx = \frac{\pi}{2^b\,1^{a/1}} \sum_0^\infty (-1)^n \binom{b}{n} (b-2c-2n)^a$, $b > 2c$;

32) $\int \frac{\mathrm{Sin}.^b x}{x^{a-1}} \mathrm{Cos}.\left\{2cx - (a-b+1)\frac{\pi}{2}\right\} dx = \frac{\pi}{2^b\,1^{a/1}} \sum_0^\infty (-1)^n \binom{b}{n} (b+2c-2n)^a$

Cauchy, P. 28. 147. P. III. § 3.

33) $\int \frac{\mathrm{Sin}.\{(2c+b)x\}.\mathrm{Sin}.^b x}{x^{a+1}}\,dx = \frac{2^{a-b-1}\pi}{(-1)^{\frac{b}{2}}\,1^{a/1}} \mathrm{Sec}.\,\frac{1}{2}a\pi\,\Delta^b.c^a$, b pair;

34) $= \frac{2^{a-b-1}\pi}{(-1)^{\frac{b+1}{2}}\,1^{a/1}} \mathrm{Cosec}.\,\frac{1}{2}a\pi\,\Delta^b.c^a$, b impair;

35) $= \frac{2^{a-b-1}\pi}{1^{a/1}} \mathrm{Sec}.\left\{\frac{a+b}{2}\pi\right\} \Delta^b.c^a$, tout b;

36) $\int \frac{\mathrm{Cos}.\{(2c+b)x\}.\mathrm{Sin}.^b x}{x^{a+1}}\,dx = \frac{2^{a-b-1}\pi}{(-1)^{\frac{b}{2}}\,1^{a/1}} \mathrm{Cosec}.\,\frac{1}{2}a\pi\,\Delta^b.c^a$, b pair;

37) $= \frac{2^{a-b-1}\pi}{(-1)^{\frac{b+1}{2}}\,1^{a/1}} \mathrm{Sec}.\,\frac{1}{2}a\pi\,\Delta^b.c^a$, b impair;

38) $= -\frac{2^{a-b-1}\pi}{1^{a/1}} \mathrm{Cosec}.\left\{\frac{a+b}{2}\pi\right\} \Delta^b.c^a$, tout b;

(33–38): , où $a < b$, $c > 0$; Cauchy, P. 28. 147. P. III. § 2.

39) $\int \mathrm{Cos}.(bx\sqrt{a}) \left(\frac{\mathrm{Sin}.\,x}{x}\right)^a dx = \frac{\pi}{2^a\,1^{a/1}} \sum_0^\infty (-1)^n \binom{a}{n} (a + b\sqrt{a} - 2n)^{a-1}$ Laplace, Mém. Inst. 1809. 353. § 10.

40) $\int \mathrm{Sin}.^a \frac{1}{2}x.\,\mathrm{Sin}.\left(\frac{2p+q}{2}x + \frac{1}{2}a\pi\right)\frac{dx}{x^{q+1}} = \frac{\pi}{2^{a+1}\Gamma(q+1)} \mathrm{Cosec}.\left\{\frac{q+1}{2}\pi\right\} \Delta^a.p^q$

41) $\int \mathrm{Sin}.^a \frac{1}{2}x.\,\mathrm{Cos}.\left(\frac{2p+q}{2}x + \frac{1}{2}a\pi\right)\frac{dx}{x^{q+1}} = \frac{\pi}{2^{a+1}\Gamma(q+1)} \mathrm{Sec}.\left\{\frac{q+1}{2}\pi\right\} \Delta^a.p^q$

Cauchy, P. 28. 147. P. III. § 2.

1) $\int \frac{Sin.\, q\, x}{1+x}\, dx = Sin.\, q.\, Ci.\, (q) + Cos.\, q.\left\{\frac{1}{2}\pi - Si.\, (q)\right\}$

2) $\int \frac{Cos.\, q\, x}{1+x}\, dx = -\, Cos.\, q.\, Ci.\, (q) + Sin.\, q.\left\{\frac{1}{2}\pi - Si.\, (q)\right\}$

Arndt, Gr. 10. 225. — Schlömilch, St. II. 21.

3) $\int \frac{Sin.\, k\, x}{1+x}\, dx = 0$

4) $\int \frac{Cos.\, k\, x}{1+x}\, dx = 0$

, $k = \infty$; Raabe, Int. 202.

5) $\int \frac{Sin.\, p\, x}{x+q}\, dx = \frac{1}{2}\pi\, Cos.\, p\, q + (A + l\, p\, q)\, Sin.\, p\, q + Cos.\, p\, q \sum_1^\infty (-1)^n \frac{(p\, q)^{2n-1}}{(2n-1)\, 1^{2n-1/1}} + Sin.\, p\, q \sum_1^\infty (-1)^n \frac{(p\, q)^{2n}}{2\, n.\, 1^{2n/1}}$

6) $\int \frac{Cos.\, p\, x}{x+q}\, dx = \frac{1}{2}\pi\, Sin.\, p\, q - (A + l\, p\, q)\, Cos.\, p\, q + Sin.\, p\, q \sum_1^\infty (-1)^n \frac{(p\, q)^{2n-1}}{(2\, n-1)\, 1^{2n-1/1}} - Cos.\, p\, q \sum_1^\infty (-1)^n \frac{(p\, q)^{2n}}{2\, n.\, 1^{2n/1}}$

Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 27.

7) $\int \frac{Sin.\, p\, x}{x+q}\, dx = Sin.\, p\, q.\, Ci.\, (p\, q) + Cos.\, p\, q.\left\{\frac{1}{2}\pi - Si.\, (pq)\right\}$

8) $\int \frac{Cos.\, p\, x}{x+q}\, dx = -\, Cos\, p\, q.\, Ci.\, (p\, q) + Sin.\, pq.\left\{\frac{1}{2}\pi - Si.\, (pq)\right\}$

Arndt, Gr. 10. 225. — Schlömilch, Stud. II. 21.

9) $\int \frac{Sin.\, p\, x}{x-q}\, dx = \frac{1}{2}\pi\, Cos.\, p\, q - (A + l\, p\, q)\, Sin.\, p\, q - Cos\, p\, q \sum_1^\infty (-1)^n \frac{(p\, q)^{2n-1}}{(2\, n-1)\, 1^{2n-1/1}} - Sin.\, p\, q \sum_1^\infty (-1)^n \frac{(p\, q)^{2n}}{2\, n.\, 1^{2n/1}}$

10) $\int \frac{Cos\, p\, x}{x-q}\, dx = -\frac{1}{2}\pi\, Sin.\, pq - (A + l\, pq)\, Cos.\, pq + Sin.\, pq \sum_1^\infty (-1)^n \frac{(p\, q)^{2n-1}}{(2\, n-1)\, 1^{2n-1/1}} - Cos.\, p\, q \sum_1^\infty (-1)^n \frac{(p\, q)^{2n}}{2\, n.\, 1^{2n/1}}$

Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 28.

11) $\int \frac{Sin.\, p\, x}{x-q}\, dx = -\, Sin.\, p\, q.\, Ci.\, (pq) + Cos.\, pq.\left\{\frac{1}{2}\pi + Si.\, (pq)\right\} + Sin.\, pq.\, l\, \alpha$

12) $\int \frac{Cos.\, p\, x}{x-q}\, dx = -\, Cos.\, p\, q.\, Ci.\, (pq) - Sin.\, pq.\left\{\frac{1}{2}\pi + Si.\, (pq)\right\} + Cos.\, pq.\, l\, \alpha$

, où α arbitraire; Arndt, Gr. 10. 240.

1) $\int \frac{Cos. px}{1+x^2} dx = \frac{1}{2}\pi e^p, p < 0;$ Fourier, Chal. 358.

2) $= \frac{1}{2}\pi e^{-p}, p \overline{>} 0;$

3) $\int \frac{x\, Sin. px}{1+x^2} dx = \frac{1}{2}\pi e^{-p}, p > 0;$

(2, 3) Laplace, Bull. Soc. Phil. Avr. 1811. — Id., Prob. L. 1. N°. 26. — Poisson, P. 19. 60. — Id., P. 16. 215. N°. 7. — Cauchy, Sav. Etr. 1827. 124. Note 18. — Id., Sav. Etr. 1824. 599. P. II. § 7. 1. — Legendre, Exerc. 3. 42. — Schlömilch, Gr. 5. 204. — Arndt, Gr. 11. 70. — Fourier, Chal. 358.

Sur la form. (2) seule voyez: Serret, L. 8. 1. — Id., L. 8. 489. — Poisson, P. 17. 612. N°. 19.

4) $= -\frac{1}{2}\pi e^p, p < 0;$ Fourier, Chal. 358.

5) $= \frac{1}{2}\pi$, p très-petit;

6) $= 0 \quad , p = 0;$

(5, 6) Cauchy, Sav. Etr. 1827. 599. Suppl. 2.

7) $\int \frac{Sin. px}{1+x^2} dx = \frac{1}{2}\{e^{-p} li.(e^p) - e^p li.(e^{-p})\}$

8) $\int \frac{x\, Cos. px}{1+x^2} dx = -\frac{1}{2}\{e^p li.(e^{-p}) + e^{-p} li.(e^p)\}$

(7, 8) Schlömilch, Gr. 5. 204.

9) $\int \frac{x\, Tang. px}{1+x^2} dx = \pi \frac{e^{-p}}{e^p + e^{-p}}$ Legendre, Exerc. 5. 35.

10) $\int \frac{x\, Cot. px}{1+x^2} dx = \pi \frac{e^{-p}}{e^p - e^{-p}}$ Cauchy, Sav. Etr. 1827. 599. S. 2. — Legendre, Exerc. 5. 33.

11) $\int \frac{Cos. px}{1+qx^2} dx = \frac{\pi}{2\sqrt{q}} e^{-\frac{p}{\sqrt{q}}}$ Poisson, P. 19. 404. N°. 56.

12) $\int \frac{Sin.\{(a+k)x\}.Cos.\{(a-k)x\}}{1+x^2} x\, dx = \frac{1+e^{-2a}}{4}\pi$, k très petit;

13) $= \frac{1}{4}\pi e^{-2a} \quad , k = 0;$

(12, 13) Cauchy, Sav. Etr. 1827. 599. S. 2.

14) $\int Sin. \left(\frac{1}{2}a\pi - qx\right) \frac{x^{a-1}}{1+x^2} dx = \frac{1}{2}\pi e^{-q}, a < 3;$ Svanberg, L. 11. 197.

15) $\int Cos. \left(\frac{1}{2}p\pi - qx\right) \frac{x^p}{1+x^2} dx = \frac{1}{2}\pi e^{-q}, -1 < p < 1;$ Liouville, Cr. 11. 1.

16) $\int \left\{ Cos. \left(q x - \frac{1}{2} p \pi \right) - Cos. \frac{1}{2} p \pi \right\} \frac{x^p}{1+x^2} dx = \frac{1-e^q}{2 e^q} \pi, \; -2 < p \leqq -1;$ Liouville, C r. 11. 1.

17) $\int \frac{Cos. x + x Sin. x}{1+x^2} dx = \frac{\pi}{e}$ Legendre, Exerc. 3. 41. — Laplace, Probab. L. 1. 33.

18) $\int \frac{Cos. q x + x Sin. q x}{1+x^2} dx = \pi e^{-q}$ Bidone, Mém. Turin. 1812. 231. Art. 2. 22.

19) $\int Sin. \left(\frac{1}{2} p x - q x \right) \frac{x^{p-1}}{1+x^2} dx = \frac{1}{2} \pi e^{-q}, \; p < 1;$ Cauchy, Cours. Leç. 39. — Svanberg, L. 11. 197.

20) $\int \frac{x Sin. p x}{1+q^2 x^2} dx = \frac{\pi}{2 q^2} e^{-\frac{p}{q}}$ Raabe, Int. 169.

21) $\int \frac{Cos. q x}{1-x^2} dx = \frac{1}{2} \pi Sin. q$ Cauchy, Lim. Imag. Add. 17. — Id., Sav. Etr. 1827. 124. Note 18.

22) $\int \frac{x Sin. q x}{1-x^2} dx = -\frac{1}{2} \pi Cos. q$ Cauchy, Sav. Etr. 1827. 124. Note 18.

23) $\int \frac{Sin. q x . Sin. \pi x}{1-x^2} dx = \frac{1}{2} \pi Sin. q, \; 0 < q < \pi;$

24) $= 0, \; q > \pi;$

(23, 24) Fourier, Chaleur. 358.

F. Alg. rat. fract. à dén. $a^2 + x^2$.
Circ. Dir. en num. à une fonct. TABLE 205. Lim. 0 et ∞.

1) $\int \frac{Cos. x}{q^2+x^2} dx = \frac{\pi}{2 q} e^{-q}$ Poisson, P. 17. 612. N°. 20. — Id., P. 18. 295. N°. 32. — Id., P. 19. 404. N°. 75. — Liouville, Cr. 13. 219. — Schlömilch, Cr. 36. 271.

2) $\int \frac{x Sin. x}{q^2+x^2} dx = \frac{1}{2} \pi e^{-q}$ Poisson, P. 18. 295. N°. 32. — Id., P. 19. 404. N°. 75. — Schlömilch, Cr. 36. 271.

3) $\int \frac{x Tang. x}{q^2+x^2} dx = \frac{\pi}{e^{2q}+1}$

4) $\int \frac{x Cot. x}{q^2+x^2} dx = \frac{\pi}{e^{2q}-1}$

(3, 4) Schlömilch, Gr. 10. 440.

5) $\int \frac{Cos. p x}{q^2+x^2} dx = \frac{\pi}{2 q} e^{-pq}, \; p \geqq 0;$

6) $\int \frac{x Sin. p x}{q^2+x^2} dx = \frac{1}{2} \pi e^{-pq}, \; p > 0;$

(5, 6) Laplace, Nouv. Bull. de la Soc. Phil. N°. 43, 49. — Poisson, Chal. 135. — Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 21, 22. — Plana, Mém. Turin. 1818. 7. I. 6. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 55. — Cauchy, Lim. Imag. Add. 14. — Id., P. 19. 511. — Id., P. 28. 147. I. § 3. — Id., Sav. Etr. 1827. 124. Note 6. — Id., Sav. Etr. 1827. 599. P. II. § 7. — Id., Exerc 1826. p. 95. — Id., ib. 1827. p. 141. — Legendre, Exerc. 3. 42. — v. Schmidten, Cr. 5. 388. — Schlömilch, Gr. 5. 204. — Id., Gr. 9. 379. — Id., Gr. 10. 440. — Id., Beitr. II. § 3. — Id., Stud. II. 14. — Arndt, Gr. 11. 70. — Schellbach, Cr. 48. 207. — Sur la form. 5) seule voyez: Schlömilch, Cr. 36. 268.

7) $\int \frac{Cos. px}{q + x^2} dx = \frac{\pi}{2\sqrt{q}} e^{-p\sqrt{q}}$ Poisson, P. 16. 215. N°. 7.

8) $\int \frac{Sin. px}{q^2 + x^2} dx = -\frac{e^{pq} - e^{-pq}}{2q}(A + lpq) + \frac{e^{pq} + e^{-pq}}{2q} \sum_1^\infty \frac{(pq)^{2n-1}}{(2n-1)\, 1^{2n-1/1}} - \frac{e^{pq} - e^{-pq}}{2q} \sum_1^\infty \frac{(pq)^{2n}}{2n.\, 1^{2n/1}}$

9) $\int \frac{x Cos. px}{q^2 + x^2} dx = -\frac{e^{pq} + e^{-pq}}{2}(A + lpq) + \frac{e^{pq} - e^{-pq}}{2} \sum_1^\infty \frac{(pq)^{2n-1}}{(2n-1)\, 1^{2n-1/1}} - \frac{e^{pq} + e^{-pq}}{2} \sum_1^\infty \frac{(pq)^{2n}}{2n.\, 1^{2n/1}}$

(8, 9) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 27.

10) $\int \frac{Sin. px}{q^2 + x^2} dx = \frac{1}{2q} \left\{ e^{-pq} Ei.(pq) - e^{pq} Ei.(-pq) \right\}$

11) $\int \frac{x Cos. px}{q^2 + x^2} dx = -\frac{1}{2} \left\{ e^{pq} Ei.(-pq) + e^{-pq} Ei.(pq) \right\}$

(10, 11) Schlömilch, Gr. 5. 204 (où la form. (10) est fautive). — Id., Stud. II. 20. — Arndt, Gr. 11. 70. — Sur la formule (10) seule voyez: Schlömilch, Gr. 11. 174.

12) $\int \frac{x^2 Cos. px}{q^2 + x^2} dx = -\frac{1}{2} pq e^{-pq}$ Plana, Mém. Turin. 1818. 7. IV. 17. La valeur en est infinie.

13) $\int \frac{x Tang. px}{q^2 + x^2} dx = \frac{1}{2}\pi \quad , z < 1;$

14) $\quad = \frac{\pi}{e^{2pq} + 1} \quad , z > 1;$

(13, 14), dans le cas de x complexe $= y + zi$; Poisson, P. 18. 295. N°. 42.

15) $\quad = \frac{\pi}{e^{2pq} + 1}$ Legendre, Exerc. 4. 131. — Bidone, Mém. Turin. 1812. 231. Art. 3. 39. — Schlömilch, Gr. 10. 440. — Id., Beitr. II. 4.

16) $\int \frac{x Cot. px}{q^2 + x^2} dx = \frac{\pi}{e^{2pq} - 1}$ Cauchy, Exerc. 1827. p. 141. — Legendre, Exerc. 4. 131. — Bidone, Mém. Turin. 1812. 231. Art. 3. 39. — Schlömilch, Gr. 10. 440. — Id., Beitr. II. 4.

17) $\int \frac{x Tang. \frac{1}{2}x}{q^2 + x^2} dx = \frac{\pi}{e^q + 1}$

18) $\int \frac{x Cot. \frac{1}{2}x}{q^2 + x^2} dx = \frac{\pi}{e^q - 1}$

(17, 18) Schlömilch, Stud. II. 18. — Id., Cr. 36. 271.

19) $\int \frac{x Sin. 2x}{q^2 + x^2} dx = \frac{\pi}{2} e^{-2q}$

20) $\int \frac{Cos. 2x}{q^2 + x^2} dx = \frac{\pi}{2q} e^{-2q}$

(19, 20) Schlömilch, Gr. 10. 440.

21) $\int \frac{Sin.^2 x}{q^2 + x^2} dx = \frac{\pi}{4} \frac{1 - e^{-2q}}{q}$ V. T. 205. N°. 20, 22.

22) $\int \frac{Cos.^2 x}{q^2 + x^2} dx = \frac{\pi}{4} \frac{1 + e^{-2q}}{q}$ Schlömilch, Gr. 10. 440.

23) $\int \frac{Cos.^2 px}{q^2+x^2} dx = \frac{\pi}{2q} \frac{1+e^{-2pq}}{2}$ Dienger, Gr. 12. 97.

24) $\int Sin\left(\frac{1}{2} r\pi - px\right) \frac{x^{r-1}}{q^2+x^2} dx = \frac{1}{2} \pi q^{r-2} e^{-pq}, \; r < 2;$ Cauchy, Lim. Imag. Add. N°. 22. — Id., P. 19. 511. — Id., Exerc. 1826. p. 95.

25) $\int Cos.\left(\frac{1}{2} a\pi - x\right) \frac{x^a}{q^2+x^2} dx = \frac{1}{2} \pi q^{a-1} e^{-q}$ Cayley, L. 12. 231.

26) $\int \frac{x^{2a} Cos. px}{q^2+x^2} dx = (-1)^a \frac{1}{2} \pi q^{2a-1} e^{-pq}$

27) $\int \frac{x^{2a+1} Sin. px}{q^2+x^2} dx = (-1)^a \frac{1}{2} \pi q^{2a} e^{-pq}$

(26, 27) Schlömilch, Cr. 33. 353. — Arndt, Gr. 11. 70. les trouve indéterminées.— Elles sont infinies pour $a > 0$.

28) $\int \frac{Cos. px}{(r+qi)^2+x^2} dx = \frac{1}{2} \frac{\pi}{r+qi} e^{-p(r+qi)}, \; p \geqq 0;$

29) $\int \frac{x \, Sin. px}{(r+qi)^2+x^2} dx = \frac{1}{2} \pi e^{-p(r+qi)}, \; p > 0;$

(28, 29) Poisson, P. 18. 295. N°. 44.

F. Alg. rat. fract. à dén. $a^2 - x^2$. Circ. Dir. en num. à une fonct. TABLE 206. Lim. 0 et ∞.

1) $\int \frac{x \, Sin. px}{q^2-x^2} dx = -\frac{1}{2} \pi \, Cos. pq$

2) $\int \frac{Cos. px}{q^2-x^2} dx = \frac{\pi}{2q} Sin. pq$

(1, 2) Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 29. — Cauchy, Sav. Etr. 1827. 124. Note 6. — Id., Sav. Etr. 1827. 599. P. 2. § 7. — Id., Lim. Imag. Add. 15. — Plana, Mém. Turin. 1818. 7. I. 3. — Schlömilch, Cr. 33. 316. — Id., Gr. 7. 270.— Id., Stud. II. 15. — Mosta, Gr. 10. 449.

Sur la form. (1) voyez: Cisa de Grésy, Mém. Turin. 1821. 209. II. 57.

3) $\int \frac{x \, Sin. px}{q^2-x^2} dx = -\frac{1}{2} Sin. pq \, l\alpha - \frac{1}{2} \pi \, Cos. pq$

4) $\int \frac{Cos \, px}{q^2-x^2} dx = \frac{1}{2q} \{\pi \, Sin. pq - Cos. pq \, l\alpha\}$

(3, 4), où α est indéterminé; Arndt, Gr. 10. 240. — Cauchy, P. 19. 511. trouve ces formules pour valeurs générales, mais dans (4) il a $\pi \, l \, \alpha$ au lieu de $l \, \alpha$.

5) $\int \frac{x \, Sin. px}{q^2-x^2} dx = -\frac{1}{2} \pi \, Cos. pq + \frac{1}{2} Sin. pq \, l(-1)$

6) $\qquad = -\frac{1}{2} \pi e^{-pqi}$

7) $\int \frac{Cos. px}{q^2-x^2} dx = \frac{\pi}{2q} Sin \, pq + \frac{1}{2q} Cos. pq \, l(-1)$

(5–7) Poisson, P. 18. 295. N°. 38.

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8) $\int \frac{Cos.px}{q^2-x^2} dx = \frac{\pi}{2qi} e^{-pqi}$ Poisson, P. 18. 295. N°. 38.

9) $\int \frac{Sin.px}{q^2-x^2} dx = \frac{1}{q}\{Ci.(pq).Sin.pq - Si.(pq).Cos.pq\}$

10) $\int \frac{x\,Cos.px}{q^2-x^2} dx = Ci.(pq).Cos.pq + Si.(pq).Sin.pq$

(9, 10) Schlömilch, Cr. 33. 316. — Id., Stud. II. 21.

11) $\int \frac{Sin.px}{q^2-x^2} dx = -\frac{1}{q} Si.(pq).Cos.pq - \frac{1}{q} Sin.pq.\left\{\frac{1}{2} l\alpha^2 - Ci.(pq)\right\}$

12) $\int \frac{x\,Cos.px}{q^2-x^2} dx = Si.(pq).Sin.pq - Cos.pq.\left\{\frac{1}{2} l\alpha^2 - Ci.(pq)\right\}$

(11, 12), où α arbitraire; Arndt, Gr. 10. 240.

13) $\int x\,Sin.\frac{b\pi x}{a} \frac{dx}{a^2-x^2} = \frac{1}{2}\pi\,Cos.b\pi$

14) $\int Cos.\left\{\frac{2b-1}{2a}\pi x\right\} \frac{dx}{a^2-x^2} = -\frac{\pi}{2a} Sin.\left\{\left(b-\frac{1}{2}\right)\pi\right\}$

(13, 14) Poisson, P. 18. 295. N°. 38.

15) $\int \frac{x\,Tang.px}{q^2-x^2} dx = -\frac{1}{2}\pi$ Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 39.

16) $= -\frac{1}{2}\pi + \frac{1}{2} Tang.\frac{1}{2}pq.l(-1)$

17) $\int Tang.\frac{2b\pi x}{a} \frac{x}{a^2-x^2} dx = -\frac{1}{2}\pi$

(16, 17) Poisson, P. 18. 295. N°. 43.

18) $\int \frac{x\,Cosec.px}{q^2-x^2} dx = 0$

19) $\int \frac{x\,Cot.px}{q^2-x^2} dx = \frac{1}{2}\pi$

(18, 19) Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 39.

20) $\int Sin.\left(\frac{1}{2}a\pi - px\right) \frac{x^{a-1}}{q^2-x^2} dx = \frac{1}{2}\pi q^{a-2} Cos.\left(\frac{1}{2}a\pi - pq\right)$ Cauchy. Lim. Imag. 23. — Id., P. 19. 511.

21) $\int \frac{Cos.^2 px}{q^2-x^2} dx = \frac{\pi}{4q} Sin.2pq$ Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 31.

1) $\int \frac{x \, Tang. \, px}{x^4 + q^4} dx = \frac{\pi e^{-pq\sqrt{2}}}{q^2} \frac{Sin. (pq\sqrt{2})}{1 + 2e^{-pq\sqrt{2}} Cos. (pq\sqrt{2}) + e^{-2pq\sqrt{2}}}$

2) $\int \frac{x \, Cot. \, px}{x^4 + q^4} dx = \frac{\pi e^{-pq\sqrt{2}}}{q^2} \frac{Sin. (pq\sqrt{2})}{1 - 2e^{-pq\sqrt{2}} Cos. (pq\sqrt{2}) + e^{-2pq\sqrt{2}}}$

3) $\int \frac{x \, Cosec. \, px}{x^4 + q^4} dx = \frac{\pi e^{-\frac{1}{2}pq\sqrt{2}}}{q^2} \frac{1 + e^{-pq\sqrt{2}}}{1 - 2e^{-pq\sqrt{2}} Cos. (pq\sqrt{2}) + e^{-2pq\sqrt{2}}} Sin. \left(\frac{1}{2} pq\sqrt{2}\right)$

Plana, Mém. Turin. 1818. 7. II. 10.

4) $\int \frac{x^2 \, Cos. \, px}{x^4 + 4q^4} dx = \frac{\pi}{4q} e^{-pq} (Cos. \, pq - Sin. \, pq)$ Schlömilch, Stud. II. 16.

5) $\int \frac{Cos. \, 2px}{q^4 + x^4} dx = \frac{\pi}{4q^3} \sqrt{2} . e^{-pq\sqrt{2}} \{Cos. (pq\sqrt{2}) + Sin. (pq\sqrt{2})\}$

6) $\int \frac{x^2 \, Cos. \, 2px}{q^4 + x^4} dx = \frac{\pi}{4q} \sqrt{2} . e^{-pq\sqrt{2}} \{Cos. (pq\sqrt{2}) - Sin. (pq\sqrt{2})\}$

7) $\int \frac{x \, Sin. \, 2px}{q^4 + x^4} dx = \frac{\pi}{2q^2} e^{-pq\sqrt{2}} Sin. (pq\sqrt{2})$

8) $\int \frac{x^3 \, Sin. \, 2px}{q^4 + x^4} dx = \frac{\pi}{2} e^{-pq\sqrt{2}} Cos. (pq\sqrt{2})$

Helmling, Transf. II. S. 63.

9) $\int \frac{Sin. \, px}{q^4 - x^4} dx = \frac{1}{4q^3} \{2Ci.(pq).Sin.pq - 2Si.(pq).Cos.pq + e^{-pq}Ei.(pq) - e^{pq}Ei.(-pq)\}$ V. T. 205. N°. 10 et T. 206. N°. 9.

10) $\int \frac{x \, Sin. \, px}{q^4 - x^4} dx = \frac{\pi}{4q^2} (e^{-pq} - Cos. \, pq)$ V. T. 205. N°. 6 et T. 206. N°. 1.

11) $\int \frac{x^2 \, Sin. px}{q^4 - x^4} dx = \frac{1}{4q} \{2Ci.(pq).Sin.pq - 2Si.(pq).Cos.pq - e^{-pq}Ei.(pq) + e^{pq}Ei.(-pq)\}$ V. T. 205. N°. 10 et T. 206. N°. 9.

12) $\int \frac{x^3 \, Sin. \, px}{q^4 - x^4} dx = -\frac{\pi}{4}(e^{-pq} + Cos. \, pq)$ V. T. 205. N°. 6 et T. 206. N°. 1.

13) $\int \frac{Cos. \, px}{q^4 - x^4} dx = \frac{\pi}{4q^3} (e^{-pq} + Sin. \, pq)$ V. T. 205. N°. 5 et T. 206. N°. 2.

14) $\int \frac{x \, Cos. px}{q^4 - x^4} dx = \frac{1}{4q^2} \{2Ci.(pq).Cos.pq + 2Si.(pq).Sin.pq - e^{-pq}Ei.(pq) - e^{pq}Ei.(-pq)\}$ V. T. 205. N°. 11 et T. 206. N°. 10.

15) $\int \frac{x^2 \, Cos. \, px}{q^4 - x^4} dx = \frac{\pi}{4q} (Sin. \, pq - e^{-pq})$ V. T. 205. N°. 5 et T. 206. N°. 2.

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16) $\int \frac{x^3 Cos.px}{q^4 - x^4} dx = \frac{1}{4}\{2Ci.(pq).Cos.pq + 2Si(pq).Sin.pq + e^{-pq}Ei.(pq) + e^{pq}Ei(-pq)\}$ V. T. 205. N°. 11 et T. 206. N°. 10.

17) $\int \frac{x\, Sin.\, a x}{1 + x^{2b}} dx = \frac{\pi}{2b} e^{-a} - \frac{\pi}{b} \sum_{1}^{\frac{b-1}{2}} e^{-a Cos. \frac{n\pi}{b}} Cos.\left(\frac{n\pi}{b} - a\, Sin. \frac{n\pi}{b}\right)$, b impair;

18) $= \frac{\pi}{b} \sum_{1}^{\frac{b}{2}-1} e^{-a Cos.\left(\frac{2n+1}{2b}\pi\right)} Cos.\left\{\frac{2n+1}{2b}\pi - a\, Sin.\left(\frac{2n+1}{2b}\pi\right)\right\}$, b pair; Plana, Mém. Turin. 1818. 7. II. 16, 15.

19) $\int \frac{Cos.\, a x}{1 + x^{2b}} dx = \frac{\pi}{2b} e^{-a} - \frac{\pi}{b} \sum_{1}^{\frac{b-1}{2}} e^{-a Cos. \frac{n\pi}{b}} Cos.\left(\frac{n\pi}{b} - a\, Sin. \frac{n\pi}{b}\right)$, b impair;

20) $= \frac{\pi}{b} \sum_{1}^{\frac{b}{2}-1} e^{-a Cos.\left(\frac{2n+1}{2b}\pi\right)} Cos.\left\{\frac{2n+1}{2b}\pi - a\, Sin.\left(\frac{2n+1}{2b}\pi\right)\right\}$, b pair; Poisson, P. 16.215. N°. 4, 5. — Plana, Mém. Turin. 1818. 7. II. 16, 15.

21) $\int \frac{Cos\, p x}{1 + x^b} x^{a-1} dx = \frac{\pi}{b} \sum_{1}^{\frac{1}{2}b} e^{-p Sin.\left(\frac{2n-1}{b}\pi\right)} Sin.\left\{\frac{2n-1}{b} a\pi + p Cos.\left(\frac{2n-1}{b}\pi\right)\right\}$, a impair;

22) $= 0$, a pair;

Dans (21) et (22) b est pair et $a < b + 1$.

23) $\int \frac{Cos.\, p x}{b^c + x^c} x^{a-1} dx = \frac{\pi}{c b^{c-a}} \sum_{1}^{\frac{1}{2}c} e^{-bp Sin.\left(\frac{2n-1}{c}\pi\right)} Sin.\left\{\frac{2n-1}{c} a\pi + bp Cos.\left(\frac{2n-1}{c}\pi\right)\right\}$

24) $\int \frac{Cos.\, p x}{b^c - x^c} x^{a-1} dx = \frac{\pi}{c b^{c-a}} \sum_{0}^{\frac{1}{2}c-1} e^{-bp Sin. \frac{2n\pi}{c}} Sin.\left(\frac{2na\pi}{c} + b p\, Cos. \frac{2n\pi}{c}\right)$

Dans (23) et (24) a est impair, c est pair, et $a < c + 1$. Schlömilch, Stud. II. 16.

25) $\int \frac{Cos.(p x^2) - Sin.(p x^2)}{1 - x^4} dx = \frac{1}{4}\pi(Sin.p + Cos.p)$ Cauchy, Sav. Etr. 1827. 124. Note 18.

26) $\int \frac{Cos.\, 2 p x}{1 + x^4} dx = \frac{\pi}{2} e^{-p\sqrt{2}} \{Sin.(p\sqrt{2}) + Cos.(p\sqrt{2})\}$ Helmling, Transf. II. S. 131.

27) $\int \frac{Sin.(p\pi - r^a x^a)}{q^2 + x^{2a}} dx = \frac{1}{2} e^{-qr^a} q^{2(p-1)} p\pi(1 + Cot.\, p\pi)$

28) $\int \frac{Sin.(p\pi - r^a x^a)}{q^2 e^{2\lambda i} + x^{2a}} dx = \frac{1}{2} e^{-(f+gi)} e^{(p-1)2\lambda i} q^{2(p-1)} p\pi(1 + Cot.\, p\pi)$

, où $f = q r^a Cos.\lambda$, $g = q r^a Sin.\lambda$; Helmling, Transf. II. S. 114.

1) $\int \frac{b^2 (p+x)^2 + a(a+1)}{(p+x)^{a+2}} Sin. b\, x\, dx = \frac{b}{p^a}$

2) $\int \frac{b^2 (p+x)^2 + a(a+1)}{(p+x)^{a+2}} Cos. b\, x\, dx = \frac{a}{p^{a+1}}$

Meyer, Int. Déf. 368.

3) $\int \frac{x\, Sin. p\, x}{(q^2+x^2)^2} dx = \frac{\pi}{4q} p\, e^{-pq}$ Legendre, Exerc. 3. 43. — Helmling, Transf. II. S. 62.

4) $\int \frac{x^3\, Sin. p\, x}{(q^2+x^2)^2} dx = \frac{2-pq}{4} \pi\, e^{-pq}$ Helmling, Transf. II. S. 62.

5) $\int \frac{x\, Sin. p\, x}{(q^2+x^2)^3} dx = \frac{p^2 q + p}{16 q^3} \pi\, e^{-pq}$

6) $\int \frac{x\, Sin. p\, x}{(q^2+x^2)^4} dx = \frac{3p + 3p^2 q + p^3 q^2}{96 q^5} \pi\, e^{-pq}$

Legendre, Exerc. 3. 43.

7) $\int \frac{Cos. p\, x}{(q^2+x^2)^2} dx = \frac{1+pq}{4q^3} \pi\, e^{-pq}$ Legendre, Exerc. 3. 43. — Plana, Mém. Turin. 1818. 7. I. 6. — Helmling, Transf. II. S. 62.

8) $\int \frac{x^2\, Cos. p\, x}{(q^2+x^2)^2} dx = \frac{1-pq}{4q} \pi\, e^{-pq}$ Plana, Mém. Turin. 1818. 7. I. 6.

9) $\int \frac{Cos. p\, x}{(q^2+x^2)^3} dx = \frac{3 + 3pq + p^2 q^2}{16 q^5} \pi\, e^{-pq}$ Legendre, Exerc. 3. 43.

10) $\int \frac{x\, Sin. x}{(q^2+x^2)^{a+1}} dx = \frac{\pi\, e^{-q}}{1^{a/1}\, 2\, (2q)^a} \sum_0^\infty \frac{(a-n)^{2n/1}}{1^{n/1} (2q)^n}$

11) $\int \frac{Cos. x}{(q^2+x^2)^{a+1}} dx = \frac{\pi\, e^{-q}}{1^{a/1}\, 2\, (2q)^{a+1}} \sum_0^\infty \frac{(a-n+1)^{2n/1}}{1^{n/1} (2q)^n}$

Schlömilch, Cr. 33. 268.

12) $\int \frac{x\, Sin. p\, x}{(q^2+x^2)^{a+1}} dx = \frac{\pi}{1^{a/1}} \frac{p\, e^{-pq}}{2^{a+1}} \sum_0^\infty \frac{(a-n)^{2n/1}}{2^{n/2}} \frac{p^{a-n-1}}{q^{a+n}}$

13) $\int \frac{Cos. p\, x}{(q^2+x^2)^a} dx = \frac{\pi}{1^{a-1/1}} \frac{e^{-pq}}{2^a} \sum_0^\infty \frac{(a-n+1)^{2n/1}}{2^{n/2}} \frac{p^{a-n-1}}{q^{a+n}}$

Legendre, Exerc. 3. 43. — Schlömilch, Cr. 33. 353. — Id., Stud. II. 14. — Id., Beitr. II. 3.

14) $\int \frac{Cos. p\, x}{(q+x^2)^a} dx = \frac{(-1)^a \pi}{2} \frac{d^a}{dq^a} \cdot \frac{e^{-p\sqrt{q}}}{\sqrt{q}}$ Poisson, P. 16. 215. N°. 7.

15) $= \frac{(-1)^a \pi}{1^{a-1/1}} \frac{d^a}{p\, dq^a} \cdot e^{-p\sqrt{q}}$ Schlömilch, Int. 27.

16) $\displaystyle\int \frac{x\, Sin.\, p\, x}{(q + x^2)^a}\, dx = \frac{(-1)^{a-1}}{1^{a-1|1}} \frac{\pi}{2} \frac{d^{a-1}}{dq^{a-1}} . e^{-p\sqrt{q}}$ Schlömilch, Int. 27.

17) $\displaystyle\int \frac{Cos.\, p\, x}{(q^2 - x^2)^2}\, dx = \frac{\pi}{4 q^3} (Sin.\, p\, q - p\, q\, Cos.\, p\, q)$ Plana, Mém. Turin. 1818. 7. I. 3.

F. Alg. rat. fract. à dén. binôme. Circ. Dir. en num. à plus. fonctions. TABLE 209. Lim. 0 et ∞.

1) $\displaystyle\int \frac{Sin.\, a\, x.\, Sin.\, b\, x}{q^2 + x^2}\, dx = \pi e^{-aq} \frac{e^{bq} - e^{-bq}}{4 q}, \quad 0 < b < a;$

2) $\displaystyle\int \frac{x\, Sin.\, a\, x.\, Cos.\, b\, x}{q^2 + x^2}\, dx = \pi e^{-aq} \frac{e^{-bq} + e^{bq}}{4}, \quad 0 < b < a;$

3) $\displaystyle = \pi e^{-bq} \frac{e^{-aq} - e^{aq}}{4}, \quad a < b < \infty;$

4) $\displaystyle\int \frac{Cos.\, a\, x.\, Cos.\, b\, x}{q^2 + x^2}\, dx = \pi e^{-aq} \frac{e^{bq} + e^{-bq}}{4 q}, \quad 0 < b < a;$

5) $\displaystyle = \pi e^{-bq} \frac{e^{aq} + e^{-aq}}{4 q}, \quad a < b < \infty;$

(Formules 1) à 5)): Boncompagni, Cr. 25. 74. — Arndt, Gr. 11. 70. — Dienger, Gr. 12. 97 (les form. 2) à 4)). — Schlömilch, Stud. I. 18. (les form. 1) à 4))

6) $\displaystyle\int \frac{Sin.^2\, a\, x.\, Cos.^2\, b\, x}{q^2 + x^2}\, dx = \frac{\pi}{8q} \left\{ -\frac{1}{2} e^{-2q(a+b)} + e^{-2bq} - \frac{1}{2} e^{-2q(a-b)} - e^{-2aq} + 1 \right\}, \quad a > b;$

7) $\displaystyle = \frac{\pi}{16\, q} (1 - e^{-4aq}) \quad , a = b;$

8) $\displaystyle = \frac{\pi}{8q} \left\{ -\frac{1}{2} e^{-2q(a+b)} + e^{-2bq} - \frac{1}{2} e^{-2q(b-a)} - e^{-2aq} + 1 \right\}, \quad a < b;$

9) $\displaystyle\int \frac{x\, Sin.\, a\, x.\, Cos.^2\, b\, x}{q^2 + x^2}\, dx = \frac{\pi}{4} \left\{ \frac{1}{2} e^{-q(a+2b)} + e^{-q(a-2b)} + e^{-aq} \right\}, \quad a > 2\, b;$

10) $\displaystyle = \frac{\pi}{4} \left\{ \frac{1}{2} e^{-3aq} + e^{-aq} \right\} \quad , a = 2\, b;$

11) $\displaystyle = \frac{\pi}{4} \left\{ \frac{1}{2} e^{-q(a+2b)} + e^{-q(2b-a)} + e^{-aq} \right\}, \quad a < 2\, b;$

(Formules 6) à 11)): Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 22.

12) $\displaystyle\int \frac{Cos.\, \lambda - Cos.\, b\, \lambda\, x}{q^2 + x^2}\, Cos.\, a\, x\, dx = \frac{\pi}{2\, q} e^{-aq} \left\{ Cos.\, \lambda - \frac{1}{2} (e^{bq\lambda} + e^{-bq\lambda}) \right\}, \quad 0 < \lambda < \frac{a}{b};$ Arndt, Gr. 11. 70.

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13) $\int \frac{Cos.\lambda - Cos.b\lambda x}{q^2+x^2} Cos.a x\, dx = \frac{\pi}{2q} e^{-aq} Cos.\lambda - \frac{\pi}{4q} e^{-bq\lambda}(e^{aq}+e^{-aq}) \;,\; \frac{a}{b} < \lambda < \infty;$

14) $\int \frac{Cos.\lambda - Cos.b\lambda x}{q^2+x^2} x Sin.a x\, dx = \frac{1}{2}\pi e^{-aq}\left\{Cos.\lambda - \frac{1}{2}(e^{bq\lambda}+e^{-bq\lambda})\right\} \;,\; 0 < \lambda < \frac{a}{b};$

15) $= \frac{1}{2}\pi e^{-aq} Cos.\lambda - \frac{1}{4}\pi e^{-bq\lambda}(e^{aq}-e^{-aq}) \;,\; \frac{a}{b} < \lambda < \infty;$

Arndt, Gr. 11. 70.

16) $\int \frac{x\, Sin.x + q\, Cos.x}{q^2+x^2} dx = \pi e^{-p}$ Poisson, P. 19. 404. N°. 68.

17) $\int \frac{x\, Sin.p x - q\, Cos.px}{q^2+x^2} dx = 0$ Poisson, Chal. 153.

18) $\int \frac{Cos.\lambda x - Cos.q\lambda}{q^2-x^2} dx = \frac{\pi}{2q} Sin.q\lambda$ Poisson, P. 18. 295. N°. 38. — Schellbach, Cr. 48. 207.

19) $\int \frac{(1-x^2)\, Cos.2x + 2x\, Sin.2x}{(1+x^2)^2} dx = \frac{2\pi}{e^2}$ Legendre, Exerc. 3. 41.

20) $\int \frac{Cos.(a^2x^2) - Sin.(a^2x^2)}{q^4+x^4} dx = \frac{\pi}{2q^3\sqrt{2}} e^{-a^2q^2}$ Schlömilch, Gr. 11. 174.

F. Alg. rat. fract. à dén. trinôme. Circ. Dir. en num. monôme. TABLE 210. Lim. 0 et ∞ .

1) $\int \frac{a+bx}{g+2hx+x^2} Sin.p x\, dx = \left\{b\, Cos.p h - \frac{a-bh}{\sqrt{(g^2-h^2)}} Sin.p h\right\} \pi e^{-p\sqrt{(g^2-h^2)}} \;,\; g > h;$

2) $\int \frac{a+bx}{g+2hx+x^2} Cos.p x\, dx = \left\{\frac{a-bh}{\sqrt{(g^2-h^2)}} Cos.p h + b\, Sin.p h\right\} \pi e^{-p\sqrt{(g^2-h^2)}}$

Laplace, Prob. L. 1. 26.

3) $\int \frac{(e^{ac}+e^{-ac})\, Cos.a x - (e^{ac}-e^{-ac})\, i\, Sin.a x}{b^2+(x+ci)^2} dx = \frac{\pi}{b}(e^{-ab}-e^{ab}) \;,\; c > b;$

4) $= \frac{2\pi}{b} e^{-ab} \;,\; c < b;$

Poisson, P. 18. 295. N°. 40.

5) $\int \frac{a^2+x^2}{b^2+(a^2+x^2)^2} Sin.2ax dx = \frac{\pi e^{-2aq}}{2b^2\sqrt{(b^2+a^4)}}\{(pa^2+pq^2-p^2q)\, Cos.2ap + (a^2q-p^3-q^3)\, Sin.2ap\}$

6) $\int \frac{Cos.2ax}{b^2+(a^2+x^2)^2} dx = \frac{\pi e^{-2aq}}{2b\sqrt{(b^2+a^4)}}(q\, Sin.2ap + p\, Cos.2ap)$

, où $2p = \sqrt{\{\sqrt{(b^2+a^4)}+b\}} - \sqrt{\{\sqrt{(b^2+a^4)}-b\}}, 2q = \sqrt{\{\sqrt{(b^2+a^4)}+b\}} + \sqrt{\{\sqrt{(b^2+a^4)}-b\}};$

Cauchy, P. 28. 147. P. 1. § 4.

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F. Alg. rat. fract. à dén. trinome. Circ. Dir. en num. monôme.	TABLE 210 suite.	Lim. 0 et ∞.

$$7)\int \frac{x\,Sin.\,a\,x}{x^4+2\,b^2\,x^2\,Cos.\,2\,\lambda+b^4}\,dx=\frac{\pi}{2\,b^2}\,e^{-ab\,Cos.\lambda}\,Cosec.\,2\,\lambda.\,Sin.\,(a\,b\,Sin.\,\lambda)$$

$$8)\int \frac{Cos.\,a\,x}{x^4+2\,b^2\,x^2\,Cos.\,2\,\lambda+b^4}\,dx=\frac{\pi}{2\,b^3}\,e^{-ab\,Cos.\lambda}\,Cosec.\,2\,\lambda.\,Sin.(\lambda+ab\,Sin.\,\lambda)$$

Plana, Mém. Turin. 1818. 7. I. 7. — Legendre, Exerc. 3. 44. — Helmling, Transf. 64, 63.

$$9)\int \frac{x^3\,Sin.\,a\,x}{x^4+2\,b^2\,x^2\,Cos.\,2\,\lambda+b^4}\,dx=\frac{\pi}{2}\,e^{-ab\,Cos.\lambda}\,Cosec.\,2\,\lambda.\,Sin.\,(2\,\lambda-a\,b\,Sin.\lambda)$$

Legendre, Exerc. 3. 44. — Helmling, Transf. II. S. 62.

$$10)\int \frac{x^2\,Cos.\,a\,x}{x^4+2\,b^2\,x^2\,Cos.\,2\,\lambda+b^4}\,dx=\frac{\pi}{2\,b}\,e^{-ab\,Cos.\lambda}\,Cosec.\,2\,\lambda.\,Sin.\,(\lambda-a\,b\,Sin.\lambda)$$

Legendre, Exerc. 3. 44. — Helmling, Transf. 65.

$$11)\int \frac{x\,Tang.\,a\,x}{x^4+2\,b^2\,x^2\,Cos.\,2\,\lambda+b^4}\,dx=\frac{\pi}{b^2}\,\frac{e^{-2ab\,Cos.\lambda}\,Sin.\,(2\,a\,b\,Sin.\,\lambda).\,Cosec.\,2\,\lambda}{1+2\,e^{-2ab\,Cos.\lambda}\,Cos.\,(2\,a\,b\,Sin.\,\lambda)+e^{-4ab\,Cos.\lambda}}$$

$$12)\int \frac{x\,Cot.\,a\,x}{x^4+2\,b^2\,x^2\,Cos.\,2\,\lambda+b^4}\,dx=\frac{\pi}{b^2}\,\frac{e^{-2ab\,Cos.\lambda}\,Sin.\,(2\,a\,b\,Sin.\,\lambda).\,Cosec.\,2\,\lambda}{1-2\,e^{-2ab\,Cos.\lambda}\,Cos.\,(2\,a\,b\,Sin.\,\lambda)+e^{-4ab\,Cos.\lambda}}$$

$$13)\int \frac{x\,Cosec.\,a\,x}{x^4+2\,b^2\,x^2\,Cos.\,2\,\lambda+b^4}\,dx=\frac{\pi}{b^2}\,\frac{(1+e^{-2ab\,Cos.\lambda})\,Cosec.\,2\,\lambda}{1-2\,e^{-2ab\,Cos.\lambda}\,Cos.\,(2\,a\,b\,Sin.\,\lambda)+e^{-4ab\,Cos.\lambda}}$$

Plana, Mém. Turin. 1818. 7. II. 10.

F. Alg. rat. fract. à dén. quadrinôme. Circ. Dir. en num. monôme.	TABLE 211.	Lim. 0 et ∞.

$$1)\int \frac{Sin.\,p\,x}{q^3+q^2\,x+q\,x^2+x^3}\,dx=\frac{1}{4\,q^2}\,\{e^{-pq}\,Ei.\,(p\,q)-e^{pq}\,Ei.\,(-p\,q)+$$
$$+2\,Ci.\,(p\,q).\,Sin\,p\,q-2\,Si.\,(p\,q).\,Cos.\,pq-\pi\,(e^{-pq}-Cos.\,pq)\}$$

$$2)\int \frac{x\,Sin.\,p\,x}{q^3+q^2\,x+q\,x^2+x^3}\,dx=\frac{1}{4\,q}\,\{e^{-pq}\,Ei.\,(p\,q)-e^{pq}\,Ei.\,(-p\,q)-$$
$$-2\,Ci.\,(p\,q).\,Sin.\,p\,q+2\,Si.\,(p\,q).\,Cos.\,pq+\pi\,(e^{-pq}-Cos.\,pq)\}$$

$$3)\int \frac{x^2\,Sin.\,p\,x}{q^3+q^2\,x+q\,x^2+x^3}\,dx=\frac{1}{4}\,\{-e^{-pq}\,Ei.\,(p\,q)+e^{pq}\,Ei.\,(-p\,q)+$$
$$+2\,Ci.\,(p\,q).\,Sin.\,p\,q-2\,Si.\,(p\,q).\,Cos.\,p\,q+\pi\,(e^{-pq}+Cos.\,pq)\}$$

V. T. 203. N°. 7 et T. 205 N°. 6, 10.

$$4)\int \frac{Cos.\,p\,x}{q^3+q^2\,x+q\,x^2+x^3}\,dx=\frac{1}{4\,q^2}\,\{e^{-pq}\,Ei.\,(p\,q)+e^{pq}\,Ei.\,(-p\,q)-$$
$$-2\,Ci.\,(p\,q).\,Cos.\,p\,q-2\,Si.\,(p\,q).\,Sin.\,pq+\pi\,(e^{-pq}+Sin.\,pq)\}$$

$$5)\int \frac{x\,Cos.\,p\,x}{q^3+q^2\,x+q\,x^2+x^3}\,dx=\frac{1}{4\,q}\,\{-e^{-pq}\,Ei.(p\,q)-e^{pq}\,Ei\,(-p\,q)+$$
$$+2\,Ci.\,(p\,q).\,Cos.\,p\,q+2\,Si.\,(p\,q).\,Sin\,p\,q+\pi\,(e^{-pq}-Sin.\,pq)\}$$

V. T. 203. N°. 8 et T. 205. N°. 5, 11.

6) $\int \frac{x^2 \, Cos. p x}{q^3 + q^2 x + q x^2 + x^3} dx = \frac{-1}{4} \{e^{-pq} Ei.(pq) + e^{pq} Ei.(-pq) + 2 \, Ci.(pq). Cos. pq + 2 \, Si.(pq). Sin. pq + \pi (e^{-pq} - Sin. pq)\}$ V. T. 203. N°. 8 et T. 205. N°. 5, 11.

7) $\int \frac{Sin. p x}{q^3 - q^2 x + q x^2 - x^3} dx = \frac{1}{4 q^2} \{e^{-pq} Ei.(pq) - e^{pq} Ei.(-pq) + 2 \, Ci.(pq). Sin. pq - 2 \, Si.(pq). Cos. pq + \pi (e^{-pq} - Cos. pq)\}$

8) $\int \frac{x \, Sin. p x}{q^3 - q^2 x + q x^2 - x^3} dx = \frac{1}{4 q} \{- e^{-pq} Ei.(pq) + e^{pq} Ei.(-pq) + 2 \, Ci.(pq). Sin. pq - 2 \, Si.(pq). Cos. pq + \pi (e^{-pq} - Cos. pq)\}$

9) $\int \frac{x^2 \, Sin. p x}{q^3 - q^2 x + q x^2 - x^3} dx = \frac{1}{4} \{- e^{-pq} Ei.(pq) + e^{pq} Ei.(-pq) + 2 \, Ci.(pq). Sin. pq - 2 \, Si.(pq). Cos. pq - \pi (e^{-pq} + Cos. pq)\}$

(7–9) V. T. 203. N°. 11 et T. 205 N°. 6, 10.

10) $\int \frac{Cos. p x}{q^3 - q^2 x + q x^2 - x^3} dx = \frac{1}{4 q^2} \{- e^{-pq} Ei.(pq) - e^{pq} Ei.(-pq) + 2 \, Ci.(pq). Cos. pq + 2 \, Si.(pq). Sin. pq + \pi (e^{-pq} + Sin. pq)\}$

11) $\int \frac{x \, Cos. p x}{q^3 - q^2 x + q x^2 - x^3} dx = \frac{1}{4 q} \{- e^{-pq} Ei.(pq) - e^{pq} Ei.(-pq) + 2 \, Ci.(pq). Cos. pq + 2 \, Si.(pq). Sin. pq - \pi (e^{-pq} - Sin. pq)\}$

12) $\int \frac{x^2 \, Cos. p x}{q^3 - q^2 x + q x^2 - x^3} dx = \frac{1}{4} \{e^{-pq} Ei.(pq) + e^{pq} Ei(-pq) + 2 \, Ci.(pq). Cos. pq + 2 \, Si.(pq). Sin. pq - \pi (e^{-pq} - Sin. pq)\}$

(10–12) V. T. 203. N°. 12 et T. 205. N°. 5, 11.

F. Alg. rat. fract. à dén. prod. de mon. et bin. Circ. Dir. en num. TABLE 212. Lim. 0 et ∞.

1) $\int \left\{Cos. x - \frac{1}{1 + x}\right\} \frac{dx}{x} = - A$ Arndt, Gr. 10. 225. — Id., Gr. 10. 233.

2) $\int \left\{\frac{Sin. x}{x} - \frac{1}{1 + x}\right\} \frac{dx}{x} = 1 - A$

3) $\int \left\{\frac{Cos. x - 1}{x^2} + \frac{1}{2(1 + x)}\right\} \frac{dx}{x} = \frac{1}{2} A - \frac{3}{4}$

(2–3) Arndt, Gr. 10. 233.

4) $\int \frac{Sin. p x}{1 + x^2} \frac{dx}{x} = \frac{1}{2} \pi (1 - e^{-p})$ Legendre, Exerc. 3. 46. — Cauchy, Sav. Etr. 1827. 599. P. II. § 7. — Poisson, P. 16. 215. N°. 7. — Serret, L. 8. 1.

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5) $\int \frac{Tang.\, p\, x}{1+x^2} \frac{dx}{x} = \frac{1}{2} \pi \frac{e^p - e^{-p}}{e^p + e^{-p}}$ Legendre, Exerc. 5. 35. — Cauchy, Sav. Etr. 1827. 599. Suppl. 2.

6) $\int \left\{ Cos.\, x - \frac{1}{1+x^2} \right\} \frac{dx}{x} = - A$ Arndt, Gr. 10. 225.

7) $\int \frac{Cos.\, q\, x - Cos.\, p\, x}{1+x^2} \frac{dx}{x^2} = \frac{1}{2} \pi (e^{-p} - e^{-q}) + \frac{1}{2} \pi (p - q)$ Poisson, P. 16. 215. N°. 7.

8) $\int \frac{Cos.\, q\, x}{1+x^2} \frac{dx}{x^{2-p}} = \frac{1}{4} (-1)^p \pi e^q\, Cosec. \left(\frac{p-1}{2} \pi \right)$, $0 < p$; Meyer, Int. Déf. 156.

9) $\int \frac{Sin.\, q\, x}{1+x^2} \frac{dx}{x^{1-p}} = \frac{1}{4} (-1)^{p-1} \pi e^q\, Cosec. \left(\frac{p-1}{2} \pi \right)$ V. T. 212. N°. 8.

10) $\int \frac{Sin.\, q\, x}{1-x^2} \frac{dx}{x^{2-p}} = - \frac{1}{8} \pi\, Cos. \left(\frac{p-1}{2} \pi - q \right) . Cosec. \left(\frac{p-1}{2} \pi \right)$ Meyer, Int. Déf. 156.

11) $\int \frac{Cos\, q\, x}{1-x^2} \frac{dx}{x^{1-p}} = \frac{1}{8} \pi\, Sin. \left(\frac{p-1}{2} \pi - q \right) . Cosec. \left(\frac{p-1}{2} \pi \right)$ V. T. 212. N°. 10.

12) $\int \frac{Sin.\, p\, x}{q^2 + x^2} \frac{dx}{x} = \frac{\pi}{2 q^2} (1 - e^{-pq})$ Cauchy, P. 19. 511. — Id., P. 28. 147. I. § 5. — Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 22. — Schlömilch, Stud. II. 14. — Schellbach, Cr. 48. 207.

13) $\int \frac{1 - Cos.\, p\, x}{q^2 + x^2} \frac{dx}{x^2} = \frac{\pi}{2 q^2} \left\{ p - \frac{1 - e^{-pq}}{q} \right\}$ Minding, Taf. 11.

14) $\int \frac{Sin.\, p\, x}{q^2 + x^2} \frac{dx}{x^{2a-1}} = (-1)^a \frac{\pi}{2 q^{2a}} e^{-pq}$

15) $\int \frac{Cos.\, p\, x}{q^2 + x^2} \frac{dx}{x^{2a}} = (-1)^a \frac{\pi}{2 q^{2a+1}} e^{-pq}$

(14, 15) Meyer, Int. Déf. 274.

16) $\int \frac{Cos. (px + \frac{1}{2} r \pi)}{q^2 + x^2} \frac{dx}{x^r} = \frac{\pi e^{-pq}}{4 q^{r+1}}$ Schlömilch, Gr. 11. 174.

17) $\int \frac{Sin.\, p\, x}{q^2 - x^2} \frac{dx}{x} = \frac{\pi}{2 q^2} (1 - Cos.\, p\, q)$ Cauchy, P. 19. 511. — Schlömilch, Stud. II. 15.

F. Alg. rat. fract. à dén. prod. de polyn. Circ. Dir. en num. monôme. TABLE 213. Lim. 0 et ∞.

1) $\int \left\{ \frac{b+x}{a^2 + (b+x)^2} - \frac{b-x}{a^2 + (b-x)^2} \right\} Sin.\, q\, x\, dx = \pi e^{-bq}\, Cos.\, b\, q$

2) $\int \left\{ \frac{b+x}{a^2 + (b+x)^2} + \frac{b-x}{a^2 + (b-x)^2} \right\} Cos.\, q\, x\, dx = \pi e^{-bq}\, Sin.\, b\, q$

(1, 2) Cauchy, Sav. Etr. 1827. 124. Note 6.

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$$3)\int \frac{x\,Sin.\,p\,x}{(q^2+x^2)(q^4-x^4)}\,dx = \frac{\pi}{8q^4}\left\{(1+pq)\,e^{-pq} - Cos.\,p\,q\right\}$$

$$4)\int \frac{x^3\,Sin.\,p\,x}{(q^2+x^2)(q^4-x^4)}\,dx = \frac{\pi}{8q^2}\left\{(1-pq)\,e^{-pq} - Cos.\,p\,q\right\}$$

$$5)\int \frac{x^5\,Sin.\,p\,x}{(q^2+x^2)(q^4-x^4)}\,dx = \frac{\pi}{8}\left\{(pq-3)\,e^{-pq} - Cos.\,p\,q\right\}$$

V. T. 203. N°. 7, 11 et T. 205. N°. 6, 10.

$$6)\int \frac{Cos.\,p\,x}{(q^2+x^2)(q^4-x^4)}\,dx = \frac{\pi}{8q^5}\left\{Sin.\,p\,q + (pq+2)\,e^{-pq}\right\}$$

$$7)\int \frac{x^2\,Cos.\,p\,x}{(q^2+x^2)(q^4-x^4)}\,dx = \frac{\pi}{8q^3}\left\{Sin.\,p\,q - pq\,e^{-pq}\right\}$$

$$8)\int \frac{x^4\,Cos.\,p\,x}{(q^2+x^2)(q^4-x^4)}\,dx = \frac{\pi}{8q}\left\{Sin.\,p\,q + (pq-2)\,e^{-pq}\right\}$$

V. T. 203. N°. 8, 12 et T. 205. N°. 5, 11.

$$9)\int \left\{\frac{b^2}{(a+x)^c} + \frac{c(c+1)}{(a+x)^{c+2}}\right\} Sin.\,b\,x\,dx = b\,a^{-c}$$

$$10)\int \left\{\frac{b^2}{(a+x)^c} + \frac{c(c+1)}{(a+x)^{c+2}}\right\} Cos.\,b\,x\,dx = \frac{c}{a^{c+1}}$$

Schlömilch, Beitr. III. § 4.

$$11)\int \frac{(a-xi)^{-p} - (a+xi)^{-p}}{2i}\,Sin.\,q\,x\,dx = \frac{\pi}{2\,\Gamma(p)}\,q^{p-1}\,e^{-aq}$$

$$12)\int \frac{(a-xi)^{-p} + (a+xi)^{-p}}{2}\,Cos.\,q\,x\,dx = \frac{\pi}{2\,\Gamma(p)}\,q^{p-1}\,e^{-aq}$$

Cauchy, Lim. Imag. N°. 107. — Id., P. 28. 147. P. 1. § 3. — Id., Sav. Etr. 1827. 124. Note 6. — Id., Exerc. 1827. p. 141.

$$13)\int \frac{Sin.\,a\,x}{x(x^2+2^2)(x^2+4^2)\ldots(x^2+4b^2)}\,dx = \frac{\pi(-1)^b}{2^{2b-1}\,1^{2b|1}}\sum_0^b (-1)^n \binom{2b}{n} e^{2(n-b)a}$$

$$14)\int \frac{Cos.\,a\,x}{(x^2+1^2)(x^2+3^2)\ldots(x^2+(2b+1)^2)}\,dx = \frac{\pi(-1)^b}{2^{2b}\,1^{2b+1|1}}\sum_0^b (-1)^n \binom{2b+1}{n} e^{(2n-2b-1)a}$$

Schlömilch, Gr. 7. 38.

$$15)\int \frac{(a-xi)^{-p} + (a+xi)^{-p}}{2}\,x^{2c-1}\,Sin.\,qx\,dx = \frac{(-1)^{\frac{2c-1}{2}}}{2}\,\frac{\pi}{\Gamma(p)}\,\frac{d^{2c-1}}{dq^{2c-1}}\,.\,q^{p-1}\,e^{-aq}$$

$$16)\int \frac{(a-xi)^{-p} + (a+xi)^{-p}}{2}\,x^{2c}\,Cos.\,q\,x\,dx = \frac{(-1)^c}{2}\,\frac{\pi}{\Gamma(p)}\,\frac{d^{2c}}{dq^{2c}}\,.\,q^{p-1}\,e^{-aq}$$

$$17)\int \frac{(a-xi)^{-p} - (a+xi)^{-p}}{2i}\,x^{2c}\,Sin.\,q\,x\,dx = \frac{(-1)^c}{2}\,\frac{\pi}{\Gamma(p)}\,\frac{d^{2c}}{dq^{2c}}\,.\,q^{p-1}\,e^{-aq}$$

Cauchy, P. 28. 147. P. 1. § 3.

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18) $\int \frac{(a-xi)^{-p}-(a+xi)^{-p}}{2i} x^{2c-1} Cos. qx\, dx = \frac{(-1)^{\frac{2c-1}{2}}}{2} \frac{\pi}{\Gamma(p)} \frac{d^{2c-1}}{dq^{2c-1}} . q^{p-1} e^{-aq}$ Cauchy, P. 28. 147. P. 1. § 3.

19) $\int \left\{ \frac{(b-xi)^{-a}+(b+xi)^{-a}}{2} Cos. cx + \frac{(b-xi)^{-a}-(b+xi)^{-a}}{2i} Sin. cx \right\} dx = \frac{\pi c^a}{1^{a/1}} e^{-bc}, c > 0;$

20) $= 0 \quad , c < 0;$ Cauchy, P. 28. 147. P. III. § 3.

21) $\int \frac{(q-xi)^{-a}-(q+xi)^{-a}}{2i} x^b Sin. \left(\frac{1}{2} b\pi + px\right) dx = \frac{\pi}{2.1^{a/1}} \frac{d^b}{dp^b} p^{a-1} e^{-pq}$

22) $\int \frac{(q-xi)^{-a}+(q+xi)^{-a}}{2} x^b Cos. \left(\frac{1}{2} b\pi + px\right) dx = \frac{\pi}{2.1^{a/1}} \frac{d^b}{dp^b} p^{a-1} e^{-pq}$

Cauchy, Exerc. 1827. p. 141.

F. Alg. rat. fract. à dén. x. Circ. Dir. en dén. mon. $Cos. x$. Val. princ. TABLE 214. Lim. 0 et ∞.

1) $\int \frac{Sin. ax}{Cos. bx} \frac{dx}{x} = 0 \quad , a < b;$

2) $= \frac{1 - Cos. h\pi}{2} \pi , a = 2bh + c;$

Legendre, Exerc. 5. 37, 39. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 59.

3) $\int \frac{Sin. \{(b-a)x\}}{Cos. bx} \frac{dx}{x} = 0 \quad , a$ très-petit;

4) $= \frac{1}{2}\pi , a = 0;$

Cauchy, Sav. Etr. 1827. 599. S. 2.

5) $\int \frac{Sin. \{(2a+1)bx\}}{Cos. bx} \frac{dx}{x} = \frac{1}{2}\pi$ Legendre, Exerc. 5. 40.

6) $\int \frac{dx}{x\, Tang. x} = \infty$ Plana, Mém. Turin. 1818. 7. 2. N°. 13.

F. Alg. rat. fract. à dén $1 + x^2$. Circ. Dir. en dén. mon. $Sin. x$. Val. princ. TABLE 215. Lim. 0 et ∞.

1) $\int \frac{Sin. bx}{Sin. ax} \frac{dx}{1+x^2} = \frac{1}{2}\pi \frac{e^b - e^{-b}}{e^a - e^{-a}} \quad , b < a;$

2) $\int \frac{Cos. bx}{Sin. ax} \frac{x\,dx}{1+x^2} = \frac{1}{2}\pi \frac{e^b + e^{-b}}{e^a - e^{-a}}$

Cauchy, Sav. Etr. 1827. 599. P. II. § 5. — Id., Sav. Etr. 1827. 599. Suppl. 2. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 60. — Legendre, Exerc. 5. 29. — Boncompagni, Cr. 25. 74.

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3) $\int \frac{Sin.\, b x}{Sin.\, a x} \frac{dx}{1+x^2} = \frac{1}{2}\pi \frac{e^{ar}+e^{-ar}-2e^{-b}}{e^a-e^{-a}}$

4) $\int \frac{Cos.\, b x}{Sin.\, a x} \frac{x\, dx}{1+x^2} = \frac{1}{2}\pi \frac{e^{ar}-e^{-ar}+2e^{-a}}{e^a-e^{-a}}$

où $\frac{b}{2a}$ est égal à un nombre entier $+\frac{1}{2}r$; si r est négatif, il faut changer les signes de e^{ar} et de e^{-ar} dans la formule (4). Cauchy, Sav. Etr. 1827. 599. P. II. § 7.

5) $\int \frac{Sin.\{(c+2ha)x\}}{Sin.\, a x} \frac{dx}{1+x^2} = \frac{1}{2}\pi \frac{e^c+e^{-c}-2e^{-(c+2ha)}}{e^a-e^{-a}}$ Legendre, Exerc. 5. 31.

6) $\int \frac{Sin.\{(2h+1)ax\}}{Sin.\, a x} \frac{dx}{1+x^2} = \frac{1}{2}\pi \frac{e^{2a}+1-2e^{-2ha}}{e^{2a}-1}$

7) $\int \frac{Sin.\, 2hax}{Sin.\, a x} \frac{dx}{1+x^2} = \pi \frac{1-e^{-2ha}}{e^a-e^{-a}}$

Legendre, Exerc. 5. 36. — Cauchy, Sav. Etr. 1827. 599. S. 1.

8) $\int \frac{Cos.\{(c+2ha)x\}}{Sin.\, a x} \frac{x\, dx}{1+x^2} = \frac{1}{2}\pi \frac{e^c-e^{-c}+2e^{-(c+2ha)}}{e^a-e^{-a}}$ Legendre, Exerc. 5. 32.

9) $\int \frac{Cos.\{(2h+1)ax\}}{Sin.\, a x} \frac{x\, dx}{1+x^2} = \pi \frac{e^{-(2h+1)a}}{e^a-e^{-a}}$

10) $\int \frac{Cos.\, 2hax}{Sin.\, a x} \frac{x\, dx}{1+x^2} = \pi \frac{e^{-2ha}}{e^a-e^{-a}}$

Legendre, Exerc. 5. 33, 36. — Cauchy, Sav. Etr. 1827. 599. S. 2.

11) $\int \frac{Cos.\{(a-b)x\}}{Sin.\, a x} \frac{x\, dx}{1+x^2} = \frac{1}{2}\pi \frac{e^a+e^{-a}}{e^a-e^{-a}}$, b très-petit;

12) $= \pi \frac{e^{-a}}{e^a-e^{-a}}$, $b=0$;

13) $\int \frac{Cos.\{(a+b)x\}}{Sin.\, a x} \frac{x\, dx}{1+x^2} = \frac{\pi e^{-a}}{e^a-e^{-a}} - \frac{1}{2}\pi$, b très-petit;

14) $= \frac{\pi e^{-a}}{e^a-e^{-a}}$, $b=0$;

15) $\int \frac{Cos.\{(2c+1)a \pm b\}x}{Sin.\, a x} \frac{x\, dx}{1+x^2} = \pi \frac{e^{-(2c+1)a}}{e^a-e^{-a}} \mp \frac{\pi}{2}$, b très-petit;

16) $= \pi \frac{e^{-(2c+1)a}}{e^a-e^{-a}}$, $b=0$;

17) $\int \frac{Sin.\, bx\; Sin.\{(2c+1)ax\}}{Sin.\, a x} \frac{x\, dx}{1+x^2} = \frac{1}{2}\pi$, b très-petit;

18) $= 0$, $b=0$;

Cauchy, Sav. Etr. 1827. 599. S. 2.

19) $\int \frac{Sin.\{(2c+1)ax\} - e^{-a} Sin. 2acx}{Sin. ax} \frac{dx}{1+x^2} = \frac{1}{2}\pi$

20) $\int \frac{Sin.\{(a+b)x\} - e^{-a} Sin. bx}{Sin. ax} \frac{dx}{1+x^2} = \frac{1}{2}\pi e^{-ar}$

} , où $\frac{b}{2a}$ est égal à un nombre pair $+ \frac{1}{2} r$; Cauchy, Sav. Etr. 1827. 599. Suppl. 1.

21) $\int \frac{x}{Sin. ax} \frac{dx}{1+x^2} = \frac{\pi}{e^a - e^{-a}}$ Legendre, Exerc. 4. 132. — Cauchy, Sav. Etr. 1827. 599. P. II. § 5. — Id., Sav. Etr. 1827. 599. S. 2.

F. Alg. rat. fract. à dén. $1 + x^2$. Circ. Dir. en dén. mon. *Cos. x.* } Val. princ. TABLE 216. Lim. 0 et ∞.

1) $\int \frac{1}{Cos. ax} \frac{dx}{1+x^2} = \frac{\pi}{e^a + e^{-a}}$ Cauchy, Sav. Etr. 1827. 599. P. II. § 5.

2) $\int \frac{Sin.\{(c+2ha)x\}}{Cos. ax} \frac{x\,dx}{1+x^2} = -\frac{1}{2}\pi\, Cos. h\pi \frac{e^c + e^{-c}}{e^a + e^{-a}} + \pi \frac{e^{-(c+2ha)}}{e^a + e^{-a}}$ Legendre, Exerc. 5. 37.

3) $\int \frac{Sin.\{(2h+1)ax\}}{Cos. ax} \frac{x\,dx}{1+x^2} = \pi \frac{e^{-(2h+1)a}}{e^a + e^{-a}}$

4) $\int \frac{Sin. 2hax}{Cos. ax} \frac{x\,dx}{1+x^2} = \pi \frac{e^{-ha} - Cos. h\pi}{e^a + e^{-a}}$

} Legendre, Exerc. 5. 37, 41.

5) $\int \frac{Cos. bx}{Cos. ax} \frac{dx}{1+x^2} = \frac{1}{2}\pi \frac{e^b + e^{-b}}{e^a + e^{-a}}, \ a > b$; Cauchy, Sav. Etr. 1827. 599. P. II. § 5. — Legendre, Exerc. 5. 29. — Cisa de Grésy, Mém. de Turin. 1821. 209. II. 60. — Boncompagni, Cr. 25. 74.

6) $= \frac{1}{2}\pi \frac{e^{ar} - e^{-ar} + 2e^{-b}}{e^a + e^{-a}}$, où $\frac{b}{2a}$ est égal à un nombre pair $+ \frac{1}{2} r$: si ce nombre est impair, ou si r est négatif, il faut changer les signes de e^{ar} et de e^{-ar}; Cauchy, Sav. Etr. 1827. 599. P. II. § 7.

7) $\int \frac{Cos.\{(c+2ha)x\}}{Cos. ax} \frac{dx}{1+x^2} = \frac{1}{2}\pi\, Cos. h\pi \frac{e^c - e^{-c}}{e^a + e^{-a}} + \pi \frac{e^{-(c+2ha)}}{e^a + e^{-a}}, \ a < c$; Legendre, Exerc. 5. 34.

8) $\int \frac{Cos.\{(2h+1)ax\}}{Cos. ax} \frac{dx}{1+x^2} = \frac{1}{2}\pi \frac{(e^{2a} - 1)\, Cos. h\pi + 2e^{-2ha}}{e^{2a} + 1}$

9) $\int \frac{Cos. 2hax}{Cos. ax} \frac{dx}{1+x^2} = \pi \frac{e^{-2ha}}{e^a + e^{-a}}$

} Legendre, Exerc. 5. 36.

1) $\int \frac{1}{Sin.\, a x} \frac{x\, dx}{q^2 + x^2} = \frac{\pi e^{aq}}{e^{2aq} - 1}$ Legendre, Exerc. 4. 133. — Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 39. — Schlömilch, Beitr. II. § 4.

2) $\int \frac{Sin.\, b x}{Sin.\, a x} \frac{dx}{q^2 + x^2} = \frac{\pi}{2q} \frac{e^{bq} - e^{-bq}}{e^{aq} - e^{-aq}}$

3) $\int \frac{Cos.\, b x}{Sin.\, a x} \frac{x\, dx}{q^2 + x^2} = \frac{\pi}{2} \frac{e^{bq} + e^{-bq}}{e^{aq} - e^{-aq}}$

(2, 3) Cauchy, Lim. Imag. Add. 33. — Boncompagni, Cr. 25. 74.

4) $\int \frac{Sin.\, k x}{Sin.\, x} \frac{x\, Tang.\, x}{p^2 + 4 x^2} dx = 0$

5) $\int \frac{Sin.\, k x}{Sin.\, x} \frac{Tang.\, x .\, Sin.\, x}{p^2 + x^2} dx = 0$

(4, 5) , $k = \infty$; Schlömilch, Beitr. II. 4.
Elles sont fautives: au lieu de k mettez $2k + 1$; leurs valeurs sont alors ∞.

6) $\int \frac{Cos.\, k x}{Sin.\, x} \frac{x\, Cot.\, x}{p^2 + 4 x^2} dx = 0, k = \infty$; Meyer, Int. Déf. 221.
Elle est fautive: mettez $2k$ au lieu de k, alors la valeur en est ∞.

7) $\int \frac{Cos.\, k x}{Sin.\, x} \frac{x}{p^2 + 4 x^2} dx = 0$

8) $\int \frac{Cos.\, k x}{Cos.\, x} \frac{dx}{p^2 + x^2} = 0$

9) $\int \frac{1}{Cos.\, x} \frac{dx}{q^2 + x^2} = \frac{\pi}{q} \frac{1}{e^q + e^{-q}}$

(7, 8, 9) , $k = \infty$;
Schlömilch, Beitr. II. 4.

10) $\int \frac{1}{Cos.\, a x} \frac{dx}{x^2 + q^2} = \frac{\pi}{q} \frac{1}{e^{aq} + e^{-aq}}$ Schlömilch, Beitr. II. § 4.

11) $\int \frac{Sin.\, b x}{Cos.\, a x} \frac{x}{x^2 + q^2} dx = -\frac{1}{2} \pi \frac{e^{bq} - e^{-bq}}{e^{aq} + e^{-aq}}$

12) $\int \frac{Cos.\, b x}{Cos.\, a x} \frac{dx}{x^2 + q^2} = \frac{\pi}{2q} \frac{e^{bq} + e^{-bq}}{e^{aq} + e^{-aq}}$

(11, 12) Cauchy, Lim. Imag. Add. 33. — Boncompagni, Cr. 25. 74.

13) $\int \frac{1}{Cos.\, a x} \frac{dx}{1 + x^4} = \frac{\pi}{\sqrt{2}} \frac{(e^{\frac{1}{2}a\sqrt{2}} + e^{-\frac{1}{2}a\sqrt{2}}) Cos. \left(\frac{a}{\sqrt{2}}\right) + (e^{\frac{1}{2}a\sqrt{2}} - e^{-\frac{1}{2}a\sqrt{2}}) Sin. \left(\frac{a}{\sqrt{2}}\right)}{e^{a\sqrt{2}} + e^{-a\sqrt{2}} + 2 Cos.(a\sqrt{2})}$

14) $\int \frac{x}{Sin.\, a x} \frac{dx}{1 + x^4} = \pi\, Sin. \left(\frac{a}{\sqrt{2}}\right) \frac{e^{\frac{1}{2}a\sqrt{2}} + e^{-\frac{1}{2}a\sqrt{2}}}{e^{a\sqrt{2}} + e^{-a\sqrt{2}} - 2 Cos.(a\sqrt{2})}$

(13, 14) Cauchy, Sav. Etr. 1827. 599. P. II. § 5.

F. Alg. rat. fract. à dén. $(a^2+x^2)x$. Circ. Dir. en dén. monôme. } Val. princ. **TABLE 218.** Lim. 0 et ∞.

1) $\int \frac{Sin.\, bx}{Cos.\, ax} \frac{1}{x} \frac{dx}{1+x^2} = \frac{1}{2}\pi \frac{e^b - e^{-b}}{e^a + e^{-a}}, a > b;$ Cauchy, Sav. Etr. 1827. 599. P. II. § 5. — Legendre, Exerc. 5. 29. — Cisa de Grésy, Mem. Turin. 1821. 209. II. 60. — Boncompagni, Cr. 25. 74.

2) $= \frac{1}{2}\pi \frac{e^{ar} + e^{-ar} - 2e^{-b}}{e^a + e^{-a}}$, où $\frac{b}{2a}$ est égal à un nombre pair $+ \frac{1}{2}r$; si le nombre est impair, ou si r est négatif, il faut changer les signes de e^{ar} et de e^{-ar}; Cauchy, Sav. Etr. 1827. 599. P. II § 7.

3) $\int \frac{Sin.\{(c+2ha)x\}}{Cos.\, ax} \frac{1}{x} \frac{dx}{1+x^2} = \frac{1}{2}\pi(1 - Cos.\, h\pi) - \frac{\pi e^{-(c+2ha)}}{e^a + e^{-a}} + \frac{1}{2}\pi\, Cos.\, h\pi \frac{e^c + e^{-c}}{e^a + e^{-a}}$

4) $\int \frac{Sin.\{(2h+1)ax\}}{Cos.\, ax} \frac{1}{x} \frac{dx}{1+x^2} = \frac{1}{2}\pi - \pi \frac{e^{-(2h+1)a}}{e^a + e^{-a}}$

5) $\int \frac{Sin.\, 2hax}{Cos.\, ax} \frac{1}{x} \frac{dx}{1+x^2} = \frac{1}{2}\pi \frac{e^a + e^{-a} - 2e^{ha}}{e^a + e^{-a}} - \frac{1}{2}\pi\, Cos.\, h\pi \frac{e^a + e^{-a} - 2}{e^a + e^{-a}}$

} $a < c$; Legendre, Exerc. 5. 35, 36.

6) $\int \frac{Cos.\, bx}{Sin.\, ax} \frac{1}{x} \frac{dx}{q^2 + x^2} = \frac{\pi}{2q^2} \frac{e^{bq} + e^{-bq}}{e^{aq} - e^{-aq}}$

7) $\int \frac{Sin.\, bx}{Cos.\, ax} \frac{1}{x} \frac{dx}{q^2 + x^2} = \frac{\pi}{2q^2} \frac{e^{bq} - e^{-bq}}{e^{aq} + e^{-aq}}$

} $b < a$; Boncompagni, Cr. 25. 74.

F. Alg. rat. fract. à dén. x. Circ. Dir. en dén. trinôme. **TABLE 219.** Lim. 0 et ∞.

1) $\int \frac{Sin\, x}{1 + 2p\, Cos.\, 2x + p^2} \frac{dx}{x} = \frac{1}{2}\pi \frac{1}{1-p^2}, p < 1;$

2) $= \frac{1}{2}\pi \frac{1}{p^2 - 1}, p > 1;$

3) $\int \frac{Tang.\, x}{1 + 2p\, Cos.\, 2x + p^2} \frac{dx}{x} = \frac{1}{2}\pi \frac{1}{1-p^2}, p < 1;$

4) $= \frac{1}{2}\pi \frac{1}{p^2 - 1}, p > 1;$

} Schlömilch, Gr. 4. 316.

5) $\int \frac{Sin.\, ax}{1 - 2p\, Cos.\, ax + p^2} \frac{dx}{x} = \frac{1}{2} \frac{\pi}{1-p}, p < 1;$

6) $= \frac{1}{2p} \frac{\pi}{1-p}, p > 1;$

7) $\int \frac{Sin.\, ax}{1 + 2p\, Cos.\, ax + p^2} \frac{dx}{x} = \frac{1}{2} \frac{\pi}{1+p}, p < 1;$

8) $= \frac{1}{2p} \frac{\pi}{1+p}, p > 1;$

} Plana, Mém. Turin. 1818. 7. II. 13.

1) $\int\frac{Sin.\,x}{1+2p\,Cos.\,x+p^2}\frac{x}{q^2+x^2}dx=\frac{\pi}{2}\frac{1}{e^q+p}$, $p^2<1$;

2) $\int\frac{1+p\,Cos.\,x}{1+2p\,Cos.\,x+p^2}\frac{dx}{q^2+x^2}=\frac{\pi}{2q}\frac{1}{1+pe^{-q}}$

Schlömilch, Stud. II. 18.

3) $\int\frac{Sin.\,rx}{1+2p\,Cos.\,rx+p^2}\frac{x}{q^2+x^2}dx=\frac{1}{2}\frac{\pi}{e^{qr}+p}$, $p<1$; Legendre, Exerc. 4. 131.

4) $=\frac{1}{2p}\frac{\pi}{pe^{qr}+1}$, $p>1$; Ohm, Ausw. 26.

5) $\int\frac{1}{e^{-2ac}+2\,Cos.\,2ax+e^{2ac}}\frac{dx}{q^2+x^2}=\frac{1}{e^{2ac}-e^{-2ac}}\left\{\frac{\pi}{2q}-\frac{\pi}{q}\frac{e^{-2ac}}{e^{2ac}+e^{-2ac}}\right\}$

6) $\int\frac{Sin.\,2ax}{e^{-2ac}+2\,Cos.\,2ax+e^{2ac}}\frac{x}{q\;\;+x^2}dx=\frac{\pi}{2}\frac{e^{-2ac}}{e^{2ac}+e^{-2ac}}$

Poisson, P. 18. 295. N°. 42. il a faut. dans 6) pour dénom. $e^{-2ac}-2\,Cos.\,2ax+e^{2ac}$.

7) $\int\frac{Cos.\,2ax}{1+2p\,Cos.\,2ax+p^2}\frac{dx}{q^2+x^2}=\frac{\pi}{2q}\frac{1}{e^{2aq}+p}$ Legendre, Exerc. 4. 134.

8) $\int\frac{Sin.\,ax.\,Sin.\,bx}{1+2p\,Cos.\,ax+p^2}\frac{dx}{q^2+x^2}=\frac{\pi}{4pq}e^{-bq}\left\{\frac{pe^{aq}}{1+pe^{aq}}-\frac{pe^{-aq}}{1+pe^{-aq}}\right\}$, $p^2<1$;

9) $\int\frac{Sin.\,ax.\,Cos.\,bx}{1+2p\,Cos.\,ax+p^2}\frac{x}{q^2+x^2}dx=\frac{\pi}{4p}e^{-bq}\left\{\frac{pe^{-aq}}{1+pe^{-aq}}-\frac{pe^{aq}}{1+pe^{aq}}\right\}$

Boncompagni, Cr. 25. 74.

10) $\int\frac{Sin.\,ax}{1+2p\,Cos.\,ax+p^2}\frac{dx}{1+x^{2b}}=\frac{\pi}{2b}\frac{e^{-a}}{1+pe^{-a}}-\frac{\pi}{b}\sum_{1}^{\frac{b-1}{2}}\frac{e^{-a\,Cos.\frac{n\pi}{b}}\,Cos.\frac{2n\pi}{b}\cdot Cos.\left(a\,Sin.\frac{n\pi}{b}\right)+pe^{-a\,Cos.\frac{n\pi}{b}}}{1+2pe^{-a\,Cos.\frac{n\pi}{b}}\,Cos.\left(a\,Sin.\frac{n\pi}{b}\right)+p^2e^{-2a\,Cos.\frac{n\pi}{b}}}$

$-\frac{\pi}{b}\sum_{1}^{\frac{b-1}{2}}\frac{e^{-a\,Cos.\frac{n\pi}{b}}\,Sin.\frac{2n\pi}{b}\cdot Sin.\left(a\,Sin.\frac{n\pi}{b}\right)}{1+2pe^{-a\,Cos.\frac{n\pi}{b}}\,Cos.\left(a\,Sin.\frac{n\pi}{b}\right)+p^2e^{-2a\,Cos.\frac{n\pi}{b}}}$, b impair;

11) $=\frac{\pi}{b}\sum_{1}^{\frac{b}{2}-1}\frac{e^{-a\,Cos.\left(\frac{2n+1}{2b}\pi\right)}\,Cos.\left(a\,Sin.\left\{\frac{2n+1}{2b}\pi\right\}\right)+pe^{-a\,Cos.\left(\frac{2n+1}{2b}\pi\right)}}{1+2pe^{-a\,Cos.\left(\frac{2n+1}{2b}\pi\right)}\,Cos.\left(a\,Sin.\left\{\frac{2n+1}{2b}\pi\right\}\right)+p^2e^{-2a\,Cos.\left(\frac{2n+1}{2b}\pi\right)}}Cos.\left(\frac{2n+1}{b}\pi\right)$

$+\frac{\pi}{b}\sum_{0}^{\frac{b}{2}-1}\frac{e^{-a\,Cos.\left(\frac{2n+1}{2b}\pi\right)}\,Sin.\left(a\,Sin.\left\{\frac{2n+1}{2b}\pi\right\}\right).Sin.\left(\frac{2n+1}{b}\pi\right)}{1+2pe^{-a\,Cos.\left(\frac{2n+1}{2b}\pi\right)}\,Cos.\left(a\,Sin.\left\{\frac{2n+1}{2b}\pi\right\}\right)+p^2e^{-2a\,Cos.\left(\frac{2n+1}{2b}\pi\right)}}$, b pair;

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12) $$\int \frac{p+Cos\,ax}{1+2pCos.ax+p^2}\frac{dx}{1+x^{2b}} = \frac{\pi}{2b}\frac{e^{-a}}{1+pe^{-a}} - \frac{\pi}{b}\sum_{1}^{\frac{b-1}{2}} \frac{e^{-aCos.\frac{n\pi}{b}}Sin.\frac{n\pi}{b}.\ Sin.\left(a\,Sin.\frac{n\pi}{b}\right)}{1+2pe^{-aCos.\frac{n\pi}{b}}Cos.\left(aSin.\frac{n\pi}{b}\right)+p^2e^{-2aCos.\frac{n\pi}{b}}}$$

$$-\frac{\pi}{b}\sum_{1}^{\frac{b-1}{2}} Cos.\frac{n\pi}{b}\,\frac{e^{-aCos.\frac{n\pi}{b}}Cos.\left(aSin.\frac{n\pi}{b}\right)+pe^{-2aCos.\frac{n\pi}{b}}}{1+2pe^{-aCos.\frac{n\pi}{b}}Cos.\left(aSin.\frac{n\pi}{b}\right)+p^2e^{-2aCos.\frac{n\pi}{b}}}$$, b impair;

13) $$= \frac{\pi}{b}\sum_{1}^{\frac{b}{2}-1} \frac{e^{-aCos\left(\frac{2n+1}{2b}\pi\right)}Sin.\left(a\,Sin.\left\{\frac{2n+1}{2b}\pi\right\}\right).\ Sin.\left(\frac{2n+1}{2b}\pi\right)}{1+2pe^{-aCos.\left(\frac{2n+1}{2b}\pi\right)}Cos.\left(aSin.\left\{\frac{2n+1}{2b}\pi\right\}\right)+p^2e^{-2aCos.\left(\frac{2n+1}{2b}\pi\right)}}$$

$$+\frac{\pi}{b}\sum_{1}^{\frac{b}{2}-1} Cos.\left(\frac{2n+1}{2b}\pi\right)\frac{e^{-aCos.\left(\frac{2n+1}{2b}\pi\right)}Cos.\left(aSin.\left\{\frac{2n+1}{2b}\pi\right\}\right)+p^2e^{-2aCos.\left(\frac{2n+1}{2b}\pi\right)}}{1+2pe^{-aCos.\left(\frac{2n+1}{2b}\pi\right)}Cos.\left(aSin.\left\{\frac{2n+1}{2b}\pi\right\}\right)+p^2e^{-2aCos.\left(\frac{2n+1}{2b}\pi\right)}}$$, b pair;

Les formules (10) à (13) valent pour $p < 1$; voyez: Plana, Mém. Turin. 1818. 7 III. 16.

F. Alg. rat. fract. à dén. binôme. Circ. Dir. en dén. trin. $a - b\,Cos.x + c$. TABLE 221. Lim. 0 et ∞.

1) $$\int \frac{Sin.ax.\,Sin.bx}{1-2p\,Cos.ax+p^2}\frac{dx}{1+x^2} = \frac{1}{4}\pi\frac{e^b-e^{-b}}{e^a-p}$$, $b < a$;

2) $$= \frac{1}{4}\pi\left\{\frac{p^ce^q-e^{-b}}{e^a-p}+\frac{e^{-b}-p^ce^{-q}}{e^{-a}-p}\right\}$$, $b = ac+q$;

3) $$= \frac{1}{4}\pi\left\{\frac{p^re^{b-ar}-e^{-b}}{e^a-p}+e^{-b}\frac{1-p^re^{ar}}{e^{-a}-p}\right.$$, $b = ra+q$ en tous cas;

4) $$\int \frac{Cos\,ax-p}{1-2pCos.ax+p^2}\frac{Cos.bx}{1+x^2}dx = \frac{1}{4}\pi\left\{\frac{p^re^{b-ra}+e^{-b}}{e^a-p}+e^{-b}\frac{1-p^re^{ra}}{e^{-a}-p}\right\}$$, $b = ra+q$;

(1 à 4) , $p < 1$; Poisson, P. 17.612 N°. 19.

5) $$\int \frac{Sin.x}{1-2p\,Cos.x+p^2}\frac{x}{q^2+x^2}dx = \frac{\pi}{2}\frac{1}{e^q-p}$$, $p^2 \leqq 1$; Schlömilch, Beitr. II. § 4.

6) $$\int \frac{1}{1-2p\,Cos.x+p^2}\frac{dx}{q^2+x^2} = \frac{\pi}{2q}\frac{1}{1-p^2}\frac{e^q+p}{e^q-p}$$

7) $$\int \frac{Cos.x}{1-2p\,Cos.x+p^2}\frac{dx}{q^2+x^2} = \frac{\pi}{2q}\frac{1}{1-p^2}\frac{1-pe^q}{e^q-p}$$

(6, 7) , $p^2 < 1$; V. T. 19. N°. 2 et T. 221. N°. 8.

8) $\displaystyle\int \frac{1 - p\,Cos.x}{1 - 2p\,Cos.x + p^2}\,\frac{dx}{q^2 + x^2} = \frac{\pi}{2q}\,\frac{e^q}{e^q - p}$ Schlömilch, Beitr. II. 4.

9) $\displaystyle\int \frac{Sin.rx}{1 - 2p\,Cos.rx + p^2}\,\frac{x}{q^2 + x^2}\,dx = \frac{\pi}{2}\,\frac{1}{e^{qr} - p}$, $p < 1$; Legendre, Exerc. 4. 131. — Boncompagni, Cr. 25. 74.

10) $\displaystyle = \frac{\pi}{2q}\,\frac{1}{p\,e^{qr} - 1}$, $p > 1$;

11) $\displaystyle\int \frac{Cos.rx - p}{1 - 2p\,Cos.rx + p^2}\,\frac{dx}{q^2 + x^2} = \frac{\pi}{2q}\,\frac{1}{e^{rq} - p}$, $p^2 < 1$; Ohm, Ausw. 26.

12) $\displaystyle = \frac{\pi}{2q}\,\frac{1}{e^{-rq} - p}$, $p^2 > 1$;

13) $\displaystyle\int \frac{1 - p\,Cos.rx}{1 - 2p\,Cos.rx + p^2}\,\frac{dx}{q^2 + x^2} = \frac{\pi}{2q}\,\frac{1}{1 - p\,e^{-rq}}$, $p^2 < 1$; Boncompagni, Cr. 25. 74.

14) $\displaystyle = \frac{\pi}{2q}\,\frac{1}{1 - p\,e^{rq}}$, $p^2 > 1$; Ohm, Ausw. 26.

15) $\displaystyle\int \frac{Sin.rx}{1 - 2p\,Cos.rx + p^2}\,\frac{x}{q^2 + x^2}\,dx = \frac{1}{2}\,\frac{\pi}{1 + p}\,\frac{e^{rq}}{e^{2qr} - p}$, $p < 1$; Legendre, Exerc. 4. 132.

16) $\displaystyle = \frac{1}{2}\,\frac{\pi}{1 + p}\,\frac{e^{qr}}{p\,e^{2qr} - 1}$, $p > 1$; Ohm, Ausw. 26.

17) $\displaystyle\int \frac{Cos.2rx - p}{1 - 2p\,Cos.2rx + p^2}\,\frac{dx}{q^2 + x^2} = \frac{\pi}{2q}\,\frac{1}{e^{2qr} - p}$, $p^2 < 1$; Legendre, Exerc. 4. 134.

18) $\displaystyle\int \frac{Sin.ax.\,Sin.bx}{1 - 2p\,Cos.ax + p^2}\,\frac{dx}{q^2 + x^2} = \frac{\pi}{4pq}\,e^{-bq}\left\{\frac{p\,e^{aq}}{1 - p\,e^{aq}} - \frac{p\,e^{-aq}}{1 - e^{-aq}}\right\}$

19) $\displaystyle\int \frac{Sin.ax.\,Cos.bx}{1 - 2p\,Cos.ax + p^2}\,\frac{x}{q^2 + x^2}\,dx = \frac{\pi}{4p}\,e^{-bq}\left\{\frac{p\,e^{-aq}}{1 - p\,e^{-aq}} - \frac{p\,e^{aq}}{1 - e^{aq}}\right\}$

(18, 19): $p^2 < 1$; Boncompagni, Cr. 25. 74.

20) $\displaystyle\int \frac{Sin.2ax}{1 - 2p\,Cos.2ax + p^2}\,\frac{x}{q^2 - x^2}\,dx = -\frac{\pi}{4p} + \frac{1}{4p}\,\frac{(1 - p^2)\pi - 2p\,Sin\,2aq.l(-1)}{1 - 2p\,Cos.2aq + p^2}$ Poisson, P. 18. 295. N°. 43 (où il y a faut. aq au lieu de $2aq$).

F. Alg. rat. fract. à dén. polynôme. Circ. Dir. en dén. trinôme. TABLE 222. Lim. 0 et ∞.

1) $\displaystyle\int \frac{Sin.ax}{1 + 2p\,Cos.ax + p^2}\,\frac{x}{x^4 + 2q^2x^2\,Cos.2\lambda + q^4}\,dx =$

$$= \frac{\pi}{2q^2}\,\frac{e^{-aq\,Cos.\lambda}}{1 + 2p\,e^{-aq\,Cos.\lambda}\,Cos.(aq\,Sin.\lambda) + p^2\,e^{-2aq\,Cos.\lambda}}\,\frac{Sin.(aq\,Sin.\lambda)}{Sin.2\lambda}$$

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2) $\int \frac{Cos.\,a\,x + c}{1 + 2\,p\,Cos.\,a\,x + p^2} \frac{dx}{x^4 + 2\,q^2\,x^2\,Cos.\,2\,\lambda + q^4} =$

$$= \frac{\pi}{4q^3} \frac{e^{-aq\,Cos.\lambda}}{1 + 2\,p\,e^{-aq\,Cos.\lambda}\,Cos.(a\,q\,Sin.\,\lambda) + p^2\,e^{-2aq\,Cos.\lambda}} \left\{ \frac{Cos.(a\,q\,Sin.\,\lambda) + c\,e^{-q}}{Cos.\,\lambda} + \frac{Sin.\,(a\,q\,Sin.\,\lambda)}{Sin.\,\lambda} \right\}$$

3) $\int \frac{Sin.\,a\,x}{1 - 2\,p\,Cos.\,a\,x + p^2} \frac{x}{x^4 + 2\,q^2\,x^2\,Cos.\,2\,\lambda + q^4}\,dx =$

$$= \frac{\pi}{2\,q} \frac{e^{-aq\,Cos.\lambda}}{1 - 2\,p\,e^{-aq\,Cos.\lambda}\,Cos.\,(a\,q\,Sin.\,\lambda) + p^2\,e^{-2aq\,Cos.\lambda}} \frac{Sin.\,(a\,q\,Sin.\,\lambda)}{Sin.\,2\,\lambda}$$

4) $\int \frac{Sin.\,a\,x}{1 - 2\,p\,Cos.\,2\,a\,x + p^2} \frac{x}{x^4 + 2\,q^2\,x^2\,Cos.\,2\,\lambda + q^4}\,dx =$

$$\frac{\pi}{2\,q^2} \frac{e^{-aq\,Cos\,\lambda}\,Sin.\,(a\,q\,Sin.\,\lambda)}{1 - 2\,p\,e^{-2aq\,Cos\,\lambda}\,Cos.\,(2\,a\,q\,Sin.\,\lambda) + p^2\,e^{-4aq\,Cos.\lambda}} \frac{1 + p\,e^{-2aq\,Cos.\lambda}}{(1+p)\,Sin.\,2\,\lambda}$$

Les formules (1) à (4) se trouvent Plana, Mém. Turin. 1818. 7. II. 10.

5) $\int \frac{Sin.\,2\,a\,x}{e^{-2ac} + 2\,Cos.\,2\,a\,x + e^{2ac}} \frac{x}{x^2 + (b+c)^2}\,dx = \frac{\pi}{2} \frac{e^{-2ac}}{e^{2a(b+c)} + e^{-2ac}}$

6) $\int \frac{e^{-2ac} - e^{2ac}}{e^{-2ac} + 2\,Cos.\,2\,a\,x + e^{2ac}} \frac{dx}{x^2 + (b+c)^2} = \frac{1}{2} \frac{\pi}{b+c} - \frac{\pi}{b+c} \frac{e^{-2ac}}{e^{2a(b+c)} + e^{-2ac}}$

7) $\int \frac{Sin.\,2\,a\,x}{e^{-2ac} + 2\,Cos.\,2\,a\,x + e^{2ac}} \frac{x}{x^2 + (b-c)^2}\,dx = \frac{\pi}{e^{2ab} + 1}$, $b > c$;

8) $= \frac{\pi\,e^{-2ac}}{e^{2a(c-b)} - e^{-2ac}}$, $b < c$;

9) $\int \frac{e^{2ac} - e^{-2ac}}{e^{-2ac} + 2\,Cos.\,2\,a\,x + e^{2ac}} \frac{dx}{x^2 + (b-c)^2} = \frac{1}{2} \frac{\pi}{b-c} - \frac{\pi}{b-c} \frac{1}{e^{2ab} + 1}$, $b > c$;

10) $= \frac{1}{2} \frac{\pi}{c-b} + \frac{\pi}{b-c} \frac{e^{-2ac}}{e^{2a(c-b)} - e^{-2ac}}$, $b < c$;

11) $\int \frac{(b^2 + c^2 + x^2)\,2\,x\,Sin.\,2\,a\,x - c\,(b^2 - c^2 + x^2)\,(e^{2ac} - e^{-2ac})}{e^{-2ac} + 2\,Cos.\,2\,a\,x + e^{2ac}}\,.$

$$\frac{dx}{\{x^2 + (b-c)^2\}\{x^2 + (b+c)^2\}} = \pi \qquad , c > b;$$

12) $= \frac{2\pi}{e^{2ab} + 1}$, $c < b$;

(5)–(12): Poisson, P. 18. 295. N°. 42.

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13) $\int \frac{Sin.\{(a+b)x\}.\,Sin.\,2ax}{1+Cos.\,2ax}\,\frac{dx}{q^2+x^2} = \frac{\pi}{2q}\,e^{-(a+b)q}\left\{\frac{e^{2ar}}{1+e^{2ar}} - \frac{e^{-2ar}}{1+e^{-2ar}}\right\}$

14) $\int \frac{Sin.\{(a+b)x\}.\,Sin.\,2ax}{1-Cos.\,2ax}\,\frac{dx}{q^2+x^2} = \frac{\pi}{2q}\,e^{-(a+b)q}\left\{\frac{e^{2ar}}{1-e^{2ar}} - \frac{e^{-2ar}}{1-e^{-2ar}}\right\}$

(13, 14) Boncompagni, Cr. 25. 74.

15) $\int \frac{Sin.\,2ax}{1+p\,Cos.\,2ax+p^2}\,\frac{dx}{x(1+x^2)} = \frac{1}{2}\,\frac{\pi}{1+p}\,\frac{e^a-e^{-a}}{e^a+p\,e^{-a}},\ p<1;$ Cauchy, Sav. Etr. 1827. 599. S. 2.

F. Alg. irrat. ent. Circ. Dir. — TABLE 223. — Lim. 0 et ∞ .

1) $\int Sin.\,qx.\,dx\sqrt{x} = \frac{1}{4q}\sqrt{\frac{2\pi}{q}}$

2) $\int Sin.\,qx.\,x\,dx\sqrt{x} = -\frac{3}{8q^2}\sqrt{\frac{2\pi}{q}}$

3) $\int Sin.\,qx.\,x^2\,dx\sqrt{x} = -\frac{15}{16q^3}\sqrt{\frac{2\pi}{q}}$

4) $\int Cos.\,qx.\,dx\sqrt{x} = \frac{1}{4q}\sqrt{\frac{2\pi}{q}}$

5) $\int Cos.\,qx.\,x\,dx\sqrt{x} = \frac{3}{8q^2}\sqrt{\frac{2\pi}{q}}$

6) $\int Cos.\,qx.\,x^2\,dx\sqrt{x} = \frac{15}{16q^3}\sqrt{\frac{2\pi}{q}}$

(1–6) Oettinger, Cr. 38. 216.

7) $\int Sin.\,x.\,x^{2a-1}\,dx\sqrt{x} = (-1)^a\,\Gamma\left(\frac{4a+1}{2}\right)\frac{1}{\sqrt{2}}$

8) $\int Sin.\,x.\,x^{2a}\,dx\sqrt{x} = (-1)^a\,\Gamma\left(\frac{4a+3}{2}\right)\frac{1}{\sqrt{2}}$

9) $\int Cos.\,x.\,x^{2a-1}\,dx\sqrt{x} = (-1)^a\,\Gamma\left(\frac{4a+1}{2}\right)\frac{1}{\sqrt{2}}$

10) $\int Cos.\,x.\,x^{2a}\,dx\sqrt{x} = (-1)^{a-1}\,\Gamma\left(\frac{4a+3}{2}\right)\frac{1}{\sqrt{2}}$

(7–10) Cauchy, Sav. Etr. 1827. 124. Note 3.

11) $\int Sin.\,ax.\,x^{2b-1}\,dx\sqrt[3]{x} = (-1)^b\,\frac{4^{2b-1/3}\,1^{b/1}}{3^{2b-1}\,2\,a^{2b}\sqrt[3]{a}}$

12) $\int Sin.\,ax.\,x^{2b}\,dx\sqrt[3]{x} = (-1)^b\,\frac{4^{2b}\,3\,1^{b/1}\sqrt{3}}{3^{2b}\,2\,a^{2b+1}\sqrt[3]{a}}.$

(11, 12) Oettinger, Cr. 38. 216.

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F. Alg. irrat. ent. Circ. Dir.	TABLE 225 suite.	Lim. 0 et ∞.

13) $\int Cos.\, a x . x^{2b-1}\, dx \sqrt[3]{x} = (-1)^b \frac{4^{2b-1/3}\, 1^{b/1} \sqrt{3}}{3^{2b-1}\, 2\, a^{2b} \sqrt[3]{a}}$

14) $\int Cos.\, a x . x^{2b}\, dx \sqrt[3]{x} = (-1)^{b+1} \frac{4^{2b/3}\, 1^{b/1}}{3^{2b}\, 2\, a^{2b+1} \sqrt[3]{a}}$

Oettinger, Cr. 38. 216.

F. Alg. irrat. fract. à dén. $\sqrt{x}$. Circ. Dir. en num. mon. à un fact. circ. de x.	TABLE 224.	Lim. 0 et ∞.

1) $\int Sin.\, x \frac{dx}{\sqrt{x}} = \sqrt{\frac{\pi}{2}}$

2) $\int Cos.\, x \frac{dx}{\sqrt{x}} = \sqrt{\frac{\pi}{2}}$

Euler, Calc. Int. 4. S. 5. § 127. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 2, 24. — Fourier, Chal. 360. — Laplace, P. 15. 229. — Cauchy, Sav. Etr. 1827. 124. Note 16. — Id., Sav. Etr. 1827. 599. P. 1. § 6. — Boncompagni, Cr. 25. 74. — Schlömilch, Beitr. III. 4. — Id., Stud. I. 13.

3) toutes deux $= \sqrt{2\pi}$ (fautives par faute de calcul) Mascheroni, Adn. p. 57, 58.

$= \frac{1}{2} \sqrt{\pi}$ (fautives) Fusz, Mém. Pétersb. 1830.

4) $\int Cos.\, p x \frac{dx}{\sqrt{x}} = \sqrt{\frac{\pi}{2p}}$

5) $\int Sin.\, p x \frac{dx}{\sqrt{x}} = \sqrt{\frac{\pi}{2p}}$

Legendre, Exerc. 3. 55. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 53. — Plana, Mém. Brux. 1837. — Oettinger, Cr. 38. 216.

6) $= 0 \quad , p = 0;$

7) $= - \sqrt{\frac{\pi}{2p}} \quad , p < 0;$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.

8) $\int Tang.\, p x \frac{dx}{\sqrt{x}} = \sqrt{\frac{\pi}{p}} \sum_1^\infty \frac{(-1)^{n+1}}{\sqrt{n}}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 38.

9) $\int Sin.^2\, p x \frac{dx}{\sqrt{x}} = \sqrt{\infty} - \frac{1}{4} \sqrt{\frac{\pi}{p}} = \infty$

10) $\int Sin.^3\, p x \frac{dx}{\sqrt{x}} = \frac{3\sqrt{3} - 1}{4\sqrt{3}} \sqrt{\frac{\pi}{2p}}$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 10, 16.

11) $\int Cos.^2\, p x \frac{dx}{\sqrt{x}} = \infty$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 17.

12) $\int Cos.^3\, p x \frac{dx}{\sqrt{x}} = \frac{3\sqrt{3} + 1}{4\sqrt{3}} \sqrt{\frac{\pi}{2p}}$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 17.

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13) $\int Cos.^5 px \frac{dx}{\sqrt{x}} = \frac{1}{16}\left(10 + \frac{5}{\sqrt{3}} + \frac{1}{\sqrt{5}}\right)\sqrt{\frac{\pi}{2p}}$

14) $\int Sin.^{2b+1} px \frac{dx}{\sqrt{x}} = \frac{1}{2^{2b}}\sqrt{\frac{\pi}{2p}}\sum_0^b (-1)^n \binom{2b+1}{b+n+1}\frac{1}{\sqrt{(2n+1)}}$

15) $\int Cos.^{2b+1} px \frac{dx}{\sqrt{x}} = \frac{1}{2^{2b}}\sqrt{\frac{\pi}{2p}}\sum_0^b \binom{2b+1}{b+n+1}\frac{1}{\sqrt{(2n+1)}}$

(13–15:) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 17, 16.

16) $\int x^{2a} Sin.\, qx \frac{dx}{\sqrt{x}} = (-1)^a \sqrt{\frac{2\pi}{q}}\frac{1^{2a/2}}{2^{2a+1} q^{2a}}$

17) $= (-1)^a \binom{-\frac{1}{2}}{2a}\frac{1^{2a/1}}{q^{2a}}\sqrt{\frac{\pi}{2q}}$

18) $\int x^{2a+1} Sin.\, qx \frac{dx}{\sqrt{x}} = (-1)^a \sqrt{\frac{2\pi}{q}}\frac{1^{2a+1/2}}{2^{2a+2} q^{2a+1}}$

19) $= (-1)^{a+1} \binom{-\frac{1}{2}}{2a+1}\frac{1^{2a+1/1}}{q^{2a+1}}\sqrt{\frac{\pi}{2q}}$

20) $\int x^{2a} Cos.\, qx \frac{dx}{\sqrt{x}} = (-1)^a \sqrt{\frac{2\pi}{q}}\frac{1^{2a/2}}{2^{2a+1} q^{2a}}$

21) $= (-1)^a \binom{-\frac{1}{2}}{2a}\frac{1^{2a/1}}{q^{2a}}\sqrt{\frac{\pi}{2q}}$

22) $\int x^{2a+1} Cos.\, qx \frac{dx}{\sqrt{x}} = (-1)^{a+1} \sqrt{\frac{2\pi}{q}}\frac{1^{2a+1/2}}{2^{2a+2} q^{2a+1}}$

23) $= (-1)^a \binom{-\frac{1}{2}}{2a+1}\frac{1^{2a+1/1}}{q^{2a+1}}\sqrt{\frac{\pi}{2q}}$

Sur les intégrales 16, 18, 20, 22 voyez: Oettinger, Cr. 38. 216.
Sur les intégrales 17, 19, 21, 23 voyez: Raabe, Int. 167.

F. Alg. irrat. fract. à dén. $x^a\sqrt{x}$.
Circ. Dir. en num. mon. à un fact. circ. de x. TABLE 225. Lim. 0 et ∞.

1) $\int \frac{Sin.\, x}{x\sqrt{x}} dx = \sqrt{2\pi}$ Laplace, P. 15. 229. — Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 4. — Plana, Mém. Brux. 1837.

2) $\int \frac{Sin.\, px}{x\sqrt{x}} dx = \sqrt{2p\pi}$ Cisa de Grésy, Mém. Turin. 1821. 209. II. 53.

3) $= -\sqrt{2p\pi}$ Oettinger, Cr. 38. 216. (faut.).

4) $\int \frac{Sin.\, p x}{x^2 \sqrt{x}} dx = \infty$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 7.

5) $= \frac{2p}{3} \sqrt{2 p \pi}$

6) $\int \frac{Sin.\, p x}{x^3 \sqrt{x}} dx = -\frac{4 p^2}{15} \sqrt{2 p \pi}$

7) $\int \frac{Sin.\, p x}{x^{2a} \sqrt{x}} dx = (-1)^a \frac{(2p)^{2a-1}}{1^{2a/2}} \sqrt{2 p \pi}$

8) $\int \frac{Sin.\, p x}{x^{2a+1} \sqrt{x}} dx = (-1)^a \frac{(2p)^{2a}}{1^{2a+1/2}} \sqrt{2 p \pi}$

9) $\int \frac{Sin.\, p x}{x^{2a} \sqrt[3]{x}} dx = (-1)^a \frac{1^{-\frac{1}{3}/1}\, 3^{2a}\, p^{2a-1} \sqrt[3]{p}}{2\,.\,1^{2a/3}} \sqrt{3}$

10) $\int \frac{Sin.\, p x}{x^{2a+1} \sqrt[3]{x}} dx = (-1)^a \frac{1^{-\frac{1}{3}/1}\, 3^{2a+1}\, p^{2a} \sqrt[3]{p}}{2\,.\,1^{2a+1/3}}$

(5–10) Oettinger, Cr. 38. 216.

11) $\int \frac{Sin.^2\, p x}{x \sqrt{x}} dx = \sqrt{p \pi}$

12) $\int \frac{Sin.^2\, p x}{x^2 \sqrt{x}} dx = \frac{4}{3} p \sqrt{p \pi}$

13) $\int \frac{Sin.^3\, p x}{x \sqrt{x}} dx = \frac{3 - \sqrt{3}}{4} \sqrt{2 p \pi}$

14) $\int \frac{Sin.^3\, p x}{x^2 \sqrt{x}} dx = \frac{\sqrt{3} - 1}{2} p \sqrt{2 p \pi}$

15) $\int \frac{Sin.^4\, p x}{x \sqrt{x}} dx = \frac{4 - \sqrt{2}}{4} \sqrt{p \pi}$

16) $\int \frac{Sin.^4\, p x}{x^2 \sqrt{x}} dx = \frac{4 - 2\sqrt{2}}{2} p \sqrt{p \pi}$

17) $\int \frac{Sin.^6\, p x}{x^5 \sqrt{x}} dx = \frac{5 - 32\sqrt{2} + 27\sqrt{3}}{315} 16 p^4 \sqrt{p \pi}$

18) $\int \frac{Sin.^{2b}\, p x}{x^{a+\frac{1}{2}}} dx = \frac{p^{a-\frac{1}{2}} \sqrt{\pi}}{1^{a/2}\, 2^{2b-2a}} \sum_1^b (-1)^n \binom{2b}{b+n} n^{a-\frac{1}{2}}$, a de la forme $4h$ et $4h+1$;

19) $= \frac{p^{a-\frac{1}{2}} \sqrt{\pi}}{1^{a/2}\, 2^{2b-2a}} \sum_1^b (-1)^{n-1} \binom{2b}{b+n} n^{a-\frac{1}{2}}$, a de la forme $4h+2$ et $4h+3$;

(11–19) Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 13, 16.

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20) $\int \frac{Sin.^{2b+1} px}{x^{a+\frac{1}{2}}} dx = \frac{p^{a-\frac{1}{2}} \sqrt{\pi}}{1^{a/2} 2^{2b-a+\frac{1}{2}}} \sum_1^{b+1} (-1)^{n-1} \binom{2b+1}{b+n} (2n-1)^{a-\frac{1}{2}}$, a de la forme $4h$ et $4h+1$; Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 13, 16.

21) $= \frac{p^{a-\frac{1}{2}} \sqrt{\pi}}{1^{a/2} 2^{2b-a+\frac{1}{2}}} \sum_1^{b+1} (-1)^n \binom{2b+1}{b+n} (2n-1)^{a-\frac{1}{2}}$, a de la forme $4h+2$ et $4h+3$;

22) $\int \frac{Cos. x}{x \sqrt{x}} dx = -\sqrt{2\pi}$ Plana, Mém. Brux. 1837.

23) $= \infty$ (faut.) Cisa de Grésy, Mém. Turin. 1821. 209. II. 53.

24) $\int \frac{Cos. px}{x \sqrt{x}} dx = -\sqrt{2p\pi}$ Oettinger, Cr. 38. 216.

25) $\int \frac{Cos. px}{x^2 \sqrt{x}} dx = \infty$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 8. — Cisa de Grésy, Mém. Turin. 1821. 209. II. 53.

26) $= \frac{2p}{3} \sqrt{2p\pi}$

27) $\int \frac{Cos. px}{x^3 \sqrt{x}} dx = \frac{4p^2}{15} \sqrt{2p\pi}$

28) $\int \frac{Cos. px}{x^4 \sqrt{x}} dx = \frac{8p^3}{105} \sqrt{2p\pi}$

29) $\int \frac{Cos. px}{x^{2a} \sqrt{x}} dx = (-1)^{a+1} \frac{(2p)^{2a-1}}{1^{2a/2}} \sqrt{2p\pi}$

30) $\int \frac{Cos. px}{x^{2a+1} \sqrt{x}} dx = (-1)^{a+1} \frac{(2p)^{2a}}{1^{2a+1/2}} \sqrt{2p\pi}$

31) $\int \frac{Cos. px}{x^{2a} \sqrt[3]{x}} dx = (-1)^a \frac{1^{-\frac{1}{3}/1} 3^{2a} p^{2a-1} \sqrt[3]{p}}{2 . 1^{2a/3}}$

32) $\int \frac{Cos. px}{x^{2a+1} \sqrt[3]{x}} dx = (-1)^a \frac{1^{-\frac{1}{3}/1} 3^{2a-1} p^{2a-2} \sqrt[3]{p}}{2 . 1^{2a-1/3}}$

(26–32: Oettinger, Cr. 38. 216.)

33) $\int \frac{Tang. px}{x \sqrt{x}} dx = 4 \sqrt{p\pi} \sum_1^{\infty} (-1)^{n+1} \sqrt{n}$ Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 38.

F. Alg. irrat. fract. à dén. monôme. Circ. Dir. en num. mon. à deux fact. circ. de x.	TABLE 226.	Lim. 0 et ∞.

1) $\int \frac{Sin.\, a x.\ Cos.\, b x}{\sqrt{x}}\, d x = \left\{\frac{1}{2\sqrt{(a+b)}} + \frac{1}{2\sqrt{(a-b)}}\right\}\sqrt{\frac{\pi}{2}}, \ a > b;$

2) $= \frac{1}{4}\sqrt{\frac{\pi}{a}} \qquad , a = b;$

3) $= \left\{\frac{1}{2\sqrt{(a+b)}} - \frac{1}{2\sqrt{(b-a)}}\right\}\sqrt{\frac{\pi}{2}}, \ a < b;$

4) $\int \frac{Sin.^2\, a x.\ Cos.^3\, b x}{\sqrt{x}}\, d x = \frac{1}{8}\sqrt{\frac{\pi}{2}}\left\{-\frac{1}{2\sqrt{(2a+3b)}} + \frac{1}{\sqrt{3b}} - \frac{1}{2\sqrt{(2a-3b)}} - \right.$
$\left. - \frac{3}{2\sqrt{(2a+b)}} + \frac{3}{\sqrt{b}} - \frac{3}{2\sqrt{(2a-b)}}\right\}, \ 2a > 3b;$

5) $= \frac{1}{8}\sqrt{\frac{\pi}{2}}\left\{-\frac{1}{2\sqrt{(2a+3b)}} + \frac{1}{\sqrt{3b}} + \frac{1}{2\sqrt{(3b-2a)}} - \right.$
$\left. - \frac{3}{2\sqrt{(2a+b)}} + \frac{3}{\sqrt{b}} - \frac{3}{2\sqrt{(2a-b)}}\right\}, \ 3b > 2a > b;$

6) $= \frac{1}{8}\sqrt{\frac{\pi}{2}}\left\{-\frac{1}{2\sqrt{(2a+3b)}} + \frac{1}{\sqrt{3b}} + \frac{1}{2\sqrt{(3b-2a)}} - \right.$
$\left. - \frac{3}{2\sqrt{(2a+b)}} + \frac{3}{\sqrt{b}} + \frac{3}{2\sqrt{(b-2a)}}\right\}, \ b > 2a;$

7) $\int \frac{Sin.\, a x.\ Cos.\, b x}{x\sqrt{x}}\, d x = \sqrt{\left(\pi\frac{a+b}{2}\right)} + \sqrt{\left(\pi\frac{a-b}{2}\right)}, \ a > b;$

8) $= \sqrt{a\pi} \qquad , a = b;$

9) $= \sqrt{\left(\pi\frac{a+b}{2}\right)} - \sqrt{\left(\pi\frac{b-a}{2}\right)}, \ a < b;$

Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 19.

F. Alg. irrat. fract. à dén. monôme. Circ. Dir. en numér. binôme.	TABLE 227.	Lim. 0 et ∞.

1) $\int \frac{Sin.^2\, a x - Sin.^2\, b x}{\sqrt{x}}\, d x = \frac{1}{4}\left(\sqrt{\frac{\pi}{b}} - \sqrt{\frac{\pi}{a}}\right)$

2) $\int \frac{Sin.^4\, a x - Sin.^4\, b x}{\sqrt{x}}\, d x = \frac{1}{4}\left(\sqrt{\frac{\pi}{b}} - \sqrt{\frac{\pi}{a}}\right) - \frac{1}{16}\left(\sqrt{\frac{\pi}{2b}} - \sqrt{\frac{\pi}{2a}}\right)$

3) $\int \frac{Cos.^2\, a x - Sin.^2\, b x}{\sqrt{x}}\, d x = \frac{1}{4}\left(\sqrt{\frac{\pi}{a}} + \sqrt{\frac{\pi}{b}}\right)$

Bidone, Mém. Turin. 1812. 231. Tableau.

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4) $\int \frac{Cos.^4 ax - Sin.^4 bx}{\sqrt{x}} dx = \frac{1}{4}\left(\sqrt{\frac{\pi}{a}} + \sqrt{\frac{\pi}{b}}\right) + \frac{1}{16}\left(\sqrt{\frac{\pi}{2a}} + \sqrt{\frac{\pi}{2b}}\right)$

5) $\int \frac{Cos.^2 ax - Cos.^2 bx}{\sqrt{x}} dx = \frac{1}{4}\left(\sqrt{\frac{\pi}{a}} - \sqrt{\frac{\pi}{b}}\right)$ Bidone, Mém. Turin. 1812. 231. Tableau.

6) $\int \frac{Cos.^4 ax - Cos.^4 bx}{\sqrt{x}} dx = \frac{1}{4}\left(\sqrt{\frac{\pi}{a}} - \sqrt{\frac{\pi}{b}}\right) + \frac{1}{16}\left(\sqrt{\frac{\pi}{2a}} - \sqrt{\frac{\pi}{2b}}\right)$

7) $\int \frac{Sin.(a-x) + Cos.(a-x)}{\sqrt{x}} dx = Sin.\, a\sqrt{2\pi}$ Cauchy, Sav. Etr. 1827. 124. Note 16.

8) $\int \frac{Sin. x - x\, Cos. x}{x^2\sqrt{x}} dx = \frac{1}{3}\sqrt{2\pi}$ Bidone, Mém. Turin. 1812. 231. Art. 1. N°. 9.

9) $\int \frac{Cos.(bx\sqrt{a}) + Sin.(bx\sqrt{a})}{\sqrt{x}}\left(\frac{Sin. x}{x}\right)^a dx = \frac{\sqrt{2\pi}}{1^{a,2}}\sum_0^\infty (-1)^n \binom{a}{n}(a + b\sqrt{a} - 2n)^{a-\frac{1}{2}}$, a de la forme $4h$ et $4h+3$;

10) $\int \frac{Cos.(bx\sqrt{a}) - Sin.(bx\sqrt{a})}{\sqrt{x}}\left(\frac{Sin. x}{x}\right)^a dx = \frac{\sqrt{2\pi}}{1^{a/2}}\sum_0^\infty (-1)^n \binom{a}{n}(a - b\sqrt{a} - 2n)^{a-\frac{1}{2}}$, a de la forme $4h+1$ et $4h+2$;

, où $0 \leqq 2a < 4b + 1$; voyez sur ces deux intégrales: Laplace, Mém. Inst. 1809. 353. § 10.

F. Alg. irrat. fract. à dén. monôme. Circ. Dir. en num. circ. de $x - \frac{1}{x}$. TABLE 228. Lim. 0 et ∞.

1) $\int Sin.\left\{a\left(x - \frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}} = e^{-2a}\sqrt{\frac{\pi}{2a}}$ Cauchy, P. 28. 147. P. 1. § 3.

2) $\int \left(x - \frac{1}{x}\right) Sin.\left\{a\left(x - \frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}} = \frac{1+4a}{2a} e^{-2a}\sqrt{\frac{\pi}{2a}}$ V. T. 228 N°. 7.

3) $\int \left(x - \frac{1}{x}\right)^2 Sin.\left\{a\left(x - \frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}} = -\frac{3 + 8a + 16a^2}{4a^2} e^{-2a}\sqrt{\frac{\pi}{2a}}$ V. T. 228. N°. 1.

4) $\int \left(x - \frac{1}{x}\right)^3 Sin.\left\{a\left(x - \frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}} = -\frac{15 + 36a + 48a^2 + 64a^3}{8a^3} e^{-2a}\sqrt{\frac{\pi}{2a}}$ V. T. 228. N°. 7.

5) $\int \left(x - \frac{1}{x}\right)^{2b} Sin.\left\{a\left(x - \frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}} = (-1)^b\sqrt{\frac{\pi}{2}}\frac{d^{2b}}{da^{2b}}.\frac{e^{-2a}}{\sqrt{a}}$ V. T. 228. N°. 1.

6) $\int \left(x - \frac{1}{x}\right)^{2b-1} Sin.\left\{a\left(x - \frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}} = (-1)^{b-1}\sqrt{\frac{\pi}{2}}\frac{d^{2b-1}}{da^{2b-1}}.\frac{e^{-2a}}{\sqrt{a}}$ V. T. 228. N°. 7.

F. Alg. irrat. fract. à dén. monôme. Circ. Dir. en num. circ. de $x-\frac{1}{x}$.	TABLE 228 suite.	Lim. 0 et ∞ .

7) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}}=e^{-2a}\sqrt{\frac{\pi}{2a}}$ Cauchy, P. 28. 147. P. 1. § 3.

8) $\int\left(x-\frac{1}{x}\right)Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}}=-\frac{1+4a}{2a}e^{-2a}\sqrt{\frac{\pi}{2a}}$ V. T. 228. N°. 1.

9) $\int\left(x-\frac{1}{x}\right)^2Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}}=-\frac{3+8a+16a^2}{4a^2}e^{-2a}\sqrt{\frac{\pi}{2a}}$ V. T. 228. N°. 7.

10) $\int\left(x-\frac{1}{x}\right)^3Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}}=\frac{15+36a+48a^2+64a^3}{8a^3}e^{-2a}\sqrt{\frac{\pi}{2a}}$ V. T. 228. N°. 1.

11) $\int\left(x-\frac{1}{x}\right)^{2b}Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}}=(-1)^b\sqrt{\frac{\pi}{2}}\frac{d^{2b}}{da^{2b}}\cdot\frac{e^{-2a}}{\sqrt{a}}$ V. T. 228. N°. 7.

12) $\int\left(x-\frac{1}{x}\right)^{2b-1}Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{dx}{\sqrt{x}}=(-1)^b\sqrt{\frac{\pi}{2}}\frac{d^{2b-1}}{da^{2b-1}}\cdot\frac{e^{-2a}}{\sqrt{a}}$ V. T. 228. N°. 1.

F. Alg. irrat. fract. à autre dén. irrat. Circ. Dir. en num.	TABLE 229.	Lim. 0 et ∞ .

1) $\int\frac{Sin.\,x}{\sqrt[4]{x^3}}dx=-p\,Sin.\frac{1}{8}\pi$ ⎫ , où $p=3{,}625608$;

2) $\int\frac{Cos.\,x}{\sqrt[4]{x^3}}dx=p\,Cos.\frac{1}{8}\pi$ ⎭ Laplace, P. 15. 229.

3) $\int\frac{Sin.\,px}{\sqrt[q]{x^{q-1}}}dx=\frac{\Gamma\left(\frac{1}{q}\right)}{\sqrt[q]{p}}Sin.\frac{\pi}{2q}$ ⎫

4) $\int\frac{Cos.\,px}{\sqrt[q]{x^{q-1}}}dx=\frac{\Gamma\left(\frac{1}{q}\right)}{\sqrt[q]{p}}Cos.\frac{\pi}{2q}$ ⎭ , $q\geqq 1$; Raabe, Int. 416.

5) $\int\frac{Sin.\,px}{a+bx}\frac{dx}{\sqrt{x}}=\frac{-\pi}{\sqrt{ab}}Sin.\frac{ap}{b}+\frac{1}{a}\sqrt{\frac{\pi}{2b}}\sum_1^\infty\frac{1}{1^{n\,2}}\left(\frac{2ap}{b}\right)^n$ ⎫

6) $\int\frac{Cos.\,px}{a+bx}\frac{dx}{\sqrt{x}}=\frac{\pi}{\sqrt{ab}}Cos.\frac{ap}{b}+\frac{1}{b}\sqrt{\frac{\pi}{2b}}\sum_1^\infty\frac{(-1)^n}{1^{n\,2}}\left(\frac{2ap}{b}\right)^n$ ⎭ Bidone, Mém. Turin. 1812. 231. Art. 2. 25.

7) $\int\frac{Cos.\,bx-Sin.\,bx}{a^2+x^2}\frac{dx}{\sqrt{x}}=\frac{\pi}{4a}e^{-ab}\sqrt{\frac{2}{a}}$ Schlömilch, Gr. 11. 174.

8) $\int \frac{Sin.\,bx - Cos.\,bx}{1+x^2}\, x\,dx\sqrt{x} = \frac{1}{2}\pi e^{-b}$

9) $\int \frac{Cos.\,bx - Sin.\,bx}{(q^2+x^2)^2}\, dx\sqrt{x} = \frac{\pi}{4q} e^{-bq}\left(1+\frac{1}{q^2}\right)$

10) $\int \frac{Sin.\,bx - Cos.\,bx}{(q^2+x^2)^2}\, x\,dx\sqrt{x} = \frac{\pi}{4\sqrt{q}} e^{-bq}\left(b-\frac{1}{2q}\right)$

(8–10) Helmling, Transf. II. S. 116, 117.

F. Alg. irrat. fract. à dén. binôme. Circ. Dir. en num. circ. de $x-\frac{1}{x}$. — TABLE 230. — Lim. 0 et ∞.

1) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3+x}{\left(x+\frac{1}{x}\right)^2}\,dx\sqrt{x} = e^{-2a}\sqrt{2\pi}$ Cauchy, P. 28. 147. P. 1. § 3.

2) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3-x}{\left(x+\frac{1}{x}\right)^2}\left(x-\frac{1}{x}\right)dx\sqrt{x} = -\frac{1-4a}{2}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 230. N°. 7.

3) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3+x}{\left(x+\frac{1}{x}\right)^2}\left(x-\frac{1}{x}\right)^2 dx\sqrt{x} = \frac{1+8a-16a^2}{4a}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 230. N°. 1.

4) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3-x}{\left(x+\frac{1}{x}\right)^2}\left(x-\frac{1}{x}\right)^3 dx\sqrt{x} = \frac{3+12a+48a^2-64a}{8a^2}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 230. N°. 7.

5) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3+x}{\left(x+\frac{1}{x}\right)^2}\left(x-\frac{1}{x}\right)^{2b} dx\sqrt{x} = (-1)^b\sqrt{2\pi}\frac{d^{2b}}{da^{2b}}.\,e^{-2a}\sqrt{a}$ V. T. 230. N°. 1.

6) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3-x}{\left(x+\frac{1}{x}\right)^2}\left(x-\frac{1}{x}\right)^{2b+1} dx\sqrt{x} = (-1)^{b-1}\sqrt{2\pi}\frac{d^{2b+1}}{da^{2b+1}}.\,e^{-2a}\sqrt{a}$ V. T. 230. N°. 7.

7) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3-x}{\left(x+\frac{1}{x}\right)^2}\,dx\sqrt{x} = e^{-2a}\sqrt{2\pi}$ Cauchy, P. 28. 147. P. 1. § 3.

8) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3+x}{\left(x+\frac{1}{x}\right)^2}\left(x-\frac{1}{x}\right)dx\sqrt{x} = \frac{1-4a}{2}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 230. N°. 1.

9) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3-x}{\left(x+\frac{1}{x}\right)^2}\left(x-\frac{1}{x}\right)^2 dx\sqrt{x} = \frac{1+8a-16a^2}{4a}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 230. N°. 7.

10) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3+x}{\left(x+\frac{1}{x}\right)^2}\left(x-\frac{1}{x}\right)^3 dx\sqrt{x} = -\frac{3+12a+48a^2-64a^3}{8a^2}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 230. N°. 1.

11) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3-x}{\left(x+\frac{1}{x}\right)^2}\left(x-\frac{1}{x}\right)^{2b} dx\sqrt{x} = (-1)^{b-1}\sqrt{2\pi}\frac{d^{2b}}{da^{2b}}.e^{-2a}\sqrt{a}$ V. T. 230. N°. 7.

12) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{3+x}{\left(x+\frac{1}{x}\right)^2}\left(x-\frac{1}{x}\right)^{2b+1} dx\sqrt{x} = (-1)^{b-1}\sqrt{2\pi}\frac{d^{2b+1}}{da^{2b+1}}.e^{-2a}\sqrt{a}$ V. T. 230. N°. 1.

F. Alg. irrat. fract. Circ. Dir. en dén. **TABLE 231.** Lim. 0 et ∞ .

1) $\int \frac{Cos.2px}{Sin.px+Cos.px}\frac{x\,dx\sqrt{x}}{1+x^2} = \frac{1}{2}\pi e^{-p}$

2) $\int \frac{Cos.2px}{Sin.px+Cos.px}\frac{dx\sqrt{x}}{q^2+x^2} = \frac{\pi}{2q}e^{-pq}$

3) $\int \frac{Cos.2px}{Sin.px+Cos.px}\frac{dx\sqrt{x}}{(q^2+x^2)^2} = \frac{1+q^2}{q^2}\frac{\pi}{4q}e^{-pq}$

4) $\int \frac{Cos.2px}{Sin.px+Cos.px}\frac{x\,dx\sqrt{x}}{(q^2+x^2)^2} = \left(p-\frac{1}{2q}\right)\frac{\pi}{4\sqrt{q}}e^{-pq}$

(1–4) Helmling, Transf. 87, 88, 92, 93.

5) $\int \frac{Sin.x}{\sqrt{(1-p^2 Sin.^2 x)}}\frac{dx}{x} = F'(p)$

6) $\int \frac{Sin.x}{\sqrt{(1-p^2 Cos.^2 x)}}\frac{dx}{x} = F'(p)$

7) $\int \frac{Sin.2x}{\sqrt{(1-p^2 Cos.^2 x)}}\frac{dx}{x} = \frac{2}{p^2}\{F'(p)-E'(p)\}$

(5–7) , $p<1$; Raabe, Cr. 25. 160.

1) $\int \frac{Sin.\, a x}{x}\, dx = \pi$ Poisson, P. 18. 295. N°. 42.

2) $\int \frac{Sin.\, p x}{x + q}\, dx = \pi\, Cos.\, p q$ Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 32.

3) $\int \frac{Sin.\, p x}{x + q i}\, dx = \pi\, e^{-pq}$

4) $\int \frac{Sin.\, p x}{x + (q i - r)}\, dx = \pi\, e^{-p(r+qi)}$

Ohm, Ausw. 23.

5) $\int \frac{Sin.\, p x}{x - q}\, dx = \pi\, Cos.\, p q$ Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 32.

6) $\int \frac{Sin.\, p x}{x - q i}\, dx = \pi\, e^{-pq}$

7) $\int \frac{Sin.\, p x}{x - (q i + r)}\, dx = \pi\, e^{-p(r-qi)}$

Ohm, Ausw. 23.

8) $\int \frac{Sin.\, x}{(a + x i)^{1-p}}\, dx = - e^{-a}\, \Gamma(p)\, i\, Sin.\, p\pi$

9) $\int \frac{Sin.\, x}{(a - x i)^{1-p}}\, dx = e^{-a}\, \Gamma(p)\, i\, Sin.\, p\,\pi$

Cayley, L. 12. 231.

10) $\int \frac{Sin.\, p x}{1 + x^2}\, dx = 0$ Moigno, Int. 133.

11) $\int \frac{Sin.\, p x}{q^2 + x^2}\, dx = 0$ Lejeune-Dirichlet, Cr. 4. 94.

12) $\int \frac{Sin.\{a(b - x)\}}{1 + x^2}\, dx = \pi\, e^{-a}\, Sin.\, a b$ Poisson, P. 19. 404. N°. 66.

13) $\int \frac{x\, Sin.\, p x}{q^2 + x^2}\, dx = \pi\, e^{-pq}$

14) $\int \frac{p + q x}{r + 2 s x + x^2}\, Sin.\, t x\, dx = \left(\frac{p - q s}{\sqrt{(r - s^2)}}\, Sin.\, r t + q\, Cos.\, r t\right) \pi\, e^{-t\sqrt{(r - s^2)}}$

Ohm, Ausw. 25.

15) $\int \frac{x\, Sin.\, p x}{x^2 + (q i + r)^2}\, dx = \pi\, e^{-p(r+qi)}$

16) $\int \frac{x\, Sin.\, p x}{x^2 + (q i - r)^2}\, dx = \pi\, e^{-p(r-qi)}$

Ohm, Ausw. 23.

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17) $\int \frac{Sin.px}{q^2+x^2}\frac{dx}{x^{2a}} = 0$

18) $\int \frac{Sin.px}{q^2+x^2}\frac{dx}{x^{2a-1}} = (-1)^a \frac{\pi}{q^{2a}} e^{-pq}$

} Meyer, Int. Déf. 274.

19) $\int \frac{Sin.px}{x^2+(qi+r)^2}\frac{dx}{x} = \frac{\pi}{(r+qi)^2}\{1-e^{-p(r+qi)}\}$

20) $\int \frac{Sin.px}{x^2+(qi-r)^2}\frac{dx}{x} = \frac{\pi}{(r-qi)^2}\{1-e^{-p(r-qi)}\}$

} Ohm, Ausw. 23.

1) $\int \frac{Cos.px}{x+q} dx = \pi \, Sin \, pq$ Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 32.

2) $\int \frac{Cos.px}{x+qi} dx = -i\pi e^{-pq}$

3) $\int \frac{Cos.px}{x+(qi-r)} dx = -i\pi e^{-p(r+qi)}$

} Ohm, Ausw. 23.

4) $\int \frac{Cos.px}{1-x} dx = \pi \, Sin.p$ Schlömilch, Stud. II. 16.

5) $\int \frac{Cos.px}{x-q} dx = -\pi \, Sin.pq$ Bidone, Mém. Turin. 1812. 231. Art. 2. N°. 32.

6) $\int \frac{Cos.px}{x-qi} dx = i\pi e^{-pq}$

7) $\int \frac{Cos.px}{x-(r+qi)} dx = i\pi e^{-p(r-qi)}$

} Ohm, Ausw. 23.

8) $\int \frac{Cos.x}{(a+xi)^{1-p}} dx = e^{-a}\Gamma(p) \, Sin.p\pi$

9) $\int \frac{Cos.x}{(a-xi)^{1-p}} dx = e^{-a}\Gamma(p) \, Sin.p\pi$

} Cayley, L. 12. 231.

10) $\int \frac{Cos.px}{1+x^2} dx = \pi e^{-p}$ Cauchy, Cours. Leç. 39. — Moigno, Int. 183.

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11) $\int \frac{Cos\{a(b-x)\}}{1+x^2} dx = \pi e^{-a} Cos. ab$ Poisson, P. 19. 404. N°. 66.

12) $\int \frac{Cos. px}{x^2+q^2} dx = \frac{\pi}{q} e^{-pq}$ Ohm, Ausw. 25.

13) $= -\pi \frac{e^{-pq} - e^{pq}}{2q}$, $q < z$; } , dans le cas, où $x = y + zi$;

14) $= \frac{\pi}{q} e^{-pq}$, $q > z$; } Poisson, P. 18. 295. N°. 40.

15) $\int \frac{x\, Cos. px}{x^2+q^2} dx = 0$

16) $\int \frac{Cos. px}{x^2+(qi+r)^2} dx = \frac{\pi}{r+qi} e^{-p(r+qi)}$ } Ohm, Ausw. 23.

17) $\int \frac{Cos. px}{x^2+(qi-r)^2} dx = \frac{\pi}{r-qi} e^{-p(r-qi)}$

18) $\int \frac{x^{a-1}}{1+x^{2b}} Cos. px dx = \frac{\pi}{b} \sum_1^b e^{-p Sin.\left(\frac{2n-1}{2b}\pi\right)} Sin. \left\{\frac{2n-1}{2b} a\pi + p Cos.\left(\frac{2n-1}{2b}\pi\right)\right\}$ } , $a < 2b+1$;

19) $\int \frac{x^{a-1}}{1-x^{2b}} Cos. px\, dx = \frac{\pi}{b} \sum_0^{b-1} e^{-p Sin. \frac{n\pi}{b}} Sin. \left(\frac{na\pi}{b} + p\, Cos. \frac{n\pi}{b}\right)$ } Schlömilch, Stud. II. 16.

20) $\int \frac{p+qx}{r+2sx+x^2} Cos. tx\, dx = \left(q\, Sin. rt - \frac{p-qs}{\sqrt{(r-s^2)}} Cos. rt\right) \pi e^{-t\sqrt{(r-s^2)}}$ Ohm, Ausw. 25.

21) $\int \frac{p+qx}{a^2-2bx+x^2} Cos. cx\, dx = \pi e^{-c\sqrt{(a^2-b^2)}} \left\{\frac{p+bq}{\sqrt{(a^2-b^2)}} Cos. bc - q\, Sin. bc\right\}$ } Schlömilch, Stud. II. 16.

22) $\int \frac{Cos.\{(b-c)\lambda\} - x Cos. b\lambda}{1-2x\, Cos. \lambda + x^2} Cos. cx\, dx = \pi e^{-c Sin. \lambda} Sin. (b\lambda + c\, Cos. \lambda)$

23) $\int \frac{a+bx}{p^2 - 2px\, Cos. \lambda + x^2} Cos. cx\, dx = \pi e^{-cp Sin. \lambda}$
$\left\{\frac{a - bp\, Cos. \lambda}{p\, Sin. \lambda} Cos. (pc\, Cos. \lambda) + b\, Sin. (pc\, Cos. \lambda)\right\}$ } Laplace, Prob. 26. — Plana, Mém. Turin. 1818. 7. II. 12.

24) $\int \frac{Cos. px}{q^2+x^2} \frac{dx}{x^{2a}} = (-1)^a \frac{\pi}{q^{a+1}} e^{-pq}$ } Meyer, Int. Déf. 274.

25) $\int \frac{Cos. px}{q^2+x^2} \frac{dx}{x^{2a-1}} = 0$

F. Alg. rat. fract. Circ. Dir. en num. d'autre forme.	TABLE 234.	Lim. $-\infty$ et ∞.

1) $\int \frac{Tang.\, p\, x}{x}\, dx = \pi$

2) $\int \frac{x\, Tang.\, p\, x}{x^2 + q^2}\, dx = \pi \quad , q < z;$

3) $\qquad = \frac{2\pi}{e^{2pq} + 1}, q > z;$, où $x = y + zi$;

4) $\int \frac{x\, Tang.\, p\, x}{x^2 - q^2}\, dx = \pi$

(1–4) Poisson, P. 18. 295. N°. 42.

5) $\int \frac{Cos.\, a\, x - Cos.\, a\, \lambda}{x^2 - \lambda^2}\, dx = -\frac{\pi}{\lambda} Sin.\, a\, \lambda$ Poisson, P. 18. 295. N°. 38.

6) $\int \frac{(e^{ac} + e^{-ac})\, Cos.\, a\, x - (e^{ac} - e^{-ac})\, i\, Sin\, a\, x}{b^2 + x^2 - c^2 + 2\, c\, x\, i}\, dx = \pi \frac{e^{-ab} - e^{ab}}{b}, c > b;$

7) $\qquad = \frac{2\pi}{b} e^{-ab} \quad , c < b;$

8) $\int \frac{(b^2 + c^2 + x^2)\, 2\, x\, Sin.\, 2\, a\, x - c\,(b^2 - c^2 - x^2)\,(e^{2ac} - e^{-2ac})}{e^{2ac} + 2\, Cos.\, 2\, a\, x + e^{-2ac}} \frac{dx}{\{x^2 + (b-c)^2\}\, \{x^2 + (b+c)^2\}} = \pi \quad , c > b;$

9) $\qquad = \frac{2\pi}{e^{2ab} + 1}, c < b;$

(6–9) Poisson, P. 18. 295. N°. 41.

F. Alg. fract. Circ. Dir. *Sin. x*.	TABLE 235.	Lim. 1 et ∞.

1) $\int \frac{Sin.\, \{p\,(x-1)\}}{x}\, dx = Ci.\,(p).\, Sin.\, p + Cos.\, p \left\{\frac{1}{2}\pi - Si.\,(p)\right\}$ Arndt, Gr. 10. 225.

2) $\int Sin. \left\{a\left(x - \frac{1}{x}\right)\right\} \left(x - \frac{1}{x}\right) dx \sqrt{x} = e^{-2a} \sqrt{\frac{\pi}{2a}}$ Cauchy, P. 28. 147. P. 1. § 3.

3) $\int Sin. \left\{a\left(x - \frac{1}{x}\right)\right\} \left(x - \frac{1}{x}\right) \left(x + \frac{1}{x}\right) dx \sqrt{x} = \frac{1 + 4a}{2a} e^{-2a} \sqrt{\frac{\pi}{2a}}$ V. T. 236. N°. 3.

4) $\int Sin. \left\{a\left(x - \frac{1}{x}\right)\right\} \left(x - \frac{1}{x}\right)^3 dx \sqrt{x} = -\frac{3 + 8a + 16a^2}{4a^2} e^{-2a} \sqrt{\frac{\pi}{2a}}$ V. T. 235. N°. 2.

5) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\left(x-\frac{1}{x}\right)^3\left(x+\frac{1}{x}\right)dx\sqrt{x}=-\frac{15+36a+48a^2+64a^3}{8a^3}e^{-2a}\sqrt{\frac{\pi}{2a}}$ V. T. 236. N°. 3.

6) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\left(x-\frac{1}{x}\right)^{2b-1}\left(x+\frac{1}{x}\right)dx\sqrt{x}=(-1)^{b-1}\sqrt{\frac{\pi}{2}}\frac{d^{2b-1}}{da^{2b-1}}\cdot\frac{e^{-2a}}{\sqrt{a}}$ V. T. 236. N°. 3.

7) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\left(x-\frac{1}{x}\right)^{2b+1}dx\sqrt{x}=(-1)^b\sqrt{\frac{\pi}{2}}\frac{d^{2b}}{da^{2b}}\cdot\frac{e^{-2a}}{\sqrt{a}}$ V. T. 235. N°. 2.

8) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4+x+\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)\frac{dx}{x}=e^{-2a}\sqrt{2a\pi}$ Cauchy, P. 28. 147. P. 1. § 3.

9) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4-x-\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)\frac{dx}{x}=-\frac{1-4a}{2}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 236. N°. 9.

10) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4+x+\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)^2\frac{dx}{x}=\frac{1+8a-16a^2}{4a}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 235. N°. 8.

11) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4-x-\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)^3\frac{dx}{x}=\frac{3+12a+48a^2-64a^3}{8a^2}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 236. N°. 9.

12) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4+x+\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)^{2b}\frac{dx}{x}=(-1)^b\sqrt{2\pi}\frac{d^{2b}}{da^{2b}}\cdot e^{-2a}\sqrt{a}$ V. T. 235. N°. 8.

13) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4-x-\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)^{2b-1}\frac{dx}{x}=(-1)^b\sqrt{2\pi}\frac{d^{2b-1}}{da^{2b-1}}\cdot e^{-2a}\sqrt{a}$ V. T. 236. N°. 9.

14) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}-\{x+1+(x-1)i\}^{-b}}{2i}\left(x+\frac{1}{x}\right)x^{\frac{b}{2}-1}dx=\frac{\pi a^{\frac{1}{2}b-1}e^{-2a}}{2^{\frac{1}{2}b+1}\Gamma(\frac{1}{2}b)}$ Cauchy, P. 28. 147. P. 1. § 3.

F. Alg. fract. Circ. Dir. *Sin. x.*	TABLE 235 suite.	Lim. 1 et ∞.

15) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}+\{x+1+(x-1)i\}^{-b}}{2}$

$\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)x^{\frac12 b-1}\,dx=-\frac{\pi a^{\frac12 b-2}e^{-2a}}{2^{\frac12 b+1}\Gamma(\frac12 b)}\left(\frac{b-2}{2}-2a\right)$ V. T. 236. N°. 15.

16) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}-\{x+1+(x-1)i\}^{-b}}{2i}$

$\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)^2x^{\frac12 b-1}\,dx=-\frac{\pi a^{\frac12 b-2}e^{-2a}}{2^{\frac12 b+1}\Gamma(\frac12 b)}\left\{\frac14(b-4)(b-2)-(b-2)2a+4a^2\right\}$ V. T. 235. N°. 14.

17) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}+\{x+1+(x-1)i\}^{-b}}{2}\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)^3x^{\frac12 b-1}dx=$

$=\frac{\pi a^{\frac12 b-2}e^{-2a}}{2^{\frac12 b+1}\Gamma(\frac12 b)}\left\{\frac18(b-6)(b-4)(b-2)+\frac32(b-4)(b-2)a+6(b-2)a^2-8a^3\right\}$ V. T. 236. N°. 15.

18) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}+\{x+1+(x-1)i\}^{-b}}{2}$

$\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)^{2c-1}x^{\frac12 b-1}\,dx=\frac{(-1)^c\pi}{2^{\frac12 b+1}\Gamma(\frac12 b)}\frac{d^{2c-1}}{d\,a^{2c-1}}\,.\,a^{\frac12 b-1}e^{-2a}$ V. T. 236. N°. 15.

19) $\int Sin.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}-\{x+1+(x-1)i\}^{-b}}{2i}$

$\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)^{2c}x^{\frac12 b-1}\,dx=\frac{(-1)^c\pi}{2^{\frac12 b+1}\Gamma(\frac12 b)}\frac{d^{2c}}{d\,a^{2c}}\,.\,a^{\frac12 b-1}e^{-2a}$ V. T. 235. N°. 14.

20) $\int Sin.\,ax\,dx\sqrt{\frac{x}{x^2-1}}=(Cos.\,a+Sin.\,a)\sqrt{\frac{\pi}{4a}}$, pour a très-petit; V. T. 77. N°. 1.

F. Alg. fract. Circ. Dir. *Cos. x.*	TABLE 236.	Lim. 1 et ∞.

1) $\int Cos.\,px\frac{dx}{x}=-Ci.(p)$

2) $\int\frac{Cos.\{p(x-1)\}}{x}dx=-Ci.(p).Cos.p+Sin.p\left\{\frac12\pi-Si.(p)\right\}$

(1) and 2): Arndt, Gr. 10. 225.)

3) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\left(x+\frac{1}{x}\right)dx\sqrt{x}=e^{-2a}\sqrt{\frac{\pi}{2a}}$ Cauchy, P. 28. 147. P. 1. § 3.

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4) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\left(x-\frac{1}{x}\right)^2 dx\sqrt{x} = -\frac{1-4a}{2a}e^{-2a}\sqrt{\frac{\pi}{2a}}$ V. T. 235. N°. 2.

5) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\left(x-\frac{1}{x}\right)^2\left(x+\frac{1}{x}\right) dx\sqrt{x} = -\frac{3+8a+16a^2}{4a^2}e^{-2a}\sqrt{\frac{\pi}{2a}}$ V. T. 236. N°. 3.

6) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\left(x-\frac{1}{x}\right)^4 dx\sqrt{x} = \frac{15+36a+48a^2+64a^3}{8a^3}e^{-2a}\sqrt{\frac{\pi}{2a}}$ V. T. 235. N°. 2.

7) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\left(x-\frac{1}{x}\right)^{2b}\left(x+\frac{1}{x}\right) dx\sqrt{x} = (-1)^b\sqrt{\frac{\pi}{2}}\frac{d^{2b}}{da^{2b}}\cdot\frac{e^{-2a}}{\sqrt{a}}$ V. T. 236. N°.

8) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\left(x-\frac{1}{x}\right)^{2b} dx\sqrt{x} = (-1)^b\sqrt{\frac{\pi}{2}}\frac{d^{2b-1}}{da^{2b-1}}\cdot\frac{e^{-2a}}{\sqrt{a}}$ V. T. 235. N°. 2.

9) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4-x-\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\frac{dx}{x} = e^{-2a}\sqrt{2a\pi}$ Cauchy, P. 28. 147. P. 1. § 3.

10) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4+x+\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)\frac{dx}{x} = \frac{1-4a}{2}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 235. N°. 8.

11) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4-x-\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)^2\frac{dx}{x} = \frac{1+8a-16a^2}{4a}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 236. N°. 9.

12) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4+x+\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)^3\frac{dx}{x} = -\frac{3+12a+48a^2-64a^3}{8a^2}e^{-2a}\sqrt{\frac{2\pi}{a}}$ V. T. 235. N°. 8.

13) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4-x-\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{a}+\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)^{2b}\frac{dx}{x} = (-1)^b\sqrt{2\pi}\frac{d^{2b}}{da^{2b}}\cdot e^{-2a}\sqrt{a}$ V. T. 236. N°. 9.

14) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{4+x+\frac{1}{x}}{\left(x+\frac{1}{x}\right)^2}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)^{2b+1}\frac{dx}{x} = (-1)^b\sqrt{2\pi}\frac{d^{2b+1}}{da^{2b+1}}\cdot e^{-2a}\sqrt{a}$ V. T. 235. N°. 8.

15) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}+\{x+1+(x-1)i\}^{-b}}{2}\left(x+\frac{1}{x}\right)x^{\frac12 b-1}dx=\frac{\pi a^{\frac12 b-1}e^{-2a}}{2^{\frac12 b+1}\Gamma(\frac12 b)}$ Cauchy, P. 28. 147. P. 1. § 3.

16) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}-\{(x+1)+(x-1)i\}^{-b}}{2i}\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)x^{\frac12 b-1}dx=$

$=\frac{\pi a^{\frac12 b-2}e^{-2a}}{2^{\frac12 b+1}\Gamma(\frac12 b)}\left\{\frac{1}{2}(b-2)-2a\right\}$ V. T. 235. N°. 14.

17) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}+\{x+1+(x-1)i\}^{-b}}{2}\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)^2x^{\frac12 b-1}dx=$

$=-\frac{\pi a^{\frac12 b-2}e^{-2a}}{2^{\frac12 b+1}\Gamma(\frac12 b)}\left\{\frac{1}{4}(b-4)(b-2)-(b-2)2a+4a^2\right\}$ V. T. 236. N°. 15.

18) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}-\{x+1+(x-1)i\}^{-b}}{2i}\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)^3x^{\frac12 b-1}dx=$

$=-\frac{\pi a^{\frac12 b-2}e^{-2a}}{2^{\frac12 b+1}\Gamma(\frac12 b)}\left\{\frac{1}{8}(b-6)(b-4)(b-2)-\frac{3}{2}(b-4)(b-2)a+6(b-2)a^2-8a^3\right\}$ V. T. 235. N°. 14.

19) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}+\{x+1+(x-1)i\}^{-b}}{2}\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)^{2c}x^{\frac12 b-1}dx=$

$=\frac{(-1)^c\pi}{2^{\frac12 b+1}\Gamma(\frac12 b)}\frac{d^{2c}}{da^{2c}}.a^{\frac12 b-1}e^{-2a}$ V. T. 236. N°. 15.

20) $\int Cos.\left\{a\left(x-\frac{1}{x}\right)\right\}\frac{\{x+1-(x-1)i\}^{-b}-\{x+1+(x-1)i\}^{-b}}{2i}\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)^{2c+1}x^{\frac12 b-1}dx=$

$=\frac{(-1)^c\pi}{2^{\frac12 b+1}\Gamma(\frac12 b)}\frac{d^{2c+1}}{da^{2c+1}}.a^{\frac12 b-1}e^{-2a}$ V. T. 235. N°. 14.

21) $\int Cos.\,ax\,dx\sqrt{\frac{x}{x^2-1}}=(Cos.\,a-Sin.\,a)\sqrt{\frac{\pi}{4a}}$, pour a très-petit; V. T. 77. N°. 2.

F. Alg. rat. ent. Circ. Dir. — **TABLE 237.** — Lim. 0 et $\frac{\pi}{4}$.

1) $\int x\,Tang.\,x\,dx=-\frac{\pi}{8}l\,2+\frac{1}{2}\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 304. N°. 1.

2) $\int x\,Cot.\,x\,dx=\frac{\pi}{8}l\,2+\frac{1}{2}\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 303. N°. 1.

3) $\int x\,Tang.^2\,x\,dx=\frac{1}{4}\pi-\frac{1}{32}\pi^2-\frac{1}{2}l\,2$ V. T. 237. N°. 5.

4) $\int \frac{x}{Sin. 2x} dx = \frac{1}{2} \sum_0^\infty (-1)^n \frac{1}{(2n+1)^2}$ V. T. 305. N°. 1.

5) $\int \frac{x}{Cos.^2 x} dx = \frac{1}{4}\pi - \frac{1}{2} l 2$ V. T. 46. N°. 1.

6) $\int \frac{x^2}{Sin.^2 x} dx = \frac{1}{4}\pi l 2 - \frac{1}{16}\pi^2 + \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 238. N°. 4.

7) $\int \frac{x^2 Tang. x}{Cos.^2 x} dx = \frac{1}{2} l 2 - \frac{1}{4}\pi + \frac{1}{16}\pi^2$ V. T. 237. N°. 3.

8) $\int \frac{x^{p+1}}{Sin.^2 x} dx = -\left(\frac{1}{4}\pi\right)^{p+1} + \frac{p+1}{2}\left(\frac{\pi}{4}\right)^p \left\{1 - \sum_1^\infty \frac{4}{p+2m} \sum_1^\infty \frac{1}{(4n)^{2m}}\right\}$ V. T. 238. N°. 20.

9) $\int \frac{x\, Sin.^{a-1} x}{Cos.^{a+1} x} dx = \frac{\pi}{4} + \sum_0^\infty \frac{(-1)^{n-1}}{a+2n+1}$ V. T. 46. N°. 2.

10) $\int \frac{(x - \frac{1}{2}\pi) Tang.^2 x + x}{Cos. 2x} \frac{dx}{Cos.^2 x} = \frac{1}{4}\pi l 2$ V. T. 258. N°. 28.

11) $\int \frac{x}{(Cos. x + p\, Sin. x)^2} dx = \frac{1}{1+p^2} l \frac{\sqrt{2}}{1+p} + \frac{\pi}{4} \frac{1-p}{(1+p)(1+p^2)}$ V. T. 48. N°. 1.

12) $\int \frac{x\, Cos. 2x}{(1 + Sin. x. Cos. x)^2} dx = \pi \frac{2-\sqrt{3}}{6\sqrt{3}}$ V. T. 48. N°. 3.

13) $\int \frac{x\, Cos. 2x}{(1 - Sin. x. Cos. a)^2} dx = \pi \frac{3\sqrt{3}-4}{6\sqrt{3}}$ V. T. 48. N°. 2.

14) $\int \frac{x\, Sin. 4x}{(1 - Sin.^2 x. Cos.^2 x)^2} dx = \pi \frac{\sqrt{3}-1}{3\sqrt{3}}$ V. T. 48. N°. 4.

15) $\int \frac{x}{Sin. x + Cos. x} \frac{dx}{Cos. x} = \frac{1}{8}\pi l 2$ V. T. 306. N°. 1.

16) $\int \frac{1 - 2 Cos.\lambda. Sin. 2x. Sin.^2 x}{(1 - Cos.\lambda. Sin. 2x)^2} \frac{x}{Cos.^2 x} dx = \frac{\pi}{4(1-Cos.\lambda)} + \frac{\lambda - \pi}{2 Tang.\lambda} + l\left\{2 Sin. \frac{1}{2}\lambda\right\}$ V. T. 48. N°. 8.

17) $\int \frac{\sqrt{Tang. x} - \sqrt{Cot. x}}{Sin. 2x} x\, dx = \frac{1}{2}\pi(1-\sqrt{2})$ V. T. 50. N°. 15.

18) $\int \frac{x\, Tang.^3 x}{\sqrt{Cos. 2x}} dx = \frac{\{\Gamma(\frac{1}{4})\}^2}{8\sqrt{2\pi}} - \frac{\pi\sqrt{2\pi}}{2\{\Gamma(\frac{1}{4})\}^2}$ V. T. 50. N°. 1.

19) $\int \frac{x}{Sin. x \sqrt{Cos. 2x}} dx = \frac{1}{2}\pi l (1 + \sqrt{2})$ V. T. 261. N°. 14.

1) $\int x \, Cos. \, x \, dx = \frac{1}{2}\pi - 1$ V. T. 108. N°. 3.

2) $\int x \, Sin. 2x \, dx = \frac{1}{2}\pi \, l \, 2$ V. T. 267. N°. 22.

3) $\int x \, Cot. \, x \, dx = \frac{1}{2}\pi \, l \, 2$ Legendre, Exerc. 5. 61. — Poisson, P. 17. 612. N°. 16. — Cauchy, Sav. Etr. 1827. 599. P. 2. § 5, 7. — Mosta, Gr. 10. 449.

4) $\int x \, Cot. \frac{1}{2}x \, dx = \frac{1}{2}\pi \, l \, 2 + 2 \sum_{0}^{\infty} \frac{(-1)^n}{(2n+1)^2}$ Legendre, Exerc. 5. 64.

5) $\int \left(\frac{\pi}{2} - x\right) Tang. \, x \, dx = \frac{1}{2}\pi \, l \, 2$ Cauchy, Sav. Etr. 1827. 599. Suppl. 1.

6) $\int x \, Tang. \, x \, dx = \infty$ V. T. 333. N°. 1.

7) $\int x \, Cos.^p x \, . \, Tang. \, x dx = \frac{\pi}{p \, . \, 2^{p+1}} \frac{\Gamma(p+1)}{\{\Gamma(\frac{1}{2}p+1)\}^2}$ V. T. 53. N°. 21.

8) $\int Cos.^{a-1} x . \, Sin. \{(a+1)x\} . \, x \, dx = \frac{\pi}{a \, . \, 2^{a+1}}$

9) $\int (2 \, Cos. \, x)^{p-1} . \, Sin. \, q \, x \, . \, x \, dx = \frac{1}{4}\pi \, \Gamma(p) \frac{Z'\left(\frac{p+q+1}{2}\right) - Z'\left(\frac{p-q+1}{2}\right)}{\Gamma\left(\frac{p+q+1}{2}\right)\Gamma\left(\frac{p-q+1}{2}\right)}$

(8–9) Kummer, Cr. 20. 1.

10) $\int Sin. (p \, Tang. \, x) . x \, dx = \frac{1}{4}\pi e^{-p} \{A + l \, 2 \, p + e^{2p} Ei. (-2p)\}$ V. T. 431. N°. 5.

11) $\int Cos. (p \, Tang. \, x) \, Tang. \, x \, . \, x \, dx = -\frac{1}{4}\pi e^{-p} \{A + l \, 2 \, p + e^{2p} Ei. (-2p)\}$ V. T. 431. N°. 7.

12) $\int x^2 \, Sin. \, x \, dx = \pi - 2$ V. T. 238. N°. 1.

13) $\int x^2 \, Cot. \, x \, dx = \frac{1}{4}\pi^2 \, l \, 2 - 2 \sum_{1}^{\infty} \left\{\frac{1}{n^3} - \frac{(-1)^{n-1}}{n^3} - 2\frac{(-1)^{n-1}}{(2n)^3}\right\}$

14) $\int x^3 \, Cot. \, x \, dx = \frac{1}{8}\pi^3 \, l \, 2 - 6\pi \sum_{1}^{\infty} \frac{(-1)^{n-1}}{(2n)^3}$

15) $\int (9\pi x^2 - 14x^3) \, Cot. \, x \, dx = \frac{1}{2}\pi^3 \, l \, 2$

(13–15) Legendre, Exerc. 5. 61.

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16) $\int x^a \,Tang. \frac{1}{2} x \, dx = -\left(\frac{1}{2}\pi\right)^a l2 + 2\, Cos. \frac{1}{2} a\pi . 1^{a/1} \sum_1^\infty \frac{(-1)^{n-1}}{n^{a+1}} +$

$+ 2 \sum_1^\infty (-1)^{n-1} \left\{ a^{2n-1/-1} \left(\frac{\pi}{2}\right)^{a-2n+1} \sum_0^\infty \frac{(-1)^m}{(2m+1)^{2n}} + a^{2n/-1} \left(\frac{\pi}{2}\right)^{a-2n} \sum_0^\infty \frac{(-1)^{m-1}}{(2m)^{2n+1}} \right\}$

17) $\int x^a \, Cot. \frac{1}{2} x \, dx = \left(\frac{1}{2}\pi\right)^a l2 + 2\, Cos. \frac{1}{2} a\pi . 1^{a/1} \sum_1^\infty \frac{1}{n^{a+1}} +$

$+ 2 \sum_1^\infty (-1)^{n-1} \left\{ a^{2n-1/-1} \left(\frac{\pi}{2}\right)^{a-2n+1} \sum_0^\infty \frac{(-1)^m}{(2m+1)^{2n}} - a^{2n/-1} \left(\frac{\pi}{2}\right)^{a-2n} \sum_0^\infty \frac{(-1)^{m-1}}{(2m)^{2n+1}} \right\}$

Legendre, Exerc. 5. 58, 59.

18) $\int x^p \, Cot. x \, dx = \left(\frac{\pi}{2}\right)^p \left\{ 1 - \sum_1^\infty \frac{2}{p+2m} \sum_1^\infty \frac{1}{(2n)^{2m}} \right\}$, où p fraction;

19) $\int x^p \, Cot. \frac{1}{2} x \, dx = \left(\frac{\pi}{2}\right)^p \left\{ 1 - \sum_1^\infty \frac{4}{p+2m} \sum_1^\infty \frac{1}{(4n)^{2m}} \right\}$ Legendre, Exerc. 5. 63.

1) $\int \frac{x}{Sin. x} dx = 2 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$

2) $\int \frac{x^2}{Sin. x} dx = -2 \sum_1^\infty \left\{ \frac{1}{n^3} + \frac{(-1)^{n-1}}{n^3} + \pi \frac{(-1)^{n-1}}{(2n-1)^2} \right\}$

3) $\int \frac{x^3}{Sin. x} dx = 3 \sum_1^\infty (-1)^n \left\{ \frac{1}{2}\pi^2 \frac{1}{(2n-1)^2} - \frac{4}{(2n-1)^4} \right\}$

Legendre, Exerc. 5. 61.

4) $\int \frac{x^a}{Sin. x} dx = Cos. \frac{1}{2} a\pi . 1^{a/1} \sum_1^\infty \left\{ \frac{1}{n^{a+1}} + \frac{(-1)^{n-1}}{n^{a+1}} \right\} + 2 \sum_1^\infty (-1)^{n-1} a^{2n-1/-1} \left(\frac{\pi}{2}\right)^{a-2n-1} \sum_1^\infty \frac{(-1)^{2n-1}}{(2m-1)^{2n}}$ Legendre, Ex. 5. 60.

5) $\int \frac{x^p}{Sin. x} dx = \left(\frac{\pi}{2}\right)^p \left\{ 1 + \sum_1^\infty \frac{1}{2^{m-2}} \frac{2^{2m-1}-1}{p+2m} \sum_1^\infty \frac{1}{(4n^2)^m} \right\}$, où p fraction; Legendre, Exerc. 5. 63.

6) $\int \frac{x \, Cos. x}{Sin. x} dx = \frac{1}{2} \pi l 2$ Legendre, Exerc. Suppl. 28. — Cauchy, Exerc. 1826. p. 205.

7) $\int \frac{x}{Sin.^2 x} dx = \frac{1}{2} \pi l 2$ V. T. 264. N°. 2.

8) $\int \frac{x^2}{Sin.^2 x} dx = \pi l 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. — Mosta, Gr. 10. 449.

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9) $\int \frac{x^{p+1}}{Sin.^2 x} dx = (p+1)\left(\frac{1}{2}\pi\right)^p \left\{1 - \sum_1^\infty \frac{2}{p+2m} \sum_1^\infty \frac{1}{(4n^2)^m}\right\}$ V. T. 238. N°. 18.

10) $\int \frac{x^2 Cos. x}{Sin.^3 x} dx = -\frac{1}{4}\pi^2 + 4\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 239. N°. 1.

11) $\int \frac{x^3 Cos. x}{Sin.^3 x} dx = -\frac{1}{16}\pi^3 + \frac{3}{2}\pi l 2$ V T. 239. N°. 8.

12) $\int \frac{1 - x Cot. x}{Sin.^2 x} dx = \frac{1}{4}\pi$ Legendre, Exerc. Suppl. 17.

13) $\int \frac{4x^2 Cos. x + (2\pi - x) x}{Sin. x} dx = \pi^2 l 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

14) $\int \frac{x}{Sin. 2x} dx = \infty$ Cauchy, Exerc 1826. p. 205.

15) $\int \frac{x^a}{Tang. x} dx = \left(\frac{1}{2}\pi\right)^a l 2 + Cos. \frac{1}{2} a\pi . 1^{a/1} \sum_1^\infty \left\{\frac{1}{n^{a+1}} + \frac{(-1)^n}{n^{a+1}} + \right.$
$\left. + (-1)^n 2a \sum_1^\infty (-1)^{m-1} (a-1)^{2m-1/-1} \left(\frac{\pi}{2}\right)^{a-2m} \frac{1}{(2n-1)^{2m+1}}\right\}$ Legendre, Exerc. 5. 60.

16) $\int Sin. (q\, Cot. x) \frac{x}{Sin.^2 x} dx = \frac{e^{-q} - 1}{2q}\pi$ V. T. 374. N°. 1.

17) $\int Sin. (q\, Tang. x) \frac{x}{Cos.^2 x} dx = \frac{\pi}{2q} e^{-q}$ V. T. 374. N°. 2.

18) $\int Cos. (q\, Tang. x) \frac{x}{Sin. 2x} dx = -\frac{1}{4}\pi\, Ei. (-q)$ V. T. 431. N°. 1.

19) $\int \frac{x}{Cos.^2 x . Sin. x} dx = -\infty$ V. T. 334. N°. 1.

20) $\int \frac{x\, Sin.^p x}{Tang. x} dx = \frac{\pi}{2p} - 2^{p-2} \frac{\{\Gamma(\frac{1}{2}p)\}^2}{\Gamma(p+1)}$ V. T. 53. N°. 18.

21) $\int \frac{x}{Tang. x . Cos. 2x} dx = \frac{1}{4}\pi l 2$ V. T. 265. N°. 13.

22) $\int \frac{x}{Tang.^p x . Sin. 2x} dx = \frac{\pi}{p} Sec. p\pi$, $p < 1$; V. T. 63. N°. 4.

1) $\int \frac{x \, Sin. x}{Cos.^2 \lambda - Sin.^2 x} dx = -2 \, Cosec. \lambda \sum_0^\infty \frac{Sin. \{(2n+1)\lambda\}}{(2n+1)^2}$ Legendre, Exerc. 5. 85.

2) $\int \frac{x \, Sin. 2x}{1 + Tang.^2 \lambda . Cos.^2 x} dx = -\pi \, Cot.^2 \lambda \, l \left(Cos.^2 \frac{1}{2} \lambda . Sec. \lambda \right)$ V. T. 334. N°. 9.

3) $\int \frac{x^2}{1 - Cos. x} dx = \pi l 2 - \frac{\pi^2}{4} + \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 238. N°. 4.

4) $\int \frac{x^{p+1}}{1 - Cos. x} dx = -\left(\frac{1}{2}\pi\right)^{p+1} + (p+1)\left(\frac{1}{2}\pi\right)^p \left\{ 1 - \sum_1^\infty \frac{4}{p+2m} \sum_1^\infty \frac{1}{(4n)^{2m}} \right\}$ V. T. 238. N°. 19.

5) $\int \frac{x^a Sin. x}{Cos. x + Cos. \lambda} dx = -\left(\frac{1}{2}\pi\right)^a l(2 Cos. \lambda) + 2 . 1^{a/1} Cos. \frac{1}{2} a\pi \sum_1^\infty \frac{Cos. n\lambda}{n^{a+1}} + 2 \sum_1^\infty (-1)^{n-1}$

$\left\{ Cos. \{(2n-1)\lambda\} \sum_1^\infty (-1)^{m-1} \frac{a^{2m-1|-1}}{(2n-1)^{2m}} \left(\frac{\pi}{2}\right)^{a+1-2m} + Cos. 2n\lambda \sum_1^\infty (-1)^{m-1} \frac{a^{2m|-1}}{(2n)^{2m+1}} \left(\frac{\pi}{2}\right)^{a-2m} \right\}$ Legendre, Exerc. 5. 57.

6) $\int \frac{x^a Sin. x}{Cos. x - Cos. \lambda} dx = -\left(\frac{1}{2}\pi\right)^a l(2 Cos. \lambda) - 2 . 1^{a/1} Cos. \frac{1}{2} a\pi \sum_1^\infty \frac{Cos. n\lambda}{n^{a+1}} - 2 \sum_1^\infty (-1)^{n-1}$

$\left\{ Cos. \{(2n-1)\lambda\} \sum_1^\infty (-1)^{m-1} \frac{a^{2m-1|-1}}{(2n-1)^{2m}} \left(\frac{\pi}{2}\right)^{a+1-2m} - Cos. 2n\lambda \sum_1^\infty (-1)^{m-1} \frac{a^{2m|-1}}{(2n)^{2m+1}} \left(\frac{\pi}{2}\right)^{a-2m} \right\}$ Legendre, Exerc. 5. 55.

7) $\int \frac{x^a Sin. x}{q + Cos. x} dx = 2 Cos. \frac{1}{2} a\pi . 1^{a/1} \sum_1^\infty (-1)^{n-1} \frac{c^n}{n^{a+1}} - 2 \sum_1^\infty \left\{ c^{2n} \sum_0^\infty \binom{a}{2m} (-1)^m \left(\frac{1}{2}\pi\right)^{a-2m} \frac{1}{(2n)^{2m+1}} - \right.$

$\left. - c^{2n-1} \sum_0^\infty \binom{a}{2m+1} (-1)^m \left(\frac{\pi}{2}\right)^{a-2m-1} \frac{1}{(2n)^{2m+2}} \right\}$

8) $\int \frac{x^a Sin. x}{q - Cos. x} dx = 2 Cos. \frac{1}{2} a\pi . 1^{a/1} \sum_1^\infty \frac{c^n}{n^{a+1}} + 2 \sum_1^\infty \left\{ c^{2n} \sum_0^\infty \binom{a}{2m} (-1)^m \left(\frac{1}{2}\pi\right)^{a-2m} \frac{1}{(2n)^{2m+1}} + \right.$

$\left. + c^{2n-1} \sum_0^\infty \binom{a}{2m+1} (-1)^m \left(\frac{1}{2}\pi\right)^{a-2m-1} \frac{1}{(2n)^{2m+2}} \right\}$

Sur les form. 7), 8), où $c = a - \sqrt{(a^2 - 1)}$, voyez: Legendre, Exerc. 5. 73.

9) $\int \frac{x \, Sin. 2x}{p + Cos. 2x} dx = -\frac{1}{4}\pi l \{2(1-p)\}$, $p < 1$;

10) $= \frac{1}{4}\pi l \frac{p + \sqrt{(p^2 - 1)}}{2(p-1)}$, $p > 1$;

11) $\int \frac{x \, Sin. 2x}{p - Cos. 2x} dx = \frac{1}{4}\pi l \{2(1+p)\}$, $p < 1$;

Cauchy, Sav. Etr. 1827. 599. S. 1.

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12) $\int \frac{x \, Sin. 2x}{p - Cos. 2x} dx = \frac{1}{4}\pi l \frac{2(1+p)}{p + \sqrt{(p^2-1)}}, p > 1;$ Cauchy, Sav. Etr. 1827. 599. S. 1.

13) $\int \frac{x \, Sin. 2x}{1 - Sin.^2\lambda . Cos.^2 x} dx = -2\pi \, Cosec.^2\lambda . l \, Cos. \frac{1}{2}\lambda$ V. T. 334. N°. 13.

14) $\int \frac{x \, Sin. 2x}{1 + q^2 Cos.^2 x} dx = \frac{\pi}{q^2} l \frac{1 + \sqrt{(1+q)}}{2}$ V. T. 334. N°. 8.

15) $\int \frac{x \, Sin. 2x}{p^2 - Cos.^2 2x} dx = \frac{\pi}{8p} l \frac{1+p}{1-p}, p < 1;$ ⎫ V. T. 240. N°. 9—12.

16) $= \frac{\pi}{8p} l \frac{p+1}{p-1}, p > 1;$ ⎭

17) $\int \frac{x \, Sin. 4x}{p^2 - Cos.^2 2x} dx = \frac{1}{4}\pi l \{4(1-p^2)\}, p < 1;$ ⎫ V. T. 240. N°. 9—12.

18) $= \frac{1}{2}\pi l \frac{2\sqrt{(p^2-1)}}{p + \sqrt{(p^2-1)}}, p > 1;$ ⎭

19) $\int \frac{x \, Sin. 2x}{1 - Sin.^2\lambda . Cos.^4 x} dx = \frac{1}{2}\pi \, Cosec.\lambda \, l \frac{1 + Sin. \frac{1}{2}\lambda}{Cos. \frac{1}{2}\lambda}$ V. T. 334. N°. 22.

20) $\int \frac{x \, Tang. x}{p^2 Sin.^2 x + Cos.^2 x} dx = -\frac{\pi}{p^2} l(1+p)$ V. T. 334. N°. 14.

F. Alg. rat. ent.
Circ. Dir. en dén. d'autre forme. TABLE 241. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{x \, Sin. x}{1 - 2p \, Cos \, x + p^2} dx = \frac{\pi}{p} l(1-p)$ V. T. 356. N°. 2.

2) $\int \frac{x \, Cos. x}{1 + 2p \, Sin. x + p^2} dx = \frac{\pi}{2p} l(1+p) - \frac{1}{2p} \sum_1^\infty \frac{1}{2n-1} \frac{2^{n/2}}{3^{n/2}} \left(\frac{2p}{1+p^2}\right)^{2n-1}$ V. T. 334. N°. 19.

3) $\int \frac{x \, Sin. 2x}{1 - 2p \, Cos. 2x + p^2} dx = \frac{\pi}{4p} l(1+p), p^2 < 1;$ ⎫ Cauchy, Lim. Imag. 115, 121. — Poisson, P. 19. 404. N°. 76, 171.

4) $= \frac{\pi}{4p} l \frac{1+p}{p}, p^2 > 1;$ ⎭

5) $\int \frac{x \, Cot. x}{1 - (1-q) Cos. 2x + q} dx = \frac{\pi}{8q} l \frac{1 + \sqrt{(1+q)}}{2}$ V. T. 353. N°. 16.

6) $\int \frac{x}{1-q^4 \, Tang.^4 x} \frac{dx}{Sin. 2x} = \frac{1}{16}\pi \, l \frac{(1+q)^2 (1+q^2)}{q^4}$ V. T. 265. N°. 15.

7) $\int \frac{Sin. x}{1-q^4 \, Tang.^4 x} \frac{x}{Cos.^3 x} dx = \frac{\pi}{8q^2} l \frac{1+q^2}{(1+q)^2}$ V. T. 265. N°. 14.

8) $\int \frac{x \, Cos. x}{(1+Sin. x. Cos. \lambda)^2} dx = 2\lambda \, Cosec. 2\lambda - \frac{1}{2\, Cos. \lambda} \frac{\pi}{1+Cos. \lambda}$ V. T. 65. N°. 6.

9) $\int \frac{x \, Cos. 2x}{(1+Sin. x. Cos. x)^2} dx = \frac{2}{9}\pi \sqrt{3} - \frac{1}{2}\pi$ V. T. 65. N°. 4.

10) $\int \frac{x \, Cos. x}{(1-Sin. x. Cos. \lambda)^2} dx = 2(\lambda-\pi) \, Cosec. 2\lambda + \frac{\pi}{2\, Cos. \lambda (1-Cos. \lambda)}$ V. T. 65. N°. 5.

11) $\int \frac{x \, Cos. 2x}{(1-Sin. x. Cos. x)^2} dx = \frac{1}{2}\pi - \frac{4}{9}\pi \sqrt{3}$ V. T. 65. N°. 3.

12) $\int \frac{x \, Sin. 2x}{(1-Cos.^2 \lambda. Sin.^2 x)^2} dx = -2\pi \, Cosec.^2 2\lambda (1-Sin. \lambda)$ V. T. 66. N°. 8.

13) $\int \frac{x}{(Sin. x \pm q \, Cos. x)^2} dx = \pm \frac{\pi}{2} \frac{q}{1+q^2} - \frac{1}{1+q^2} l \, q$ V. T. 65. N°. 7, 8.

14) $\int \frac{x \, Sin. x}{(p+q \, Cos. x)^2} dx = \frac{\pi}{2p} - \frac{1}{\sqrt{(p^2-q^2)}} Arccos. \frac{q}{p} \quad , q < p;$

15) $\quad = \frac{\pi}{2p} + \frac{1}{\sqrt{(q^2-p^2)}} l \frac{p}{q+\sqrt{(q^2-p^2)}}, q > p;$

(14, 15) V. T. 65. N°. 9, 10.

16) $\int \frac{x \, Sin. 4x}{(1-Sin.^2 x. Cos.^2 x)^2} dx = \frac{2}{\sqrt{3}}\pi$ V. T. 66. N°. 6.

17) $\int \frac{x \, Sin. 2x}{(Cos.^2 x + p^2 \, Sin.^2 x)^2} dx = \frac{\pi}{2p^2(1+p)}$ V. T. 66. N°. 2.

18) $\int \frac{x \, Sin. 2x}{(p^2 \, Sin.^2 x + q^2 \, Cos.^2 x)^3} dx = \frac{\pi}{8p^4 q^3} \frac{p^2+pq+2q^2}{p+q}$ V. T. 67. N°. 7.

19) $\int \frac{x \, Sin. 2x}{(p^2 \, Sin.^2 x + q^2 \, Cos.^2 x)^4} dx = \frac{\pi}{48 p^6 q^5} \frac{3p^4+3p^3 q+5p^2 q^2+5pq^3+8q^4}{p+q}$ V. T. 67. N°. 10.

20) $\int \frac{x \, Sin. 2x}{(p^2 Sin.^2 x+q^2 Cos.^2 x)^5} dx = \frac{\pi}{128 p^8 q^7} \frac{5p^6+5p^5 q+8p^4 q^2+8p^3 q^3+11p^2 q^4+11pq^5+16q^6}{p+q}$ V. T. 67. N°. 13.

21) $\int \frac{Cos.^2 \lambda + Sin.^2 x}{(Cos.^2 \lambda - Sin.^2 x)^2} x^2 \, Cos. x \, dx = -\frac{\pi^2}{4\, Sin.^2 \lambda} + 4\, Cosec. \lambda \sum_0^\infty \frac{Sin. \{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 240. N°. 1.

22) $\int \frac{Tang.^2 x}{(p^2 + Tang.^2 x)^2} \frac{x}{Sin. 2 x} dx = \frac{\pi}{8 p (p+1)}$ V. T. 66. N°. 11.

23) $\int \frac{x}{(Tang. x + Cot. x)^3} \frac{dx}{Tang. 2x. Sin. 2x} = -\frac{\pi}{128}$ V. T. 67. N°. 24.

24) $\int \frac{Sin. x. Cos. x}{1 - Sin.^2 \lambda. Cos.^2 x} \frac{x}{1 - Sin.^2 \mu. Cos.^2 x} dx = \frac{\pi}{Cos.^2 \lambda - Cos.^2 \mu} l\left(Cos. \frac{1}{2}\lambda. Sec. \frac{1}{2}\mu\right)$ Lobatschewsky, Mém. Kasan. 1835. 1.

25) $\int \frac{Sin. 2x}{Sin.^2 \lambda - Sin.^2 \mu. Cos.^2 x} \frac{x}{1 - Sin.^2 \mu. Cos.^2 x} dx = \frac{\pi}{Sin.^2 \mu. Cos.^2 \lambda} l \frac{Cot. \frac{1}{2}\mu}{Sin. \lambda}. Tang. \left(\frac{1}{2} Arcsin. \frac{Sin. \mu}{Sin. \lambda}\right)$ V. T. 334. N°. 24.

26) $\int \left[\frac{p^2 x Sin. 2px}{Cos. p\pi - Cos. 2px} - \frac{(1-p)^2 x - (1-p)\frac{1}{2}\pi}{Cos. p\pi - Cos. \{(1-p)2x\}} Sin. \{2(1-p)x\}\right] dx = \frac{\pi}{4} l \{2(1 + Cos. p\pi)\}$ Cauchy, Sav. Etr. 1827. 599. S. I.

F. Alg. rat. ent.
Circ. Dir. sous forme irrat. à dén. mon. TABLE 242. Lim. 0 et $\frac{\pi}{2}$.

1) $\int x Sin. 2x dx \sqrt{(1 - p^2 Sin.^2 x)} = \frac{2}{3p^2}\left[\frac{4 - 2p^2}{3} E'(p) - \frac{1 - p^2}{3} F'(p) - \frac{\pi}{2}\sqrt{(1-p^2)^3}\right]$ V. T. 72. N°. 10.

2) $\int x Tang. x dx \sqrt[3]{Cos. x} = \sqrt[4]{27}\left\{(1 - \sqrt{3}) F'\left(Cos. \frac{\pi}{12}\right) + 2\sqrt{3} E'\left(Cos. \frac{\pi}{12}\right)\right\}$ V. T. 72. N°. 22.

3) $\int \frac{x dx}{Tang. x} \sqrt[3]{Sin. x} = \frac{3}{2}\pi + \sqrt[4]{27}\left\{(\sqrt{3} - 1) F'\left(Cos. \frac{\pi}{12}\right) - 2\sqrt{3} E'\left(Cos. \frac{\pi}{12}\right)\right\}$ V. T. 72. N°. 20.

4) $\int \frac{\sqrt{Tang. x} - \sqrt{Cot. x}}{Sin. 2 x} x dx = -\infty$ V. T. 73. N°. 4.

5) $\int \frac{x Cos. x}{\sqrt{Sin.^3 x}} dx = -\pi + 2\sqrt{2}. F'\left(Sin. \frac{\pi}{4}\right)$ V. T. 73. N°. 3.

6) $\int \frac{x Sin. x}{\sqrt{Cos.^3 x}} dx = -\infty$ V. T. 73. N°. 8.

7) $\int \frac{x Cos. x}{\sqrt[3]{Sin. x}} dx = \frac{3}{4}\pi + \frac{9}{2}\sqrt[4]{3}\left\{\frac{3 + \sqrt{3}}{2} F'\left(Sin. \frac{\pi}{12}\right) - 3 E'\left(Sin. \frac{\pi}{12}\right)\right\}$ V. T. 72. N°. 21.

8) $\int \frac{x Sin. x}{\sqrt[3]{Cos. x}} dx = \frac{3}{2}\sqrt[4]{3}\left\{3 E'\left(Sin. \frac{\pi}{12}\right) - \frac{3 + 3\sqrt{3}}{2} F'\left(Sin. \frac{\pi}{12}\right)\right\}$ V. T. 72. N°. 23.

9) $\int \frac{x Tang. x}{\sqrt[3]{Cos. x}} dx = \infty$ V. T. 73. N°. 9.

10) $\int \frac{x \, Tang. x}{\sqrt[3]{Cos.^2 x}} dx = \infty$ V. T. 73. N°. 10.

11) $\int \frac{x}{Tang. x \sqrt[3]{Sin. x}} dx = \frac{3}{\sqrt[4]{3}} F'\left(Cos. \frac{\pi}{12}\right) - \frac{3}{2}\pi$ V. T. 73. N°. 7.

12) $\int \frac{x}{Tang. x \sqrt[3]{Sin.^2 x}} dx = \frac{3}{2} \sqrt[4]{27} \,.\, F'\left(Sin. \frac{\pi}{12}\right) - \frac{3}{4}\pi$ V. T. 73. N°. 8.

13) $\int \frac{x \, dx \sqrt[q]{Sin.^{-2} x}}{Cos.^{\frac{2q-2}{q}} x} = \frac{q\pi}{2(2-q)} Cosec. \frac{\pi}{q}$ V. T. 73. N°. 12.

F. Alg. rat. ent.
Circ. Dir. sous forme irrat. à dén. polyn. TABLE 243. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{x \, Sin. 2x}{\sqrt{(1-p^2 Sin.^2 x)}} dx = -\frac{\pi}{p^2}\sqrt{(1-p^2)} + \frac{2}{p^2} E'(p)$ V. T. 72. N°. 2.

2) $\int \frac{x \, Sin. 2x}{\sqrt{(1-p^2 Cos.^2 x)}} dx = \frac{\pi}{p^2} - \frac{2}{p^2} E'(p)$ V. T. 72. N°. 11.

3) $\int \frac{x \, Sin.^2 x . Sin. 2x}{\sqrt{(1-p^2 Sin.^2 x)}} dx = \frac{2}{3p^4}\left\{\frac{2p^2+5}{3} E'(p) + \frac{1-p^2}{3} F'(p) - \frac{2+p^2}{2}\pi\sqrt{(1-p^2)}\right\}$ V. T. 72. N°. 1, 4.

4) $\int \frac{x \, Sin. x}{\sqrt{(p^2+Cos.^2 x)^3}} dx = \frac{1}{p^2} Arccot. p$ V. T. 75. N°. 8.

5) $\int \frac{x \, Cos. x}{\sqrt{(p^2+Sin.^2 x)^3}} dx = \frac{1}{p^2}\left\{-Arccot. p + \frac{\pi}{2\sqrt{(1+p^2)}}\right\}$ V. T. 75. N°. 5.

6) $\int \frac{x \, Sin \, 2x}{\sqrt{(1+Sin.^2 x)^3}} dx = \frac{\pi}{2} - \frac{1}{2}\sqrt{2} . F'\left(Sin. \frac{\pi}{4}\right)$ V. T. 75. N°. 1.

7) $\int \frac{x \, Sin. 2x}{\sqrt{(1+Cos.^2 x)^3}} dx = \pi - \sqrt{2} . F'\left(Sin. \frac{\pi}{4}\right)$ V. T. 74. N°. 5.

8) $\int \frac{x \, Sin. x}{\sqrt{(a+b \, Cos. x)^3}} dx = \frac{1}{b}\left[\frac{\pi}{\sqrt{a}} - \frac{4}{\sqrt{(a+b)}} F\left(\frac{1}{4}\pi, \sqrt{\frac{2b}{a+b}}\right)\right]$ V. T. 74. N°. 1.

9) $\int \frac{x \, Sin. x}{\sqrt{(a-b \, Cos. x)^3}} dx = \frac{1}{b}\left[\frac{-\pi}{\sqrt{a}} + \frac{4}{\sqrt{(a+b)}}\left\{F\left(\frac{1}{2}\pi, \sqrt{\frac{2b}{a+b}}\right) - F\left(\frac{1}{4}\pi, \sqrt{\frac{2b}{a+b}}\right)\right\}\right]$ V. T. 74. N°. 2.

10) $\int \frac{x \, Sin. 2x}{\sqrt{(a-b\,Cos.x)^3}} dx = \frac{4}{b^2}\left[-\pi\sqrt{a}+\frac{2a}{\sqrt{(a+b)}}\left\{F\left(\frac{\pi}{2},\sqrt{\frac{2b}{a+b}}\right)-F\left(\frac{1}{4}\pi,\sqrt{\frac{2b}{a+b}}\right)\right\}+\right.$
$\left.+2a\sqrt{(a+b)}\left\{E\left(\frac{1}{2}\pi,\sqrt{\frac{2b}{a+b}}\right)-E\left(\frac{1}{4}\pi,\sqrt{\frac{2b}{a+b}}\right)\right\}\right]$ V. T. 74. N°. 2 et 3.

11) $\int \frac{x Sin\, 2x}{\sqrt{(a+b Cos.x)^3}} dx = \frac{4}{b^2}\left[-\pi\sqrt{a}+\frac{2}{\sqrt{(a+b)}}\left\{(a+b)E\left(\frac{\pi}{4},\sqrt{\frac{2b}{a+b}}\right)+aF\left(\frac{\pi}{4},\sqrt{\frac{2b}{a+b}}\right)\right]\right\}$ V. T. 74. N°. 1 et 4.

12) $\int \frac{x\, Cos.x}{\sqrt{(1-p^2\, Sin.^2 x)^3}} dx = \frac{\pi}{2\sqrt{(1-p^2)}} + \frac{1}{2p} l \frac{1-p}{1+p}, p < 1;$ V. T. 75. N°. 10.

13) $\int \frac{x\, Sin.\, 2x}{\sqrt{(1-p^2\, Sin.^2 x)^3}} dx = \frac{1}{p^2}\left\{\frac{\pi}{\sqrt{(1-p^2)}} - 2F'(p)\right\}, p < 1;$ V. T. 75. N°. 9.

14) $\int \frac{x\, Sin.\, 2x}{\sqrt{(1-p^2\, Sin.^2 x)^5}} dx = \frac{2}{3p^2}\frac{1}{1-p^2}\left\{\frac{\pi}{2\sqrt{(1-p^2)}} - E'(p)\right\}, p<1;$ V. T. 75. N°. 18.

15) $\int \frac{x\, Sin.^2 x.\, Cos.\, x}{\sqrt{(1-p^2\, Sin.^2 x)^5}} dx = \frac{1}{3p^2(1-p^2)}\left[-1-\frac{p^2\pi}{\sqrt{(1-p^2)}}+\frac{1-p^2}{2p} l \frac{1+p}{1-p}\right]$ V. T. 75. N°. 10 et 19.

16) $\int \frac{x\, Sin.^2 x.\, Sin.\, 2x}{\sqrt{(1-p^2\, Sin.^2 x)^5}} dx = \frac{1}{3p^4}\left[\pi\frac{3p^2-2}{\sqrt{(1-p^2)^3}} - \frac{2}{1-p^2}E'(p) + 6F'(p)\right]$ V. T. 75. N°. 9 et 20.

17) $\int \frac{x}{Sin.\, x + Cos.\, x}\frac{dx}{\sqrt{Sin.\, 2x}} = \frac{1}{8}\pi^2\sqrt{2}$ V. T. 268. N°. 1.

18) $\int \frac{1 - x\, Cot.\, x}{\sqrt{(1 - Cos.^2 \lambda.\, Sin.^2 x)}}\frac{dx}{Sin.\, x} = \frac{1}{2}\frac{\pi}{1+Cos.\,\lambda} + \frac{\lambda\, Cot.\,\lambda - 1}{Sin.\,\lambda}$ Legendre, Exerc. Suppl. 49.

19) $\int \frac{Sin.\, 2x}{\sqrt{(1-p^2\, Sin.^2 x)}}\frac{x dx}{\sqrt{Sin. x}} = \frac{2}{p^2}\left[-\pi\sqrt{(1-p^2)}+4\frac{cF'(c)+bF'(b)}{(b+c)^2}+4\frac{b-c}{(b+c)^2}\{E'(b)-E'(c)\}\right]$
, où $2b^2 = \frac{(1+\sqrt{p})^2}{1+p}, 2c^2 = \frac{(1-\sqrt{p})^2}{1+p};$ V. T. 78. N°. 13.

20) $\int \frac{x}{\sqrt[3]{Sin.^2 x} + \sqrt[3]{Cos.^2 x}}\frac{dx}{\sqrt[3]{Sin.^2 x.\, Cos.^2 x}} = \frac{3}{8}\pi^2$ V. T. 268. N°. 4.

F. Alg. rat. ent.
Circ. Dir. ent.

TABLE 244.

Lim. 0 et π.

1) $\int x\, Cos.\, ax\, dx = \frac{1}{a^2}(Cos.\, a\pi - 1)$ Dienger, Cr. 34. 75. — Schlömilch, Höh. An. 80. — Id., Beitr. I. § 8.

2) $\int x\, Sin.\, a\, x\, dx = \frac{\pi}{a} Cos. \{(a+1)\pi\}$ Schlömilch, Höh. Anal. 80. — Id., Beitr. I. § 8.

3) $\int x \, Sin. \left\{\frac{2a-1}{2} x\right\} dx = \frac{4}{(2a-1)^2} Sin. \left\{\frac{2a-1}{2}\pi\right\}$ Dienger, Cr. 34. 75.

4) $\int x \, Cot. \frac{1}{2} x \, dx = 2\pi l 2$ Legendre, Exerc. 5. 68.

5) $\int x \, Tang. x \, dx = -\pi l 2$ V. T. 346. N°. 6.

6) $\int x \, Sin.^q x \, dx = \frac{\pi^2}{2^{q+1}} \frac{\Gamma(q+1)}{\{\Gamma(\frac{1}{2}q+1)\}^2}$ Lobatschewsky, Mém. Kasan. 1835. 211. — Grunert, Gr. 4. 113.

7) $\int x \, Sin. x . Cos. a x \, dx = (-1)^{a+1} \frac{\pi}{a^2-1}$

8) $\int x \, Sin. a x . Cos. x \, dx = (-1)^a \frac{a\pi}{a^2-1}$

(7, 8) Schlömilch, Beitr. I. § 8, 10.

9) $\int x \, Sin. x . Cos.^{2a} x \, dx = \frac{\pi}{2a+1}$ Poisson, P. 17. 612. N°. 17.

10) $\int x \, Tang. x . Sec. x \, dx = -\pi$ V. T. 81. N°. 1.

11) $\int \left(\frac{\pi}{2} - x\right) Tang. x \, dx = \pi l 2$ V. T. 271. N°. 3.

12) $\int x^2 \, Sin. q x \, dx = \frac{1}{q^2} \{ (2 - q^2\pi^2) Cos. q\pi - 2 \}$ V. T. 244. N°. 1.

13) $\int x^2 \, Cos. a x \, dx = \frac{2\pi}{a^3} Cos. a\pi$ Dienger, Cr. 34. 75.

14) $\int x^2 \, Cot. \frac{1}{2} x \, dx = 2\pi^2 l 2 - 4 \sum_1^\infty \left\{\frac{1}{n^3} + \frac{(-1)^{n-1}}{n^3}\right\}$

15) $\int x^3 \, Cot. \frac{1}{2} x \, dx = 2\pi^3 l 2 - 9\pi \sum_1^\infty \frac{1}{n^3}$

16) $\int x^4 \, Cot. \frac{1}{2} x \, dx = 2\pi^4 l 2 + 24 \sum_1^\infty \left\{\pi^2 \frac{(-1)^n}{n^3} + \frac{2}{n^3} + 2\frac{(-1)^{n-1}}{n^3}\right\}$

17) $\int x^5 \, Cot. \frac{1}{2} x \, dx = 2\pi^5 l 2 + 40 \sum_1^\infty \left\{\pi^3 \frac{(-1)^n}{n^3} + 6\pi \frac{(-1)^{n-1}}{n^5}\right\}$

(14–17) Legendre, Exerc. 5. 68.

1) $\int \frac{x \, Sin.\, x}{1 + Cos.\, x} dx = -\infty$ V, T. 353. N°. 7.

2) $\int \frac{x \, Sin.\, x}{2 + Cos.\, x} dx = -2\pi l(\sqrt{3} - 1)$ Poisson, P. 17. 612. N°. 17.

3) $\int \frac{x}{p + Cos.\, x} dx = \frac{\pi^2}{2\sqrt{(p^2-1)}} + \frac{4}{\sqrt{(p^2-1)}} \sum_0^\infty \frac{\{p - \sqrt{(p^2-1)}\}^{2n+1}}{(2n+1)^2}, p > 1$; Legendre, Exerc. 5. 84.

4) $\int \frac{x \, Sin.\, x}{p + Cos.\, x} dx = -\pi l\{2(1-p)\}$, $p < 1$;

5) $= -2\pi l\{1 - p + \sqrt{(p^2-1)}\}$, $p > 1$; Legendre, Exerc. 5. 75.

6) $\int \frac{x \, Sin.\, x}{i + Cos.\, x} dx = -2\pi l\{1 - (1 - \sqrt{2}) i\}$ Poisson, P. 17. 612. N°. 17.

7) $\int \frac{x \, Sin.\, x}{1 + p \, Cos.\, x} dx = \frac{\pi}{p} l \frac{1 + \sqrt{(1-p^2)}}{2(1-p)}, p < 1$; V. T. 353. N°. 9.

8) $\int \frac{x}{Cos.\, x + Cos.\, \lambda} dx = -4 \, Cosec.\, \lambda \sum_0^\infty \frac{Sin.\{(2n+1)\lambda\}}{(2n+1)^2}$ Legendre, Exerc. 5. 85.

9) $\int \frac{x^a \, Sin.\, x}{Cos.\, x + Cos.\, \lambda} dx = -\pi^a l\{2(1 - Cos.\lambda)\} + 2.1^{a/1} Cos.\frac{1}{2} a\pi \sum_0^\infty (-1)^{n-1} \frac{Cos.\, n\lambda}{n^{a+1}} -$

$- 2 \sum_1^\infty \left\{ \frac{Cos.\, n\lambda}{n} \sum_1^\infty (-1)^m a^{2m/-1} \pi^{a-2m} \frac{1}{n^{2m}} \right\}$ Legendre, Exerc. 5. 66.

10) $\int \frac{x^p \, Sin.\, x}{q + Cos.\, x} dx = 2 \, Cos.\frac{1}{2} p\pi \, \Gamma(1+p) \sum_1^\infty (-1)^{n-1} \frac{c^n}{n^{p+1}} - 2\pi^p \, l(1-c) -$

$- 2 \sum_1^\infty \left\{ \frac{c^n}{n} \sum_1^\infty (-1)^{m-1} p^{2m/-1} \pi^{a-2m} \frac{1}{n^{2m}} \right\}$ Legendre, Exerc. 5. 74.

, où $c = q - \sqrt{(q^2 - 1)}$;

11) $\int \frac{x \, Cos.\, x}{1 + 2q^2 \, Sin.\, x} dx = \frac{\pi}{q^2} l \frac{2}{1 + \sqrt{(1 + 4q^2)}}$ V. T. 327. N°. 10.

12) $\int \frac{x \, Sin.\, x}{1 + Cos.^2 x} dx = \frac{1}{4}\pi^2$ Poisson, P. 17. 612. N°. 17. — Grunert, Gr. 4. 118.

13) $\int \frac{p \, Cos.\, x + q}{Cos.^2 x + Cot.^2 \lambda} x \, Sin.\, x \, dx = 2p\pi l \, Cos.\frac{1}{2}\lambda + \pi q \, Tang.\, \lambda$ Legendre, Exerc. 5. 77.

1) $\int \frac{x^2}{1-Cos.x} dx = 4\pi l 2$ V. T. 238. N°. 3.

2) $\int \frac{x\, Sin.x}{1-Cos.x} dx = 2\pi l 2$

3) $\int \frac{x\, Sin.x}{2-Cos.x} dx = 2\pi l (3-\sqrt{3})$ } Poisson, P. 17. 612. N°. 16, 17.

4) $\int \frac{x}{p-Cos.x} dx = \frac{\pi^2}{2\sqrt{(p^2-1)}} - \frac{4}{\sqrt{(p^2-1)}} \sum_0^\infty \frac{\{p-\sqrt{(p^2-1)}\}^{2n+1}}{(2n+1)^2}, p>1;$

5) $\int \frac{x}{Cos.x - Cos.\lambda} dx = -4\, Cosec.\lambda \sum_0^\infty \frac{Sin.\{(2n+1)\lambda\}}{(2n+1)^2}$ } Legendre, Exerc. 5. 84, 85.

6) $\int \frac{x\, Sin.x}{p-Cos.x} dx = \pi l \{2(1+p)\}$ $, p<1;$

7) $= 2\pi l \{1+p-\sqrt{(p^2-1)}\}$ $, p>1;$ } Legendre, Exerc. 5. 75.

8) $\int \frac{x\, Sin.x}{Cos.\lambda - Cos.x} dx = 2\pi l\{1+Cos.\lambda - i\, Sin.\lambda\}$ } Poisson, P. 18. 295. N°. 37.

9) $\int \frac{x\, Sin.x}{e^p+e^{-p}-2\, Cos.x} dx = 2\pi l\{1+e^{-p}\}$ } , qui a faut. dans 9) $l(1-Cos.\lambda+i\, Sin.\lambda)$ et dans 10) $l(1-e^{-p})$;

10) $\int \frac{x\, Sin.x}{i-Cos.x} dx = 2\pi l\{1+(1-\sqrt{2})i\}$ Poisson, P. 17. 612. N°. 17.

11) $\int \frac{x\, Sin.x}{1-p\, Cos.x} dx = \frac{\pi}{p} l \frac{2(1+p)}{1+\sqrt{(1-p^2)}}, p<1;$ V. T. 353. N°. 9.

12) $\int \frac{x^a\, Sin.x}{Cos.x - Cos.\lambda} dx = -\pi^a l\{2(1+Cos.\lambda)\} - 2.1^{a/1}\, Cos.\frac{1}{2}a\pi \sum_0^\infty \frac{Cos.n\lambda}{n^{a+1}} +$

$+ 2\sum_1^\infty \left\{\frac{Cos.n\lambda}{n}(-1)^n \sum_1^\infty (-1)^m a^{2m/-1} \pi^{a-2m} \frac{1}{n^{2m}}\right\}$ Legendre, Exerc. 5. 66.

13) $\int \frac{x^p\, Sin.x}{q-Cos.x} dx = 2\, Cos.\frac{1}{2}p\pi\, \Gamma(1+p) \sum_1^\infty \frac{c^n}{n^{p+1}} + 2\pi^p l(1+c) +$

$+ 2\sum_1^\infty (-1)^n \frac{c^n}{n} \sum_1^\infty (-1)^{m-1} p^{2m/-1} \frac{\pi^{a-2m}}{n^{2m}}$ } , où $c = q - \sqrt{(q^2-1)}$; Legendre, Ex. 5. 74.

14) $\int \frac{x}{p^2 - Cos.^2 x} dx = \frac{\pi^2}{2p\sqrt{(p^2-1)}}, p>1;$ V. T. 245. N°. 3 et T. 246. N°. 4.

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15) $\int \frac{x}{Cos.^2\lambda - Cos.^2 x} dx = 0$ V. T. 245. N°. 8 et T. 246. N°. 5.

16) $\int \frac{x\, Sin.\, x}{p^2 - Cos.^2 x} dx = \frac{\pi}{2p} l \frac{1+p}{1-p}, p < 1;$

17) $= \frac{\pi}{2p} l \frac{p+1}{p-1}, p > 1;$

V. T. 245. N°. 4, 5 et T. 246. N° 6, 7.

18) $\int \frac{x\, Cos.\, x}{p^2 - Cos.^2 x} dx = \frac{-4}{\sqrt{(p^2-1)}} \sum_0^\infty \frac{\{p - \sqrt{(p^2-1)}\}^{2n+1}}{(2n+1)^2}, p > 1;$ V. T. 245. N°. 8 et T. 246. N°. 4.

19) $\int \frac{x\, Cos.\, x}{Cos.^2\lambda - Cos.^2 x} dx = 4\, Cosec.\, \lambda \sum_0^\infty \frac{Sin.\{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 245. N°. 8 et T. 246. N°. 5.

20) $\int \frac{x\, Sin.\, 2x}{p^2 - Cos.^2 x} dx = \pi l \{4(1-p^2)\}, p < 1;$

21) $= 2\pi l [2\{(p^2 - 1 + p\sqrt{p^2-1})\}], p > 1;$

V. T. 245. N°. 4, 5 et T. 246. N°. 6, 7.

F. Alg. rat. ent.
Circ. Dir. en dén. puiss. de binôme.

TABLE 247.

Lim. 0 et π.

1) $\int \frac{x\, Sin.\, x}{(1 - Cos.\lambda . Cos.\, x)^2} dx = \frac{1}{2}\pi\sqrt{2}.\, Cosec.\lambda .\, Sec.\frac{1}{2}\lambda .\, Cosec.\frac{\pi + 2\lambda}{4}$ Lobatschewsky, Mém. Kasan. 1835. 1.

2) $\int \frac{x\, Cos.\, x}{(1 + Cos.\lambda . Sin.\, x)^2} dx = 4\lambda\, Cosec.\, 2\lambda - \frac{\pi}{Cos.\,\lambda}$ V. T. 82. N°. 5.

3) $\int \frac{x\, Cos.\, x}{(1 - Cos.\lambda . Sin.\, x)^2} dx = 4(\lambda - \pi)\, Cosec.\, 2\lambda + \frac{\pi}{Cos.\,\lambda}$ V. T. 82. N°. 4.

4) $\int \frac{x\, Cos.\, x}{\left(1 - Sin.\, x .\, Cos.\frac{2a\pi}{b}\right)^2} dx = \pi\, Sec.\frac{2a\pi}{b} - \frac{2}{b} Cosec.\frac{4a\pi}{b} \sum_1^{b-1} Sin.\frac{2na\pi}{b} .\, Cot.\frac{n\pi}{b}$ V. T. 82. N°. 3.

5) $\int \frac{x\, Sin.\, x}{(Cos.\, x - a)^2} dx = \infty$ Legendre, Exerc. 5. 80.

6) $\int \frac{x^2\, Sin.\, x}{(p + Cos.\, x)^2} dx = \frac{-\pi^2}{\sqrt{(p^2-1)}} + \frac{\pi^2}{p-1} - \frac{8}{\sqrt{(p^2-1)}} \sum_0^\infty \frac{\{p - \sqrt{(p^2-1)}\}^{2n+1}}{(2n+1)^2}, p > 1;$ V. T. 245. N°. 3.

7) $\int \frac{x^2\, Sin.\, x}{(Cos.\, x + Cos.\, \lambda)^2}\, dx = \frac{-\pi^2}{1 - Cos.\, \lambda} + 8\, Cosec.\, \lambda \sum_0^\infty \frac{Sin.\{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 245. N°. 8.

8) $\int \frac{x^2\, Sin.\, x}{(p - Cos.\, x)^2}\, dx = \frac{\pi^2}{\sqrt{(p^2-1)}} - \frac{\pi^2}{p+1} - \frac{8}{\sqrt{(p^2-1)}} \sum_0^\infty \frac{\{p - \sqrt{(p^2-1)}\}^{2n+1}}{(2n+1)^2}, p > 1;$ V. T. 246. N°. 4.

9) $\int \frac{x^2\, Sin.\, x}{(Cos.\, x - Cos.\, \lambda)^2}\, dx = -\frac{\pi^2}{1 + Cos.\, \lambda} + 8\, Cosec.\, \lambda \sum_0^\infty \frac{Sin.\{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 246. N°. 5.

10) $\int \frac{p\, Cos.\, x + 1}{(p + Cos.\, x)^2}\, x^2\, dx = 4\pi\, l\, \{1 - p - \sqrt{(p^2-1)}\}\ , p \geqq 1;$

11) $= 2\pi\, l\, \{2(1-p)\}\ , p < 1;$ V. T. 245. N°. 4, 5.

12) $\int \frac{p\, Cos.\, x - 1}{(p - Cos.\, x)^2}\, x^2\, dx = -2\pi\, l\, \{2(1+p)\}\ , p < 1;$

13) $= -4\pi\, l\, \{1 + p - \sqrt{(p^2-1)}\}\ , p \geqq 1;$ V. T. 246. N°. 6, 7.

14) $\int \frac{x\, Sin.\, x}{(p + q\, Cos.\, x)^2}\, dx = \frac{\pi}{q}\left\{\frac{1}{p-q} - \frac{1}{\sqrt{(p^2-q^2)}}\right\}, p^2 > q^2;$ V. T. 82. N°. 6

15) $\int \frac{q\, Cos.\, 2x + Sin.^2\, x}{(q - Sin.^2\, x)^2}\, dx = \pi\, l(-4q)\ , q < 0;$

16) $\int \frac{q\, Cos.\, 2x - Sin.^2\, x}{(q + Sin.^2\, x)^2}\, dx = -2\pi\, l\left[2\{-q + \sqrt{q(q+1)}\}\right], q > 0;$ V. T. 246. N°. 20 21.

17) $\int \frac{p^2 - 1 - Sin.^2\, x}{(p^2 - Cos.^2\, x)^2}\, Cos.\, x\, dx = \frac{\pi}{p}\, l\, \frac{1-p}{1+p}, p < 1;$

18) $= \frac{\pi}{p}\, l\, \frac{p-1}{p+1}, p > 1;$ V. T. 246. N°. 16, 17.

19) $\int \frac{x^2\, Sin.\, 2x}{(p^2 - Cos.^2\, x)^2}\, dx = \frac{\pi^2}{p}\, \frac{\sqrt{(p^2-1)} - p}{p^2 - 1}, p > 1;$ V. T. 246. N°. 14.

20) $\int \frac{x\, Sin.\, 2x}{(1 - Cos.^2\, \lambda.\, Sin.^2\, x)^2}\, dx = \frac{Sin.\, \lambda - 1}{Cos.\, \lambda.\, Sin.\, 2\lambda}\, 2\pi$ V. T. 83. N°. 5.

21) $\int \frac{x^2\, Sin.\, 2x}{(Cos.^2\, \lambda - Cos.^2\, x)^2}\, dx = \pi^2\, Cosec.^2\, \lambda$ V. T. 246. N°. 15.

22) $\int \frac{1 + Cos.^2 \lambda . Sin.^2 x}{(1 - Cos.^2 \lambda\ Sin.^2 x)^2} x\, Cos.\, x\, dx = 2\, Cosec.\, 2\lambda (2\lambda - \pi)$ V. T. 83. N°. 6.

23) $\int \frac{x\, Sin.\, 2x}{(p^2 Sin.^2 x + q^2 Cos.^2 x)^2} dx = \frac{-\pi}{p q^2 (q + p)}$ V. T. 83. N°. 7.

24) $\int \frac{x\, Sin.\, 2x}{(p^2 Sin.^2 x + q^2 Cos.^2 x)^3} dx = \frac{-\pi}{4 p^3 q^4} \frac{p^2 + pq + q^2}{q + p}$ V. T. 83. N°. 8.

25) $\int \frac{x\, Sin.\, x}{(Cos.\, x + Cos.\, \lambda)^a} dx = \infty$
26) $\int \frac{x\, Sin.\, x}{(Cos.\, x - Cos.\, \lambda)^a} dx = \infty$
} $, a \geqq 2;$ Legendre, Exerc. 5. 79.

27) $\int \frac{Cos.\, kx}{(q + 2p\, Cos.\, x)^a} dx = \frac{a^{k/1}}{1^{a/2}} (q^2 - 4p^2)^{-\frac{a}{2}} \left\{ \frac{-4p}{q + \sqrt{(q^2 - 4p^2)}} \right\}^k \sqrt{\frac{\pi}{k}}, k = \infty;$ Jacobi, Cr. 15. 1.

F. Alg. rat. ent. Circ. Dir. en dén. trin. $1 - a\, Cos.\, x + b$. TABLE 248. Lim. 0 et π.

1) $\int \frac{x\, Sin.\, x}{1 - 2p\, Cos.\, x + p^2} dx = \frac{\pi}{p} l (1 + p), p < 1;$
2) $= \frac{\pi}{p} l \frac{1 + p}{p}, p > 1;$
} Poisson, P. 17. 612. N°. 16.

3) $\int \frac{x\, Sin.\, x}{1 - Sin.^2 \lambda . Cos.\, x + Cos^2 \lambda} dx = -4\pi\, Cosec.^2 \lambda\, l\, Cos.\, \frac{1}{2}\lambda$ V. T. 334. N°. 13.

4) $\int \frac{Cos.\, bx}{1 - 2p\, Cos.\, x + p^2} x^{\pm 2a} dx = (-1)^a \pi \frac{p^b}{1 - p^2} (lp)^{\pm 2a}$

5) $\int \frac{Cos.\, bx . Cos.\, x}{1 - 2p\, Cos.\, x + p^2} x^{\pm 2a} dx = (-1)^a \frac{1}{2} \pi \frac{1 + p^2}{1 - p^2} p^{b-1} (lp)^{\pm 2a}$

6) $\int \frac{Cos.\, bx . Sin.\, x}{1 - 2p\, Cos.\, x + p^2} x^{\pm(2a+1)} dx = \pm (-1)^a \frac{1}{2} \pi p^{b-1} (lp)^{\pm(2a+1)}$

7) $\int \frac{Sin.\, bx}{1 - 2p\, Cos.\, x + p^2} x^{\pm(2a+1)} dx = \pm (-1)^{a+1} \pi \frac{p^b}{1 - p^2} (lp)^{\pm(2a+1)}$

8) $\int \frac{Sin.\, bx . Sin.\, x}{1 - 2p\, Cos.\, x + p^2} x^{\pm 2a} dx = (-1)^a \frac{1}{2} \pi p^{b-1} (lp)^{\pm 2a}$

9) $\int \frac{Sin.\, bx . Cos.\, x}{1 - 2p\, Cos.\, x + p^2} x^{\pm(2a+1)} dx = \pm (-1)^{a+1} \frac{1}{2} \pi \frac{1 + p^2}{1 - p^2} p^{b-1} (lp)^{\pm(2a+1)}$

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10) $\int \frac{Cos.\{(2b-1)x\}}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a}\,dx = 0$

11) $\int \frac{Sin.\{(2b-1)x\}}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a+1}\,dx = 0$

12) $\int \frac{Cos.\,2bx.\,Cos.\,x}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a}\,dx = 0$

13) $\int \frac{Cos.\,2bx.\,Sin.\,x}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a+1}\,dx = 0$

14) $\int \frac{Sin.\,2bx.\,Sin.\,x}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a}\,dx = 0$

15) $\int \frac{Sin.\,2bx.\,Cos.\,x}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a+1}\,dx = 0$

16) $\int \frac{Cos.\{(2b-1)x\}.\,Cos.2x}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a}\,dx = 0$

17) $\int \frac{Cos.\{(2b-1)x\}.\,Sin.\,2x}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a+1}\,dx = 0$

18) $\int \frac{Sin.\{(2b-1)x\}.\,Sin.2x}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a}\,dx = 0$

19) $\int \frac{Sin.\{(2b-1)x\}.\,Cos.2x}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a+1}\,dx = 0$

20) $\int \frac{Cos.\{(2b+1)x\}.\,Cos.x}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a}\,dx = (-1)^a \frac{\pi}{2^{\pm 2a+2}} \frac{q^b}{1-q} (lq)^{\pm 2a}$

21) $\int \frac{Cos.\{(2b+1)x\}.\,Sin.x}{1-2q\,Cos.\,2x+q^2} x^{\pm(2a+1)}\,dx = \pm(-1)^a \frac{\pi}{2^{\pm(2a+1)+2}} \frac{q^b}{1+q} (lq)^{\pm(2a+1)}$

22) $\int \frac{Sin.\{(2b+1)x\}.\,Sin.x}{1-2q\,Cos.\,2x+q^2} x^{\pm 2a}\,dx = (-1)^a \frac{\pi}{2^{\pm 2a+2}} \frac{q^b}{1+q} (lq)^{\pm 2a}$

23) $\int \frac{Sin.\{(2b+1)x\}.\,Cos.x}{1-2q\,Cos.\,2x+q^2} x^{\pm(2a+1)}\,dx = \pm(-1)^{a+1} \frac{\pi}{2^{\pm(2a+1)+2}} \frac{q^b}{1-q} (lq)^{\pm(2a+1)}$

Les intégrales (4) à (23) où $p^2 \leqq 1$, $0 \leqq q < 1$, se trouvent: Bierens de Haan, Gr. 13. 193.

24) $\int \frac{x\,Sin.\,2x}{1-2p\,Cos.\,2x+p^2}\,dx = \frac{\pi}{2p} l(1-p)$, $p < 1$; V. T. 353. N°. 23.

1) $\int \frac{x\, Sin.\, x}{1+2p\, Cos.\, x+p^2}\, dx = -\frac{\pi}{p}\, l\,(1-p)\,,\ p \leqq 1;$

2) $\qquad = \frac{\pi}{p}\, l\, \frac{p}{p-1}\qquad ,\ p > 1;$ V. T. 353. N°. 19, 20.

3) $\int \frac{x\, Sin.\, x}{2+p\, Cos.\, x+p}\, dx = \frac{2\pi}{p}\, l\, \frac{1+\sqrt{(1+p)}}{2}$ V. T. 334. N°. 8.

4) $\int \frac{Cos.\, x+q\, Cos.\, \lambda}{q^2+2q Cos.\lambda. Cos.x+Cos.^2 x} x Sin.x dx = -\pi l\{1-2q Cos.\lambda+2h Cos.\theta+ +q^2+h^2-2gh Cos.(\lambda-\theta)\}$

5) $\int \frac{x Sin.x}{q^2+2q Cos.\lambda. Cos.x+Cos.^2 x} dx = \frac{2}{q}\pi Cosec.\lambda. Arctang. \frac{h Sin.\theta-q Sin.\lambda}{1-q Cos.\lambda+h Cos.\theta}$

6) $\int \frac{p Cos.x+r}{q^2+2q Cos.\lambda. Cos.x+Cos.^2 x} x Sin.x dx = -2\pi p l\{1-2q Cos.\lambda+2h Cos\,\theta+ +q^2+h^2-2gh Cos.(\lambda-\theta)\}+\frac{r-pq Cos.\lambda}{q Sin.\lambda} 2\pi Arctang. \frac{h Sin\,\theta-q Sin.\lambda}{1-q Cos.\lambda+h Cos\,\theta}$

Legendre, Exerc. 5. 77., où $Tang\ 2\theta = \frac{q^2 Sin.2\lambda}{q^2 Cos.2\lambda-1}$, $h^4 = 1-2q^2\, Cos.2\lambda+q^4$;

7) $\int \frac{x\, Sin.\, x}{(Cos.\, x-q)^2-k^2}\, dx = -\frac{\pi}{1+q}$, k infiniment petit > 0; Legendre, Exerc. 5. 81.

8) $\int \frac{\frac{\pi}{2}+x}{Sin.^2\, x+(a\, Sin.\, x+b\, Cos.\, x)^2}\, dx = \frac{\pi}{b}\left[\frac{1}{2} Arctang.\left(\frac{2ab}{1+a^2-b^2}\right) - - Arctang. \frac{Tang.\left\{\frac{1}{2} Arctang.\left(\frac{2ab}{1+a^2-b^2}\right)\right\}}{a Tang.\left\{\frac{1}{2} Arctang.\left(\frac{2ab}{1+a^2-b^2}\right)\right\}-b}\right]$ V. T. 271. N°. 2.

9) $\int \frac{Sin.\, x}{1-Cos.\lambda. Cos.x}\, \frac{x}{1-Cos.\mu. Cos.x}\, dx = \pi Cosec.\left\{\frac{1}{2}(\lambda+\mu)\right\}. Cosec.\left\{\frac{1}{2}(\lambda-\mu)\right\}. l\, \frac{1+Tang.\frac{1}{2}\lambda}{1+Tang.\frac{1}{2}\mu}$ Lobatschewsky, Mém. Kasan. 1835. 1.

10) $\int \frac{(1+p^2)\, Cos.\, x-2p}{(1-2p\, Cos.\, x+p^2)^2}\, x^2\, dx = 2\frac{\pi}{p}\, l\, \frac{p}{1+p}\qquad ,\ p \geqq 1;$

11) $\qquad = -2\frac{\pi}{p}\, l\,(1+p)\,,\ p < 1;$ V. T. 248. N°. 1, 2.

12) $\int \frac{x\, Sin.\, x}{(1+p^2-2p\, Cos.\, x)^2}\, dx = \frac{\pi}{(1-p)(1+p)^2}\,,\ p < 1;$ V. T. 84. N°. 1.

13) $\int \frac{x \, Sin. x}{(1+p^2-2p \, Cos. x)^2} dx = \frac{\pi}{p(p-1)(1+p)^2}, p > 1$; V. T. 84. N°. 2.

14) $\int \frac{x \, Sin. x}{(1+p^2+2p \, Cos. x)^2} dx = \frac{\pi}{(1+p)(1-p)^2}, p < 1$;

15) $= \frac{\pi}{p(1+p)(p-1)^2}, p > 1$; V. T. 84. N°. 9.

16) $\int \frac{x \, Sin. x}{(1+p^2-2p \, Cos. x)^3} dx = \frac{p^2-p+2}{2(1+p)^4(1-p)^3}, p < 1$;

17) $= \frac{2p^2-p+1}{2p(1+p)^4(p-1)^3}, p > 1$; V. T. 85. N°. 6, 7.

18) $\int \frac{x \, Sin. x}{(1+p^2-2p \, Cos. x)^{a+1}} dx = \frac{\pi}{2pa}\left\{\frac{-1}{(1+p)^{2a}} + \frac{1}{(1-p^2)^{2a-1}} \sum_0^{a-1} \binom{a-1}{n}^2 p^{2n}\right\}, p^2 < 1$;

19) $= \frac{\pi}{2pa}\left\{\frac{-1}{(1+p)^{2a}} + \frac{1}{(p^2-1)^{2a-1}} \sum_0^{a-1} \binom{a-1}{n}^2 p^{2n}\right\}, p^2 > 1$; V. T. 85. N°. 12, 13.

20) $\int \frac{x \, Tang. x}{(1+p^2-2p \, Cos. x)^2} \frac{dx}{Cos. x} = -\pi \frac{1+p+3p^2-p^3}{(1+p)^2(1+p^2)^2(1-p)}, p^2 < 1$;

21) $= \pi \frac{1-3p-p^2-p^3}{(1+p)^2(1+p^2)^2(p-1)}, p^2 > 1$; V. T. 85. N°. 4, 5, d'après Schlömilch; elles sont ∞.

22) $\int \frac{x \, Sin. x}{\sqrt{(1-2p \, Cos. x+p^2)}} dx = \frac{1+p}{p}\pi + 2\frac{1-p^2}{p} F'(p) - \frac{4}{p} E'(p), p < 1$; Ramus, Danske Afh. 6. 265.

23) $\int \frac{p \, Cos.^2 x - (1+p^2) Cos. x + p}{\sqrt{(1-2p \, Cos. x+p^2)^3}} x^2 dx = \frac{1+p}{p} 2\pi + 4\frac{1-p^2}{p} F'(p) - \frac{8}{p} E'(p), p < 1$; V. T. 249. N°. 22.

24) $\int \frac{Sin. x}{x} Cos. \frac{2a\pi x}{b} dx = \frac{1}{2}\pi, a < b$; Schaar, Mém. Cour. Brux. T. 22.

F. Alg. rat. Circ. Dir. TABLE 250. Lim. 0 et 2π.

1) $\int x \, Sin. x \, dx = -2\pi$

2) $\int x \, Cos. x \, dx = 0$ Raabe, Cr. 15. 355.

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3) $\int \frac{Sin.ax}{1+pCos.x} xdx = \frac{2\pi}{\sqrt{(1-p^2)}}\left[\frac{\{-\sqrt{(1-p^2)}-1\}^a - \{\sqrt{(1-p^2)}-1\}^a}{p^a} l \frac{2\sqrt{(1+p)}}{\sqrt{(1+p)}+\sqrt{(1-p)}} + \sum_1^{a-1} \frac{1}{a-n} \frac{\{-\sqrt{(1-p^2)}-1\}^n - \{\sqrt{(1-p^2)}-1\}^n}{a^n}\right], p < 1;$ Ohm, Ausw. 26.

4) $\int \frac{Sin.ax}{1-pCos.x} xdx = \frac{2\pi}{\sqrt{(1-p^2)}}\left[\frac{\{1+\sqrt{(1-p^2)}\}^a - \{1-\sqrt{(1-p^2)}\}^a}{p^a} l \frac{2\sqrt{(1-p)}}{\sqrt{(1+p)}+\sqrt{(1-p)}} + \sum_1^{a-1} \frac{1}{a-n} \frac{\{1+\sqrt{(1-p^2)}\}^n - \{1-\sqrt{(1-p^2)}\}^n}{a^n}\right], p < 1;$ Raabe, Int. 173, — Ohm, Ausw. 26.

5) $\int \frac{Cos.\,a\,x}{1+p\,Cos.\,x} x\,dx = \frac{2\pi^2}{\sqrt{(1-p^2)}}\left\{\frac{1-\sqrt{(1-p^2)}}{p}\right\}^a, p < 1;$

6) $\int \frac{Cos.\,a\,x}{1-p\,Cos.\,x} x\,dx = \frac{2\pi^2}{\sqrt{(1-p^2)}}\left\{\frac{\sqrt{(1-p^2)}-1}{p}\right\}^a$ Raabe, Int. 173. — Ohm, Ausw. 26.

7) $\int \frac{x\,Sin.\,x}{1-2p\,Cos.x+p^2} dx = \frac{2\pi}{p} l(1-p), p<1;$ Raabe, Int. 173.

8) $\int \frac{x\,Sin.\,a\,x}{1-2p\,Cos.\,x+p^2} dx = \frac{2\pi}{1-p^2}\left\{(p^{-a}-p^a)l(1-p) + \sum_1^{a-1} \frac{p^{-n}-p^n}{a-n}\right\}$ Raabe, Int. 173.

9) $\int \frac{Cos.\,b\,x}{1-2p\,Cos.\,x+p^2} x^{\pm 2a} dx = (-1)^a \pi \frac{p^b}{1-p^2}(l\,p)^{\pm 2a}$

10) $\int \frac{Cos.\,b\,x.\,Cos.\,x}{1-2p\,Cos.\,x+p^2} x^{\pm 2a} dx = (-1)^a \frac{1}{2}\pi \frac{1+p^2}{1-p^2} p^{b-1}(l\,p)^{\pm 2a}$

11) $\int \frac{Cos.\,b\,x.\,Sin.\,x}{1-2p\,Cos.\,x+p^2} x^{\pm(2a+1)} dx = \pm(-1)^a \frac{1}{2}\pi p^{b-1}(l\,p)^{\pm(2a+1)}$

12) $\int \frac{Sin.\,b\,x}{1-2p\,Cos.\,x+p^2} x^{\pm(2a+1)} dx = \pm(-1)^{a-1} \pi \frac{p^b}{1-p^2}(l\,p)^{\pm(2a+1)}$

13) $\int \frac{Sin.\,b\,x.\,Sin.\,x}{1-2p\,Cos.\,x+p^2} x^{\pm 2a} dx = (-1)^a \frac{1}{2}\pi p^{b-1}(l\,p)^{\pm 2a}$

14) $\int \frac{Sin.\,b\,x.\,Cos.\,x}{1-2p\,Cos.\,x+p^2} x^{\pm(2a+1)} dx = (-1)^{a+1} \frac{1}{2}\pi \frac{1+p^2}{1-p^2} p^{b-1}(l\,p)^{\pm(2a+1)}$

(9–14) $, 0 \leq p \leq 1;$ Bierens de Haan, Gr. 13. 193.

15) $\int \frac{Cos.\,a\,x - p\,Cos.\{(a+1)\,x\}}{1-2p\,Cos.\,x+p^2} x\,dx = 2\pi^2 p^a, p<1;$ Raabe, Int. 173.

1) $\int x\, Sin.(2a\pi x^2)\, dx = \frac{1}{2a\pi}\left\{1 - Cos.(2a\pi r^2)\right\}$

2) $\int x\, Cos.(2a\pi x^2)\, dx = \frac{1}{2a\pi} Sin.(2a\pi r^2)$ Abria, L. 4. 248.

3) $\int \frac{Sin.x}{x} dx = Si.(r)$ Arndt, Gr. 10. 225. — Schlömilch, Gr. 11. 389.

4) $\int \frac{Sin.qx}{x} dx = Si.(qr)$ Schlömilch, Gr. 11. 389.

5) $\int \frac{1 - Cos.rx}{x^2} dx = \frac{1}{2} r\pi \quad , r > 0;$

6) $\qquad = -\frac{1}{2} r\pi, r < 0;$

7) $\int \frac{Sin.rx - rx\, Cos.rx}{x^3} dx = \frac{1}{4} r^2 \pi \quad , r > 0;$

8) $\qquad = -\frac{1}{4} r^2 \pi, r < 0;$ Poisson, Mém. Inst. 1816. 71.

9) $\int \frac{\frac{x}{r} Sin.\frac{r\lambda}{x} - \frac{r}{x} Sin.\frac{\lambda x}{r}}{\frac{x}{r} - \frac{r}{x}} \frac{dx}{x} = \frac{1}{2}\pi(1 - Cos.\lambda)$

10) $\int \frac{\left(\frac{x}{r}\right)^{1-p} Sin.\left(\frac{1}{2}p\pi - \frac{r\lambda}{x}\right) - \left(\frac{r}{x}\right)^{1-p} Sin.\left(\frac{1}{2}p\pi - \frac{\lambda x}{r}\right)}{\frac{x}{r} - \frac{r}{x}} \frac{dx}{x} = \frac{1}{2}\pi\, Cos.\left(\frac{1}{2}p\pi - \lambda\right)$ Cauchy, P. 29. 511.

11) $\int \frac{x}{Cos.x.\, Cos.(r - x)} dx = r\, Cosec.r.\, l\, Sec.r, r < \frac{1}{2}\pi;$ Lindman, Gr. 16. 94.

12) $\int \frac{x^2\, Sin.(2x - r)}{Cos.^2 x.\, Cos.^2 (r - x)} dx = r^2\, Sec.r. + 2r\, Cosec.r.\, l\, Cos.r, r < \frac{1}{2}\pi;$ V. T. 251. N°. 11.

13) $\int \frac{x Sin.x}{(Cos.x - Cos.\mu)^3} dx = \frac{-\frac{1}{2}r}{Cos.r - Cos.\mu} + \frac{Sin.r}{2Sin.^2\mu(Cos.r - Cos.\mu)} + \frac{Cos.\mu}{2Sin.^3\mu}\, l\frac{Sin.\left\{\frac{1}{2}(\mu + r)\right\}}{Sin.\left\{\frac{1}{2}(\mu - r)\right\}}, r < \mu;$ Legendre, Exerc. 5. 83.

F. Alg. rat. Circ. Dir. sous forme irrat.	TABLE 252.	Lim. 0 et λ.

1) $\int x\, Sin.x\, dx \sqrt{(Sin.^2\lambda - Sin.^2 x)} = \frac{1}{8}\pi\, Sin.^2\lambda + \frac{1}{4}\pi\, Cos.^2\lambda.\, l\, Cos.\lambda$

2) $\int \frac{x\,dx\sqrt{(Sin.^2\lambda - Sin.^2 x)}}{Sin.x} = \pi\frac{1+Sin\,\lambda}{4}\, l(1+Sin.\lambda) + \pi\frac{1-Sin.\lambda}{4}\, l(1-Sin.\lambda)$

3) $\int \frac{x\,Sin.x}{\sqrt{(Sin.^2\lambda - Sin.^2 x)}}\, dx = \frac{1}{2}\pi\, l\, Sec.\lambda$

4) $\int \frac{x\,Sin.^3 x}{\sqrt{(Sin.^2\lambda - Sin.^2 x)}}\, dx = -\pi\frac{1+Sin.^2\lambda}{4}\, l\, Cos.\lambda - \frac{1}{8}\pi\, Sin.^2\lambda$

Legendre, Exerc. Suppl. 18, (1–4)

5) $\int \frac{x}{Sin.x\sqrt{(Sin.^2\lambda - Sin.^2 x)}}\, dx = \frac{1}{4}\pi\, Cosec.\lambda.\, l\frac{1+Sin.\lambda}{1-Sin.\lambda}$ Legendre, Exerc. Suppl. 23.

6) $\int \frac{x\,Sin.x}{Cos^2 x\sqrt{(Sin.^2\lambda - Sin.^2 x)}}\, dx = \frac{1}{2}\pi\, Sec.^2\lambda\,(1-Cos.\lambda)$ Legendre, Exerc. Suppl. 23. — Lobatschewsky, Mém. Kasan. 1835. 1. — Id., Mém. Kasan. 1836. 1. I. form. (82). — Id., ib. II. form. (18).

7) $\int \frac{x\,Sin.x}{1-Sin.^2\mu.Sin.^2 x}\frac{dx}{\sqrt{(Sin.^2\lambda - Sin.^2 x)}} = \frac{\pi}{2Cos.\mu}\frac{1}{1-Sin.^2\lambda.Sin.^2\mu}\, l\frac{Cos.\mu+\sqrt{(1-Sin.^2\lambda.Sin.^2\mu)}}{2Cos.\lambda.Sin.^2\frac{1}{2}\mu}$

Lobatschewsky, Mém. Kasan. 1835. 1. — Id., Mém. Kasan. 1836. 1. I form. (79). — Id., ib. II. form. (17).

8) $\int \frac{x\,Sin.x}{Sin.^2\mu - Sin.^2 x}\frac{dx}{\sqrt{(Sin.^2\lambda - Sin.^2 x)}} = \frac{\pi\, Sec.\mu}{2\sqrt{(Sin.^2\mu - Sin.^2\lambda)}}\left\{r') - Arccos.\frac{Cos.\mu}{Cos.\lambda}\right\}$

Lobatschewsky, Mém. Kasan. 1835. I. — Id., Mém. Kasan. 1836. 1. I form. (81). — Id, ib. II. form. (19).

9) $\int \frac{1-x\,Cot\,x}{Sin.^2 x\sqrt{(Sin.^2\lambda - Sin.^2 x)}}\, Cos.x\, dx = \frac{1}{4}\pi\, Cosec.^3\lambda - \frac{1}{8}\pi\, Cos.^2\lambda.Cosec.^3\lambda.\, l\frac{1+Sin.\lambda}{1-Sin.\lambda}$ Legendre, Exerc. Suppl. 17.

F. Alg. rat. Circ. Dir. sous forme irrat.	TABLE 253.	Lim. λ et μ.

1) $\int \frac{x}{\sqrt{(Sin.^2 x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2 x)}}\, dx = \frac{1}{2}\pi\, Sec.\lambda.\, Cosec.\mu\, F'(c, \mu)$

2) $\int \frac{x}{Sin.^2 x\sqrt{(Sin.^2 x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2 x)}}\, dx = \pi\frac{Sin.\lambda - Sin.\mu}{2\, Sin.^2\lambda.\, Sin.^2\mu} +$

$+ \frac{\pi}{2\, Cos.\lambda.\, Sin.\mu} F(c, \mu) + \frac{\pi\, Cos.\lambda}{2\, Sin.^2\lambda.\, Sin.\mu} E(c, \mu)$

Legendre, Exerc. Suppl. 15. — Roberts, L. 11. 157. (1–2)

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3) $(2a+1)\,Sin.^2\lambda.\,Sin.^2\mu \int_\lambda^\mu \frac{x}{Sin.^{2a+2}x\sqrt{(Sin.^2x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2x)}}\,dx =$

$$2a(Sin.^2\lambda + Sin.^2\mu + Sin.^2\lambda.Sin.^2\mu)\int_\lambda^\mu \frac{x\,dx}{Sin.^{2a}x\sqrt{(Sin.^2x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2x)}}$$

$$-(2a-1)(1+Sin.^2\lambda+Sin.^2\mu)\int_\lambda^\mu \frac{x\,dx}{Sin.^{2a-2}x\sqrt{(Sin.^2x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2x)}}$$ Legendre, Exerc. S. 15.

$$+(2a-2)\int_\lambda^\mu \frac{x\,dx}{Sin.^{2a-4}x\sqrt{(Sin.^2x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2x)}}$$

$$-\frac{\pi(Sin.^2\mu - Sin.^2\lambda)^2}{4\,Sin.^{2a-1}\lambda.\,Sin.^4\mu}\sum_0^\infty(-1)^n\binom{a-2}{n}\frac{3^{n/2}}{4^{n+1/2}}\left(\frac{Sin.^2\mu - Sin.^2\lambda}{Sin.^2\mu}\right)^n$$

4) $\int \frac{x}{Cos.^2x\sqrt{(Sin.^2x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2x)}}\,dx = \frac{\pi}{2}\,\frac{Cos.\mu - Cos.\lambda}{Cos.^2\lambda.Cos.\mu} +$

$$+\frac{\pi}{2\,Cos.\lambda.\,Sin.\mu}\,F(c,\mu) + \frac{\pi\,Sin.\mu}{2\,Cos.\lambda.\,Cos.^2\mu}\,E(c,\mu)$$ Legendre, Exerc. Suppl. 19. — Roberts, L. 11. 157.

5) $(2a+1)\,Cos.^2\lambda.\,Cos.^2\mu \int_\lambda^\mu \frac{x}{Cos.^{2a+2}x\sqrt{(Sin.^2x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2x)}}\,dx =$

$$2a(Cos.^2\lambda + Cos.^2\mu + Cos.^2\lambda.\,Cos.^2\mu)\int_\lambda^\mu \frac{x\,dx}{Cos.^{2a}x\sqrt{(Sin.^2x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2x)}}$$

$$-(2a-1)(1+Cos.^2\lambda+Cos.^2\mu)\int_\lambda^\mu \frac{x\,dx}{Cos.^{2a-2}x\sqrt{(Sin.^2x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2x)}}$$ Legendre, Exerc. S. 21.

$$+(2a-2)\int_\lambda^\mu \frac{x\,dx}{Cos.^{2a-4}x\sqrt{(Sin.^2x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2x)}}$$

$$+\frac{\pi\,Cos.\mu(Cos.^2\lambda - Cos.^2\mu)^2}{4\,Cos.^{2a-1}\mu.\,Cos.^4\lambda}\sum_0^\infty(-1)^n\binom{a-2}{n}\frac{1^{n+1/2}}{4^{n+1/2}}\left(\frac{Cos.^2\lambda - Cos.^2\mu}{Cos.^2\lambda}\right)^n$$

6) $\int \frac{x\,Sin.^2x}{\sqrt{(Sin.^2x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2x)}}\,dx = \frac{\pi}{2\,Cos.\lambda.\,Sin.\mu}\,F(c,\mu) -$

$$-\frac{\pi\,Cos.^2\mu}{2\,Cos.\lambda.\,Sin.\mu}\,\Pi(-Sin.^2\theta, c, \mu) - \frac{1}{4}\pi\,l(1+Sin.^2\mu - Sin.^2\lambda)$$ Legendre, Exerc. S. 27.

7) $\int \frac{x \,Sin.^4 x}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)}} dx = \frac{Sin.^2 \mu - Sin.^2 \lambda}{8} \pi +$

$+ \frac{\pi}{4} \frac{2 - Cos.^2\lambda . Cos.^2\mu}{Cos.\lambda . Sin.\mu} F(c,\mu) - \frac{\pi}{4} Cos.\lambda . Sin.\mu E(c,\mu) - \frac{1 + Sin.^2\lambda + Sin.^2\mu}{8} \pi l(1 - Sin.^2\lambda + Sin.^2\mu)$

$- \frac{\pi \,Cos.^2 \mu}{4 \,Cos.\lambda . Sin.\mu} (1 + Sin.^2 \lambda + Sin.^2 \mu) \Pi (- Sin.^2 \theta, c, \mu)$

8) $2a \int_\lambda^\mu \frac{x \,Sin.^{2a+2} x}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)}} dx =$

$(2a - 1)(1 + Sin.^2 \lambda + Sin.^2 \mu) \int_\lambda^\mu \frac{x \,Sin.^{2a} x \,dx}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)}}$

$- (2a - 2)(Sin.^2 \lambda + Sin.^2 \mu + Sin.^2 \lambda . Sin.^2 \mu) \int_\lambda^\mu \frac{x \,Sin.^{2a-2} x \,dx}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)}}$

$+ (2a - 3) Sin.^2 \lambda . Sin.^2 \mu \int_\lambda^\mu \frac{x \,Sin.^{2a-4} x \,dx}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)}}$

$- \frac{\pi}{4} \frac{(Sin.^2 \mu - Sin.^2 \lambda)^2}{Sin.^{3-2a} \mu} \sum_0^\infty (-1)^n \binom{a-2}{n} \frac{1^{n+1/2}}{4^{n+1/2}} \left(\frac{Sin.^2 \mu - Sin.^2 \lambda}{Sin.^2 \mu}\right)^n$

Legendre, Exerc. Suppl. 27.

9) $\int x \,dx \sqrt{\frac{Sin.^2 \mu - Sin.^2 x}{Sin.^2 x - Sin.^2 \lambda}} = \frac{1}{4} \pi l (1 - Sin.^2 \lambda + Sin.^2 \mu) +$

$+ \frac{\pi \,Cos.^2 \mu}{2 \,Cos.\lambda . Sin.\mu} \Pi (- Sin.^2 \theta, c, \mu) - \frac{\pi \,Cos.^2 \mu}{2 \,Cos.\lambda . Sin.\mu} F(c, \mu)$ Legendre, Exerc. Suppl. 25.

10) $\int x \,dx \sqrt{\frac{Sin.^2 x - Sin.^2 \lambda}{Sin.^2 \mu - Sin.^2 x}} = - \frac{1}{4} \pi l (1 - Sin.^2 \lambda + Sin.^2 \mu) -$

$- \frac{\pi \,Cos.^2 \mu}{2 \,Cos.\lambda . Sin.\mu} \Pi (- Sin.^2 \theta, c, \mu) + \frac{\pi \,Cos.\lambda}{2 \,Sin.\mu} F(c, \mu)$ Legendre, Exerc. Suppl. 26.

11) $\int \frac{x \,Tang.^2 x}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)}} dx = \frac{\pi \,Sin.\mu}{2 Cos.\lambda . Cos.^2 \mu} \{E(c,\mu) - Cot.\mu + Cot\,\mu . Cos.\mu . Sec.\lambda\}$

12) $\int \frac{x}{Tg.^2 x \sqrt{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin^2 x)}} dx = \frac{\pi}{2 Sin.\lambda . Tg.\lambda . Sin.\mu} \left\{E(c,\mu) + \frac{Sin.\lambda - Sin.\mu}{Cos.\lambda}\right\}$

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13) $$\int \frac{x}{Cos.^4 x \sqrt{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)}} dx = \frac{\pi}{Cos.^2 \lambda . Cos.^2 \mu} \left\{ \frac{Cos.^2 \lambda + Cos.^2 \mu + Cos.^2 \lambda . Cos.^2 \mu}{3 Cos.^2 \lambda} \right.$$
$$- \frac{3 Cos.^2 \lambda + 3 Cos.^2 \mu + 4 Cos.^2 \lambda . Cos.^2 \mu + 2 Cos. \lambda . Cos \mu}{12 Cos. \lambda . Cos. \mu} + \frac{2 Cos.^2 \lambda . Cos.^2 \mu + Cos.^2 \lambda + Cos.^2 \mu - 1}{6 Cos. \lambda . Sin. \mu} F(c, \mu)$$
$$\left. + \frac{Cos.^2 \lambda + Cos.^2 \mu + Cos.^2 \lambda . Cos.^2 \mu}{3 Cos. \lambda . Cos. \mu} Tang. \mu E(c, \mu) \right\}$$

14) $$\int \frac{Sin. x . Cos. x}{1 - p^2 Sin.^2 x} \frac{dx}{\sqrt{\{Sin.^2 x - Sin.^2 \lambda\}\{Sin.^2 \mu - Sin.^2 x\}}} = \frac{\pi}{2 \sqrt{\{(1 - p^2 Sin.^2 \lambda)(1 - p^2 Sin.^2 \mu)\}}}$$

Sur les formules (11) à (14) voyez: Roberts, L. 11. 157.

Partout on a ici: $Cos. \theta = Cos. \mu . Sec. \lambda$, $c = Sin. \theta . Cosec. \mu$.

F. Alg. rat. fract. Circ. Dir. — TABLE 254. — Lim. diverses p et $\pm \infty$.

1) $$\int_1^\infty \frac{Cos. p x}{x} dx = - Ci. (p)$$

2) $$= - A - \frac{1}{2} l p^2 - \sum_1^\infty (-1)^n \frac{p^{2n}}{2n . 1^{2n/1}}$$

(1, 2) Arndt, Gr. 10. 225.

3) $$\int_1^\infty \frac{Cos. k x}{x} dx = 0, \quad k = \infty;$$ Raabe, Int. 202.

4) $$\int_p^\infty \frac{Sin. x}{x} dx = \frac{1}{2} \pi - Si. (p)$$ Schlömilch, Stud. II. 21.

5) $$\int_p^\infty \frac{Cos. x}{x} dx = - Ci. (p)$$

6) $$= - A - \frac{1}{2} l p^2 + \sum_1^\infty (-1)^{n-1} \frac{1}{2n} \frac{p^{2n}}{1^{2n/1}}$$

(5, 6) Arndt, Gr. 10. 225. — Schlömilch, Gr. 11. 389.

7) $$\int_p^\infty \frac{Cos. q x}{x} dx = - Ci. (p q)$$ Schlömilch, Gr. 11. 389.

8) $$\int_p^\infty \frac{Sin. x}{x^{2a}} dx = \frac{(-1)^a}{1^{2a-1/1}} \left(A - \sum_1^{2a-1} \frac{1}{n} \right) - \sum_1^{a-1} \frac{(-1)^n}{1^{2n-1/1}} \frac{1}{2(a-n) p^{2(a-n)}} + \frac{(-1)^a}{1^{2a-1/1}} l p + \sum_1^\infty \frac{(-1)^{a+n}}{1^{2a+2n-1/1}} \frac{p^{2n-1}}{2n-1}$$

9) $$\int_p^\infty \frac{Cos. x}{a^{2a+1}} dx = \frac{(-1)^{a+1}}{1^{2a/1}} \left(A - \sum_1^{2a} \frac{1}{n} \right) - \sum_1^{a} \frac{(-1)^n}{1^{2n-2/1}} \frac{1}{2(a+1-n) p^{2(a-n+1)}} + \frac{(-1)^{a-1}}{1^{2a/1}} l p - \sum_1^\infty \frac{(-1)^{a+n}}{1^{2a+2n+1/1}} \frac{p^{2n}}{2n}$$

(8, 9) Arndt, Gr. 10. 233.

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10) $\int_p^{\infty} \frac{Sin.\, p\, x}{x - q}\, d\, x = \frac{1}{2}\pi\, Cos.\, p\, q - (\Lambda + l\, q + l\, 0)\, Sin.\, p\, q$

11) $\int_{-\infty}^{p} \frac{Sin.\, p\, x}{x - q}\, d\, x = \frac{1}{2}\pi\, Cos.\, p\, q + (\Lambda + l\, p + l\, 0)\, Sin.\, p\, q$

Bidone, Mém. Turin. 1812. 231. N°. 82.

F. Alg. rat. Circ. Dir. — **TABLE 255.** — Lim. diverses.

1) $\int_0^{2a\pi} x^b\, Cos.\, q\, x\, d\, x = -\sum_0^{b-1} \frac{1^{n/1}}{q^{n+1}} \binom{b}{n} (2\, a\, \pi)^{b-n}\, Cos.\left\{\frac{n+1}{2}\pi\right\}$ Hoppe, Cr. 40. 139.

2) $\int_{b\pi}^{c\pi} x\, Cos.^{2a} x\, d\, x = \frac{c^2 - b^2}{2}\pi^2 \frac{1^{a/2}}{2^{a/2}}$ Arndt, Gr. 6. 187.

3) $\int_{\lambda}^{\frac{\pi}{2}} \frac{x\, Sin.\, x}{(Cos.x - Cos.\mu)^3}\, dx = -\frac{1}{2}\frac{\pi}{(1 + Cos.\mu)^2} + \frac{1}{2}\frac{\lambda}{(Cos.\lambda - Cos\mu)^2} - \frac{Sin.\,\lambda}{2 Sin.^2\mu(Cos.\lambda - Cos.\mu)}$
$- \frac{Cos.\,\mu}{2 Sin.^2\mu}\, l \frac{Sin.\left\{\frac{1}{2}(\lambda + \mu)\right\}}{Sin.\left\{\frac{1}{2}(\lambda - \mu)\right\}}, \ \mu < \lambda;$ Legendre, Exerc. 5. 83.

4) $\int_{\lambda}^{\frac{\pi}{2}} \frac{x\, Cos.\, x}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)}}\, d\, x = \frac{1}{2}\pi\, l(1 + Cos.\,\lambda)$ Legendre, Exerc. Suppl. 29.

5) $\int_{\lambda}^{\frac{\pi}{2}} \frac{x\, Cos.\, x}{Sin.^2 x \sqrt{(Sin.^2 x - Sin.^2 \lambda)}}\, d\, x = \frac{1}{2}\pi\, Cosec.\,\lambda \left(1 - Tang.\, \frac{1}{2}\lambda\right)$ Legendre, Exerc. Suppl. 17.

6) $\int_0^{\infty} \frac{Sin.\, p\, x}{x}\, d\, x = \frac{1}{2}\pi$ Fourier, Chaleur. 415.

F. Alg. rat. ent. Circ. Inv. de x. — **TABLE 256.** — Lim. 0 et 1.

1) $\int x\, Arcsin.\, x\, d\, x = \frac{1}{8}\pi$ V. T. 9. N°. 4.

2) $\int x^{2a}\, Arcsin.\, x\, d\, x = \frac{1}{2\, a + 1}\left\{\frac{1}{2}\pi - \frac{2^{a/2}}{1^{a+1/2}}\right\}$ V. T. 12. N°. 13.

F. Alg. rat. ent. Circ. Inv. de x. **TABLE 256 suite.** Lim. 0 et 1.

3) $\int x^{2a-1}\, Arcsin.\, x\, dx = \frac{\pi}{4a}\left\{1 - \frac{3^{a-1/2}}{2^{a/2}}\right\}$ V. T. 12. N°. 12.

4) $\int (1-x^2)^{a-1}\, x\, Arcsin.\, x\, dx = \frac{\pi}{2^{2a+1}}\, \frac{(a+1)^{a-1/1}}{1^{a/1}}$ V. T. 9. N°. 5.

5) $\int Arccos.\, x.\, x^{2a-1}\, dx = \frac{\pi}{4a}\, \frac{3^{a-1/2}}{2^{a/2}}$ V. T. 12. N°. 12.

6) $\int Arccos.\, x.\, x^{2a}\, dx = \frac{2^{a-1/2}}{1^{a+1/2}}$ V. T. 12. N°. 13.

7) $\int x^{p-1}\, Arctang.\, x\, dx = \frac{1}{4p}\left\{\pi + Z'\left(\frac{p+1}{4}\right) - Z'\left(\frac{p+3}{4}\right)\right\}$ V. T. 3. N°. 13.

8) $\int Arccot.\, x.\, x^{p-1}\, dx = \frac{1}{4p}\left\{\pi + Z'\left(\frac{p+3}{4}\right) - Z'\left(\frac{p+1}{4}\right)\right\}$ V. T. 3. N°. 13.

F. Alg. rat. fract. à dén. monôme. Circ. Inv. de x. **TABLE 257.** Lim. 0 et 1.

1) $\int Arcsin.\, x\, \frac{dx}{x} = \frac{1}{2}\pi\, l\, 2$ Euler, N. A. Petr. 14. 129. — Arndt, Gr. 6. 187.

2) $\int (Arcsin.\, x)^p\, \frac{dx}{x} = \left(\frac{1}{2}\pi\right)^p \left\{1 - \sum_1^\infty \frac{2}{p+2m} \sum_1^\infty \frac{1}{(2n)^{2m}}\right\}$ V. T. 238. N°. 18.

3) $\int Arctang.\, x\, \frac{dx}{x} = \sum_0^\infty (-1)^n \frac{1}{(2n+1)^2}$ V. T. 152. N°. 11.

4) $\int Arctang.\, x\, \frac{dx}{x^2} = \infty$ V. T. 110. N°. 2.

5) $\int Arctang.\, x\, \frac{x^p - x^{-p}}{x}\, dx = \frac{\pi}{2p}\left(1 - Sec.\frac{1}{2}p\pi\right)$ V. T. 5. N°. 11.

6) $\int Arccot.\, x\, \frac{(x^p - x^{-p})}{x}\, dx = \frac{\pi}{2}\left\{1 + Sec.\frac{1}{2}p\pi\right\}$ V. T. 5. N°. 11.

7) $\int Arctang.\, x\, \frac{(p-q)(x^{p-q} - x^{q-p}) - (p+q)(x^{p+q} - x^{-p-q})}{x}\, dx = \frac{2\pi Sin.\frac{1}{2}p\pi. Sin.\frac{1}{2}q\pi}{Cos. p\pi + Cos. q\pi}$, $p+q<1, q<p<1$; V. T. 5. N°. 9.

8) $= \infty$, $p+q>1$;

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9) $\int Arctang.\, x \frac{(p+q)(x^{p+q}-x^{-p-q})+(p-q)(x^{p-q}-x^{q-p})}{x} dx =$

$= \pi \frac{Cos.\, p\pi + Cos.\, q\pi - 2\, Cos.\, \frac{1}{2} p\pi .\, Cos.\, \frac{1}{2} q\pi}{Cos.\, p\pi + Cos.\, q\pi}$, $p+q<1, q<p<1$;

10) $= \infty$, $p+q>1$; V. T. 5. N°. 10.

11) $\int (Arcsin.\, x)^2 \frac{dx}{x^2} = -\frac{1}{4}\pi^2 + 4 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 261. N°. 6.

12) $\int (Arctang.\, x)^2 \frac{dx}{x^2} = -\frac{1}{16}\pi^2 + \frac{1}{4}\pi\, l\, 2 + \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 260. N°. 4.

13) $\int (Arcsin.\, x)^p \frac{dx}{x^2} = p\left(\frac{\pi}{2}\right)^{p-1}\left[1 + \sum_1^\infty \left\{\frac{1}{4^{m-1}} \frac{2^{2m-1}-1}{p+2m-1} \sum_1^\infty \frac{1}{(2n)^{2m}}\right\}\right] - \frac{1}{p}\left(\frac{\pi}{2}\right)^p$ V. T. 261. N°. 24.

14) $\int (Arctang.\, x)^{p+1} \frac{dx}{x^2} = -\left(\frac{\pi}{4}\right)^{p+1} + \frac{p+1}{2^{2p+2}}\pi^p \left[1 - \sum_1^\infty \frac{4}{p+2m} \sum_1^\infty \frac{1}{(4n)^{2m}}\right]$ V. T. 260. N°. 20.

15) $\int (Arcsin.\, x)^3 \frac{dx}{x^3} = \frac{3}{2}\pi\, l\, 2 - \frac{1}{16}\pi^3$ V. T. 261. N°. 22.

F. Alg. rat. fract. à dén. polynôme. Circ. Inv. de x à un fact. TABLE 258. Lim. 0 et 1.

1) $\int Arcsin.\, x \frac{x}{1 - x^2 Sin.^2 \lambda} dx = \frac{1}{2}\pi\, Cosec.^2 \lambda\, l \left(Cos.^2 \frac{1}{2}\lambda .\, Sec.\, \lambda\right)$ V. T. 165. N°. 22.

2) $\int Arcsin.\, x \frac{x}{1+qx^2} dx = \frac{\pi}{2q} l \frac{2\sqrt{(1+q)}}{1+\sqrt{(1+q)}}$ V. T. 165. N°. 6.

3) $\int Arcsin.\, x \frac{x}{1 + x^2 Tang.^2 \lambda} dx = -\pi\, Cot.^2 \lambda\, l\, Cos.\, \frac{1}{2}\lambda$ V. T. 165. N°. 7.

4) $\int Arcsin.\, x \frac{x}{1 - x^4 Sin.^2 \lambda} dx = -\frac{1}{4}\pi\, Cosec.\, \lambda\, l \left\{\frac{1+Sin.\,\frac{1}{2}\lambda}{1+Sin.\,\lambda} \frac{Cos.\,\lambda}{Cos.\,\frac{1}{2}\lambda}\right\}$ V. T. 166. N°. 1.

5) $\int Arcsin.\, x \frac{x}{\frac{1}{2}(p+1) - x^2} dx = -\frac{\pi}{4} l \{2(1-p)\}$, $p^2 < 1$;

6) $= \frac{\pi}{4} l \frac{p + \sqrt{(p^2-1)}}{2(p-1)}$, $p^2 > 1$; V. T. 240. N°. 9, 10.

7) $\int Arcsin.\,x \frac{x}{\frac{1}{2}(p-1)+x^2}\,dx = \frac{\pi}{4} l\{2(1+p)\}$, $p^2 < 1$;

8) $= \frac{\pi}{4} l \frac{2(1+p)}{p+\sqrt{(p^2-1)}}$, $p^2 > 1$; V. T. 240. N°. 11, 12.

9) $\int Arcsin.\,x \frac{x}{(1-p)^2+4px^2}\,dx = \frac{\pi}{8p} l(1+p)$, $0 < p < 1$;

10) $= \frac{\pi}{8p} l \frac{1+p}{p}$, $p > 1$; V. T. 241. N°. 3, 4.

11) $\int Arccos.\,x \frac{dx}{1+x} = -\frac{1}{2}\pi l 2 + 2 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 13, 14.

12) $\int Arccos.\,x \frac{dx}{1-x} = \frac{1}{2}\pi l 2 + 2 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 13, 14.

13) $\int Arccos\,x \frac{dx}{1-x^2} = 2 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 239. N°. 1.

14) $\int Arccos.\,x \frac{x}{1-x^2}\,dx = \frac{1}{2}\pi l 2$ V. T. 165. N°. 11.

15) $\int Arccos.\,x \frac{dx}{Sin.^2\lambda - x^2} = 2\,Cosec.\,\lambda \sum_0^\infty \frac{Sin.\{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 240. N°. 1.

16) $\int Arccos.\,x \frac{x}{1-x^2\,Sin.^2\lambda}\,dx = -\pi\,Cosec.^2\lambda\, l\, Cos.\frac{1}{2}\lambda$ V. T. 165. N°. 14.

17) $\int Arccos.\,x \frac{x}{1+qx^2}\,dx = \frac{\pi}{2q} l \frac{1+\sqrt{(1+q)}}{2}$ V. T. 165. N°. 6.

18) $\int Arccos.\,x \frac{x}{1+x^2\,Tang.^2\lambda}\,dx = \frac{1}{2}\pi\,Cot.^2\lambda\, l\left(Cos.^2\frac{1}{2}\lambda.\,Sec.\,\lambda\right)$ V. T. 163. N°. 7.

19) $\int Arccos.\,x \frac{x}{1-x^4\,Sin.^2\lambda}\,dx = \frac{1}{4}\pi\,Cosec.\,\lambda\, l \frac{1+Sin.\frac{1}{2}\lambda}{Cos.\frac{1}{2}\lambda}$ V. T. 166. N°. 1.

20) $\int Arccos.\,x \frac{x}{\frac{1}{2}(p+1)-x^2}\,dx = \frac{\pi}{4} l\{2(1+p)\}$, $p^2 < 1$;

21) $= \frac{\pi}{4} l \frac{2(1+p)}{p+\sqrt{(p^2-1)}}$, $p^2 > 1$; V. T. 240. N°. 11, 12.

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22) $\int Arccos.\,x \frac{x}{\frac{1}{2}(p-1)+x^2}\,dx = -\frac{\pi}{4} l\{2(1-p)\}$, $p^2 < 1$;

23) $= \frac{\pi}{4} l \frac{p+\sqrt{(p^2-1)}}{2(p-1)}$, $p^2 > 1$; } V. T. 240. N°. 9, 10.

24) $\int Arccos.\,x \frac{x}{(1+p)^2-4px^2}\,dx = \frac{\pi}{8p} l(1+p)$, $0 < p < 1$;

25) $= \frac{\pi}{8p} l \frac{1+p}{p}$ $p > 1$; } V. T. 241. N°. 3, 4.

26) $\int Arctang.\,x \frac{dx}{1+x} = \frac{1}{8}\pi l 2$ V. T. 160. N°. 2.

27) $\int Arccot.\,x \frac{x}{1+x^2}\,dx = \frac{3}{8}\pi l 2 - \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 265. N°. 90 et T. 270. N°. 7.

28) $\int \left(x\,Arccot.\,x - \frac{1}{x} Arctang.\,x\right) \frac{dx}{x-\frac{1}{x}} = \frac{1}{4}\pi l 2$ Cauchy. Lim. Imag. Add. 32.

29) $\int Arcsin.\,x \frac{dx}{1+p^2+2px} = \frac{1}{2p}\left\{\pi l(1+p) - \sum_0^\infty \frac{1}{2n+1}\frac{2^{n/2}}{3^{n/2}}\left(\frac{2p}{1+p^2}\right)^{2n+1}\right\}$

30) $\int Arccos.\,x \frac{dx}{1+p^2+2px} = \frac{1}{2p}\left\{-\frac{\pi}{2}l(1+p^2) + \sum_0^\infty \frac{1}{2n+1}\frac{2^{n/2}}{3^{n/2}}\left(\frac{2p}{1+p^2}\right)^{2n+1}\right\}$ } V. T. 165. N°. 25.

1) $\int (Arccos.\,x)^2 \frac{dx}{1-x^2} = -2\sum_1^\infty \left\{\frac{1}{n^3} - \frac{(-1)^n}{n^3} - \pi\frac{(-1)^n}{(2n-1)^2}\right\}$ V. T. 239. N°. 2.

2) $\int (Arccos.\,x)^3 \frac{dx}{1-x^2} = 3\sum_1^\infty (-1)^n \left\{\frac{\pi^2}{2}\frac{1}{(2n-1)^2} - \frac{4}{(2n-1)^4}\right\}$ V. T. 239. N°. 3.

3) $\int (Arccos.\,x)^p \frac{dx}{1+x} = \left(\frac{\pi}{2}\right)^p \sum_1^\infty \left\{\frac{2^{2m}-1}{4^{m-1}}\frac{1}{p+2m}\sum_1^\infty \frac{1}{(2n)^{2m}}\right\}$

4) $\int (Arccos.\,x)^p \frac{dx}{1-x} = \left(\frac{\pi}{2}\right)^p \left[2 - \sum_1^\infty \left\{\frac{1}{4^{m-1}}\frac{1}{p+2m}\sum_1^\infty \frac{1}{(2n)^{2m}}\right\}\right]$ } V. T. 238. N°. 18. et T. 239. N°. 5.

5) $\int (Arccos.\,x)^b \frac{dx}{q+x} = -2\,Cos.\frac{1}{2}b\pi.\,1^{b\,1}\sum_1^\infty \frac{(-c)^n}{n^{b+1}} - 2\sum_1^\infty \left[c^{2n}\sum_0^\infty \binom{b}{2m}(-1)^m\left(\frac{\pi}{2}\right)^{b-2m}\frac{1}{(2n)^{2m+1}}\right.$

$\left. - c^{2n-1}\sum_0^\infty \binom{b}{2m+1}(-1)^m\left(\frac{\pi}{2}\right)^{b-2m-1}\frac{1}{(2n)^{2m+2}}\right]$ V. T 240. N°. 7.

6) $\int (Arccos.x)^b \frac{dx}{q-x} = 2\,Cos.\frac{1}{2}b\pi.1^{b/1}\sum_1^\infty \frac{c^n}{n^{b+1}} + 2\sum_1^\infty\Big[c^{2n}\sum_0^\infty \binom{b}{2m}(-1)^m\left(\frac{\pi}{2}\right)^{b-2m}\frac{1}{(2n)^{2m+1}} + c^{2n-1}\sum_0^\infty\binom{b}{2m+1}(-1)^m\left(\frac{\pi}{2}\right)^{b-2m-1}\frac{1}{(2n-1)^{2m+2}}\Big]$, où, dans les intégrales (5) et (6) $c = q - \sqrt{(q^2-1)}$; V. T. 240. N°. 8.

7) $\int (Arccos.x)^p \frac{dx}{1-x^2} = \left(\frac{\pi}{2}\right)^p\left[1 + \sum_1^\infty\left\{\frac{1}{4^{m-1}}\frac{2^{2m-1}-1}{p+2m}\sum_1^\infty\frac{1}{(2n)^{2m}}\right\}\right]$ V. T. 239. N°. 5.

8) $\int (Arccos.x)^p \frac{x}{1-x^2}dx = \left(\frac{\pi}{2}\right)^p\left[1 - \sum_1^\infty\left\{\frac{2}{p+2m}\sum_1^\infty\frac{1}{(2n)^{2m}}\right\}\right]$ V. T. 238. N°. 18.

9) $\int Arctang.x\frac{(1+x)(1+2x\,Arctang.x)-(1+x^2)}{(1+x)^2}dx = \frac{\pi}{16}(1-2l2)$ V. T. 258. N°. 26.

10) $\int Arctang.x\frac{(1+x^2)\,Arccot.x - x}{1+x^2}dx = \frac{1}{16}\pi^2 - \frac{3}{8}\pi l 2 + \frac{1}{2}\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 27.

F. Alg. rat. fract. à dén. prod. de fact. Circ. Inv. de x. TABLE 260. Lim. 0 et 1.

1) $\int Arcsin.x\frac{dx}{x(1-x^2)} = \infty$ V. T. 239. N°. 14.

2) $\int Arcsin.x\frac{x-(1+x^2)Arctang.x}{x^2}\frac{dx}{1+x^2} = \frac{1}{8}\pi^2 - \frac{1}{2}\pi l(1+\sqrt{2})$ V. T. 261. N°. 14.

3) $\int Arcsin.x\frac{qx-(1+q^2x^2)Arctg.qx}{x^2}\frac{dx}{1+q^2x^2} = \frac{1}{2}\pi\,Arctang.q - \frac{1}{2}\pi l\{q+\sqrt{(q^2+1)}\}$ V. T. 261. N°. 15.

4) $\int Arctang.x\frac{dx}{x(1+x^2)} = \frac{1}{8}\pi l 2 + \frac{1}{2}\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 238. N°. 4.

5) $\int Arctang.x\frac{x-(1+x^2)Arctang.x}{x^2}\frac{dx}{1+x^2} = \frac{1}{16}\pi^2 - \frac{1}{8}\pi l2 - \frac{1}{2}\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 260. N°. 4.

6) $\int Arcsin.x\frac{x}{Cos.^2\lambda + x^2 Sin.^2\lambda}\frac{dx}{Cos.^2\mu + x^2 Sin.^2\mu} = \frac{\pi}{Sin.(\lambda-\mu).Sin.(\lambda+\mu)}l\,Cos.\frac{1}{2}\mu.Sec.\frac{1}{2}\lambda$ V. T. 166. N°. 4.

7) $\int Arcsin.x\frac{x}{1-x^2 Sin.^2\lambda}\frac{dx}{1-x^2 Sin.^2\mu} = \frac{\pi}{Sin.^2\lambda - Sin.^2\mu}l\frac{Cos.\frac{1}{2}\lambda.\sqrt{Cos.\mu}}{Cos.\frac{1}{2}\mu.\sqrt{Cos.\lambda}}$ V. T. 166. N°. 2.

8) $\int Arcsin.x\frac{x}{Sin.^2\lambda - x^2 Sin.^2\mu}\frac{dx}{1-x^2 Sin.^2\mu} = \frac{\pi}{2Sin.^2\mu.Cos.^2\lambda}l\frac{Cos.\mu.Tang.\left\{\frac{1}{2}Arcsin.\left(\frac{Sin.\mu}{Sin.\lambda}\right)\right\}}{Cot.\frac{1}{2}\mu.\sqrt{(Sin.^2\lambda - Sin.^2\mu)}}$ V. T. 166. N°. 3.

9) $\int Arccos.x \frac{x}{Sin.^2\lambda - x^2 Sin.^2\mu} \frac{dx}{1 - x^2 Sin.^2\mu} = \frac{\pi}{2 Sin.^2\mu . Cos.^2\lambda} l \frac{Sin.\lambda . Cot.\frac{1}{2}\mu}{Tang.\left\{\frac{1}{2} Arcsin.\left(\frac{Sin.\mu}{Sin.\lambda}\right)\right\}}$ V. T 166. N°. 3.

10) $\int Arccos.x \frac{x}{Cos.^2\lambda + x^2 Sin.^2\lambda} \frac{dx}{Cos.^2\mu + x^2 Sin.^2\mu} = \frac{1}{2} \frac{\pi}{Sin.(\lambda + \mu) . Sin.(\lambda - \mu)} l \frac{1 + Sec.\lambda}{1 + Sec.\mu}$ V. T. 166. N°. 4.

11) $\int Arccos.x \frac{x}{1 - x^2 Sin.^2\lambda} \frac{dx}{1 - x^2 Sin.^2\mu} = \frac{\pi}{Sin.^2\lambda - Sin.^2\mu} l \frac{Cos.\frac{1}{2}\mu}{Cos.\frac{1}{2}\lambda}$ V. T. 166. N°. 2.

12) $\int Arcsin.x \frac{x^{2p-1}}{(1 - x^2)^{p+1}} dx = \infty$ V. T. 12. N°. 17.

13) $\int Arccos.x \frac{x^{2p-1}}{(1 - x^2)^{p+1}} dx = \frac{\pi}{4p} Sec.p\pi$ V. T. 12. N°. 17.

14) $\int Arctang.x \frac{2x + x^2}{(1 + x)^2} dx = \frac{1}{4}\pi - \frac{3}{4} l 2$ V. T. 3. N°. 16.

15) $\int Arctang.x \frac{dx}{(1 + px)^2} = \frac{1}{1 + p^2} l \frac{1 + p}{\sqrt{2}} + \frac{\pi}{4p} \frac{1 - p}{(1 + p)(1 + p^2)}$ V. T. 6. N°. 1.

16) $\int Arccot.x \frac{x + 2}{(1 + x)^3} x\, dx = \frac{3}{4} l 2$ V. T. 3. N°. 16.

17) $\int Arccot.x \frac{x}{(1 + x^2)^2} dx = \frac{1}{16}\left\{\pi + 2 + Z'\left(\frac{3}{4}\right) - Z'\left(\frac{5}{4}\right)\right\}$ V. T. 4. N°. 18.

18) $\int Arctang.x \frac{1 - x^2}{(1 + x^2)^{2p}} x^{2p-2} dx = \frac{1}{2p - 1}\left\{\frac{\pi}{2^{2p+1}} - \frac{\{\Gamma(p)\}^2}{4\Gamma(2p)}\right\}$ V. T. 5. N°. 24.

19) $\int (Arccos.x)^3 \frac{x}{(1 - x^2)^2} dx = \frac{3}{2}\pi\, l 2 - \frac{1}{16}\pi^3$ V. T. 261. N°. 25.

20) $\int (Arctang.x)^p \frac{dx}{x(1 + x^2)} = \frac{\pi^p}{2^{2p+1}}\left\{2 - \sum_1^\infty \frac{4}{p + 2m} \sum_1^\infty \frac{1}{(4n)^{2m}}\right\}$ V. T. 238. N°. 19.

21) $\int (Arctang.x)^p \frac{px - (1 + x^2) Arctang.x}{x^2} \frac{dx}{1 + x^2} = \left(\frac{1}{4}\pi\right)^{p+1} - \frac{\pi^p}{2^{2p+1}}\left\{2 - \sum_1^\infty \frac{4}{p + 2m} \sum_1^\infty \frac{1}{(4n)^{2m}}\right\}$ V. T. 260. N°. 20.

F. Alg. irrat. fract. Circ. Inv. de x. TABLE 261. Lim. 0 et 1.

1) $\int Arcsin.x \frac{dx}{\sqrt[3]{x}} = \frac{3}{2}\left\{\frac{1}{2}\pi - \frac{3\sqrt{3}}{\sqrt[4]{3}} E'\left(Sin.\frac{\pi}{12}\right) + \frac{3 + 3\sqrt{3}}{2\sqrt[4]{3}} F'\left(Sin.\frac{\pi}{12}\right)\right\}$ V. T. 12. N°. 16.

2) $\int Arcsin.x \frac{dx}{\sqrt[3]{x^2}} = 3\left\{\frac{1}{2}\pi + \frac{\sqrt{3} - 1}{\sqrt[4]{3}} F'\left(Cos.\frac{\pi}{12}\right) - 2\sqrt[4]{3} E'\left(Cos.\frac{\pi}{12}\right)\right\}$ V. T. 12. N°. 15.

3) $\int Arcsin.x \frac{dx}{x\sqrt[3]{x}} = \sqrt[4]{27}.F'\left(Cos.\frac{\pi}{12}\right) - \frac{3}{2}\pi$ V. T. 15. N°. 12.

4) $\int Arcsin.x \frac{dx}{x\sqrt[3]{x^2}} = \frac{3}{2}\sqrt[4]{27}.F'\left(Sin.\frac{\pi}{12}\right) - \frac{3}{4}\pi$ V. T. 15. N°. 11.

5) $\int Arcsin.x \frac{x}{\sqrt{(1-p^2x^2)}} dx = \frac{1}{p^2}\left\{1 - \frac{1}{2}\pi\sqrt{(1-p^2)} - \sum_1^\infty \left\{\frac{1^{n-1/2}}{2^{n/2}}\right\}^2 (2n-1)p^{2n}\right\}$ V. T. 12. N°. 14.

6) $\int Arcsin.x \frac{dx}{x\sqrt{(1-x^2)}} = 2\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 239. N°. 1.

7) $\int Arcsin.x \frac{x}{\sqrt{(1+x^2)^3}} dx = -\frac{1}{4}\pi\sqrt{2} + \frac{1}{2}\sqrt{2}.F'\left(Sin.\frac{\pi}{4}\right)$ V. T. 13. N°. 6.

8) $\int Arcsin.x \frac{dx}{\sqrt{(p^2+x^2)^3}} = \frac{1}{p^2}\left(\frac{1-p}{2p}\pi + Arctang.p\right)$ V. T. 16. N°. 6.

9) $\int Arcsin.x \frac{x}{x^2 - Cos.^2\lambda}\frac{dx}{\sqrt{(1-x^2)}} = 2\,Cosec.\lambda \sum_0^\infty \frac{Sin.\{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 240. N°. 1.

10) $\int Arccos.x \frac{dx}{x\sqrt[3]{x}} = \infty$ V. T. 15. N°. 12.

11) $\int Arccos.x \frac{dx}{x\sqrt[3]{x^2}} = \infty$ V. T. 15. N°. 11.

12) $\int Arccos.x \frac{x}{\sqrt{(1+x^2)^3}} dx = \frac{\pi}{2} - \frac{1}{2}\sqrt{2}\,F\left(Sin.\frac{\pi}{4}\right)$ V. T. 13. N°. 6.

13) $\int Arctang.x \frac{1-x}{x}\frac{dx}{\sqrt{x}} = \pi(\sqrt{2}-1)$ V. T. 15. N°. 2.

14) $\int Arctang.x \frac{dx}{x\sqrt{(1-x^2)}} = \frac{1}{2}\pi l(1+\sqrt{2})$

15) $\int Arctang.qx \frac{dx}{x\sqrt{(1-x^2)}} = \frac{1}{2}\pi l\{q+\sqrt{(1+q^2)}\}$

(14, 15) Schlömilch, Int. § 26.

16) $\int Arctang.x \frac{x^3}{\sqrt{(1-x^4)}} dx = \frac{\{\Gamma(\frac{1}{4})\}^2}{8\sqrt{2\pi}} - \frac{\pi\sqrt{2\pi}}{2\{\Gamma(\frac{1}{4})\}^2}$ V. T. 12. N°. 9.

17) $\int Arctang.x \frac{x}{\sqrt{(1-x^2)}}\frac{dx}{Tang.^2\mu + x^2} = \frac{\pi}{2}Cos.\mu.l\left\{Cos.\frac{\pi-4\mu}{8}.Cosec.\frac{\pi+4\mu}{8}\right\}$ V. T. 160. N°. 18.

18) $\int Arctang.\, x \frac{x}{1-x^2} \frac{dx}{\sqrt{(1-x^4)}} = \infty$ V. T. 13. N°. 6.

19) $\int \frac{Arcsin.\, x \sqrt{(1-x^2)} - x}{x^3} \frac{dx}{\sqrt{(1-x^2)}} = -\frac{1}{4}\pi$ V. T. 239. N°. 12.

20) $\int \frac{x\, Arccos.\, x - \sqrt{(1-x^2)}}{\sqrt{(1-x^2)^3}} dx = -\frac{1}{4}\pi$ V. T. 239. N°. 12.

21) $\int (Arcsin.\, x)^2 \frac{dx}{x\sqrt{(1-x^2)}} = 2\sum_1^\infty \left\{-\frac{1}{n^3} + \frac{(-1)^n}{n^3} - \pi \frac{(-1)^n}{(2n-1)^2}\right\}$ V. T. 239. N°. 2.

22) $\int (Arcsin.\, x)^2 \frac{dx}{x^2\sqrt{(1-x^2)}} = \pi\, l\, 2$ V. T. 239. N°. 8.

23) $\int (Arcsin.\, x)^3 \frac{dx}{x\sqrt{(1-x^2)}} = 3\sum_1^\infty \frac{(-1)^{n-1}}{(2n-1)^2} \left\{\frac{1}{2}\pi^2 - \frac{4}{(2n-1)^2}\right\}$ V. T. 239. N°. 3.

24) $\int (Arcsin.\, x)^p \frac{dx}{x\sqrt{(1-x^2)}} = \left(\frac{1}{2}\pi\right)^p \left\{1 + \sum_1^\infty \frac{1}{4^{m-1}} \frac{2^{2m-1}-1}{p+2m} \sum_1^\infty \frac{1}{(2n)^{2m}}\right\}$ V. T. 239. N°. 5.

25) $\int (Arccos.\, x)^2 \frac{dx}{\sqrt{(1-x^2)^3}} = \pi\, l\, 2$ V. T. 239. N°. 8.

F. Alg. fract.
Circ. Inv. d'autre forme.

TABLE 262.

Lim. 0 et 1.

1) $\int Arctang. \{\sqrt{(1-x^2)}\} \frac{dx}{1-x^2 Cos.^2 \mu} = \frac{\pi}{Cos.\, \mu} l \left\{Cos. \frac{\pi - 4\mu}{8} . Cosec. \frac{\pi + 4\mu}{8}\right\}$ V. T. 166. N°. 16.

2) $\int Arctang. \{p\sqrt{(1-x^2)}\} \frac{dx}{1-x^2} = \frac{1}{2}\pi\, l\, \{p + \sqrt{(1+p^2)}\}$ Raabe, Int. p. 421.

3) $\int Arctang. \{\sqrt{(1-x)}\} \frac{dx}{(1-x\, Cos.^2 \mu)\sqrt{x}} = \frac{2\pi}{Cos.\, \mu} l \left\{Cos. \frac{\pi - 4\mu}{8} . Cosec. \frac{\pi+4\mu}{8}\right\}$ V. T. 166. N°. 16.

4) $\int Arctang. \{Tang.\, \lambda. \sqrt{(1-p^2 x^2)}\} \frac{dx}{\sqrt{(1-x^2)(1-p^2x^2)}} = \frac{1}{2}\pi\, F(p,\lambda)$ V. T. 369. N°. 14.

5) $\int Arctang. \{Tang.\, \lambda. \sqrt{(1-p^2 x^2)}\} \frac{dx}{(1-p^2x^2)\sqrt{(1-x^2)(1-p^2x^2)}} =$

$= \frac{1}{2} \frac{\pi}{1-p^2} E(p,\lambda) - \frac{\pi}{2} \frac{Tang.\, \lambda}{1-p^2} \{\sqrt{(1-p^2 Sin.^2 \lambda)} - \sqrt{1-p^2)}\}$ V. T. 369. N°. 16.

6) $\int Arctang.\{Tang.\lambda.\sqrt{(1-p^2x^2)}\}dx\sqrt{\frac{1-p^2x^2}{1-x^2}} = \frac{1}{2}\pi E(p,\lambda) - \frac{1}{2}\pi Cot.\lambda.\{1-\sqrt{(1-p^2 Sin.^2\lambda)}\}$ V.T. 368. N°. 11.

7) $\int Arctang.\left\{\frac{Cot.\lambda}{\sqrt{(1-p^2x^2)}}\right\}\frac{dx}{\sqrt{(1-x^2)(1-p^2x^2)}} = \frac{1}{2}\pi F(p,\varphi)$ V. T. 369. N°. 15.

8) $\int Arctang.\left\{\frac{Cot.\lambda}{\sqrt{(1-p^2x^2)}}\right\}\frac{dx}{(1-p^2x^2)\sqrt{(1-x^2)(1-p^2x^2)}} = \frac{1}{2}\frac{\pi}{1-p^2}E(p,\varphi)$

$-\frac{\pi}{2}\frac{Tang.\lambda}{\sqrt{(1-p^2)}}\{1-\sqrt{(1-p^2 Sin.^2\varphi)}\}$ V. T. 369. N°. 17.

9) $\int Arctang.\left\{\frac{Cot.\lambda}{\sqrt{(1-p^2x^2)}}\right\}dx\sqrt{\frac{1-p^2x^2}{1-x^2}} = \frac{1}{2}\pi E(p,\varphi) - \frac{1}{2}\pi\frac{Cot.\lambda}{\sqrt{(1-p^2)}}\{\sqrt{(1-p^2 Sin.^2\varphi)} - \sqrt{(1-p^2)}\}$ V. T. 368. N°. 13.

Dans les intégrales (7) à (9) φ est donné par l'équation: $Cot.\ \varphi = Tang.\ \lambda.\sqrt{(1-p^2)}$.
Dans les intégrales (4) à (9) on a $p^2 < 1$.

F. Alg. rat. ent.
Circ. Inv. de x. TABLE 263. Lim. 0 et ∞.

1) $\int x^{p-1}\ Arccot.\ x\ dx = \frac{\pi}{2p}\ Sec.\frac{1}{2}p\pi,\ 0 < p < 1;$ V. T. 19. N°. 8.

2) $\int x^{p-2}\ Arctang.\ x\ dx = \frac{1}{1-p}\frac{\pi}{2}\ Cosec.\frac{1}{2}p\pi,\ 0 < p < 1;$ V. T. 19. N°. 6.

3) $\int (1 - x\ Arccot.\ x)\ dx = \frac{1}{4}\pi$ V. T. 239. N°. 12.

F. Alg. rat. fract. à dén. monôme.
Circ. Inv. de x. TABLE 264. Lim. 0 et ∞.

1) $\int Arctang.\frac{x}{q}\frac{dx}{x} = \infty$ V. T. 180. N°. 10.

2) $\int Arctang.\ x\frac{dx}{x^2} = \frac{1}{2}\pi l 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5. fautive: elle est ∞.

3) $\int Arctang.\ x\frac{dx}{x^p} = \frac{1}{2}\frac{\pi}{p-1}\ Sec.\left(\frac{p-1}{2}\pi\right),\ p < 1;$ V. T. 19. N°. 8.

4) $\int (Arctang.\ x)^2\frac{dx}{x^2} = \pi l 2$ Schlömilch, Gr. 4. 71. — Mosta, Gr. 10. 439.

5) $\int (Arctang.\,x)^3 \frac{dx}{x^2} = \frac{3}{4}\pi^2 l\,2 - 6\sum_1^\infty \left\{\frac{1}{n^3} + \frac{(-1)^n}{n^3} + 2\frac{(-1)^n}{(2n)^3}\right\}$ V. T. 266. N°. 8.

6) $\int (Arctang.\,x)^{p+1} \frac{dx}{x^2} = (p+1)\left(\frac{1}{2}\pi\right)^p \left\{1 - \sum_1^\infty \frac{2}{(p+2m)} \sum_1^\infty \frac{1}{(2n)^{2m}}\right\}$ V. T. 266. N°. 10.

7) $\int Arccot.\,p\,x \frac{dx}{x} = \infty$ V. T. 180. N°. 8.

8) $\int Arccot.\,x \frac{dx}{x^2} = \infty$ V. T. 109. N°. 1.

9) $\int Arccot.\,x \frac{dx}{x^p} = \frac{\pi}{2(1-p)}\,Cosec.\,\frac{1}{2}p\pi\,,\; p < 1$; V. T. 19. N°. 6.

10) $\int (Arcsec.\,x)^2 \frac{dx}{x^2} = \frac{1}{2}\pi\,l\,2$ V. T. 109. N°. 3; mais elle est fautive, avec celle-ci.

11) $\int \frac{Arctang.\,x - x}{x^3}\,dx = -\frac{1}{4}\pi$ V. T. 239. N°. 12.

12) $\int Arctang.\,p\,x.\,Arctang.\,x \frac{dx}{x^2} = \frac{1+p}{2}\pi\,l\,(1+p) - \frac{p\pi}{2}\,l\,p$ V. T. 266. N°. 2, 3.

13) $\int Arctang.\,x.\,Arctang.\,\frac{x}{q}\frac{dx}{x^2} = \frac{1}{2}\pi\left\{\frac{1}{q}\,l\,(1+q) + l\,\frac{1+q}{q}\right\}$

14) $\int Arctang.\,\frac{x}{p}.\,Arctang.\,\frac{x}{q}\frac{dx}{x^2} = \frac{1}{2}\pi\left\{\frac{1}{p}\,l\left(1+\frac{p}{q}\right) + \frac{1}{q}\,l\left(1+\frac{q}{p}\right)\right\}$

(13, 14) Schlömilch, Gr. 4. 71.

15) $\int Arctang.\,x.\,Arccos.\,x \frac{dx}{x^2} = \infty$ V. T. 109. N°. 7. (corrigée).

16) $\int Arctang.\,x.\,Arccot.\,q\,x \frac{dx}{x^2} = \infty$ V. T. 109. N°. 9. (corrigée).

17) $\int Arctang.\,q\,x.\,Arccot.\,x \frac{dx}{x^2} = \infty$ V. T. 109. N°. 8. (corrigée).

18) $\int Arctang.\,\frac{x}{p}.\,Arccot.\,\frac{x}{q}\frac{dx}{x^2} = \infty$ V. T. 109. N°. 10. (corrigée).

1) $\int Arctang.\, x \frac{dx}{1+x^2} = \frac{1}{8}\pi^2$ Schlömilch, Gr. 4. 316.

2) $\int Arctang.\, x \frac{x}{q^2+x^2} dx = \infty$ V. T. 181. N°. 2.

3) $\int Arctang.\, x \frac{x}{1+x^4} dx = \frac{1}{16}\pi^2$ V. T. 268. N°. 1.

4) $\int Arctang.\, px \frac{x}{x^4-q^4} dx = \frac{\pi}{8q^2} l \frac{(pq+1)^2}{p^2q^2+1}$ V. T. 265. N°. 14.

5) $\int Arctang.\, x \frac{dx}{(1+qx)^2} = \frac{1}{1+q^2}\left\{lq + \frac{\pi}{2q}\right\}$ V. T. 24. N°. 1.

6) $\int Arctang.\, x \frac{x}{(1+x^2)^3} dx = \frac{3}{64}\pi$ V. T. 21. N°. 4.

7) $\int Arctang.\, x \frac{x^3}{(1+x^2)^3} dx = \frac{1}{64}\pi$ V. T. 21. N°. 6.

8) $\int Arctang.\, x \frac{x}{(1+x^2)^2 - Sin.^2 2\lambda} dx = \frac{\pi}{4\, Sin.\, 2\lambda} l \frac{1+Sin.\,\lambda}{Cos.\,\lambda}$ V. T. 181. N°. 15.

9) $\int Arccot.\, x \frac{dx}{1+x^2} = \frac{1}{8}\pi^2$ V. T. 265. N°. 1.

10) $\int Arccot.\, x \frac{x}{1+x^2} dx = \frac{1}{2}\pi l 2$ Cauchy, Lim. Imag. Add. N°. 31.

11) $\int Arccot.\, x \frac{x}{1-x^2} dx = -\frac{1}{4}\pi l 2$ V. T. 265. N°. 13.

12) $\int Arccot.\, \frac{x}{p} \frac{x}{x^2+q^2} dx = \frac{1}{2}\pi l \frac{p+q}{q}$

13) $\int Arccot.\, \frac{x}{p} \frac{x}{x^2-q^2} dx = \frac{1}{4}\pi l \frac{p^2+q^2}{q^2}$

(12, 13) Cauchy, Lim. Imag. Add. N°. 26, 30.

14) $\int Arccot.\, \frac{x}{p} \frac{x}{x^4-q^4} dx = \frac{\pi}{8q^2} l \frac{p^2+q^2}{(p+q)^2}$

15) $\int Arccot.\, \frac{x}{p} \frac{x^3}{x^4-q^4} dx = \frac{1}{8}\pi l \frac{(p+q)^2(p^2+q^2)}{q^4}$

(14, 15) V. T. 265. N°. 12, 13.

16) $\int Arccot.\, px \frac{x}{1+x^2} dx = \frac{1}{2}\pi l \frac{1+p}{p}$ V. T. 266. N°. 3.

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17) $\int Arccot.\, x \frac{dx}{(1+qx)^2} = \frac{1}{1+q^2}\left\{\frac{1}{2} q\pi - lq\right\}$ V. T. 24. N°. 1.

18) $\int Arccot.\, x \frac{x}{(1+x^2)^3} dx = \frac{1}{64}\pi$ V. T. 21. N°. 4.

19) $\int Arccot.\, x \frac{x^3}{(1+x^2)^3} dx = \frac{3}{64}\pi$ V. T. 21. N°. 6.

20) $\int Arccot.\, x \frac{x}{(1+x^2)^2 - Sin.^2 2\lambda} dx = \frac{\pi}{4\, Sin.\, 2\lambda}\, l \frac{(1+Sin.\, 2\lambda)(1-Sin.\, \lambda)}{(1-Sin.\, 2\lambda)(1+Sin.\, \lambda)}$ V.T. 181. N°. 15.

21) $\int (Arctang.\, x)^q \frac{dx}{1+x^2} = \frac{1}{q+1}\left(\frac{1}{2}\pi\right)^{q+1}$

22) $\int (p + Arctang.\, x)^q \frac{dx}{1+x^2} = \frac{1}{q+1}\left\{\left(p+\frac{1}{2}\pi\right)^{q+1} - p^{q+1}\right\}$

(21, 22) Raabe, Int. 138. — Ohm, Ausw. 5.

23) $\int (p + Arctang\, x)^{q-1}.Arctang.x \frac{dx}{1+x^2} = \frac{1}{q}\left[\left(p+\frac{1}{2}\pi\right)^q \frac{\pi}{2} - \frac{1}{q+1}\left\{\left(p+\frac{1}{2}\pi\right)^{q+1} - p^{q+1}\right\}\right]$ V. T. 265. N°. 22.

24) $\int (Arccot.\, x)^2 \frac{x}{1+x^2} dx = \frac{1}{4}\pi^2 l2 - 2\sum_1^\infty \left\{\frac{1}{n^3} + \frac{(-1)^n}{n^3} + 2\frac{(-1)^n}{(2n)^3}\right\}$ V. T. 238. N°. 13.

25) $\int (Arccot.\, x)^3 \frac{x}{1+x^2} dx = \frac{1}{8}\pi^3 l2 + 6\pi \sum_1^\infty \frac{(-1)^n}{(2n)^3}$ V. T. 238. N°. 14.

26) $\int (Arccot.\, x)^p \frac{x}{1+x^2} dx = \left(\frac{1}{2}\pi\right)^p \left\{1 - \sum_1^\infty \frac{2}{p+2m} \sum_1^\infty \frac{1}{(2n)^{2m}}\right\}$ V. T. 266. N°. 10.

27) $\int (Arccot.\, x)^p \frac{dx}{1+x^2} = \frac{1}{p+1}\left(\frac{1}{2}\pi\right)^{p+1}$ V. T. 265. N°. 21.

28) $\int (q + Arccot.\, x)^p \frac{x}{1+x^2} dx = \frac{1}{p+1}\left\{\left(q+\frac{1}{2}\pi\right)^{p+1} - q^{p+1}\right\}$ V. T. 265. N°. 22.

F. Alg. rat. fract. à dén. $x^a(q^2+x^2)$. Circ. Inv. de x. TABLE 266. Lim. 0 et ∞.

1) $\int Arctang.\, x \frac{dx}{x(1+x^2)} = \frac{1}{2}\pi\, l2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

2) $\int Arctang.\, x \frac{dx}{x(q^2+x^2)} = \frac{\pi}{2q^2}\, l(1+q)$ Schlömilch, Cr. 33. 268. — Id., Gr. 4. 71.

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3) $\int Arctang.\, px \frac{dx}{x(1+x^2)} = \frac{1}{2}\pi l(1+p)$ Schlömilch, Cr. 33. 268. — Id., Gr. 4, 71.

4) $\int Arctang.\, px \frac{dx}{x(1+q^2x^2)} = \frac{1}{2}\pi l \frac{p+q}{q}$ V. T. 265. N°. 12.

5) $\int Arctang.\, \frac{x}{p} \frac{dx}{x(q^2+x^2)} = \frac{1}{2q^2}\pi l \frac{p+q}{p}$ V. T 181. N°. 10.

6) $\int Arctang.\, px \frac{dx}{x(1-q^2x^2)} = \frac{1}{4}\pi l \frac{p^2+q^2}{q^2}$ V. T. 265. N°. 13.

7) $\int Arctang.\, px \frac{x\,dx}{x^4-q^4} = \frac{\pi}{8q^2} l \frac{(pq+1)^2}{p^2q^2+1}$ V. T. 265. N°. 14.

8) $\int (Arctang.\, x)^2 \frac{dx}{x(1+x^2)} = \frac{1}{4}\pi^2 l2 - 2\sum_1^\infty \left\{\frac{1}{n^3} + \frac{(-1)^n}{n^3} + 2\frac{(-1)^n}{(2n)^3}\right\}$ V. T. 238. N°. 13.

9) $\int (Arctang\, x)^3 \frac{dx}{x(1+x^2)} = \frac{1}{8}\pi^3 l2 + \frac{3}{4}\pi \sum_1^\infty \frac{(-1)^n}{n^3}$ V. T. 238. N°. 14.

10) $\int (Arctang.\, x)^p \frac{dx}{x(1+x^2)} = \left(\frac{1}{2}\pi\right)^p \left\{1 - \sum_1^\infty \frac{2}{p+2m} \sum_1^\infty \frac{1}{(2n)^{2m}}\right\}$ V. T. 238. N°. 18.

F. Alg. rat. fract. à dén. prod. de binômes. Circ. Inv. de x. TABLE 267. Lim. 0 et ∞.

1) $\int Arctang.\, \frac{x}{q} \frac{dx}{(1+x)^2} = \frac{q}{1+q^2}\left(lq + \frac{\pi}{2q}\right)$ V. T. 24. N°. 3.

2) $\int Arctang.\, x \frac{x}{(1+x^2)^2} dx = \frac{1}{8}\pi$ Schlömilch, Gr. 4. 316.

3) $\int Arctang.\, x \frac{x}{(p^2+x^2)^2} dx = \frac{\pi}{4p(p+1)}$ V. T. 24. N°. 7.

4) $\int Arctang.\, px \frac{x}{(1+x^2)^2} dx = \frac{\pi p}{4(p+1)}$ V. T. 24. N°. 7.

5) $\int Arctang.\, \frac{x}{q} \frac{x}{(x^2+p^2)^2} dx = \frac{\pi}{4p(p+q)}$ V. T. 24. N°. 8.

6) $\int Arctang.\, \frac{x}{q} \frac{dx}{(1-x)^2} = \frac{q}{1+q^2}\left(lq - \frac{\pi}{2q}\right)$ V. T. 24. N°. 4.

7) $\int \mathit{Arctang.}\frac{x}{q}\frac{x}{(1-x^2)^2}dx = -\frac{\pi}{4(1+q^2)}$

8) $\int \mathit{Arctang.}\frac{x}{q}\frac{1+x^2}{(1-x^2)^2}dx = \frac{q}{1+q^2}lq$ V. T. 267. N°. 1, 6.

9) $\int \mathit{Arctang.}\frac{x}{q}\frac{x}{(x^2-p^2)^2}dx = -\frac{\pi}{4(p^2+q^2)}$ V. T. 24. N°. 11.

10) $\int \mathit{Arctang.}\,x\frac{x}{(q^2-x^2)^2}dx = -\frac{\pi}{4(1+q^2)}$ V. T. 24. N°. 13.

11) $\int \mathit{Arctang.}\,x\frac{x}{1+x^2}\frac{dx}{1+x^2\,\mathit{Cos.}^2\lambda} = \frac{1}{2}\pi\,\mathit{Cosec.}^2\lambda\,l\left(\mathit{Cos.}^2\frac{1}{2}\lambda.\,\mathit{Sec.}\,\lambda\right)$ V. T. 181. N°. 12.

12) $\int \mathit{Arctang.}\,x\frac{x}{1+x^2}\frac{dx}{x^2+\mathit{Cos.}^2\lambda} = \pi\,\mathit{Cosec.}^2\lambda\,l\,\mathit{Sec.}\frac{1}{2}\lambda$ V. T. 178. N°. 13.

13) $\int \mathit{Arctang.}\,x\frac{x}{\mathit{Cos.}^2\mu-\mathit{Cos.}^2\lambda+x^2\,\mathit{Sin.}^2\lambda}\frac{dx}{x^2+\mathit{Cos.}^2\mu} = \frac{\pi}{2\mathit{Cos.}^2\lambda.\,\mathit{Sin.}^2\mu}\times$
$\times\, l\left[\mathit{Sin.}\,\lambda.\,\mathit{Cot.}\frac{1}{2}\mu.\,\mathit{Cot.}\left\{\frac{1}{2}\,\mathit{Arcsin.}\left(\frac{\mathit{Sin.}\,\mu}{\mathit{Sin.}\,\lambda}\right)\right\}\right]$ V. T. 181. N°. 16.

14) $\int \mathit{Arctang.}\,x\frac{x}{x^2+\mathit{Cos.}^2\lambda}\frac{dx}{x^2+\mathit{Cos.}^2\mu} = \frac{\pi}{\mathit{Cos.}^2\lambda-\mathit{Cos.}^2\mu}\,l\left(\mathit{Cos.}\frac{1}{2}\lambda.\,\mathit{Sec.}\frac{1}{2}\mu\right)$ V. T. 241. N°. 24.

15) $\int \mathit{Arccot.}\,qx\frac{dx}{(1+x)^2} = \frac{q}{1+q^2}\left\{\frac{\pi}{2q}+lq\right\}$ V. T. 24. N°. 3.

16) $\int \mathit{Arccot.}\,qx\frac{dx}{(1-x)^2} = \frac{q}{1+q^2}\left\{lq-\frac{\pi}{2q}\right\}$ V. T. 24. N°. 4.

17) $\int \mathit{Arccot.}\,x\frac{x}{(1+x^2)^2}dx = \frac{1}{8}\pi$ V. T. 267. N°. 2.

18) $\int \mathit{Arccot.}\frac{x}{q}\frac{x}{(p^2+x^2)^2}dx = \frac{\pi q}{4p^2(p+q)}$ V. T. 24. N°. 8.

19) $\int \mathit{Arccot.}\,x\frac{x}{(q^2-x^2)^2}dx = \frac{-\pi}{4q^2(1+q^2)}$ V. T. 24. N°. 14.

20) $\int \mathit{Arccot.}\,x\frac{x}{1+x^2}\frac{dx}{1+x^2\,\mathit{Cos.}^2\lambda} = \pi\,\mathit{Cosec.}^2\lambda\,l\,\mathit{Sec.}\frac{1}{2}\lambda$ V. T. 181. N°. 12.

21) $\int \mathit{Arccot.}\,x\frac{x}{1+x^2}\frac{dx}{x^2+\mathit{Cos.}^2\lambda} = \frac{1}{2}\pi\,\mathit{Cosec.}^2\lambda\,l\left(\mathit{Sec.}\,\lambda.\,\mathit{Cos.}^2\frac{1}{2}\lambda\right)$ V. T. 181. N°. 13.

22) $\int Arccot.\, x \frac{x}{1+x^2 Cos.^2 \lambda} \frac{dx}{1+x^2 Cos.^2 \mu} = \frac{\pi}{Cos.^2 \lambda - Cos.^2 \mu}\, l\left(Cos. \frac{1}{2}\lambda\ Sec. \frac{1}{2}\mu\right)$ V. T. 241. N°. 24.

23) $\int Arccot.\, x \frac{x}{x^2 + Cos.^2 \lambda} \frac{dx}{x^2 + Cos.^2 \mu} = \frac{1}{2} \frac{\pi}{Cos.^2 \lambda - Cos.^2 \mu}\, l \frac{Cos.^2 \frac{1}{2}\mu .\ Cos.\lambda}{Cos.^2 \frac{1}{2}\lambda .\ Cos.\mu}$ V. T. 181. N°. 14.

24) $\int Arccot. x \frac{x}{Cos.^2\mu - Cos.^2\lambda + x^2 Sin.^2\lambda} \frac{dx}{x^2 + Cos.^2\mu} = \frac{\pi}{2 Sin.^2\lambda . Sin.^2\mu}\, l \frac{Cos.\mu . Tang.\left\{\frac{1}{2} Arcsin. \frac{Sin.\mu}{Sin.\lambda}\right\}}{Cot.\frac{1}{2}\mu \sqrt{(Cos.^2\mu - Cos.^2\lambda)}}$ V. T. 181. N°. 16.

25) $\int Arccot. \frac{x}{p} \frac{(b - xi)^{-a} - (b + xi)^{-a}}{2i} dx = \frac{\pi}{2(a-1)}\left\{\left(\frac{1}{b}\right)^{a-1} - \left(\frac{1}{b+p}\right)^{a-1}\right\}$ Cauchy, Lim. Imag. Add. N°. 29. où les puissances a de $\frac{1}{b}$ et de $\frac{1}{b+p}$ sont fautives.

26) $\int (Arctang. x)^2 \frac{1 - x^2}{(1+x^2)^2} dx = \frac{\pi - 1}{4}\pi$ V. T. 267. N°. 2.

1) $\int Arctang. x \frac{dx}{(1+x)\sqrt{x}} = \frac{1}{4}\pi^2$ Schlömilch, Gr. 4. 316.

2) $\int Arctang. x \frac{dx}{x\sqrt{(1+x^2)}} = 2 \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 239. N°. 1.

3) $\int Arctang. x \frac{x}{Cos.^2\lambda - x^2 Sin.^2\lambda} \frac{dx}{\sqrt{(1+x^2)}} = -2\, Cosec.\lambda \sum_0^\infty \frac{Sin.\{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 240. N°. 1.

4) $\int Arctang. x \frac{dx}{\sqrt[3]{x^2} + \sqrt[3]{x^4}} = \frac{3}{8}\pi^2$ Schlömilch, Gr. 4. 316.

5) $\int Arctang. \frac{x}{q} \frac{x}{\sqrt{(p^2+x^2)^3}} dx = \frac{1}{\sqrt{(p^2-q^2)}} Arctang. \frac{\sqrt{(p^2-q^2)}}{q}, q < p;$

6) $= \frac{1}{\sqrt{(q^2-p^2)}}\, l \frac{q + \sqrt{(q^2-p^2)}}{p}, q > p;$ V. T. 28. N°. 9, 10.

7) $\int Arctg. \frac{x}{p} \frac{x^2 + 2p^2 - q^2}{\sqrt{(p^2+x^2)}} \frac{x\,dx}{(q^2+x^2)^2} = -\frac{\pi}{2} + \frac{p}{q\sqrt{(p^2-q^2)}} Arctg. \frac{\sqrt{(p^2-q^2)}}{q}, p < q;$

8) $= -\frac{\pi}{2} + \frac{p}{q\sqrt{(q^2-p^2)}}\, l \frac{q + \sqrt{(q^2-p^2)}}{p}, p < q;$ V. T. 28. N°. 9, 10.

9) $\int Arccot. x \frac{dx}{(1+x)\sqrt{x}} = \frac{1}{4}\pi^2$ V. T. 268. N°. 1.

10) $\int Arccot.\, x \frac{dx}{\sqrt{(1+x^2)}} = 2\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 239. N°. 1.

11) $\int Arccot.\frac{x}{q} \frac{x}{\sqrt{(p^2+x^2)^3}} dx = \frac{1}{2}\left[\frac{\pi}{2p} - \frac{1}{\sqrt{(p^2-q^2)}} Arctang. \frac{\sqrt{(p^2-q^2)}}{q}\right], p > q;$ V. T. 28. N°. 9, 10.

12) $= \frac{1}{2}\left[\frac{\pi}{2p} + \frac{1}{\sqrt{(q^2-p^2)}} l \frac{p}{q+\sqrt{(q^2-p^2)}}\right], p < q;$

13) $\int Arccot.\, x \frac{dx}{(Sin.^2 \lambda - x^2 Cos.^2 \lambda)\sqrt{(1+x^2)}} = 2\, Cosec.\, \lambda \sum_0^\infty \frac{Sin.\{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 240. N°. 1.

14) $\int Arccot.x \frac{x^2+2p^2-q^2}{\sqrt{(p^2+x^2)}} \frac{x\,dx}{(q^2+x^2)^2} = \frac{-p}{2q}\left[\frac{\pi}{2q} + \frac{1}{\sqrt{(p^2-q^2)}} Arctg. \frac{\sqrt{(p^2-q^2)}}{q}\right], p>q;$ V. T. 28. N°. 9, 10.

15) $= \frac{p}{2q}\left[\frac{\pi}{2q} + \frac{1}{\sqrt{(q^2-p^2)}} l \frac{q+\sqrt{(q^2-p^2)}}{p}\right], p<q;$

16) $\int (Arctang.\, x)^2 \frac{dx}{x^2\sqrt{(1+x^2)}} = -\frac{1}{4}\pi^2 + 4\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 268. N°. 2.

17) $\int (Arccot.\, x)^2 \frac{x}{\sqrt{(1+x^2)}} dx = -\frac{1}{4}\pi^2 + 4\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 268. N°. 10.

F. Alg. fract. Circ. Inv. d'autre forme. **TABLE 269.** Lim. 0 et ∞.

1) $\int Arctang.\, (x^2) \frac{dx}{1+x} = \frac{1}{8}\pi^2$ V. T. 268. N°. 1.

2) $\int Arctang.\, (x^2) \frac{dx}{1+x^2} = \frac{1}{8}\pi^2$ V. T. 268. N°. 4.

3) $\int Arctang.\, (\sqrt{x}) \frac{dx}{(1+x)^2} = \frac{1}{4}\pi$ V. T. 267. N°. 2.

4) $\int Arccot.\, (x^2) \frac{dx}{1+x^2} = \frac{1}{8}\pi^2$ V. T. 268. N°. 9.

5) $\int Arccot.\, (x^2) \frac{dx}{1+x^2} = \frac{1}{8}\pi^2$ V. T. 269. N°. 2.

6) $\int Arccot.\, (\sqrt{x}) \frac{dx}{(1+x)^2} = \frac{1}{4}\pi$ V. T. 267. N°. 17.

7) $\int \{Arccot.(\sqrt{x})\}^2 \frac{dx}{\sqrt{(1+x)}} = -\frac{1}{2}\pi^2 + 8\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 268. N°. 17.

8) $\int Arctang.\left(\frac{p}{\sqrt{(1+x^2)}}\right)\frac{dx}{\sqrt{(1+x^2)}} = \frac{1}{2}\pi l\{p+\sqrt{(1+p^2)}\}$, $p \geqq 1$;

9) $\int Arctang.\left(\sqrt{\frac{p^2-1}{1+x^2}}\right)\frac{dx}{\sqrt{(1+x^2)}} = \frac{1}{2}\pi l\{p+\sqrt{(p^2-1)}\}$ Raabe, Int. 421.

10) $\int Arctang.\left(x\sqrt{\frac{p^2-1}{1+x^2}}\right)\frac{dx}{x\sqrt{(1+x^2)}} = \frac{1}{2}\pi l\{p+\sqrt{(p^2-1)}\}$, $p \geqq 1$; V. T. 269. N°. 9.

11) $\int Arctang.\left(\frac{px}{\sqrt{(1+x^2)}}\right)\frac{dx}{x\sqrt{(1+x^2)}} = \frac{1}{2}\pi l\{p+\sqrt{(1+p^2)}\}$, $p \geqq 1$; V. T. 269. N°. 8.

12) $\int Arctang.(\sqrt{x})\left\{\frac{1}{\sqrt{(1+x)}}Arctang.\left(\sqrt{\frac{p^2-1}{1+x}}\right) - \frac{\sqrt{(p^2-1)}}{x+p^2}\right\}dx =$
$= -\pi l\{p+\sqrt{(p^2-1)}\}$ V. T. 269. N°. 9.

1) $\int Arctang.\, x\frac{dx}{x} = \infty$ V. T. 187. N°. 2.

2) $\int Arctang.\, x\frac{dx}{x(1+x^2)} = \frac{3}{8}\pi l 2 - \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 260. N°. 4 et T. 266. N°. 1.

3) $\int Arctang.\, x\frac{x-(1+x^2)\,Arctang.\,x}{x^2}\frac{dx}{1+x^2} = -\frac{1}{16}\pi^2 - \frac{3}{8}\pi l 2 + \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 2.

4) $\int Arccot.\, x\frac{dx}{x} = \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 257. N°. 3.

5) $\int Arccot.\, x\frac{dx}{x^2} = \frac{1}{4}\pi - \frac{1}{2}l2$ V. T. 108. N°. 1.

6) $\int Arccot.\frac{x}{p}\frac{dx}{x^2} = Arctang.\, p - \frac{1}{2p}l(1+p^2)$ V. T. 108. N°. 2.

7) $\int Arccot.\, x\frac{x}{1+x^2}dx = \frac{1}{8}\pi l 2 + \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 238. N°. 4.

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8) $\int Arccosec.\, x \frac{dx}{x^2} = \frac{1}{2}\pi - 1$ V. T. 108. N°. 3.

9) $\int Arccosec.\, \frac{x}{p} \frac{dx}{x^2} = Arcsin.\, p + \frac{1}{p}\sqrt{(1-p^2)} - \frac{1}{p}$ V. T. 108. N°. 4.

10) $\int (Arctang.\, x)^2 \frac{dx}{x^2} = \frac{\pi^2}{16} + \frac{3}{4}\pi l 2 - \sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 2.

11) $\int (Arccot.\, x)^p \frac{x}{1+x^2} dx = \frac{\pi^p}{2^{2p+1}}\left\{2 - \sum_1^\infty \frac{4}{p+2m} \sum_1^\infty \frac{1}{(4n)^{2m}}\right\}$ V. T. 238. N°. 19.

12) $\int Arctang.\, x \frac{(1+x^2)\, Arccot.\, x - x}{1+x^2} dx = \frac{1}{16}\pi^2 - \frac{1}{8}\pi l 2 - \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 7.

13) $\int Arctg.x(Arccot.x)^{p-1}\frac{(1+x^2)Arccot.x - px}{1+x^2}dx = \left(\frac{1}{4}\pi\right)^{p+1} - \frac{\pi^p}{2^{2p+1}}\left\{2 - \sum_1^\infty \frac{4}{p+2m}\sum_1^\infty \frac{1}{(4n)^{2m}}\right\}$ V.T. 270. N°. 11.

14) $\int Arccot.\, x\left(Arctang.\, x - \frac{x}{1+x^2}\right)\frac{dx}{x^2} = \frac{\pi^2}{16} - \frac{3\pi}{8} l 2 + \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 2.

F. Alg. fract. Circ. Inv. — TABLE 271. — Lim. diverses.

1) $\int_{-\infty}^{\infty} Arctg.(a+bx)\frac{dx}{1+x^2} = -\pi\left\{\frac{1}{2} Arctg.\frac{2ab}{1+a^2-b^2} - Arctg.\frac{Tang.\left(\frac{1}{2}Arctang.\frac{2ab}{1+a^2-b^2}\right)}{a\,Tg.\left(\frac{1}{2}Arctg.\frac{2ab}{1+a^2-b^2}\right) - b}\right\}$ Hill, Cr. 3. 101.

2) $\int_{-\infty}^{\infty} Arctg.\, x \frac{dx}{1+(a+bx)^2} = \frac{\pi}{b}\left\{\frac{1}{2} Arctg.\frac{2ab}{1+a^2-b^2} - Arctg.\frac{Tang.\left(\frac{1}{2}Arctang.\frac{2ab}{1+a^2-b^2}\right)}{a\,Tg.\left(\frac{1}{2}Arctg.\frac{2ab}{1+a^2-b^2}\right) - b}\right\}$ V. T. 271. N°. 1.

3) $\int_{-\infty}^{\infty} Arctang.\, x \frac{dx}{x(1+x^2)} = \pi l 2$ Cauchy, Sav. Etr. 1827. 599. P. 2. § 5.

4) $\int_0^{\sqrt{\frac{1}{2}}} Arcsin.\, x \frac{dx}{x} = \frac{1}{8}\pi l 2 + \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 238. N°. 4.

5) $\int_0^{\sqrt{\frac{1}{2}}} (Arcsin.\, x)^p \frac{dx}{x} = \frac{\pi^p}{2^{2p+1}}\left\{2 - \sum_1^\infty \frac{4}{p+2m}\sum_1^\infty \frac{1}{(4n)^{2m}}\right\}$ V. T. 238. N°. 19.

6) $\int_{\sqrt{\frac{1}{2}}}^{1} \text{Arccos.}\, x \frac{x}{1-x^2}\, dx = \frac{1}{8}\pi l 2 + \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 238. N°. 4.

7) $\int_{\sqrt{\frac{1}{2}}}^{1} (\text{Arccos.}\, x)^p \frac{x}{1-x^2}\, dx = \frac{\pi^p}{2^{2p+1}}\left\{2 - \sum_1^\infty \frac{4}{p+2m}\sum_1^\infty \frac{1}{(4n)^{2m}}\right\}$ V. T. 238. N°. 19.

8) $\int_{-1}^{0} \text{Arctang.}\, x \frac{dx}{1-x} = -\frac{1}{4}\pi l 2$ V. T. 188. N°. 2.

9) $\int_{-1}^{0} \text{Arccot.}\, x \frac{dx}{1-x} = -\frac{3}{4}\pi l 2$ V. T. 188. N°. 2.

10) $\int_0^p \text{Arcsin.}\, \frac{x}{p}\, x\, dx = \frac{1}{8} p^2 \pi$ V. T. 34. N°. 8.

11) $\int_0^p \text{Arcsin.}\, \frac{x}{p}\, x^{2a-1}\, dx = \frac{\pi}{4a} p^{2a}\left[1 - \frac{1^{a/2}}{2^{a/2}}\right]$ V. T. 34. N°. 10.

12) $\int_0^p \text{Arcsin.}\, \frac{x}{p}\, x^{2a}\, dx = \frac{p^{2a+1}}{2a+1}\left[\frac{\pi}{2} - \frac{2^{a/2}}{3^{a/2}}\right]$ V. T. 34. N°. 11.

13) $\int^p \text{Arccos.}\, \frac{x}{p}\, x\, dx = \frac{1}{8} p^2 \pi$ V. T. 34. N°. 8.

14) $\int_0^p \text{Arccos.}\, \frac{x}{p}\, x^{2a-1}\, dx = \frac{1^{a/2}}{2^{a/2}} p^{2a} \frac{\pi}{4a}$ V. T. 34. N°. 10.

15) $\int_0^p \text{Arccos.}\, \frac{x}{p}\, x^{2a}\, dx = \frac{p^{2a+1}}{2a+1}\frac{2^{a/2}}{3^{a/2}}$ V. T. 34. N°. 11.

16) $\int_0^p \text{Arcsin.}\, \frac{x}{p} \frac{x}{\sqrt{(q^2-x^2)^3}}\, dx = \frac{\pi}{2\sqrt{(q^2-p^2)}} - \frac{1}{q} F'\left(\frac{p}{q}\right), p < q;$

17) $\int_0^p \text{Arccos.}\, \frac{x}{p} \frac{x}{\sqrt{(q^2-x^2)^3}}\, dx = \frac{1}{q} F'\left(\frac{p}{q}\right) - \frac{\pi}{2q}, p < q;$

(16, 17) V. T. 34. N°. 12.

18) $\int_0^p \text{Arctang.}\, x \frac{dx}{1+px} = \frac{1}{2p} \text{Arctang.}\, p \,.\, l(1+p^2)$ V. T. 188. N°. 15.

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19) $\int_0^{\frac{1}{p}} \text{Arctang.}\, x \frac{dx}{p+x} = \frac{1}{2} \text{Arccot.}\, p.\, l \frac{1+p^2}{p^2}$ V. T. 188. N°. 14.

20) $\int_p^q \text{Arcsin.}\,(\sqrt{x}) \frac{r-x}{(r+x)^2} \frac{dx}{\sqrt{x}} = \frac{2\sqrt{q}}{q+r} \text{Arcsin.}\,(\sqrt{q}) - \frac{2\sqrt{p}}{p+r} \text{Arcsin.}\,(\sqrt{p}) - - \frac{1}{(\sqrt{1-r})}\, l \left\{ \frac{\sqrt{(1+r)} + \sqrt{(1-p)}}{\sqrt{(1+r)} + \sqrt{(1-q)}} \cdot \frac{\sqrt{(1+r)} - \sqrt{(1-q)}}{\sqrt{(1+r)} - \sqrt{(1-p)}} \right\}$ V. T. 35. N°. 22.

F. Algébr. Autres fonctions. — TABLE 272. — Lim. diverses.

1) $\int_0^1 li\left(\frac{1}{x}\right) x\, dx = 0$ V. T. 300. N°. 2.

2) $\int_0^1 li(x)\, x^{p-1}\, dx = \frac{1}{p}\, l(1+p)\,,\ p^2 \geqq -1;$ V. T. 300. N°. 3.

3) $\int_0^p \mathrm{E}(x) \frac{x}{1-x^2} \frac{dx}{\sqrt{(p^2-x^2)}} = \frac{p\pi}{2\sqrt{(1-p^2)}}$

4) $\int_0^p \mathrm{F}(x) \frac{x}{1-x^2} \frac{dx}{\sqrt{(p^2-x^2)}} = \frac{p\pi}{2\sqrt{(1-p^2)}}$

Roberts, L. 10. 454. Là formule 4) est fautive, et ne vaut que pour E (x) au lieu de F (x).

F. Exponentielle. Logarithme. } F. entière. — TABLE 273. — Lim. 0 et ∞.

1) $\int e^{-x}\, lx\, dx = -\mathrm{A}$

2) $\int e^{-qx}\, lx\, dx = -\frac{1}{q}(\mathrm{A} + lq)$

Schlömilch, Beitr. III. § 9. — Id., Gr. 4. 167. — Id., Gr. 9. 5.

3) $\int e^{-x^2}\, l\frac{1}{x}\, dx = \frac{1}{4}\sqrt{\pi}\,(\mathrm{A} + 2\,l2)$ Meyer, Int. déf. 373.

4) $\int e^{-qx^2}\, lx\, dx = -\frac{\sqrt{\pi q}}{4q}(\mathrm{A} + lq + 2\,l2)$ Schlömilch, Gr. 4. 167. — Id., Stud. I. 14.

5) $\int e^{-px}\, l(q+x)^2\, dx = \frac{1}{p}\{lq^2 - 2e^{pq}\, Ei.\,(-pq)\}$ Bierens de Haan, Verh. der K. Ak. van Wet. 1854. bl. 19. — Winckler, Cr. 50. I. Page 368.

6) $\int e^{-px} l(q-x)^2\, dx = \frac{1}{p}\{lq^2 - 2e^{-pq} Ei.(pq)\}$

7) $\int e^{-px} l(q^2-x^2)^2\, dx = \frac{2}{p}\{lq^2 - e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq)\}$

8) $\int e^{-px} l(q^2+x^2)^2\, dx = \frac{2}{p}\{lq^2 - 2\, Ci.(pq).Cos.pq - 2\, Si.(pq).Sin.pq + \pi\, Sin.pq\}$

9) $\int e^{-px} l(q^4-x^4)^2\, dx = \frac{2}{p}\{2lq^2 - e^{pq} Ei.(-pq) - e^{-pq} Ei.(pq) - 2\, Ci.(pq).Cos.pq$

$- 2\, Si.(pq).Sin.pq + \pi\, Sin.pq\}$

Bierens de Haan, Verh. der K. Ak. van Wet. 1854. bl. 19. — Winckler, Cr. 50. 1.

F. Exponent. polynôme en dén. Logar. en num. lx. TABLE 274. Lim. 0 et ∞ .

1) $\int lx \frac{e^{\frac{b\pi x}{c}} + e^{-\frac{b\pi x}{c}}}{e^{\pi x}+e^{-\pi x}} dx = -\frac{1}{2} A Sec. \frac{b\pi}{2c} - \sum_0^\infty (-1)^n \left\{ \frac{l\left\{(2n+1)\pi - \frac{b\pi}{c}\right\}}{(2n+1)\pi - \frac{b\pi}{c}} + \frac{l\left\{(2n+1)\pi + \frac{b\pi}{c}\right\}}{(2n+1)\pi + \frac{b\pi}{c}} \right\}$, $b < c$; Malmsten, Cr. 38. 1. la trouve fautive.

2) $\int lx \frac{e^{ax}+e^{-ax}}{e^{bx}+e^{-bx}} dx = \frac{\pi}{2b} Sec. \frac{a\pi}{2b} l2\pi + \frac{\pi}{b} \sum_1^b (-1)^{n-1} Cos.\left(\frac{2n-1}{2b}a\pi\right) l \frac{\Gamma\left(\frac{2b+2n-1}{4b}\right)}{\Gamma\left(\frac{2n-1}{4b}\right)}$, $a+b$ impair;

3) . . . . . $= \frac{\pi}{2b} Sec. \frac{a\pi}{2b} l\pi + \frac{\pi}{b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} Cos.\left(\frac{2n-1}{2b}a\pi\right) l \frac{\Gamma\left(\frac{2b-2n+1}{2b}\right)}{\Gamma\left(\frac{2n-1}{2b}\right)}$, $a+b$ pair;

V. T. 191. N°. 8, 9.

4) $\int lx \frac{e^{\frac{b\pi x}{c}} - e^{-\frac{b\pi x}{c}}}{e^{\pi x}-e^{-\pi x}} dx = -\frac{1}{2} A\, Tang. \frac{b\pi}{2c} - \sum_0^\infty \left\{ \frac{l\left\{(2n+1)\pi - \frac{b\pi}{c}\right\}}{(2n+1)\pi - \frac{b\pi}{c}} - \frac{l\left\{(2n+1)\pi + \frac{b\pi}{c}\right\}}{(2n+1)\pi + \frac{b\pi}{c}} \right\}$, $b < c$; Malmsten, Cr. 38. 1. la trouve fautive.

5) $\int lx \frac{e^x - e^{-x} - 2}{(1+e^{-x})^2} dx = l\frac{\pi}{2}$ V. T. 134. N°. 1.

6) $\int lx \frac{(q-p)\{e^{(p+q)x} + e^{-(p+q)x}\} + (p+q)\{e^{(p-q)x} + e^{(q-p)x}\}}{(e^{px}+e^{-px})^2} dx = l\, Tang. \left\{\frac{p+q}{4p}\pi\right\}$ V. T. 136. N°. 9.

7) $\int l x \frac{(2q-1)\,e^{(q+1)x} + (2q+1)\,e^{(1-q)x} + 2\,q\,(e^{qx}+e^{-qx})}{(e^x+1)^2}(e^{qx}-e^{-qx})\,d\,x = l(q\pi Cot.q\pi)$ V. T. 184. N°. 8.

8) $\int l\,x \frac{e^x}{(e^x+1)^2}\,d\,x = \frac{1}{2}Z'\left(\frac{1}{2}\right) + \frac{1}{2}\,l\,2\,\pi$ V. T. 190. N°. 7.

9) $\int l x(e^{qx}-e^{-qx}) \frac{(p+q)\,e^{-qx} - (p-q)\,e^{qx} - q\,e^{-2px}\,(e^{qx}+e^{-qx})}{(e^{px}-e^{-px})^2}\,dx = \frac{1}{2}\,l\left\{\frac{q\pi}{p}Cosec.\frac{q\,\pi}{p}\right\}$ V. T. 136. N°. 5.

10) $\int l x(e^{qx}-e^{-qx}) \frac{(p-2q)\left\{e^{(q+p)x}-e^{-(q+p)x}\right\} + (p+2q)\left\{e^{(q-p)x}-e^{(p-q)x}\right\}}{(e^{px}-e^{-px})^2}\,dx = l Sec.\frac{q\,\pi}{p}$ V. T. 136. N°. 14.

11) $\int l x \frac{d\,x}{e^x+e^x-1} = \frac{2\,\pi}{\sqrt{3}}\left\{\frac{5}{6}\,l\,2\,\pi - l\,\Gamma\left(\frac{1}{6}\right)\right\}$ V. T. 191. N°. 7.

12) $\int l x \frac{d\,x}{e^x+e^{-x}+2\,Cos.\,\lambda} = \frac{1}{2}\,\pi\,Cosec.\,\lambda\,l\,\frac{(2\,\pi)^{\frac{\lambda}{\pi}}\,\Gamma\left(\frac{1}{2}+\frac{\lambda}{2\,\pi}\right)}{\Gamma\left(\frac{1}{2}-\frac{\lambda}{2\,\pi}\right)}$ V. T. 190. N°. 9.

F. Exponent. polynôme en dén. Logar. en num. $l\,(p^2 \pm x^2)$. TABLE 275. Lim. 0 et ∞.

1) $\int l\,(1+x^2)\,\frac{d\,x}{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}} = l\,\frac{4}{\pi}$ Malmsten, Cr. 38. 1.

2) $\int l\,(1+x^2)\,\frac{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}}{(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})^2}\,d\,x = 2\sqrt{2} - \frac{8}{\pi} + \frac{2\sqrt{2}}{\pi}\,l\,\frac{\sqrt{2}+1}{\sqrt{2}-1}$ V. T. 138. N°. 19.

3) $\int l(1+x^2)\,\frac{e^{\frac{1}{4}\pi x}+e^{-\frac{1}{4}\pi x}}{(e^{\frac{1}{4}\pi x}-e^{-\frac{1}{4}\pi x})^2}\,d\,x = \frac{\pi-2}{\pi}$ V. T. 138. N°. 16.

4) $\int l\,(1+x^2)\,\frac{d\,x}{(e^{\pi x}-e^{-\pi x})^2} = \frac{1}{4\,\pi}\,(2\,A-1)$ V. T. 138. N°. 10.

5) $\int l\,(1+x^2)\,\frac{e^{\pi x}+e^{-\pi x}}{(e^{\pi x}-e^{-\pi x})^2}\,d\,x = \frac{2\,l\,2-1}{2\,\pi}$ V. T. 138. N°. 12.

6) $\int l\,(1+x^2)\,\frac{d\,x}{(e^{qx}-e^{-qx})^2} = \frac{1}{2\,q}\left\{l\frac{q}{\pi} + \frac{\pi}{2\,q} - Z'\left(\frac{\pi+q}{\pi}\right)\right\}$ V. T. 138. N°. 11.

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7) $\int l(1+x^2)\frac{\pi(e^{px}+e^{-px})(e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x})-2p(e^{px}-e^{-px})(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})}{(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})^2}dx=-4+$

$+2\pi\,Cos.p+2\,Sin.p\,l\frac{1+Sin.p}{1-Sin.p},\ 0\leqq p\leqq\frac{1}{2}\pi;$ V. T. 138. N°. 15.

8) $\int l(1+x^2)\frac{\lambda(e^{(2\pi-\lambda)x}+e^{(\lambda-2\pi)x})+(2\pi-\lambda)(e^{\lambda x}+e^{-\lambda x})}{(e^{\pi x}-e^{-\pi x})^2}dx=1+2\sum_1^\infty\frac{Cos.n\lambda}{n+1}$ V. T. 138. N°. 8.

9) $\int l(1+x^2)\frac{\pi(e^{\pi x}+e^{-\pi x})(e^{px}+e^{-px})-p(e^{px}-e^{-px})(e^{\pi x}-e^{-\pi x})}{(e^{\pi x}-e^{-\pi x})^2}dx=$ V. T. 138. N°. 6.

$=p\,Sin.p-1+Cos.p\,l\{2(1+Cos.p)\}$

10) $\int l\left(\frac{9}{4}+x^2\right)\frac{e^{\frac{2}{3}\pi x}-e^{-\frac{2}{3}\pi x}}{e^{\pi x}-e^{-\pi x}}dx=2\,Sin.\frac{1}{3}\pi\,.\,l\left(\frac{1}{2}Cot.\frac{\pi}{12}\right)$

11) $\int l(a^2+x^2)\frac{e^{\frac{b\pi x}{c}}+e^{-\frac{b\pi x}{c}}}{e^{\pi x}+e^{-\pi x}}dx=Sec.\frac{b\pi}{2c}l2c+2\sum_1^c(-1)^{n-1}Cos.\left\{(n-\tfrac{1}{2})\frac{b\pi}{c}\right\}l\frac{\Gamma\left(\frac{a+c+n-\frac{1}{2}}{2c}\right)}{\Gamma\left(\frac{a+n-\frac{1}{2}}{2c}\right)}$, $b+c$ impair;

12) $=Sec.\frac{b\pi}{2c}lc+2\sum_1^{\frac{c-1}{2}}(-1)^{n-1}Cos.\left\{(n-\tfrac{1}{2})\frac{b\pi}{c}\right\}l\frac{\Gamma\left(\frac{a+c-n+\frac{1}{2}}{c}\right)}{\Gamma\left(\frac{a+n-\frac{1}{2}}{c}\right)}$, $b+c$ pair;

13) $\int l(a^2+x^2)\frac{e^{\frac{b\pi x}{c}}-e^{-\frac{b\pi x}{c}}}{e^{\pi x}-e^{-\pi x}}dx=Tang.\frac{b\pi}{2c}l\,2c+2\sum_1^{c-1}(-1)^{n-1}Sin.\frac{nb\pi}{c}l\frac{\Gamma\left(\frac{a+c+n}{2c}\right)}{\Gamma\left(\frac{a+n}{2c}\right)}$, $b+c$ impair;

14) $=Tang.\frac{b\pi}{2c}lc+2\sum_1^{\frac{c-1}{2}}(-1)^{n-1}Sin.\frac{nb\pi}{c}l\frac{\Gamma\left(\frac{a+c-n}{c}\right)}{\Gamma\left(\frac{a+n}{c}\right)}$, $b+c$ pair;

15) $\int l\left(\frac{1}{4}c^2+x^2\right)\frac{e^{\frac{b\pi x}{c}}+e^{-\frac{b\pi x}{c}}}{e^{\pi x}+e^{-\pi x}}dx=\sum_1^c(-1)^{n-1}Cos.\left\{(n-\tfrac{1}{2})\frac{b\pi}{c}\right\}l\left\{\left(\frac{c+1}{2}-n\right)Cot.\left(\frac{\pi}{4}-\frac{2n-1}{4c}\pi\right)\right\}$, $b+c$ impair;

16) $\int l\left(\frac{1}{4}c^2+x^2\right)\frac{e^{\frac{b\pi x}{c}}-e^{-\frac{b\pi x}{c}}}{e^{\pi x}-e^{-\pi x}}dx=\sum_1^{c-1}(-1)^{n-1}Sin.\frac{nb\pi}{c}l\left\{\left(\frac{1}{2}c-n\right)Cot.\left(\frac{\pi}{4}-\frac{n\pi}{2c}\right)\right\}$, $b+c$ impair;

17) $\int l(a^2+x^2)\frac{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}}{e^{\pi x}-e^{-\pi x}}dx = 2\,l\frac{2\Gamma\left(\frac{a+3}{4}\right)}{\Gamma\left(\frac{a+1}{4}\right)}$

18) $\int l(a^2+x^2)\frac{e^{\frac{1}{3}\pi x}-e^{-\frac{1}{3}\pi x}}{e^{\pi x}-e^{-\pi x}}dx = 2\,Sin.\frac{1}{3}\pi\,l\frac{6\Gamma\left(\frac{a+4}{6}\right)\Gamma\left(\frac{a+5}{6}\right)}{\Gamma\left(\frac{a+1}{6}\right)\Gamma\left(\frac{a+2}{6}\right)}$

Les intégrales 10) à 18) sont trouvées par Malmsten, Cr. 38. 1; (où il y a plusieurs fautes).

19) $\int l(1-x^2)\frac{dx}{(e^{\pi x}-e^{-\pi x})^2} = \frac{1}{4\pi}\sum_0^\infty(-1)^{n-1}\frac{B_{2n+1}}{n+1}$ V. T. 138. N°. 21.

F. Expon. Logar. TABLE 276. Lim. — ∞ et ∞.

1) $\int lx\frac{dx}{e^x+e^{-x}} = \frac{1}{2}\pi\,l\left\{\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}\sqrt{2\pi}\right\}$ V. T. 191. N°. 1.

2) $\int lx\frac{e^{ax}-e^{-ax}}{e^{bx}-e^{-bx}}dx = \frac{\pi}{2b}Tang.\frac{a\pi}{2b}l2\pi+\frac{\pi}{b}\sum_1^{b-1}(-1)^{n-1}Sin.\frac{na\pi}{b}l\frac{\Gamma\left(\frac{b+n}{2b}\right)}{\Gamma\left(\frac{n}{2b}\right)}$, $a+b$ impair;

3) $= \frac{\pi}{2b}Tang.\frac{a\pi}{2b}l\pi+\frac{\pi}{b}\sum_1^{\frac{b-1}{2}}(-1)^{n-1}Sin.\frac{na\pi}{b}l\frac{\Gamma\left(\frac{b-n}{b}\right)}{\Gamma\left(\frac{n}{b}\right)}$, $a+b$ pair;

V. T. 191. N°. 3, 4.

4) $\int lx\frac{dx}{e^x+e^{-x}+1} = \frac{\pi}{\sqrt{3}}l\left\{\frac{\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{3})}\sqrt[3]{2\pi}\right\}$ V. T. 191. N°. 2.

5) $\int lx\frac{e^{(a-1)x}dx}{1+e^{2x}+e^{4x}+\ldots+e^{2(a-1)x}} = \frac{\pi}{2a}Tang.\frac{\pi}{2a}l2\pi+\frac{\pi}{a}\sum_1^{a-1}(-1)^{n-1}Sin.\frac{n\pi}{a}l\frac{\Gamma\left(\frac{a+n}{2a}\right)}{\Gamma\left(\frac{n}{2a}\right)}$, a pair;

6) $= \frac{\pi}{2a}Tang.\frac{\pi}{2a}l\pi+\frac{\pi}{a}\sum_1^{\frac{a-1}{2}}(-1)^{n-1}Sin.\frac{n\pi}{a}l\frac{\Gamma\left(\frac{a-n}{a}\right)}{\Gamma\left(\frac{n}{a}\right)}$, a impair;

V.T.191. N°. 5, 6.

7) $\int lx \frac{(p+1)e^{(p-1)x} + (p-1)e^{(p+1)x} - (q+1)e^{(q-1)x} - (q-1)e^{(q+1)x}}{(e^x+e^{-x})^2} dx = \pi l \left\{ Tang. \left(\frac{p+1}{4}\pi\right). Cot. \left(\frac{q+1}{4}\pi\right) \right\}$ V. T. 146. N°. 11.

8) $\int lx \frac{(p+1)e^{(p-1)x} - (p-1)e^{(p+1)x} - (q+1)e^{(q-1)x} + (q-1)e^{(q+1)x}}{(e^x-e^{-x})^2} dx = \pi l \left\{ Sin. \left(\frac{q+1}{2}\pi\right). Cosec. \left(\frac{p+1}{2}\pi\right) \right\}$ V. T. 146. N°. 12.

F. Expon. Logar. TABLE 277. Lim. diverses.

1) $\int_0^{2\pi} l(1 - pe^{xi})\, dx = 0$

2) $\int_0^{2\pi} l(1 - pe^{-xi})\, dx = 0$

(1, 2) Moigno, Calc. Int. 138.

3) $\int_{-\pi}^{\pi} e^{-qxi} l(1 - pe^{xi})\, dx = -\frac{2\pi}{q} p^q$

4) $\int_{-\pi}^{\pi} e^{qxi} l(1 - pe^{xi})\, dx = 0$

(3, 4) , $p^2 < 1$; Poisson, P. 19. 404. N°. 78.

5) $\int_0^{\infty} \frac{e^{-x}}{lx} dx = 0$ V. T. 43. N°. 4.

6) $\int_p^{\infty} e^{-x} lx\, dx = e^{-p} lp - Ei.(-p)$ V. T. 150. N°. 5.

7) $\int_p^{\infty} lx \frac{e^{qx} - e^{-qx}}{(e^{qx}+e^{-qx})^2} dx = \frac{1}{q} \frac{lp}{e^{pq}+e^{-pq}} + \frac{1}{q\pi} \sum_0^{\infty} \left[\frac{(-1)^n}{2n+1} l \left\{ 1 + \left(\frac{2n+1}{2pq}\pi\right)^2 \right\} \right]$ V. T. 150. N°. 10.

8) $\int_p^{\infty} lx \frac{e^{qx} + e^{-qx}}{(e^{qx}-e^{-qx})^2} dx = \frac{1}{q} \frac{lp}{e^{pq}-e^{-pq}} + \frac{1}{2pq^2} + \frac{1}{q\pi} \sum_1^{\infty} \left\{ \frac{(-1)^n}{n} Arctang. \left(\frac{n\pi}{pq}\right) \right\}$ V. T. 150. N°. 9.

9) $\int_{-lp}^{\infty} e^{-x} lx\, dx = p\, l\, l\frac{1}{p} - li(p)$ V. T. 150. N°. 14.

10) $\int_0^1 e^{\sqrt{x}-1} l(1 - \sqrt{x})\, dx = 2\frac{1-e}{e}$ V. T. 376. N°. 3.

1) $\int e^{-x} Sin.\, x\, dx = \frac{1}{2}$

2) $\int e^{-qx} Sin.\, q x\, dx = \frac{1}{2q}$ Oettinger, Cr. 38. 216.

3) $\int e^{-qx} Cos.\, q x\, dx = \frac{1}{2q}$

4) $\int e^{-px} Sin.\, x\, dx = \frac{1}{p^2+1}$

5) $\int e^{-px} Cos.\, x\, dx = \frac{p}{1+p^2}$ Dienger, Cr. 38. 231. — Raabe, Int. 152.

6) $\int e^{-x} Sin.\, q x\, dx = \frac{q}{1+q^2}$ Poisson, P. 19. 60. — Dienger, Cr. 46. 119. — Schlömilch, Gr. 5. 204.

7) $\int e^{-x} Cos.\, q x\, dx = \frac{1}{1+q^2}$ Dienger, Cr. 46. 119. — Schlömilch, Gr. 5. 204.

8) $\int e^{-px} Sin.\, q x\, dx = \frac{q}{p^2+q^2}$

9) $\int e^{-px} Cos.\, q x\, dx = \frac{p}{p^2+q^2}$ Poisson, P. 16. 215. N°. 2. — Cauchy, Cours. Leç. 32. — Grunert, Cr. 8. 146. — Lobatto, Cr. 11. 169. — Boncompagni, Cr. 25. 74. — Oettinger, Cr. 38. 216.

Sur la formule (9) voyez encore: Poisson, Mém. Inst. 1811. 163. N°. 25. — Id., P. 18. 295. N°. 21. — Dienger, Cr. 38. 331.

10) $\int e^{-px} Sin.\, (q x + \lambda)\, dx = \frac{1}{p^2+q^2}(q\, Cos.\, \lambda + p\, Sin.\, \lambda)$ Poisson, Chal. 158.

11) $\int e^{bx\pi} Sin.\, q x i\, dx = \frac{q i}{b^2 \pi^2 - q^2}$ Schlömilch, Gr. 3. 9.

12) $\int e^{-px} Cot.\, \frac{1}{2} q x\, dx = 2 q \sum_1^\infty \frac{n}{p^2 + q^2 n^2}$ Cauchy, Exerc. 1827. p. 141.

13) $\int e^{-x} Sin.\, (2 p \sqrt{x})\, dx = p e^{-p^2} \sqrt{\pi}$ Helmling, Transf. 14.

14) $\int e^{-x} Tang.\, (q \sqrt{x})\, dx = 2 q \sqrt{\pi} \sum_1^\infty (-1)^n n e^{-(nq)^2}$ V. T. 388. N°. 20.

15) $\int e^{-x} Cot.\, (q \sqrt{x})\, dx = -2 q \sqrt{\pi} \sum_1^\infty n e^{-(nq)^2}$ V. T. 388. N° 21.

16) $\int e^{-x} Cosec.\, (2 q \sqrt{x})\, dx = -2 q \sqrt{\pi} \sum_1^\infty (2n-1) e^{-(2n-1)^2 q^2}$ V. T. 388. N°. 22.

1) $\int e^{-px}\, Sin.^{2a} x\, dx = \frac{2a.2a-1}{p^2+(2a)^2} \cdot \frac{2a-2.2a-3}{p^2+(2a+2)^2} \cdots \frac{2.1}{p^2+2^2} \frac{1}{p}$, $a > 1$;

2) $\int e^{-px}\, Sin.^{2a+1} x\, dx = \frac{2a+1.2a}{p^2+(2a+1)^2} \cdot \frac{2a-1.2a-2}{p^2+(2a-1)^2} \cdots \frac{3.2}{p^2+3^2} \frac{1}{p^2+1}$ Dienger, Cr. 38. 331. — Schlömilch, Gr. 7. 38.

3) $\int e^{-px} Cos.^{2a} x dx = \frac{1}{p} \frac{1^{2a/1}}{p^2+2^2.p^2+4^2 \ldots p^2+(2a)^2} \left\{1 + \frac{p^2}{1.2} + \frac{p^2.p^2+2^2}{1.2.3.4} + \ldots + \frac{p^2.p^2+2^2 \ldots p^2+(2a-2)^2}{1^{2a/1}}\right\}$ Dienger, Cr. 38. 331.

4) $\int e^{-px} Cos.^{2a+1} x dx = p \frac{1^{2a/1}}{p^2+1^2.p^2+3^2 \ldots p^2+(2a+1)^2} \left\{1 + \frac{p^2+1^2}{1.2.3} + \ldots + \frac{p^2+1^2.p^2+3^2 \ldots p^2+(2a-1)^2}{1^{2a+1/1}}\right\}$

5) $\int e^{-px}\, Cos.\, qx.\, Cos.\, rx\, dx = \frac{1}{2} \frac{p}{p^2+(q-r)^2}$ p très-petit; Cauchy, Sav. Etr. 1827. 124. Note 6. — Id., P. 19. 511.

6) $= \frac{p}{2} \left\{\frac{1}{p^2+(q-r)^2} + \frac{1}{p^2+(q+r)^2}\right\}$ Poisson, P. 18. 295. No. 26.

7) $= p \frac{p^2+q^2+r^2}{(p^2+q^2+r^2)^2 - 4q^2r^2}$ Dienger, Gr. 12. 97.

8) $\int e^{-px}\, Cos.\, qx.\, Sin.\, rx\, dx = 2 \frac{p^2-q^2+r^2}{(p^2+q^2+r^2)^2 - 4q^2r^2}$

9) $= \frac{1}{2} \left\{\frac{q+r}{p^2+(q+r)^2} - \frac{q-r}{p^2+(q-r)^2}\right\}$ Cauchy, Sav. Etr. 1827. 124. Note 6.

10) $= r \frac{p^2-q^2+r^2}{\{p^2+(q-r)^2\}\{p^2+(q+r)^2\}}$ Dienger, Cr. 41. 137.

11) $\int e^{-px}\, Sin.\, qx.\, Sin.\, rx\, dx = \frac{2pqr}{\{p^2+(q-r)^2\}\{p^2+(q+r)^2\}}$

12) $= \frac{2pqr}{(p^2+q^2+r^2)^2 - 4q^2r^2}$ Dienger, Gr. 12. 97.

13) $= \frac{1}{2} p \left\{\frac{-1}{p^2+(q+r)^2} + \frac{1}{p^2+(q-r)^2}\right\}$ Cauchy, Sav. Etr. 1827. 124. Note 6.

14) $= \frac{1}{2} \frac{p}{p^2+(q-r)^2}$, p très-petit;

15) $\int e^{-px}\, Cos.\, x\, dx \sqrt{Cos.\, 2bx} = \sum_0^\infty \frac{(-2b)^n}{n^{n-1/1}} \frac{Cos.\, (n\, Arccot.\, p)}{\sqrt{(1+p^2)^n}}$ Cauchy, Sav. Etr. 1827. 124. Note 3.

16) $\int (e^{-qx}\, Cos.\, px - e^{-px}\, Sin.\, qx)\, dx = 0$ Lobatto, Cr. 11. 169.

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17) $\int (e^{-qx}\, Sin.\, px - e^{-px}\, Cos.\, qx)\, dx = 0$ Lobatto, Cr. 11. 169.

18) $\int e^{-2px}\, Sin.\, (q^2 x^2)\, dx = \frac{1}{4q}\left\{Cos.\left(\frac{p^2}{q^2}\right) + Sin.\left(\frac{p^2}{q^2}\right)\right\}\sqrt{2\pi} -$

$-\frac{1}{q^2}\left\{Cos.\left(\frac{p^2}{q^2}\right)\sum_0^\infty(-1)^n \frac{p}{(4n+1)1^{2n/1}}\left(\frac{p}{q}\right)^{4n} + Sin.\left(\frac{p^2}{q^2}\right)\sum_0^\infty(-1)^n \frac{p}{(4n-1)1^{2n-1/1}}\left(\frac{p}{q}\right)^{4n-2}\right\}$

19) $\int e^{-2px}\, Cos.\, (q^2 x^2)\, dx = \frac{1}{4q}\left\{Cos.\left(\frac{p^2}{q^2}\right) - Sin.\left(\frac{p^2}{q^2}\right)\right\}\sqrt{2\pi} -$

$-\frac{1}{q^2}\left\{Sin.\left(\frac{p^2}{q^2}\right)\sum_0^\infty(-1)^n \frac{p}{(4n+1)1^{2n/1}}\left(\frac{p}{q}\right)^{4n} - Cos.\left(\frac{p^2}{q^2}\right)\sum_0^\infty(-1)^n \frac{p}{(4n-1)1^{2n-1/1}}\left(\frac{p}{q}\right)^{4n-2}\right\}$

(18, 19:) Helmling, Transf. 18, 19.

F. Exponent. e^{-ax^2}.
Circ. Dir. ent.

TABLE 280.

Lim. 0 et ∞.

1) $\int e^{-x^2}\, Cos.\, px\, dx = \frac{1}{2} e^{-\frac{p^2}{4}} \sqrt{\pi}$ Cauchy, Sav. Etr. 1827. 124. Note 2. — Id., Cours. Leç. 40. — Id., Sav. Etr. 1827. 599. P. 1. § 2. — Id., Lim. Imag. 91. — Legendre, Exerc. 3. 48. — Bidone, Mém. Turin. 1812. 231. Art. 8. N°. 34. — Kummer, Cr. 17. 210. — Schlömilch, Stud. I. 25.

2) $\int e^{-px^2}\, Cos.\, x\, dx = \frac{1}{2} e^{-\frac{1}{4p}} \sqrt{\frac{\pi}{p}}$ Laplace,

3) $\int e^{-px^2}\, Cos.\, qx\, dx = \frac{1}{2} e^{-\frac{q^2}{4p}} \sqrt{\frac{\pi}{p}}$ Poisson, Chal. 103. — Cauchy, Exerc. 1827. p. 233. — Laplace, Probab. I. 25. — Schlömilch, Beitr. III. § 16. — Id., Stud. I. 12. — Id., Gr. 5. 90. — Id., Gr. 9. 379. — Helmling, Transf. 12. — Raabe, Cr. 48. 178.

4) $\int e^{-p^2x^2}\, Cos.\, qx\, dx = \frac{1}{2p} e^{-\frac{q^2}{4p^2}} \sqrt{\pi}$

5) $\int e^{-\frac{1}{4}q^2x^2}\, Cos.\, qx\, dx = \frac{1}{qe}\sqrt{\pi}$

(4, 5:) Oettinger, Cr. 38. 216. La valeur de (4) est fautive.

6) $\int e^{-\frac{a}{b}x^2}\, Cos.\, (bx\sqrt{a})\, dx = \frac{1}{2} e^{-\frac{1}{4}b^3} \sqrt{\frac{b\pi}{a}}$ Laplace, Mém. Inst. 1809. 353. § 3., où elle est fautive.

7) $\int e^{-x^2}\, Sin.\, ax\, dx = \frac{1}{2}\sum_0^\infty (-1)^n \frac{a^{2n+1}}{(n+1)^{n+1/1}}$ Legendre, Exerc. 3. 49.

8) $\int e^{-px^2}\, Sin.\, qx\, dx = 0$ Meyer, Int. Déf. 119. (fautive).

9) $= \frac{1}{\sqrt{2p}}\sum_0^\infty (-1)^n \frac{1}{1^{n+1|2}}\left(\frac{q}{\sqrt{2p}}\right)^{2n+1}$ Schubert, Samml. 117. (fautive)

10) $\int e^{-px^2} Sin.\, qx\, dx = \sum_0^\infty (-1)^n \frac{1}{(n+2)^{n+1/1}} \cdot \frac{q^{2n+1}}{p^{n+1}}$

11) $\int e^{-q^2x^2} Sin.\, q\, x\, dx = \frac{1}{q} \sum_0^\infty \frac{(-1)^n}{(n+2)^{n+1/1}}$

Oettinger, Cr. 38. 216.

12) $\int e^{x^2 i} Cos.\, q\, x\, dx = \frac{1+i}{2} e^{-\frac{q^2 i}{4}} \sqrt{\frac{\pi}{2}}$ Cauchy, Lim. Imag. 190.

13) $\int e^{-rx^2} Sin.\, p\, x.\, Sin.\, q\, x\, d\, x = \frac{1}{4}\sqrt{\frac{\pi}{r}} \left\{ e^{-\frac{(q-p)^2}{4r}} - e^{-\frac{(p+q)^2}{4r}} \right\}$ Poisson, Chal. 143.

14) $\int e^{-rx^2} Cos.\, p\, x.\, Cos.\, q\, x\, d\, x = \frac{1}{4}\sqrt{\frac{\pi}{r}} \left\{ e^{-\frac{(q-p)^2}{4r}} + e^{-\frac{(q+p)^2}{4r}} \right\}$ V. T. 280. N°. 3, 13.

15) $\int e^{-px^2} Sin.\, (q x^2). Cos.\, r x\, dx = \frac{1}{2}\sqrt{\frac{\pi}{p^2+q^2}} \cdot e^{-ab}(b\, Sin.\, ac + c\, Cos.\, ac)$

16) $\int e^{-px^2} Cos.\, (q x^2). Cos.\, r x\, dx = \frac{1}{2}\sqrt{\frac{\pi}{p^2+q^2}} \cdot e^{-ab}(b\, Cos.\, ac + c\, Sin.\, ac)$

17) $\int e^{-x^2} Sin.\, \left(\frac{2p^2}{x^2}\right) dx = \frac{1}{2} e^{-2p} Sin.\, (2p) \sqrt{\pi}$

18) $\int e^{-x^2} Cos.\, \left(\frac{2p^2}{x^2}\right) dx = \frac{1}{2} e^{-2p} Cos.\, (2p) \sqrt{\pi}$

où $a = \frac{r^2}{4(p^2+q^2)}$, $b = \sqrt{\frac{p+\sqrt{(p^2+q^2)}}{2}}$, $c = \sqrt{\frac{-p+\sqrt{(p^2+q^2)}}{2}}$

Helmling, Transf. 21, 22, 29, 30.

19) $\int e^{-x^2} Sin.^2\, x\, dx = \frac{e-1}{4e}\sqrt{\pi}$

20) $\int e^{-x^2} Sin.^2\, (x\sqrt{p})\, dx = \frac{1-e^{-p}}{4}\sqrt{\pi}$

Dienger, Cr. 46. 119.

21) $\int e^{bx} Sin.^a\, x\, dx = \frac{1-(-1)^a e^{b\pi}}{\Gamma\left(\frac{a+bi}{2}+1\right)\Gamma\left(\frac{a-bi}{2}+1\right)} \frac{\pi}{2a} 1^{a/1} e^{b\pi}$ Lobatschewsky, Mém. Kasan. 1835. 211.

22) $\int e^{-x^2} Cot.\, p\, x\, d\, x = \sqrt{\pi} \sum_1^\infty e^{-(np)^2}$ Cauchy, Exerc. 1827. p. 141.

23) $\int e^{-px^2} Sin.\, (q x^2)\, dx = \frac{\sqrt{\pi}}{2\sqrt[4]{(p^2+q^2)}} Sin.\, \left(\frac{1}{2} Arctang. \frac{q}{p}\right)$

24) $\int e^{-px^2} Cos.\, (q x^2)\, dx = \frac{\sqrt{\pi}}{2\sqrt[4]{(p^2+q^2)}} Cos.\, \left(\frac{1}{2} Arctang. \frac{q}{p}\right)$

Helmling, Transf. II. S. 63.

25) $\int e^{-px^2} Sin.(rx^2).Cos.2sx\,dx = \frac{\sqrt{\pi}}{2\sqrt{(p^2+r^2)}} e^{-b\varphi}(\varphi\, Sin.b\varphi - c\,Cos.b\varphi)$

26) $= \frac{\sqrt{\pi}}{a} e^{-\frac{s^2}{a^2}Cos.2\psi}.Sin.\left(\psi - \frac{s^2}{a^2} Sin.2\psi\right)$

27) $\int e^{-px^2} Cos.(rx^2).Cos.2sx\,dx = \frac{\sqrt{\pi}}{a} e^{-\frac{s^2}{a^2}Cos.2\psi}.Cos.\left(\psi - \frac{s^2}{a^2} Sin.2\psi\right)$

28) $= \frac{1}{2}\sqrt{\frac{\pi}{p^2+r^2}}\, e^{-b\varphi}(\varphi\, Cos.b\varphi + c\, Sin.b\varphi)$

Helmling, Transf. II. S. 82, 69, 70.
où $a^4 = p^2 + r^2$, $4b = \frac{s^2}{p^2+r^2}$, $c = \sqrt{\frac{-p+\sqrt{(p^2+r^2)}}{2}}$, $\varphi = \sqrt{\frac{p+\sqrt{(p^2+r^2)}}{2}}$, $\psi = \frac{1}{2} Arctang.\frac{r}{p}$.

TABLE 281.

F. Expon. en dén. binôme à exp. $e^{\pm ax}$. Circ. Dir. en num. Lim. 0 et ∞.

1) $\int \frac{Sin.px}{e^x - 1} dx = -\frac{1}{2}\pi - \frac{1}{2p} - \frac{\pi}{e^{-2p\pi} - 1}$ Plana, Mém. Turin. 1818. 7. IV. N°. 18. — Id., Mém. Turin. 1820.

2) $\int \frac{Sin.px}{e^x - e^{-x}} dx = -\frac{1}{4}\pi + \frac{1}{2}\frac{\pi}{1+e^{-p\pi}}$ Plana, Mém. Turin. 1818. 7. IV. 19.

3) $\int \frac{Cos.px}{e^x - 1} dx = \infty$ Plana, Mém. Turin. 1820.

4) $\int \frac{Cos.px}{e^{\pi x} + e^{-\pi x}} dx = \frac{1}{2}\frac{e^{\frac{1}{2}p}}{e^p + 1}$, $p < \pi$; Legendre, Exerc. 5. 45.

5) $\int \frac{Sin.pxi}{i} \frac{dx}{e^{\pi x} + 1} = \frac{1}{2} Cosec.p - \frac{1}{2p}$ Schlömilch, Gr. 3. 9.

6) $\int \frac{Cos.px}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}} dx = \frac{1}{e^p + e^{-p}}$ Schlömilch, Gr. 1. 360. — Id., Beitr. II. § 6. — Id., Stud. II. 19.

7) $\int \frac{Sin.pxi}{i} \frac{dx}{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}} = \frac{1}{2} Tang.p$ Schlömilch, Gr. 3. 9.

8) $\int \frac{Sin.px}{e^{\pi x} - e^{-\pi x}} dx = \frac{1}{4}\frac{e^p - 1}{e^p + 1}$, $p < \pi$; Legendre, Exerc. 5. 45. — Poisson, Mém. Inst. 1811. 163. N°. 28. — Plana, Mém. Turin. 1818. 7. IV. 19. — Schlömilch, Beitr. II. § 6. — Id., Stud. II. 19.

9) $\int \frac{Sin.px}{e^{2\pi x} - 1} dx = \frac{1}{4}\frac{e^p + 1}{e^p - 1} - \frac{1}{2p}$ Legendre, Exerc. 5. 48. — Poisson, P. 18. 295. N°. 25. — Id., P. 20. 222. — Id., Mém. Inst. 1811. 163. N°. 18. — Plana, Mém. Turin. 1818. 7. IV. 18. — Dienger, Gr. 14. 225.

10) $\int \frac{Sin.pxi}{i} \frac{dx}{e^{2\pi x} - 1} = \frac{1}{2p} - \frac{1}{4} Cot.\frac{1}{2}p$ Schlömilch, Gr. 3. 9.

11) $\int \frac{Sin.\, p\, x}{e^{ax} - e^{(a-1)x}}\, dx = \frac{1}{2}\pi - \frac{1}{2p} + \frac{\pi}{e^{2p\pi}-1} - \sum_0^\infty \frac{n}{n^2+(a-1)^2}$ Plana, Mém. Turin. 1818. 7. Add.

12) $\int \frac{Sin.^2\, p\, x}{e^{\pi x} + e^{-\pi x}}\, dx = \frac{1}{8}\frac{(e^p-1)^2}{e^{2p}+1}$ V. T. 38. N°. 7 et T. 281. N°. 4.

13) $\int \frac{Cos.^2\, p\, x}{e^{\pi x} + e^{-\pi x}}\, dx = \frac{1}{8}\frac{(e^p+1)^2}{e^{2p}+1}$ V. T. 38. N°. 7 et T. 281. N°. 4.

14) $\int \frac{Sin.\, 2\, p\, x .\, Sin.\, 2\, q\, x}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}\, dx = \frac{e^{2p}-e^{-2p}}{2}\,\frac{e^{2q}-e^{-2q}}{e^{4p}+e^{-4p}+e^{4q}+e^{-4q}}$

15) $\int \frac{Cos.\, 2\, p\, x .\, Cos.\, 2\, q\, x}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}\, dx = \frac{e^{2p}+e^{-2p}}{2}\,\frac{e^{2q}+e^{-2q}}{e^{4p}+e^{-4p}+e^{4q}+e^{-4q}}$

16) $\int \frac{Sin.\, 2\, p\, x .\, Sin.\, 2\, q\, x}{e^{\pi x} + e^{-\pi x}}\, dx = \frac{e^{p}-e^{-p}}{4}\,\frac{e^{q}-e^{-q}}{e^{2p}+e^{-2p}+e^{2q}+e^{-2q}}$

17) $\int \frac{Cos.\, 2\, p\, x .\, Cos.\, 2\, q\, x}{e^{\pi x} + e^{-\pi x}}\, dx = \frac{e^{p}+e^{-p}}{4}\,\frac{e^{q}+e^{-q}}{e^{2p}+e^{-2p}+e^{2q}+e^{-2q}}$

(14–17) $, p < \frac{1}{2}\pi, q < \frac{1}{2}\pi;$ Poisson, P. 17. 612. N°. 21.

18) $\int \frac{Sin.\, p\, x .\, Cos.\, q\, x}{e^{\pi x} - e^{-\pi x}}\, dx = \frac{1}{4}\frac{e^p - e^{-p}}{e^p+e^{-p}+e^q+e^{-q}}, p+q<\pi;$ Poisson, P. 18. 295. N°. 21. — Id., Mém. Inst. 1811. 163. N°. 28. — Plana, Mém. Turin. 1818. 7. IV. 20.

F. Exp. en num. et en dén. binôme à exp. $e^{\pm ax}$. Circ. Dir. en num. TABLE 282. Lim. 0 et ∞.

1) $\int \frac{e^{qx}}{e^{\pi x} - e^{-\pi x}}\, Sin.\, p\, x\, dx = \sum_1^\infty \frac{p}{\{(2n-1)\pi - q\}^2 + p^2}, q < \pi;$ Poisson, P. 18. 295. N°. 21.

2) $\int \frac{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}\, Sin.\, p\, x\, dx = \frac{2}{e^p - e^{-p}}$ Poisson, P. 18. 295. N°. 25. — Schlömilch, Beitr. II. 6. — Id., Stud. II. 19.

3) $\int \frac{e^{2px} + e^{-2px}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}\, Cos.\, q\, x\, dx = \frac{e^q + e^{-q}}{e^{2q}+e^{-2q}+2\, Cos.\, 4\, p}\, 2\, Cos.\, 2\, p, p < \frac{1}{2}\pi;$ Poisson, P. 18. 295. N°. 21.

4) $\int \frac{e^{2px} + e^{-2px}}{e^{\pi x} + e^{-\pi x}}\, Cos.\, 2\, q\, x\, dx = \frac{e^q + e^{-q}}{e^{2q}+e^{-2q}+2\, Cos.\, 4\, p}\, Cos.\, p, p < \frac{1}{2}\pi;$ Legendre, Exerc. 5. 44. — Poisson, P. 18. 295. N°. 21.

5) $\int \frac{e^{2px} - e^{-2px}}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}\, Sin.\, q\, x\, dx = \frac{e^q - e^{-q}}{e^{2q}+e^{-2q}+2\, Cos.\, 4\, p}\, 2\, Sin.\, 2\, p, p < \frac{1}{2}\pi;$ Poisson, P. 18. 295. N°. 21. — Id., Mém. Inst. 1811. 163. N°. 28.

6) $\int \frac{e^{2px} - e^{-2px}}{e^{\pi x} + e^{-\pi x}}\, Sin.\, 2\, q\, x\, dx = \frac{e^q - e^{-q}}{e^{2q}+e^{-2q}+2\, Cos.\, 2\, p}\, Sin.\, p, p < \frac{1}{2}\pi;$ Legendre, Exerc. 5. 43. — Poisson, P. 18. 295. N°. 21. — Id., Mém. Inst. 1811, 163. N°. 28.

F. Exp. en num. et en dén. binôme à exp. $e^{\pm ax}$. Circ. Dir. en num. — **TABLE 282 suite.** — Lim. 0 et ∞.

7) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} Sin.\,q\,x\,d\,x = -\frac{1}{2}\pi + \frac{\pi}{1 - e^{-q\pi}}$ Plana, Mém. Turin. 1818. 7. IV. 18.

8) $\int \frac{e^{\pi x} + e^{-\pi x}}{e^{\pi x} - e^{-\pi x}} Sin.\,q\,x\,d\,x = \frac{1}{2}\frac{e^q + 1}{e^q - 1}$ Poisson, P. 18. 295. N°. 25. — Schlömilch, Beitr. II. 6. — Id., Stud. II. 19.

9) $\int \frac{e^{\pi x} + 1}{e^{\pi x} - 1} Sin.\,q\,x\,d\,x = \frac{e^q + e^{-q}}{e^q - e^{-q}}$ Schlömilch, Gr. 1. 360.

10) $\int \frac{e^{px} + e^{-px}}{e^{\pi x} - e^{-\pi x}} Sin.qxdx = \frac{1}{2}\frac{e^q - e^{-q}}{e^q + e^{-q} + 2\,Cos.\,p}, p^2 \leqq \pi^2$; Poisson, P. 18. 295. N°. 21. — Id., P. 20. 222. — Id., Mém. Inst. 1811. 163. N°. 26. — Legendre, Exerc. 5. 42. — Plana, Mém. Turin. 1818. 7. IV. 20. — Schlömilch, Beitr. II. 6. — Id., Stud. II. 19.

11) $\int \frac{e^{px} + e^{-px}}{e^{2\pi x} - 1} Sin.\,q\,x\,d\,x = \frac{1}{2}\frac{e^q - e^{-q}}{e^q + e^{-q} - 2\,Cos.\,p} - \frac{q}{q^2 + p^2}$, $p < 2\pi$;

12) $\int \frac{e^{-px} + e^{(p-2\pi)x}}{1 - e^{-2\pi x}} Sin.\,q\,x\,d\,x = \frac{1}{2}\frac{e^q - e^{-q}}{e^q + e^{-q} - 2\,Cos.\,p}$

(11, 12) Poisson, Mém. Inst. 1811. 163. N°. 25.

13) $\int \frac{e^{qx}\,Sin.\,p\,x}{e^{2\pi x} - 1}\,dx = \sum_1^\infty \frac{p}{(2n\pi - q)^2 + p^2}$

14) $\int \frac{e^{-qx}\,Sin.\,p\,x}{e^{2\pi x} - 1}\,dx = \sum_1^\infty \frac{p}{(q + 2n\pi)^2 + p^2}$

(13, 14) Poisson, Mém. Inst. 1811. 163. N°. 25.

15) $\int \frac{e^{-qx}\,Sin.\,p\,x}{1 - e^{-x}}\,dx = \varphi - \frac{1}{2p}Sin.\,\varphi - \sum_1^\infty \frac{Sin.^{2n}\varphi.\,Sin.\,2n\varphi}{2np^{2n}} B_{2n-1}$, où $Cot.\,\varphi = \frac{q-1}{p}$; Plana, Mém. Turin. 1818. 7. Add.

16) $\int \frac{e^{px} - e^{-px}}{e^{\pi x} - e^{-\pi x}} Cos.qxdx = \frac{Sin.\,p}{e^q + e^{-q} + 2\,Cos.\,p}, p^2 \leqq \pi^2$; Legendre, Exerc. 5. 44. — Poisson, P. 18. 295. N°. 21. — Id, Mém. Inst. 1811. 163. N°. 26 (qui la trouve fautive) — Plana, Mém. Turin. 1818. 7. IV. 20. — Schlömilch, Beitr. II. 6. — Id., Stud. II. 19.

F. Exp. en num. e^{-x^2}. Circ. Dir. en dén. trinôme. — **TABLE 283.** — Lim. 0 et ∞.

1) $\int \frac{Cos.\left\{x\sqrt{l\frac{1}{q}}\right\}}{1 - 2q\,Cos.\left\{2x\sqrt{l\frac{1}{q}}\right\} + q^2} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2(1-q)\sqrt{q^3}} \sum_1^\infty q^{n^2}$

2) $\int \frac{Cos.\left\{x\sqrt{l\frac{1}{q}}\right\}}{1 + 2q\,Cos.\left\{2x\sqrt{l\frac{1}{q}}\right\} + q^2} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2(1+q)\sqrt{q^3}} \sum_1^\infty (-1)^n q^{n^2}$

(1, 2) Schlömilch, Stud. I. 25.

3) $\int \frac{e^{-px^2}}{1 - 2q\,Cos.\,x + q^2}\,dx = \frac{1}{1-q^2}\left\{\frac{1}{2} + \sum_1^\infty q^n e^{-\frac{n^2}{4p}}\right\}\sqrt{\frac{\pi}{p}}$ Poisson, P. 19. 404. N°. 51.

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4) $\int \frac{Cos.\left\{x\sqrt{l\frac{1}{q}}\right\} - q\,Cos.\left\{3x\sqrt{l\frac{1}{q}}\right\}}{1-2q\,Cos.\left\{2x\sqrt{l\frac{1}{q}}\right\}+q^2} e^{-x^2}\,dx = \frac{1}{2}\sqrt{\pi}\sqrt[4]{q}\sum_0^\infty q^{n^2}$

5) $= \frac{1}{4}\sqrt{\pi}\sqrt[4]{q}\left[1+\sqrt{\left\{\frac{2}{\pi}.F'(p)\right\}}\right]$

6) $\int \frac{Cos.\left\{x\sqrt{l\frac{1}{q}}\right\} + q\,Cos.\left\{3x\sqrt{l\frac{1}{q}}\right\}}{1+2q\,Cos.\left\{2x\sqrt{l\frac{1}{q}}\right\}+q^2} e^{-x^2}\,dx = \frac{1}{2}\sqrt{\pi}\sqrt[4]{q}\sum_0^\infty (-1)^n q^{n^2}$

7) $= \frac{1}{4}\sqrt{\pi}\sqrt[4]{q}\left[1+\sqrt{\left\{\frac{2}{\pi}.\sqrt{(1-p^2)}F'(p)\right\}}\right]$

8) $\int \frac{1-q\,Cos.\left\{2x\sqrt{l\frac{1}{q}}\right\}}{1-2q\,Cos.\left\{2x\sqrt{l\frac{1}{q}}\right\}+q^2} e^{-x^2}\,dx = \frac{1}{2}\frac{\sqrt{\pi}}{\sqrt[4]{q}}\sum_0^\infty q^{\left(\frac{2n+1}{2}\right)^2}$

9) $= \frac{1}{2}\frac{\sqrt{\pi}}{\sqrt[4]{q}}\sqrt{\left\{\frac{p}{2\pi}F'(p)\right\}}$

10) $\int \frac{1+q\,Cos.\left\{2x\sqrt{l\frac{1}{q}}\right\}}{1+2q\,Cos.\left\{2x\sqrt{l\frac{1}{q}}\right\}+q^2} e^{-x^2}\,dx = \frac{1}{2}\frac{\sqrt{\pi}}{\sqrt[4]{q}}\sum_0^\infty (-1)^n q^{\left(\frac{2n+1}{2}\right)^2}$

11) $\int \frac{Cos.\left\{2ax\sqrt{l\frac{1}{q}}\right\} - r\,Cos.\left\{2(a+1)x\sqrt{l\frac{1}{q}}\right\}}{1-2r\,Cos.\left\{2x\sqrt{l\frac{1}{q}}\right\}+r^2} e^{-x^2}\,dx = \frac{1}{2}q^{a^2}\sqrt{\pi}.\sum_0^\infty r^n q^{n^2-2an},\ r^2<1;$

12) $\int \frac{Cos.\left\{2(a-1)x\sqrt{l\frac{1}{q}}\right\} - r\,Cos.\left\{2(a+1)x\sqrt{l\frac{1}{q}}\right\}}{1-2r\,Cos.\left\{4x\sqrt{l\frac{1}{q}}\right\}+r^2} e^{-x^2}\,dx = \frac{1}{2}q^{a^2}\sqrt{\pi}\sum_0^\infty r^n q^{(2n+1)^2-2a(2n+1)},\ r^2<1;$

Kummer, Cr. 17. 210.

, où partout on trouve p par l'équation $l\frac{1}{q}F'(p) = \pi F'\{\sqrt{(1-p^2)}\}$.

F. Exp. $e^{\pm ax}$ ou $e^{\pm ax^2}$. Circ. Dir. } d'autre forme. TABLE 284. Lim. 0 et ∞.

1) $\int \frac{Cos.\,qx}{e^x+e^{-x}+2\,Cos.\,p}\,dx = \frac{1}{2}\pi\,Cosec.\,p\,\frac{e^{pq}-e^{-pq}}{e^{q\pi}-e^{-q\pi}},\ p \leqq \pi;$ Poisson, P. 18. 295. N°. 26. — Schellbach, Cr. 48. 107.

2) $\int \frac{Cos.\,qx}{e^x+e^{-x}+e^p+e^{-p}}\,dx = \frac{2\pi}{e^p-e^{-p}}\frac{Sin.\,pq}{e^{q\pi}-e^{-q\pi}},\ p \leqq \pi;$ Poisson, P. 18. 295. N°. 26.

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3) $\int \frac{e^x - e^{-x}}{e^x + e^{-x} - 2\, Cos.\, p} Sin.\, qx\, dx = \pi \frac{e^{pq} + e^{2q\pi - pq}}{e^{2q\pi} - 1}$ Plana, Mém. Turin. 1818. 7. IV. 22.

4) $\int \frac{e^x + e^{-x}}{e^x + e^{-x} + 2\, Cos.\, p} Cos.\, qx\, dx = -\pi\, Cot.\, p \frac{e^{pq} - e^{-pq}}{e^{q\pi} - e^{-q\pi}}$

5) $\int \frac{e^x - e^{-x}}{e^x + e^{-x} + 2\, Cos.\, p} Sin.\, qx\, dx = \pi \frac{e^{pq} + e^{-pq}}{e^{q\pi} - e^{-q\pi}}$

(4, 5) , $p \leqq \pi$; Poisson, P. 18. 295. N°. 26.

6) $\int \frac{e^{px} - e^{-px}}{e^{2px} + e^{-2px} + 2\, Cos.\, 2qx} Sin.\, qx\, dx = \frac{q}{p^2 + q^2} \frac{\pi}{4}$

7) $\int \frac{e^{px} + e^{-px}}{e^{2px} + e^{-2px} + 2\, Cos.\, 2qx} Cos.\, qx\, dx = \frac{p}{p^2 + q^2} \frac{\pi}{4}$

(6, 7) Raabe, Int. 144. — Ohm, Ausw. 10.

8) $\int \frac{Sin.\{(2k+1)x\}}{Sin.\, x} e^{-px}\, dx = \frac{\pi}{2} \frac{1 + e^{-p\pi}}{1 - e^{-p\pi}}$, $k = \infty$; Schlömilch, Beitr. I. § 4.

9) $\int \frac{Cos.\{(2k+1)x\}}{Cos.\, x} e^{-px}\, dx = (-1)^p \pi \frac{e^{-\frac{1}{2}p\pi}}{1 - e^{-p\pi}}$, $k = \infty$; Raabe, Int. 180.

10) $\int \frac{Sin.\{(2a+1)x\}}{Sin.\, x} e^{-2px}\, dx = \frac{1}{2p} + \sum_1^a \frac{p}{a^2 + p^2}$ Schlömilch, Beitr. I. § 4.

11) $\int \frac{Cos.\{(2a+1)x\}}{Sin.\, x} e^{-px}\, Sin.\, x\, dx = -\frac{2a+1}{p^2 + (2a+1)^2} + 2 \sum_0^a (-1)^n \frac{2n+1}{p^2 + (2n+1)^2}$

12) $= \frac{\pi e^{-\frac{1}{2}p\pi}}{1 + e^{-p\pi}}$, $a = \infty$;

(11, 12) Schlömilch, Beitr. I. § 6.

13) $\int \frac{Sin.\{(2b+1)x\}}{Sin.\, x} e^{-a^2 x^2}\, dx = \frac{\sqrt{\pi}}{a} \left\{ \frac{1}{2} + \sum_1^b e^{-\left(\frac{n}{a}\right)^2} \right\}$

14) $\int \frac{Cos.\{(4b+1)x\}}{Cos.\, x} e^{-a^2 x^2}\, dx = \frac{\sqrt{\pi}}{a} \left\{ \frac{1}{2} + \sum_1^{2b} (-1)^n e^{-\left(\frac{n}{a}\right)^2} \right\}$

(13, 14) Schlömilch, Stud. II. 3, 4.

15) $\int \frac{Sin.\, qx - p\, Sin.\{(q-r)x\}}{1 - 2p\, Cos.\, rx + p^2} \frac{dx}{e^{\pi x} - e^{-\pi x}} = \frac{1}{4(1-p)} - \frac{1}{2} \sum_0^\infty \frac{p^n}{1 + e^{q+nr}}$

16) $\int \frac{Sin.\, qx - p\, Sin.\{(q-r)x\}}{1 - 2p\, Cos.\, rx + p^2} \frac{dx}{e^{2\pi x} - 1} = \frac{1}{4(1-p)} - \frac{1}{2} \sum_0^\infty \frac{p^n}{nr + q} - \frac{1}{2} \sum_0^\infty \frac{p^n}{1 - e^{q+nr}}$

17) $\int \frac{Sin.\, qx - p\, Sin.\{(q-r)x\}}{1 - 2p\, Cos.\, rx + p^2} \frac{e^{\lambda x} + e^{-\lambda x}}{e^{\pi x} - e^{-\pi x}} dx = \frac{1}{2(1-p)} - \sum_0^\infty \frac{1 + e^{q+nr}\, Cos.\, \lambda}{1 + 2e^{q+nr}\, Cos.\, \lambda + e^{2q+2nr}} p^n$

18) $\int \frac{Cos.\, qx - p\, Cos.\{(q-r)x\}}{1 - 2p\, Cos.\, rx + p^2} \frac{e^{\lambda x} - e^{-\lambda x}}{e^{\pi x} - e^{-\pi x}} dx = \sum_0^\infty \frac{e^{q+nr}\, p^n\, Sin.\, \lambda}{1 + 2\, e^{q+nr}\, Cos.\, \lambda + e^{2q+2nr}}$

(15–18) , $\lambda < \pi$; Poisson, P. 20. 222. , où 15 était fautive;

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19) $\int \frac{1-p\,Cos.\,rx}{1-2p\,Cos.\,rx+p^2}\,\frac{e^{\lambda x}-e^{-\lambda x}}{e^{\pi x}-e^{-\pi x}}\,dx = \sum_0^\infty \frac{p^n\,Sin.\,\lambda}{e^{nr}+2\,Cos.\,\lambda+e^{-nr}}$

20) $\int \frac{e^{\lambda x}-e^{-\lambda x}}{1-2p\,Cos.\,rx+p^2}\,\frac{dx}{e^{\pi x}-e^{-\pi x}} = \frac{-1}{2(1-p^2)}\,Tang.\,\frac{1}{2}\lambda + \frac{2}{1-p^2}\sum_0^\infty \frac{p^n\,Sin.\,\lambda}{e^{nr}+2\,Cos.\,\lambda+e^{-nr}}$, $\lambda<\pi$;

21) $\int \frac{e^{\lambda x}-e^{-\lambda x}}{1-2p\,Cos.\,2rx+p^2}\,\frac{Cos.\,rx}{e^{\pi x}-e^{-\pi x}}\,dx = \frac{1}{1-p^2}\sum_0^\infty \frac{p^n\,Sin.\,\lambda}{e^{(2n+1)r}+2\,Cos.\,\lambda+e^{-(2n+1)r}}$

22) $\int \frac{(e^{\lambda x}+e^{-\lambda x})Sin.rx.Sin.\lambda-(e^{\lambda x}-e^{-\lambda x})(e^r-Cos.rx)Cos.\lambda}{e^r-2\,Cos.\,rx+e^{-r}}\,\frac{dx}{e^{\pi x}-e^{-\pi x}} = \frac{Sin.\,\lambda}{2(e^{-r}-1)}+\sum_0^\infty \frac{Sin.\,\lambda}{e^{nr}+2Cos.\lambda+e^{-nr}}$

23) $\int \frac{(e^{\lambda x}+e^{-\lambda x})Sin.rx.Sin.\lambda+(e^{\lambda x}-e^{-\lambda x})(e^r+Cos.rx)Cos.\lambda}{e^r+2\,Cos.\,rx+e^{-r}}\,\frac{dx}{e^{\pi x}-e^{-\pi x}} = \frac{Sin.\,\lambda}{2(e^{-r}+1)}-\sum_0^\infty \frac{(-1)^n\,Sin.\,\lambda}{e^{nr}+2Cos.\lambda+e^{-nr}}$

Poisson, P. 20. 222; où 21 était fautive.

F. Expon. d'autre forme. Circ. Dir. — **TABLE 285.** — Lim. 0 et ∞.

1) $\int e^{-\sqrt{2qx}}\,Sin.\,x\,dx = \frac{Sin.\,\frac{1}{2}q-Cos.\,\frac{1}{2}q}{2}\sqrt{q\pi}+\sum_0^\infty(-1)^n\frac{(2q)^{2n}}{(2n+1)^{2n/1}}$

2) $\int e^{-\sqrt{2qx}}\,Cos.\,x\,dx = \frac{Sin.\,\frac{1}{2}q+Cos.\,\frac{1}{2}q}{2}\sqrt{q\pi}-\sum_0^\infty(-1)^n\frac{(2q)^{2n+1}}{(2n+2)^{2n+1/1}}$

Cauchy, Sav. Etr. 1827. 124. Note 3.

3) $\int e^{-\frac{p^2}{x^2}}\,Sin.\,(2q^2x^2)\,dx = e^{-2pq}\sqrt{\pi}\,\frac{Sin.\,2pq+Cos.\,2pq}{4q}$

4) $\int e^{-\frac{p^2}{x^2}}\,Cos.\,(2q^2x^2)\,dx = e^{-2pq}\sqrt{\pi}\,\frac{Sin.\,2pq-Cos.\,2pq}{4q}$

5) $\int e^{-x^2-\frac{pr^2}{x^2(p^2+q^2)}}\,Sin.\left\{\frac{p^2q}{x^2(p^2+q^2)}\right\}dx = \frac{1}{2}\sqrt{\pi}\,e^{-2ap}\,Sin.\,2bp$

6) $\int e^{-x^2-\frac{pr^2}{x^2(p^2+q^2)}}\,Cos.\left\{\frac{p^2q}{x^2(p^2+q^2)}\right\}dx = \frac{1}{2}\sqrt{\pi}\,e^{-2ap}\,Cos.\,2bp$

7) $\int e^{-px^2-\frac{q^2}{x^2}}\,Sin.\,(rx^2)\,dx = \frac{1}{2}e^{-2aq}\sqrt{\frac{\pi}{p^2+r^2}}\,(b\,Cos.\,2bq+a\,Sin.\,2bq)$

8) $= \frac{1}{2}\sqrt{\frac{\pi}{p}}\,e^{-2aq}\,Sin.\left(2bq+\frac{1}{2}\varphi\right)\sqrt{Cos.\,\varphi}$

9) $\int e^{-px^2-\frac{q^2}{x^2}}\,Cos.\,(rx^2)\,dx = \frac{1}{2}e^{-2aq}\sqrt{\frac{\pi}{p^2+r^2}}\,(a\,Cos.\,2bq-b\,Sin.\,2bq)$

10) $= \frac{1}{2}\sqrt{\frac{\pi}{p}}\,e^{-2aq}\,Cos.\left(2bq+\frac{1}{2}\varphi\right)\sqrt{Cos.\,\varphi}$

Helmling, Transf. 31—38, 65*, 75, 76.

, où partout $a=\sqrt{\frac{p+\sqrt{(p^2+r^2)}}{2}}$, $b=\sqrt{\frac{-p+\sqrt{(p^2+r^2)}}{2}}$, $Tang.\,\varphi=\frac{r}{p}$.

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11) $\int e^{-p^2x^2 Cos.2\lambda-\frac{q^2}{4x^2}} Sin.(p^2x^2 Sin.2\lambda)dx = \frac{\sqrt{\pi}}{2p}e^{-pq Cos.\lambda} Sin.(\lambda + pq Sin.\lambda)$

12) $\int e^{-p^2x^2 Cos.2\lambda-\frac{q^2}{4x^2}} Cos.(p^2x^2 Sin.2\lambda)dx = \frac{\sqrt{\pi}}{2p}e^{-pq Cos.\lambda} Cos.(\lambda + pq Sin.\lambda)$

13) $\int e^{-p\frac{1+x^4}{x^2}-\frac{q^2x^2}{(1-x^2)^2}} Sin.\left(r\frac{1+x^4}{x^2}\right)dx = \frac{1}{2}\sqrt{\frac{\pi Cos.\varphi}{p}}.e^{-2(aq+p)} Sin.\left\{\frac{1}{2}(bq+r)+\frac{1}{2}\varphi\right\}$

14) $\int e^{-p\frac{1+x^4}{x^2}-\frac{q^2x^2}{(1-x^2)^2}} Cos.\left(r\frac{1+x^4}{x^2}\right)dx = \frac{1}{2}\sqrt{\frac{\pi Cos.\varphi}{p}}.e^{-2(aq+p)} Cos.\left\{\frac{1}{2}(bq+r)+\frac{1}{2}\varphi\right\}$

Helmling, Transf. 31—38, 65*, 75, 76. , où partout $a=\sqrt{\frac{p+\sqrt{(p^2+r^2)}}{2}}$, $b=\sqrt{\frac{-p+\sqrt{(p^2+r^2)}}{2}}$, $Tang.\varphi = \frac{r}{p}$.

15) $\int e^{-p\left(x^2+\frac{1}{x^2}\right)} Sin.\left\{q\left(x^2+\frac{1}{x^2}\right)\right\}dx = \frac{1}{2}\sqrt{\frac{\pi Cos.2\varphi}{p}}.e^{-2p}Sin.(\varphi+2Tang.2\varphi)$

16) $\int e^{-p\left(x^2+\frac{1}{x^2}\right)} Cos.\left\{q\left(x^2+\frac{1}{x^2}\right)\right\}dx = \frac{1}{2}\sqrt{\frac{\pi Cos.2\varphi}{p}}.e^{-2p}Cos.(\varphi+2Tang.2\varphi)$

Helmling, Transf. II. S. 64. , où $\varphi = \frac{1}{2} Arctang.\frac{q}{p}$.

17) $\int \{e^{-x} Cos.(2p\sqrt{x}) - 4pe^{-x^2} Sin.2px\}dx = 1$ Helmling, Transf. 16.

18) $\int e^{-\frac{(x+bi)^{2a}+(x-bi)^{2a}}{2}} Cos.\left\{\frac{(x+bi)^{2a}-(x-bi)^{2a}}{2}\right\}dx = \frac{1}{2a}\Gamma\left(\frac{1}{2a}\right)$ Cauchy, Sav. Etr. 1827. 599. P. 1. § 2. d'après T. 36. N°. 12.

19) $\int e^{-\left(px^2+\frac{q}{x^2}\right)} Sin.\left(rx^2+\frac{s}{x^2}\right)dx = \frac{\sqrt{\pi}}{2a}e^{-c} Sin.\varphi$

20) $\int e^{-\left(px^2+\frac{q}{x^2}\right)} Cos.\left(rx^2+\frac{s}{x^2}\right)dx = \frac{\sqrt{\pi}}{2a}e^{-c} Cos.\varphi$

21) $\int e^{-\left(px^2+\frac{q}{x^2}\right)} Sin.\left(rx^2-\frac{s}{x^2}\right)dx = \frac{\sqrt{\pi}}{2a}e^{-f} Sin.\psi$

22) $\int e^{-\left(px^2+\frac{q}{x^2}\right)} Cos.\left(rx^2-\frac{s}{x^2}\right)dx = \frac{\sqrt{\pi}}{2a}e^{-f} Cos.\psi$

23) $\int e^{-\left(px^2+\frac{q}{x^2}\right)} Sin.(rx^2). Sin.\left(\frac{s}{x^2}\right)dx = \frac{\sqrt{\pi}}{4a}(e^{-f} Cos.\psi - e^{-c} Cos.\varphi)$

24) $\int e^{-\left(px^2+\frac{q}{x^2}\right)} Sin.(rx^2). Cos.\left(\frac{s}{x^2}\right)dx = \frac{\sqrt{\pi}}{4a}(e^{-c} Sin.\varphi + e^{-f} Sin.\psi)$

25) $\int e^{-\left(px^2+\frac{q}{x^2}\right)} Cos.(rx^2). Sin.\left(\frac{s}{x^2}\right)dx = \frac{\sqrt{\pi}}{4a}(e^{-c} Sin.\varphi - e^{-f} Sin.\psi)$

26) $\int e^{-\left(px^2+\frac{q}{x^2}\right)} Cos.(rx^2). Cos.\left(\frac{s}{x^2}\right)dx = \frac{\sqrt{\pi}}{4a}(e^{-c} Cos.\varphi + e^{-f} Cos.\psi)$

Helmling, Transf. II. S. 87, 88.

où
$a^4 = p^2 + r^2$,
$b^4 = q^2 + s^2$,
$\alpha = \frac{1}{2} Arctang.\frac{r}{p}$,
$\beta = \frac{1}{2} Arctang.\frac{s}{q}$,
$c = 2ab Cos.(\alpha+\beta)$,
$f = 2ab Cos.(\alpha-\beta)$,
$\varphi = 2ab Sin.(\alpha+\beta)+\alpha$,
$\psi = 2ab Sin.(\alpha-\beta)+\alpha$.

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27) $\int e^{-px^2}(e^{2qx}+e^{-2qx})\,Sin.(rx^2)\,dx = \frac{\sqrt{\pi}}{a}e^{-\frac{q^2}{a^2}Cos.2\alpha}Sin.\left(\frac{q^2}{a^2}Sin.2\alpha\right)$

28) $\int e^{-px^2}(e^{2qx}+e^{-2qx})\,Cos.(rx^2)\,dx = \frac{\sqrt{\pi}}{a}e^{-\frac{q^2}{a^2}Cos.2\alpha}Cos.\left(\frac{q^2}{a^2}Sin.2\alpha\right)$

Helmling, Transf. II. S. 68, 69.

, où a et α ont les mêmes valeurs qu'auparavant.

29) $\int e^{-px^2}\{e^{2qx}Sin.(rx^2-2sx)+e^{-2qx}Sin.(rx^2+2sx)\}\,dx = \frac{\sqrt{\pi}}{a}e^b\,Sin.\,\varphi$

30) $\int e^{-px^2}\{e^{2qx}Cos.(rx^2-2sx)+e^{-2qx}Cos.(rx^2+2sx)\}\,dx = \frac{\sqrt{\pi}}{a}e^b\,Cos.\,\varphi$

Helmling, Transf. II. S. 65.

, où $a^4 = p^2+r^2$, $b = \frac{q^2+s^2}{\sqrt{(p^2+r^2)}}Cos.\left\{Arctang.\frac{r}{p}-2\,Arctang.\frac{s}{q}\right\}$,

$\varphi = \frac{q^2+s^2}{\sqrt{(p^2+r^2)}}Sin.\left\{Arctang.\frac{r}{p}-2\,Arctang.\frac{s}{q}\right\}+Arctang.\frac{s}{q}$.

F. Exp. Circ. Dir. ent. — TABLE 286. — Lim. $-\infty$ et ∞.

1) $\int e^{-p\sqrt{x^2}}Cos.qx.\,Cos.rx\,dx = \frac{p}{p^2+(r-q)^2}+\frac{p}{p^2+(r+q)^2}$ Cauchy, P. 19. 511.

2) $\int e^{-q^2x^2}Cos.px\,dx = \frac{1}{q}e^{-\frac{p^2}{4q^2}}\sqrt{\pi}$ Cauchy, Exerc. 1827. p. 233.

3) $\int e^{-q^2x^2}Sin.px\,dx = 0$ Cauchy, Exerc. 1827. p. 233. — Lobatto, Int. 68.

4) $\int e^{-q^2x^2}Sin.\{p(x+\lambda)\}\,dx = \frac{\sqrt{\pi}}{q}e^{-\frac{p^2}{4q^2}}Sin.p\lambda$

5) $\int e^{-q^2x^2}Cos.\{p(x+\lambda)\}\,dx = \frac{\sqrt{\pi}}{q}e^{-\frac{p^2}{4q^2}}Cos.p\lambda$

Lobatto, Int. 68. où elles sont fautives.

6) $\int e^{-x^2}Cos.2px\,dx = e^{-p^2}\sqrt{\pi}$

7) $\int e^{-qx^2}Cos.px\,dx = e^{-\frac{p^2}{4q}}\sqrt{\frac{\pi}{q}}$

Fourier, Chal. 375. — Cauchy, P. 19. 511.

8) $\int e^{-q^2(x^2-2\lambda x)}Sin.px\,dx = \frac{\sqrt{\pi}}{q}e^{-\frac{p^2}{4q^2}+q^2\lambda^2}Sin.p\lambda$

9) $\int e^{-q^2(x^2-2\lambda x)}Cos.px\,dx = \frac{\sqrt{\pi}}{q}e^{-\frac{p^2}{4q^2}+q^2\lambda^2}Cos.p\lambda$

V. T. 286. N°. 4, 5.

10) $\int e^{px^2 i} Cos. a x d x = (1+i) e^{-\frac{a^2 i}{4p}} \sqrt{\frac{\pi}{2p}}$ Cauchy, P. 19. 511. — Schaar, Mém. Brux. T. 25.

11) $\int e^{-(px^2 Cos.\lambda + 2qx Cos.\mu + r Cos.\nu)} Sin. (p x^2 Sin. \lambda + 2 q x Sin. \mu + r Sin. \nu) d x =$

$= e^{-\frac{q^2}{p} Cos.(\lambda - 2\mu) - r Cos.\nu} Sin. \left\{\frac{1}{2}\lambda + \frac{q^2}{p} Sin.(\lambda - 2\mu) + r Sin.\nu\right\} \sqrt{\frac{\pi}{p}}$

12) $\int e^{-(px^2 Cos.\lambda + 2qx Cos.\mu + r Cos.\nu)} Cos. (p x^2 Sin. \lambda + 2 q x Sin. \mu + r Sin. \nu) d x =$

$= e^{-\frac{q^2}{p} Cos.(\lambda - 2\mu) - r Cos.\nu} Cos. \left\{\frac{1}{2}\lambda + \frac{q^2}{p} Sin.(\lambda - 2\mu) + r Sin.\nu\right\} \sqrt{\frac{\pi}{p}}$

13) $\int e^{-(px^2+qx+r)} Sin.(sx^2+tx+u) dx = e^{-r+\frac{p(q^2-t^2)+2qst}{4(p^2-s^2)}} Sin. \left\{u + \frac{(q^2-t^2)s - 2pqt}{4(p^2+s^2)} + \frac{1}{2} Arctang.\frac{s}{p}\right\} \sqrt{\frac{\pi}{p^2+s^2}}$

14) $\int e^{-(px^2+qx+r)} Cos.(sx^2+tx+u) dx = e^{-r+\frac{p(q^2-t^2)+2qst}{4(p^2-s^2)}} Cos. \left\{u + \frac{(q^2-t^2)s - 2pqt}{4(p^2+s^2)} + \frac{1}{2} Arctang.\frac{s}{p}\right\} \sqrt{\frac{\pi}{p^2+s^2}}$

15) $\int e^{-px^2 Cos.\lambda} Sin. (p x^2 Sin. \lambda) d x = Sin.\frac{1}{2}\lambda \sqrt{\frac{\pi}{p}}$

16) $\int e^{-px^2 Cos.\lambda} Cos. (p x^2 Sin. \lambda) d x = Cos.\frac{1}{2}\lambda \sqrt{\frac{\pi}{p}}$

Sur form. 11) à 16) voyez: Cauchy, Exerc. 1827. p. 233.

F. Exp. $e^{\pm ax}$. Circ. Dir. — TABLE 287. — Lim. 0 et $\frac{\pi}{2}$.

1) $\int e^{(q+1)xi} Sin.^{q-1} x d x = \frac{1}{q} e^{\frac{1}{2}q\pi i}$ Kummer, Cr. 20. 1

2) $\int e^{(p+q)xi} Sin.^{q-1} x. Cos^{p-1} x d x = e^{\frac{1}{2}q\pi i} B(p, q)$ Serret, L. 8. 1.

3) $\int e^{2x} Sin.^3 x d x = \frac{1}{8}(3 e^{\pi} - 1)$ Rogner, Mat.

4) $\int (e^{2qx} + e^{-2qx}) Cos.^{2b} x dx = \frac{\pi}{2^{2b+1}} \frac{1^{2b/1}}{\Gamma(b+qi+1)\Gamma(b-qi+1)}$ Lobatschewsky, Mém. Kasan. 1835. 211. où elle est fautive.

1) $\int e^{-a Sin.x} Sin.\,2x dx = \frac{2}{a^2}\{(a-1)e^a+1)\}$ V. T. 112. N°. 1.

2) $\int e^{-Sin.^2 x} Sin.\,2\,x\,d\,x = 1-\frac{1}{e}$ V. T. 112. N°. 2.

3) $\int e^{-Cos.x} Tang.\,x\,d\,x = \infty$ V. T. 112. N°. 4.

4) $\int e^{-q Tang\,x} d\,x = Ci.(q).Sin.\,q + Cos.\,q\left\{\frac{\pi}{2} - Si.(q)\right\}$ V. T. 130. N°. 3.

5) $\int e^{-q Tang.x} Tang.\,x\,d\,x = -Ci.(q).Cos.\,q + Sin.\,q\left\{\frac{\pi}{2} - Si.(q)\right\}$ V. T. 130. N°. 5.

6) $\int (e^{q Sin.x} - e^{-q Sin.x}) Sin.\,(q\,Cos.\,x).Sin.\,2\,a\,x\,d\,x = \frac{1}{2}\pi\frac{(-1)^{a-1}q^{2a}}{1^{2a/1}}$

7) $\int (e^{q Sin.x} - e^{-q Sin.x}) Cos.(q Cos.x).Sin.\{(2a-1)x\}\,dx = \frac{1}{2}\pi\frac{(-1)^{a-1}q^{2a-1}}{1^{2a-1/1}}$

8) $\int (e^{q Sin.x} + e^{-q Sin.x}) Sin.(q Cos.x).Cos.\{(2a-1)x\}\,dx = \frac{1}{2}\pi\frac{(-1)^{a-1}q^{2a-1}}{1^{2a-1/1}}$

9) $\int (e^{q Sin.x} + e^{-q Sin.x}) Cos.\,(q Cos.\,x).\,Cos.\,2\,a\,x\,d\,x = \frac{1}{2}\pi\frac{(-1)^{a}q^{2a}}{1^{2a/1}}$

(6–9) Poisson, P. 19. 404. N°. 77.

F. Exp. à exp. de Circ. Dir. Circ. Dir. en dén. *Sin.* $2x$. TABLE 289. Lim. 0 et $\frac{\pi}{2}$.

1) $\int e^{-p Tang.x} \frac{d\,x}{Sin.\,2\,x} = \infty$ V. T. 126. N°. 2.

2) $\int e^{-Tang.x} \frac{Tang.^p x}{Sin.\,2\,x} dx = \frac{1}{2}\Gamma(p)$, $\infty > p > -1$; V. T. 113. N°. 3.

3) $\int e^{-q Tang.x} \frac{Tang.^p x}{Sin.\,2\,x} d\,x = \frac{1}{2\,q^p}\Gamma(p)$ V. T. 113. N°. 5.

4) $\int e^{-p Tang.^2 x} \frac{Tang.^{2a} x}{Sin.\,2\,x} d\,x = \frac{1}{2^{a+1}p^a} 1^{a-1|1}$ V. T. 114. N°. 9.

5) $\int e^{-p Tang.^2 x} \frac{Tang.^{2a+1} x}{Sin.\,2\,x} d\,x = \frac{1}{4}\frac{1^{a/2}}{(2\,p)^a}\sqrt{\frac{\pi}{p}}$ V. T. 114. N°. 8.

6) $\int e^{-Tang.^p x} \frac{Tang.^p x}{Sin.\,2\,x} d\,x = \frac{1}{2\,p}$ V. T. 115. N°. 3.

7) $\int e^{-Tang.^{2p} x} \frac{Tang^p x}{Sin. 2 x} dx = \frac{1}{4p} \sqrt{\pi}$ V. T. 115. N°. 5.

8) $\int e^{-q\, Tang.^a x} \frac{Tang.^{ap} x}{Sin. 2 x} dx = \frac{1}{2 a q^p} \Gamma(p)$ V. T. 115. N°. 9.

9) $\int e^{-Tang.^{ac} x} \frac{Tang.^{ab} x}{Sin. 2 x} dx = \frac{1}{2ab} 1^{\frac{b}{c}/1}$ V. T. 115. N°. 8.

10) $\int e^{-(Tang. x + q\, Cot. x)} \frac{Tang^p x}{Sin. 2 x} dx = \frac{1}{2}\Gamma(p)\, \psi(1-p, q) + \frac{1}{2}\Gamma(-p) q^p\, \psi(1+p, q)$ V. T. 116. N°. 3.

11) $\int e^{-q(Tang.^2 x + Cot.^2 x)} \frac{Tang.^{2a+1} x}{Sin. 2 x} dx = \frac{1}{4} e^{-2q} \sqrt{\frac{\pi}{q}} \sum_0^a \frac{1}{(2q)^n} \frac{(a-n+1)^{2n,1}}{2^n 1^{n/1}}$ V. T. 116. N°. 8.

12) $\int \frac{e^{-Tang. x} - Cos.^2 x}{Sin. 2 x} dx = -\frac{1}{2} A$ V. T. 133. N°. 5.

13) $\int \frac{e^{-Tang. x} - e^{-p\, Tang. x}}{Sin. 2 x} dx = \frac{1}{2} l p$ V. T. 127. N°. 3.

14) $\int \frac{e^{-p\, Tang. x} - e^{-q\, Tang. x}}{Sin. 2 x} dx = \frac{1}{2} l \frac{q}{p}$ V. T. 127. N°. 4.

15) $\int e^{-Tang.^2 x} \frac{dx}{Sin.^2 2 x} = \frac{3}{8} \sqrt{\pi}$

16) $\int e^{-Tang.^2 x} \frac{Cos. 2 x}{Sin.^2 2 x} dx = \frac{1}{8} \sqrt{\pi}$

(15, 16) V. T. 290. N°. 2, 3.

F. Exp. à exp. de Circ. Dir. Circ. Dir. en dén. à une autre fonct. mon. TABLE 290. Lim. 0 et $\frac{\pi}{2}$.

1) $\int e^{-Cot.^2 x} \frac{dx}{Sin.^2 x} = \frac{1}{2} \sqrt{\pi}$ V. T. 36. N°. 7.

2) $\int e^{-Tang.^2 x} \frac{dx}{Sin.^2 x} = \sqrt{\pi}$ V. T. 126. N°. 3.

3) $\int e^{-Tang.^2 x} \frac{dx}{Cos.^2 x} = \frac{1}{2} \sqrt{\pi}$ V. T. 36. N°. 7.

4) $\int e^{-Tang.^2 x} \frac{Tang.^{2a} x}{Cos.^2 x} dx = \frac{1^{a/2}}{2^{a+1}} \sqrt{\pi}$ V. T. 114. N°. 7.

5) $\int e^{-Cot.^2 x} \frac{dx}{Cos.^2 x} = \sqrt{\pi}$ V. T. 126. N°. 3.

6) $\int e^{-Tang.^2 x} \frac{Sin.^2 x}{Cos.^4 x} dx = \frac{1}{4}\sqrt{\pi}$ V. T. 290. N°. 3.

7) $\int e^{-p Tang.^2 x} \frac{Sin.^2 x}{Cos.^4 x} dx = \frac{1}{4p}\sqrt{\frac{\pi}{p}}$ V. T. 114. N°. 11.

8) $\int e^{-Tang.^2 x} \frac{dx}{Cos.^4 x} = \frac{3}{4}\sqrt{\pi}$

9) $\int e^{-Tang.^2 x} \frac{Cos. 2x}{Cos.^4 x} dx = \frac{1}{4}\sqrt{\pi}$

(8, 9) V. T. 290. N°. 3, 6.

10) $\int e^{-q Cot. x} \frac{dx}{Tang. x} = - Ci.(q). Cos. q + Sin. q \left\{\frac{\pi}{2} - Si.(q)\right\}$ V. T. 130. N°. 5.

11) $\int e^{-Sin. x} \frac{dx}{Tang. x} = \infty$ V. T. 112. N°. 4.

12) $\int e^{-p Tang. x} \frac{dx}{Cos. 2x} = \frac{1}{2}\left\{e^{-p} Ei.(p) - e^{p} Ei.(-p)\right\}$ V. T. 130. N°. 10.

13) $\int e^{-p Tang. x} \frac{Tang. x}{Cos. 2x} = \frac{1}{2}\left\{e^{-p} Ei.(p) + e^{p} Ei.(-p)\right\}$ V. T. 130. N°. 12.

F. Exp. à exp. de Circ. Dir.
Circ. Dir. en dén. à plus. fact. mon. TABLE 291. Lim. 0 et $\frac{\pi}{2}$.

1) $\int e^{-p Cot. x} \frac{dx}{Cos. 2x. Tang. x} = -\frac{1}{2}\left\{e^{-p} Ei.(p) + e^{p} Ei.(-p)\right\}$ V. T. 130. N°. 12.

2) $\int e^{-Cot. x} \frac{dx}{Sin. 2x. Tang.^p x} = \frac{1}{2}\Gamma(p), \infty > p > -1$ V. T. 113. N°. 3.

3) $\int e^{-q Cot. x} \frac{dx}{Sin. 2x. Tang.^p x} = \frac{1}{2 q^p}\Gamma(p)$ V. T. 113. N°. 5.

4) $\int e^{-Cot.^2 x} \frac{dx}{Sin. 2x. Tang.^{2a} x} = \frac{1}{2} 3^{a-2/1}$ V. T. 114. N°. 4.

5) $\int e^{-p Cot.^2 x} \frac{dx}{Sin. 2x. Tang.^{2a+1} x} = \frac{1}{4} \frac{1^{a/2}}{(2p)^a}\sqrt{\frac{\pi}{p}}$ V. T. 114. N°. 8.

6) $\int e^{-p Cot.^2 x} \frac{dx}{Sin. 2x. Tang.^{2a} x} = \frac{1^{a-1/1}}{2^{a+1} p^a}\sqrt{\pi}$ V. T. 114. N°. 9.

7) $\int e^{-Cot.^{2p}x} \frac{dx}{Sin.\,2x.\,Tang.^p x} = \frac{1}{4p}\sqrt{\pi}$ V. T. 115. N°. 5.

8) $\int e^{-Cot.^q x} \frac{dx}{Tang.^q x.\,Sin.\,2x} = \frac{1}{2q}$ V. T. 115 N°. 8.

9) $\int e^{-q Cot.^a x} \frac{dx}{Tang.^{ap} x.\,Sin.\,2x} = \frac{1}{2aq^p}\Gamma(p)$ V. T. 115. N°. 9.

10) $\int e^{-Cot.^{ac} x} \frac{dx}{Tang.^{ab} x.\,Sin.\,2x} = \frac{1}{2ab}\, 1^{\frac{b}{c}/1}$ V. T. 115. N°. 8.

11) $\int e^{-(q Tang.x + Cot.x)} \frac{dx}{Tang.^p x.\,Sin.\,2x} = \frac{1}{2}\Gamma(p)\psi(1-p,q) + \frac{1}{2}\Gamma(-p)q^p\psi(1+p,q)$ V. T. 116. N°. 3.

12) $\int e^{-q(Tang.^2 x + Cot.^2 x)} \frac{dx}{Tang.^{2a+1} x.\,Sin.\,2x} = \frac{1}{4} e^{-2q}\sqrt{\frac{\pi}{q}} \sum_0^a \frac{1}{(2q)^n} \frac{(a-n+1)^{2n/1}}{2^n 1^{n/1}}$ V.T. 116. N°. 8.

F. Exp. en dén. polynôme.
Circ. Dir. en num. TABLE 292. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{dx}{e^{\frac{1}{4}\pi Tang.x} + e^{-\frac{1}{4}\pi Tang.x}} = \frac{1}{2\sqrt{2}}\left\{\pi - l\frac{\sqrt{2}+1}{\sqrt{2}-1}\right\}$ V. T. 138. N°. 17.

2) $\int \frac{dx}{e^{\frac{1}{4}\pi Tang.x} + e^{-\frac{1}{4}\pi Tang.x}} = \frac{1}{2} l 2$ V. T. 138. N°. 14.

3) $\int \frac{dx}{e^{\pi Tang.x} + e^{-\pi Tang.x}} = \frac{4-\pi}{4}$ V. T. 138. N°. 2.

4) $\int \frac{e^{p Tang.x} - e^{-p Tang.x}}{e^{\pi Tang.x} - e^{-\pi Tang.x}} dx = -\frac{1}{2}p\,Cos.p + \frac{1}{2}Sin.p\, l\{2(1 + Cos.p)\}, \pi > p > 0;$ V. T. 138. N°. 4.

5) $\int \frac{Tang.\,x}{e^{\frac{1}{4}\pi Tang.x} - e^{-\frac{1}{4}\pi Tang.x}} dx = \frac{1}{4}\pi\sqrt{2} - 1 + \frac{1}{4}\sqrt{2}\, l\frac{\sqrt{2}+1}{\sqrt{2}-1}$ V. T. 138. N°. 18.

6) $\int \frac{Tang.\,x}{e^{\frac{1}{4}\pi Tang.x} - e^{-\frac{1}{4}\pi Tang.x}} dx = \frac{\pi - 2}{4}$ V. T. 138. N°. 16.

7) $\int \frac{e^{p Tang.x} + e^{-p Tang.x}}{e^{\frac{1}{4}\pi Tang.x} - e^{-\frac{1}{4}\pi Tang.x}} Tang.\,x\,dx = -1 + \frac{1}{2}\pi\,Cos.p + \frac{1}{2}Sin.p\, l\frac{1+Sin.p}{1-Sin.p}, 0 \leqq p \leqq \frac{1}{2}\pi;$ V.T. 138. N°. 15.

8) $\int \frac{e^{p Tang.x} - e^{-p Tang.x}}{e^{\frac{1}{4}\pi Tang.x} - e^{\frac{1}{4}\pi Tang.x}} dx = \frac{1}{2}\pi\,Sin.p - \frac{1}{2}Cos.p\, l\frac{1 + Sin.p}{1 - Sin.p}, \; 0 \leqq p \leqq \frac{1}{2}\pi;$ V.T. 138. N°. 13,

9) $\int \frac{e^{p\,Tang.x} + e^{-p\,Tang.x}}{e^{\pi\,Tang.x} - e^{-\pi\,Tang.x}}\,Tang.x\,dx = \frac{1}{2}(p\,Sin.p - 1) + \frac{1}{2}Cos.p\,l\{2(1+Cos.p)\}, 0 < p < \pi$; V.T. 138. N°. 6.

10) $\int \frac{Tang.x}{e^{\pi\,Tang.x} - e^{-\pi\,Tang.x}}\,dx = \frac{1}{2}\left(-\frac{1}{2} + l\,2\right)$ V. T. 138. N°. 12.

11) $\int \frac{e^{(\pi-\lambda)Tang.x} - e^{(\lambda-\pi)Tang.x}}{e^{\pi\,Tang.x} - e^{-\pi\,Tang.x}}\,dx = \sum_1^\infty \frac{Sin.n\lambda}{n+1}, \lambda^2 < \pi^2$; V. T. 138. N°. 5.

12) $\int \frac{e^{(\pi-\lambda)Tang.x} + e^{(\lambda-\pi)Tang.x}}{e^{\pi\,Tang.x} - e^{-\pi\,Tang.x}}\,Tang.x\,dx = \frac{1}{2} + \sum_1^\infty \frac{Cos.n\lambda}{n+1}, \lambda^2 \leqq \pi^2$; V. T. 138. N°. 8.

13) $\int \frac{Tang.x}{e^{2\pi\,Tang.x} - 1}\,dx = \frac{1}{2}A - \frac{1}{4}$ V. T. 138. N°. 10.

14) $\int \frac{Tang.x}{e^{2\pi q\,Tang.x} - 1}\,dx = \frac{1}{2}l\,q + \frac{1}{4q} - \frac{1}{2}Z'(1+q)$ V. T. 138. N°. 11.

1) $\int \frac{e^{\frac{\pi Cos.x}{2b}} + e^{-\frac{\pi Cos.x}{2b}}}{e^{\frac{\pi Cos.x}{b}} + 2\,Cos.\left(\frac{\pi Sin.x}{2b}\right) + e^{-\frac{\pi Cos.x}{b}}}\,Cos.\left(\frac{\pi Sin.x}{2b}\right)dx = \frac{1}{2}\pi$

2) $\int \frac{e^{\frac{\pi Cos.x}{2b}} + e^{-\frac{\pi Cos.x}{2b}}}{e^{\frac{\pi Cos.x}{b}} + 2\,Cos.\left(\frac{\pi Sin.x}{b}\right) + e^{-\frac{\pi Cos.x}{b}}}\,Cos.\left(\frac{\pi Sin.x}{2b}\right)Cos.2ax\,dx = \frac{(-1)^a b}{4.1^{2a/1}}\left(\frac{\pi}{2b}\right)^{2a+1}B_{2a}$

3) $\int \frac{Sin.\left(\frac{\pi Sin.x}{b}\right)Sin.\{(2a-1)x\}}{e^{\frac{\pi Cos.x}{b}} + 2\,Cos.\left(\frac{\pi Sin.x}{b}\right) + e^{-\frac{\pi Cos.x}{b}}}\,dx = \frac{(-1)^{a-1}}{1^{2a-1/1}}\,\frac{2^{2a}-1}{8a}\,b\left(\frac{\pi}{b}\right)^{2a}B_{2a-1}$

4) $\int \frac{e^{\frac{\pi Cos.x}{2b}} - e^{-\frac{\pi Cos.x}{2b}}}{e^{\frac{\pi Cos.x}{b}} + 2\,Cos.\left(\frac{\pi Sin.x}{b}\right) + e^{-\frac{\pi Cos.x}{b}}}\,Sin.\left(\frac{\pi Sin.x}{2b}\right)Sin.2ax\,dx = \frac{(-1)^{a-1}b}{4.1^{2a/1}}\left(\frac{\pi}{2b}\right)^{2a+1}B_{2a}$

5) $\int \frac{e^{\frac{\pi Cos.x}{b}} - e^{-\frac{\pi Cos.x}{b}}}{e^{\frac{\pi Cos.x}{b}} + 2\,Cos.\left(\frac{\pi Sin.x}{b}\right) + e^{-\frac{\pi Cos.x}{b}}}\,Cos.\{(2a-1)x\}\,dx = \frac{(-1)^{a-1}}{1^{2a-1/1}}\,\frac{2^{2a}-1}{8a}\,b\left(\frac{\pi}{b}\right)^{2a}B_{2a-1}$

Poisson; P. 19. 404. N°. 77. d'après T. 120. N°. 14, 18.

Page 391.

6) $\int \frac{Tang.^q x}{e^{Tang.x}+1} \frac{dx}{Sin.2x} = \frac{1}{2}\Gamma(q) \sum_0^\infty \frac{(-1)^n}{(n+1)^q}$ V. T. 117. N°. 16.

7) $\int \frac{Tang.^q x}{e^{Tang.x}-1} \frac{dx}{Sin.2x} = \frac{1}{2}\Gamma(q) \sum_0^\infty \frac{1}{(n+1)^q}$ V. T. 117. N°. 17.

8) $\int \frac{1}{e^{q\,Tang.x}-1} \frac{Tang.x}{Cos.2x} dx = \frac{4\pi^4}{q^4} \sum_0^\infty (-1)^n \left(\frac{2\pi}{q}\right)^{2n} \frac{B_{2n+1}}{n+1}$ V. T. 138. N°. 21.

9) $\int \frac{1}{e^{q\,Tang.x}-1} \frac{Sin.2x}{Cos.^2 2x} dx = \frac{\pi^2}{q^2} \sum_0^\infty (-1)^n \left(\frac{2\pi}{q}\right)^{2n} B_{2n+1}$ V. T. 138. N°. 22.

10) $\int \frac{1}{e^{q\,Tang.x}-1} \frac{Sin.^2 x}{Tang.x} dx = \frac{\pi^2}{q^2} \sum_0^\infty \left(\frac{2\pi}{q}\right)^{2n} B_{2n+1}$ V. T. 138. N°. 20.

11) $\int \frac{e^{-p\,Tang.x}-e^{-q\,Tang.x}}{e^{-Tang.x}+1} \frac{dx}{Sin.2x} = \frac{1}{2} l \frac{\Gamma\left(\frac{1}{2}p\right)\Gamma\left(\frac{q+1}{2}\right)}{\Gamma\left(\frac{1}{2}q\right)\Gamma\left(\frac{p+1}{2}\right)}$ V. T. 134. N°. 4.

12) $\int \frac{(e^{q\,Tang.x}-e^{-q\,Tang.x})^2}{e^{Tang.x}+1} \frac{dx}{Sin.2x} = -\frac{1}{2} l\,(q\pi Cot.q\pi)$ V. T. 134. N°. 8.

13) $\int \frac{1}{e^{Cot.x}+1} \frac{dx}{Tang.^q x.\,Sin.2x} = \frac{1}{2}\Gamma(q) \sum_0^\infty \frac{(-1)^n}{(n+1)^q}$ V. T. 117. N°. 16.

14) $\int \frac{1}{e^{Cot.x}-1} \frac{dx}{Tang.^q x.\,Sin.2x} = \frac{1}{2}\Gamma(q) \sum_0^\infty \frac{1}{(n+1)^q}$ V. T. 117. N°. 17.

15) $\int \frac{e^{q\,Tang.x}-e^{-q\,Tang.x}}{e^{p\,Tang.x}+e^{-p\,Tang.x}} \frac{dx}{Sin.2x} = \frac{1}{2} l\, Tang. \left\{\frac{p+q}{4p}\pi\right\}$ V. T. 136. N°. 9.

16) $\int \frac{e^{q\,Tang.x}+e^{-q\,Tang.x}-2}{e^{\pi\,Tang.x}-e^{-\pi\,Tang.x}} \frac{dx}{Sin.2x} = \frac{1}{2} l\, Sec.\frac{q}{2}$ V. T. 136. N°. 12.

17) $\int \frac{(e^{q\,Tang.x}-e^{-q\,Tang.x})^2}{e^{Tang.x}-e^{-Tang.x}} \frac{dx}{Sin.2x} = \frac{1}{2} l\, Cos.q\pi$ V. T. 136. N°. 3.

18) $\int \frac{Tang.^q x}{e^{Tang.x}+e^{-Tang.x}+2\,Cos.\lambda} \frac{dx}{Sin.2x} = \frac{\Gamma(q)}{2\,Sin.\lambda} \sum_1^\infty (-1)^{n-1} \frac{Sin.n\lambda}{n^q}$ V. T. 137. N°. 5.

F. Exp. en num.
Circ. Dir. en dén. trinôme.

TABLE 294.

Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{(e^{xi} Cos. x)^p + (e^{-xi} Cos. x)^p}{Cos.^2 x + q^2 Sin.^2 x} dx = \frac{\pi}{q} \left(\frac{q}{q+1}\right)^p$ Serret, L. 8. 489.

2) $\int \frac{e^{-p Tang. x}}{Sin. 2x + q Cos. 2x + q} dx = -\frac{1}{2} e^{pq} Ei. (-pq)$ V. T. 129. N°. 3.

3) $\int \frac{e^{-p Tang. x}}{Sin. 2x - q Cos. 2x - q} dx = -\frac{1}{2} e^{-pq} Ei. (pq)$ } V. T. 129. N°. 9.

4) $\int \frac{e^{-p Cot. x}}{Sin. 2x + q Cos. 2x - q} dx = -\frac{1}{2} e^{-pq} Ei. (pq)$ } V. T. 129. N°. 9.

5) $\int \frac{e^{-p Cot. x}}{Sin. 2x - q Cos. 2x + q} dx = -\frac{1}{2} e^{pq} Ei. (-pq)$ V. T. 129. N°. 3.

6) $\int \frac{e^{-p Tang. x} Sin. 2x}{(1-q^2) - 2q^2 Cos. 2x - (1+q^2) Cos.^2 2x} dx = -\frac{1}{4}\{e^{-pq} Ei.(pq) + e^{pq} Ei.(-pq)\}$ V. T. 294. N°. 2, 3.

7) $\int \frac{e^{-p Cot. x} Sin. 2x}{(1-q^2) + 2q^2 Cos. 2x - (1+q^2) Cos.^2 2x} dx = -\frac{1}{4}\{e^{-pq} Ei.(pq) + e^{pq} Ei.(-pq)\}$ V. T. 294. N°. 4, 5.

F. Expon.
Circ. Dir. de forme irrat.

TABLE 295.

Lim. 0 et $\frac{\pi}{2}$.

1) $\int e^{-Tang. x} \frac{dx \sqrt{Sin. 2x}}{Cos.^3 x} = \sqrt{\frac{\pi}{2}}$ V. T. 112. N°. 6.

2) $\int e^{-Cot. x} \frac{dx \sqrt{Sin. 2x}}{Sin.^3 x} = \sqrt{\frac{\pi}{2}}$ V. T. 112. N°. 6.

3) $\int e^{-\frac{1}{q} Cosec. 2x} \frac{\sqrt{Sin. 2x}}{Cos.^3 x} dx = \frac{1+q}{\sqrt[q]{e}} 2 \sqrt{q\pi}$ V. T. 139. N°. 7.

4) $\int e^{-q Tang. x} \frac{dx}{Cos. x \sqrt{Sin. 2x}} = \sqrt{\frac{\pi}{2q}}$ V. T. 140. N°. 2.

5) $\int e^{-\frac{1}{q} Cosec. 2x} \frac{dx}{Cos. x \sqrt{Sin. 2x}} = \frac{\sqrt{q\pi}}{\sqrt[q]{e}}$ V. T. 140. N°. 5.

6) $\int e^{-\frac{1}{q} Cosec. 2x} \frac{Tang.^b x}{Sin. x \sqrt{Sin. 2x}} dx = \frac{\sqrt{\pi q}}{\sqrt[q]{e}} \sum_0^\infty \frac{b^{2n|1}}{2^{n2}} (-q)^n$ } V. T. 140. N°. 11.

7) $\int e^{-\frac{1}{q} Cosec. 2x} \frac{dx}{Sin. 2x. Tang.^{b-1} x} = \frac{\sqrt{2\pi q}}{2 \sqrt[q]{e}} \sum_0^\infty \frac{b^{2n|1}}{2^{n|2}} (-q)^n$ } V. T. 140. N°. 11.

8) $\int e^{-\frac{1}{q}Cosec.x}\frac{dx}{Tang.x\sqrt{\{Sin.x(1-Sin.x)\}}}=\frac{\sqrt{q\pi}}{\sqrt[q]{e}}$

9) $\int e^{-\frac{1}{q}Sec.x}\frac{Tang.x}{\sqrt{\{Cos.x(1-Cos.x)\}}}dx=\frac{\sqrt{q\pi}}{\sqrt[q]{e}}$ V. T. 150. N°. 1.

10) $\int e^{-q^2(Tang.x+Cot.x)}\frac{dx}{Cos.x\sqrt{Sin.2x}}=\frac{1}{2q}e^{-2q^2}\sqrt{2\pi}$ V. T. 140. N°. 6.

11) $\int e^{-p\,Tang.x-q\,Cot.x}\frac{dx}{Cos.x\sqrt{Sin.2x}}=e^{-2\sqrt{pq}}\sqrt{\frac{\pi}{2p}}$ V. T. 140. N°. 9.

12) $\int e^{-q^2(Tang.x+Cot.x)}\frac{dx}{Sin.x\sqrt{Sin.2x}}=\frac{1}{2q}e^{-2q^2}\sqrt{2\pi}$ V. T. 140. N°. 6.

13) $\int e^{-p\,Tang.x-q\,Cot.x}\frac{dx}{Tang.^a x.\,Cos.x\sqrt{Sin.2x}}=\left(\frac{p}{q}\right)^{\frac{1}{2}a}e^{-2\sqrt{pq}}\sqrt{\frac{\pi}{2p}}\sum_0^\infty\frac{(a-n)^{2n/1}}{2^{n/2}(2\sqrt{pq})^n}$ V. T. 140. N°. 12.

14) $\int\frac{1}{e^{Tang.x}+e^{-Tang.x}}\frac{dx}{Cos.x\sqrt{Sin.2x}}=\sqrt{\frac{\pi}{2}}\sum_0^\infty\frac{(-1)^n}{\sqrt{(2n+1)}}$ V. T. 140. N°. 19.

15) $\int\frac{1}{e^{Tang.x}+e^{-Tang.x}+1}\frac{dx}{Cos.x\sqrt{Sin.2x}}=\frac{\sqrt{2\pi}}{2\,Sin.\frac{1}{3}\pi}\sum_1^\infty(-1)^{n-1}\frac{Sin.\frac{1}{3}n\pi}{\sqrt{n}}$ V. T. 140. N°. 20.

F. Exp.
Circ. Dir. TABLE 296. Lim. 0 et π.

1) $\int e^{qx}Sin.px\,dx=\frac{p}{p^2+q^2}(1-e^{q\pi}Cos.p\pi)$

2) $\int e^{qx}Cos.px\,dx=\frac{q}{p^2+q^2}(e^{q\pi}Cos.p\pi-1)$ Dienger, Cr. 34. 75. — Schlömilch, Beitr. I. § 8, 10.

3) $\int(e^{qx}+e^{-qx})Cos.px\,dx=\frac{q}{p^2+q^2}Cos.p\pi(e^{q\pi}-e^{-q\pi})$ Schlömilch, Stud. II. 6. — Id., Beitr. I. § 8.

4) $\int(e^{qx}-e^{-qx})Sin.px\,dx=(-1)^{p-1}p\frac{e^{q\pi}-e^{-q\pi}}{p^2+q^2}$ Schlömilch, Beitr. I. § 10.

5) $\int e^{ax}Sin.^b x\,dx=\frac{\pi}{2^b}\frac{e^{\frac{1}{2}a\pi}1^{b/1}}{\Gamma\left(\frac{a+bi}{2}+1\right)\Gamma\left(\frac{a-bi}{2}+1\right)}$ Lobatschewsky, Mém. Kasan. 1835. 211.

6) $\int e^{p\,Cos.x}Cos.(p\,Sin.x)\,dx=\pi,\ p^2\leqq1;$ Poisson, P. 19. 404. N°. 77. — Serret, L. 8. 489. — Schlömilch, Beitr. II. 1.

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7) $\int e^{p\,Cos.x}\,Sin.(p\,Sin.x).Sin.\,a\,x\,d\,x = \frac{1}{2}\pi\frac{p^a}{1^{a,1}}$

8) $\int e^{p\,Cos.x}\,Cos.(p\,Sin.x).Cos.\,a\,x\,d\,x = \frac{1}{2}\pi\frac{p^a}{1^{a/1}}$

9) $\int e^{p\,Cos.x}\,Sin.(p\,Sin.x)\,Tang.\frac{1}{2}x\,d\,x = \pi(1-e^{-p})$

10) $\int e^{p\,Cos.x}\,Sin.(p\,Sin.x)\,Cot.\frac{1}{2}x\,d\,x = \pi(e^p-1)$

(7–10) , $p \leqq 1$; Poisson, P. 19. 404. N°. 77. — Schlömilch, Beitr. II. 1.

11) $\int e^{p\,Cos.x}\,Sin.(p\,Sin.x)\frac{d\,x}{Sin.\,x} = \frac{1}{2}\pi(e^p-e^{-p})$

12) $\int e^{p\,Cos.x}\,Cos.(p\,Sin.x)\frac{d\,x}{Cos.\,x} = \infty$

(11–12) , $p^2 \leqq 1$; Schlömilch, Beitr. II. 1. il trouve faut. pour 12) $\pi\,Sin.\,p$.

13) $\int e^{-Cot.^2x}\frac{d\,x}{Tang.^{2a}x.\,Sin.\,2\,x} = 0$ V. T. 142. N°. 9.

14) $\int e^{-Cot.^2x}\frac{d\,x}{Tang.^{2a+1}x.\,Sin.\,2\,x} = \frac{1^{a/2}}{2^{a+1}}\sqrt{\pi}$ V. T. 142. N°. 8.

15) $\int(e^{p\,Sin.x}+e^{-p\,Sin.x})\left\{e^{\frac{b\,Sin.x}{p}}Sin.\left(x+\frac{b\,Cos.x}{p}\right)-e^{-\frac{b\,Sin.x}{p}}Sin.\left(x-\frac{b\,Cos.x}{p}\right)\right\}Cos.(p\,Cos.x)\,dx=$

$= \frac{2\,b\,\pi}{p}\left\{2+\sum_1^\infty\frac{p^{2n}}{(2\,n+1)(1^{2n/1})^2}\right\}$

16) $\int e^{a^p(Cos.px+Cos.qx)}\,Cos.(a^p\,Sin.\,px).Cos.(a^p\,Sin.\,q\,x)\,d\,x = \frac{1}{2}\pi\left\{2+\sum_1^\infty\frac{a^{np(p+q)}}{1^{np/1}\,1^{nq/1}}\right\}$

(15–16) Smaasen, Cr. 42. 222.

17) $\int e^{-p\,Tang.^2x-q\,Cot.^2x}\frac{Tang.^{2a}x}{Sin.^2x}\,d\,x = \left(\frac{q}{p}\right)^{\frac{1}{2}a}e^{-2\sqrt{pq}}\sqrt{\frac{\pi}{q}}\sum_0^\infty\frac{(a-n)^{2n/1}}{1^{n/1}(4\sqrt{pq})^n}$ V. T. 146. N°. 3.

18) $\int\frac{e^{p\,Cos.x}\,Cos.(p\,Sin.x)}{Cos.^2\frac{1}{2}x+q^2\,Sin.^2\frac{1}{2}x}\,d\,x = \frac{\pi}{q}e^{p\frac{q-1}{q+1}}$ Serret, L. 8. 489.

19) $\int\frac{Sin.(q\,Sin.x).\,Sin.\,x}{1-2\,p\,Cos.x+p^2}e^{q\,Cos.x}\,d\,x = \frac{\pi}{2\,p}(e^{pq}-1)$

20) $\int\frac{Cos.(q\,Sin.x)}{1-2\,p\,Cos.x+p^2}e^{q\,Cos.x}\,d\,x = \frac{\pi}{1-p^2}e^{pq}$

(19–20) , $p^2 < 1$; Poisson, P. 19. 404. N°. 77. — Schlömilch, Beitr. II. 2.

21) $\int\frac{1-p^b\,Cos.\,b\,x}{1-2\,p^b\,Cos.\,b\,x+p^{2b}}e^{\frac{c\,Cos.x}{p}}\,Cos.\left(\frac{c\,Sin.x}{p}\right)d\,x = \frac{1}{2}\pi\left\{2+\sum_1^\infty\frac{c^{nb}}{1^{nb/1}}\right\}$ Smaasen, Cr. 42. 222.

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F. Expon. Circ. Dir.	TABLE 296 suite.	Lim 0 et π.

22) $\int \frac{Sin.\frac{3x}{2} - p\, e^{Cos.x} Sin.\left(\frac{5x}{2} - Sin.\, x\right)}{1 - 2p\, e^{Cos.x} Cos.(x - Sin.\, x) + p^2\, e^{2Cos.x}} Sin.\frac{1}{2} x\, dx = \sum_1^\infty \frac{n^{n-1}}{1^{n/1}} q^n$ Poisson, P. 19. 404. N°. 80.

23) $\int e^{-p Cot.^2 x - q Tang.^2 x} \frac{dx}{Cos.^2 x} = \frac{1}{2} e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{q}}$ V. T. 146. N°. 2.

F. Expon. Circ. Dir.	TABLE 297.	Lim. $-\frac{\pi}{2}$ et $\frac{\pi}{2}$.

1) $\int e^{-px} Cos.^{2a} x\, dx = \frac{1^{2a/1}}{p^2 + 2^2 . p^2 + 4^2 \ldots p^2 + 4a^2} \frac{1}{p} (e^{\frac{1}{2}p\pi} - e^{-\frac{1}{2}p\pi})$

2) $\int e^{-px} Cos.^{2a+1} x\, dx = \frac{1^{2a+1/1}}{p^2 + 1^2 . p^2 + 3^2 \ldots p^2 + (2a+1)^2} (e^{\frac{1}{2}p\pi} + e^{-\frac{1}{2}p\pi})$

(1, 2) Ohm, Ausw. 13.

3) $\int e^{-Tang.^2 x} \frac{Tang.^{2a+1} x}{Sin.\, 2x} dx = \frac{1^{a/2}}{2^{a+1}} \sqrt{\pi}$ V. T. 142. N°. 8.

4) $\int e^{-Tang.^2 x} \frac{Tang.^{2a} x}{Sin.\, 2x} dx = 0$ V. T. 142. N°. 9.

5) $\int e^{-(p Tang.^2 x + q Cot.^2 x)} \frac{dx}{Sin.^2 x} = e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{q}}$ V. T. 146. N°. 2.

6) $\int e^{-(p Tang.^2 x + q Cot.^2 x)} \frac{dx}{Tang.^{2a} x . Cos.^2 x} = \left(\frac{p}{q}\right)^{\frac{a}{2}} e^{-2\sqrt{pq}} \sqrt{\frac{\pi}{p}} \sum_0^\infty \frac{(a-n)^{2n|-1}}{1^{n/1}} \left(\frac{1}{4\sqrt{pq}}\right)^n$ V. T. 146. N°. 3.

7) $\int (2\, Cos.\, x)^{q-1} e^{(q+1)xi + \frac{1}{2}be^{-xi} Sec.x} dx = \frac{\pi b^q}{e^b \Gamma(q+1)}$ Kummer, Cr. 20. 1. où il y a faut. $e^{-(q+1)xi}$.

F. Expon. Circ. Dir.	TABLE 298.	Lim. diverses.

1) $\int_0^1 \frac{e^{\frac{\pi}{b}\sqrt{(1-x^2)}} - e^{-\frac{\pi}{b}\sqrt{(1-x^2)}}}{e^{\frac{\pi}{b}\sqrt{(1-x^2)}} + e^{-\frac{\pi}{b}\sqrt{(1-x^2)}} + 2\, Cos.\left(\frac{\pi x}{b}\right)} dx = \frac{\pi^2}{16b}$ V. T. 293. N°. 5.

2) $\int_0^1 \frac{Sin.\left\{\frac{\pi}{b}\sqrt{(1-x^2)}\right\}}{e^{\frac{\pi x}{b}} + e^{-\frac{\pi x}{b}} + 2\, Cos.\left\{\frac{\pi}{b}\sqrt{(1-x^2)}\right\}} dx = \frac{\pi^2}{16b}$ V. T. 293. N°. 3.

3) $\int_0 p^{x-1}\, Sin.\, 2\, a\, \pi\, x\, d\, x = \frac{1-p}{(l p)^2 + 4 a^2 \pi^2} \frac{2 a \pi}{p}$ Kummer, Cr. 35. 1.

4) $\int_0^\lambda e^{-p\, Cos.\, x}\, Sin.\, (x - p\, Sin.\, x)\, d\, x = \frac{1}{p}\{e^{-p\, Cos.\, \lambda}\, Cos.\, (p\, Sin.\, \lambda) - e^{-p}\}$

5) $\int_0^\lambda e^{-p\, Cos.\, x}\, Cos.\, (x - p\, Sin.\, x)\, d\, x = \frac{1}{p}\{e^{-p\, Cos.\, \lambda}\, Sin.\, (p\, Sin.\, \lambda)\}$

6) $\int_0^\lambda e^{-p\, Cos.\, x}\, Sin.\, (a\, x - p\, Sin.\, x)\, d\, x = (-1)^a \frac{d^a}{d\, p^a} . \frac{e^{-p\, Cos.\, \lambda}\, Cos.\, (p\, Sin.\, \lambda) - e^{-p}}{p}$

7) $\int_0^\lambda e^{-p\, Cos.\, x}\, Cos.\, (a\, x - p\, Sin.\, x)\, d\, x = (-1)^a \frac{d^a}{d\, p^a} . \frac{e^{-p\, Cos.\, \lambda}\, Sin.\, (p\, Sin.\, \lambda)}{p}$

(4–7) Cauchy, Sav. Etr. 1827. 599. P. 1. § 4.

8) $\int_a^{a+1} e^{-a(x-1)}\, Sin.\, \{q(x-p)\}\, d\, x = q\, e^{-ap} \frac{e^q - 1}{a^2 + q^2}$ Schaar, Mém. Cour. Brux. T. 23.

9) $\int_0^{\frac{\pi}{4}} e^{-Tang\, x} \frac{d\, x}{Sin.\, 2\, x} = \infty$ V. T. 112. N°. 4.

10) $\int_0^{\frac{\pi}{4}} e^{Tang.\, x} \frac{Tang.\, x}{(Sin.\, x + Cos.\, x)^2}\, d\, x = \frac{1}{2} e - 1$ V. T. 112. N°. 5.

11) $\int_0^{\frac{\pi}{4}} e^{Cos.\, 2x} \frac{Sin.^3\, 2x}{Cos.^2\, x}\, d\, x = 2\, (e - 2)$ V. T. 112. N°. 3.

12) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{Cot.\, x} \frac{Cot.\, x}{(Cos.\, x + Sin.\, x)^2}\, d\, x = \frac{1}{2} e - 1$ V. T. 112. N°. 5.

13) $\int_{-\pi}^{\pi} e^{qx}\, Cos.\, \frac{1}{2}\, p\, x\, d\, x = \frac{e^{q\pi} - e^{-q\pi}}{4\, q^2 + p^2}\, 4\, q\, Cos.\, \frac{1}{2} p\, \pi + \frac{e^{q\pi} + e^{-q\pi}}{4\, q^2 + p^2}\, 2\, p\, Sin.\, \frac{1}{2} p\, \pi$ Dienger, Cr. 34. 75.

14) $\int_{-\pi}^{\pi} e^{qx}\, Sin.\, \frac{1}{2} p x\, d\, x = \frac{2}{4\, q^2 + p^2} \left\{(e^{q\pi} + e^{-q\pi}) 2 q\, Sin.\, \frac{1}{2} p\, \pi - (e^{q\pi} - e^{-q\pi})\, p\, Cos.\, \frac{1}{2}\, p\, \pi\right\}$ V. T. 298. N°. 13.

15) $\int_{-\pi}^{\pi} \frac{(1 - e^{-xi})(p + e^{-xi})}{1 - q\, e^{p + Cos.\, x}\, e^{(x - Sin.\, x) i}}\, d\, x = 2\, \pi \left\{p + \sum_1^\infty \frac{n^{n-1}}{1^{n/1}}\, q^n\, e^{np}\right\}$ Poisson, P. 19. 404. N°. 80.

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16) $\int_{-\pi}^{\pi} \frac{e^{-\frac{1}{2}xi}\, Sin.\, \frac{1}{2} x}{1 - q\, e^{p + Cos.x}\, e^{(x - Sin.x)i}}\, dx = \frac{\pi}{i} \sum_{1}^{\infty} \frac{n^{n-1}}{1^{n/1}} q^n e^{np}$ Poisson, P. 19. 404. N°. 80.

17) $\int_{-\pi}^{\pi} \frac{e^{bSin.x}\, Sin.\{(2a+1)x\} - Sin.\{(2a+1)x - b\, Cos.x\}}{e^{bSin.x} - 2\, Cos.(b\, Cos.x) + e^{-bSin.x}}\, dx = \left(\frac{b}{2\pi}\right)^{2a-1} \sum_{1}^{b} n^{2a}$

18) $\int_{-\pi}^{\pi} \frac{e^{bSin\,x}\, Sin.\left(x + \frac{ab}{4\pi^2} Sin.2x\right) - Sin.\left(x + \frac{ab}{4\pi^2} Sin.2x - b\,Cos.x\right)}{e^{bSin.x} - 2\, Cos.(b\, Cos.x) + e^{-bSin.x}}\, e^{\frac{ab}{4\pi^2} Cos.2x}\, dx = \frac{b}{2\pi}\left\{\frac{1}{2} + \sum_{1}^{b} e^{n^2 a}\right\}$

Sur (17) et (18) voyez: Cauchy, Exerc. 1826. p. 205.

19) $\int_{\frac{1}{2}\pi}^{\infty} e^{-px}\, Cos.^{2a} x\, dx = \frac{1^{2a/1}}{p^2 + 2^2 . p^2 + 4^2 \ldots p^2 + 4a^2} \frac{1}{p} e^{-\frac{1}{2}p\pi}$

20) $\int_{\frac{1}{2}\pi}^{\infty} e^{-px}\, Cos.^{2a+1} x dx = \frac{-\ 1^{2a+1/1}}{p^2 + 1^2 . p^2 + 3^2 \ldots p^2 + (2a+1)^2} e^{-\frac{1}{2}p\pi}$

21) $\int_{-\frac{1}{2}\pi}^{\infty} e^{-px}\, Cos.^{2a} x\, dx = \frac{1^{2a/1}}{p^2 + 2^2 . p^2 + 4^2 \ldots p^2 + 4a^2} \frac{1}{p} e^{\frac{1}{2}p\pi}$

22) $\int_{-\frac{1}{2}\pi}^{\infty} e^{-px}\, Cos.^{2a+1} x dx = \frac{1^{2a+1/1}}{p^2 + 1^2 . p^2 + 3^2 \ldots p^2 + (2a+1)^2} e^{\frac{1}{2}p\pi}$

(19)–(22): Ohm, Ausw. 23.

F. Expon.
Circ. Inv.

TABLE 299.

Lim. 0 et ∞ .

1) $\int Arctang.\frac{\pi}{q}\, e^{-px}\, dx = \frac{1}{p}\left\{Ci.(pq).Sin.pq - Si.(pq).Cos.pq + \frac{1}{2}\pi Cos.pq\right\}$ Bierens de Haan, Verh. v. K. Acad. van Wet. 1854, bl. 19.

2) $\int Arctang.\, x \frac{dx}{e^{2\pi x} - 1} = \frac{1}{2} - \frac{1}{4} l\, 2\pi$ Plana, Mém. Turin. 1820.

3) $\int Arctang.\frac{x}{p} \frac{dx}{e^{2\pi x} - 1} = \frac{1}{2} l \left\{\left(\frac{e}{p}\right)^p \Gamma(p) \sqrt{\frac{p}{2\pi}}\right\}$ Binet, P. 27. 123.

4) $\int Arctang.\, x \frac{dx}{e^{2\pi q x} - 1} = \frac{1}{2q}\left\{l\, \Gamma(q+1) - \frac{1}{2} l\, 2q\pi + q(1 - l\, q)\right\}$ V. T. 378. N°. 4.

5) $\int Arctang.\, 2x \frac{e^{\pi x} - e^{-\pi x}}{(e^{\pi x} + e^{-\pi x})^2}\, dx = \frac{1}{2\pi} l\, 2$ V. T. 138. N°. 3.

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6) $\int Arctang.\, x \frac{e^{\frac{1}{4}\pi x} - e^{-\frac{1}{4}\pi x}}{(e^{\frac{1}{4}\pi x} + e^{-\frac{1}{4}\pi x})^2} dx = \frac{\sqrt{2}}{\pi} \left\{\pi - l \frac{\sqrt{2}+1}{\sqrt{2}-1}\right\}$ V. T. 138. N°. 17.

7) $\int Arctang.\, x \frac{e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x}}{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x})^2} dx = \frac{1}{\pi} l\, 2$ V. T. 138. N°. 14.

8) $\int Arctang.\, x \frac{e^{\pi x} - e^{-\pi x}}{(e^{\pi x} + e^{-\pi x})^2} dx = \frac{4 - \pi}{4\pi}$ V. T. 138. N°. 2.

9) $\int Arctang. \frac{x}{p} \frac{e^{\pi x} - e^{-\pi x}}{(e^{\pi x} + e^{-\pi x})^2} dx = \frac{1}{4\pi}\left\{Z'\left(\frac{2q+3}{4}\right) - Z'\left(\frac{2q+1}{4}\right)\right.$ V. T. 138. N°. 1.

10) $\int Arctang.\, x \frac{\pi(e^{px} - e^{-px})(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}) - 2p(e^{px} + e^{-px})(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})}{(e^{\frac{1}{2}\pi x} - e^{-\pi x})^2} dx =$

$= \pi\, Sin.\, p - Cos.\, p\, l \frac{1 + Sin.\, p}{1 - Sin.\, p},\ 0 \leqq p \leqq \frac{1}{2}\pi;$ V. T. 138. N°. 13.

11) $\int Arctang. \frac{x}{q} \frac{p(e^{\pi x} - e^{-\pi x})(e^{(\pi - p)x} + e^{(p - \pi)x}) - 2\pi(e^{px} - e^{-px})}{(e^{\pi x} - e^{-\pi x})^2} dx = \sum_1^\infty \frac{Sin.\, np}{q+n},\ p^2 < \pi^2;$ V. T. 138. N°. 5.

12) $\int Arctang\ x \frac{\pi(e^{px} - e^{-px})(e^{\pi x} + e^{-\pi x}) - p(e^{px} + e^{-px})(e^{\pi x} - e^{-\pi x})}{(e^{\pi x} - e^{-\pi x})^2} dx =$

$- \frac{1}{2} p\, Cos.\, p + \frac{1}{2} Sin.\, p\, l\{2(1 + Cos.\, p)\}$ V. T. 138. N°. 4.

1) $\int e^{-x}\, l\, i.(e^{-x})\, dx = -l\, 2$ Schlömilch, Gr. 9. 5.

2) $\int e^{-2x}\, l\, i.(e^{x})\, dx = 0$ Schlömilch, Beitr. III. 6.

3) $\int e^{-px}\, li.(e^{-x})\, dx = -\frac{1}{p} l(1+p),\ p \geqq -1;$

4) $\int e^{-px^2}\, li.(e^{-x^2})\, dx = -\sqrt{\frac{\pi}{p}}.\, l\{\sqrt{p} + \sqrt{(1+p)}\},\ p > 0;$

5) $\int e^{px^2}\, l\, i.(e^{-x^2})\, dx = -\sqrt{\frac{\pi}{p}}\, Arcsin.(\sqrt{p}),\ p < 1;$

(3–5) Schlömilch, Beitr. III. 7.

1) $\int l\,x . Cos.\,p\,x . d\,x = -\frac{1}{p} Si.(p)$ V. T. 192. N°. 5.

2) $\int Sin.\,(q\,l\,x)\,d\,x = -\frac{q}{1+q^2}$ V. T. 278. N°. 6.

3) $\int Cos.\,(q\,l\,x)\,d\,x = \frac{1}{1+q^2}$ V. T. 278. N°. 7.

4) $\int Sin.\,2\,a\,\pi\,x . l\,Sin.\,\pi\,x\,d\,x = 0$

5) $\int Cos.\,2\,a\,\pi\,x . l\,Sin.\,\pi\,x\,d\,x = -\frac{1}{2a}$

6) $\int Cos.\{2\,b\,\pi\,(x-a)\} . l\,Sin.\,\pi\,x\,dx = -\frac{1}{2b} e^{-2\pi abi}$

(4–6) Schaar, Mém. Cour. Brux. T. 23.

7) $\int Sin.\left(2\,p\sqrt{l\frac{1}{x}}\right) d\,x = p\,e^{-p^2}\sqrt{\pi}$ V. T. 388. N°. 1.

8) $\int Sin.\,(l\,x)\,d\,x\sqrt{l\frac{1}{x}} = -Sin.\frac{3}{8}\pi . \frac{\sqrt{\pi}}{\sqrt[4]{8}}$ V. T. 397. N°. 1.

9) $\int Cos.\,(l\,x)\,d\,x\sqrt{l\frac{1}{x}} = Cos.\frac{3}{8}\pi . \frac{\sqrt{\pi}}{\sqrt[4]{8}}$ V. T. 397. N°. 2.

10) $\int Sin.\,(l\,x) . d\,x\,l\,l\frac{1}{x} = -\frac{1}{8}\pi + \frac{1}{4}l\,2 + \frac{1}{2}\mathrm{A}$ V. T. 439. N°. 1.

11) $\int Sin.\,(l\,x)\frac{d\,x}{l\,x} = \frac{1}{4}\pi$

12) $\int Sin.\,(q\,l\,x)\frac{d\,x}{l\,x} = Arctang.\,q$

13) $\int Sin.\,(p\,l\,x) . Sin.\,(q\,l\,x)\frac{d\,x}{l\,x} = \frac{1}{4}l\frac{1+(p-q)^2}{1+(p+q)^2}$

14) $\int Sin.\,(p\,l\,x) . Cos.\,(q\,l\,x)\frac{d\,x}{l\,x} = \frac{1}{2}Arctang.\left\{\frac{2\,p}{1-p^2+q^2}\right\}$

(11–14) Euler, N. C. Petr. 20. 59.

15) $\int Sin.^2\left(\frac{1}{2}p\,l\,x\right)\frac{d\,x}{l\,x} = \frac{1}{4}l(1+p^2)$ V. T. 392. N°. 9.

16) $\int Sin.\,(l\,x)\frac{d\,x}{\sqrt{l\frac{1}{x}}} = -\sqrt{\left(\frac{\pi}{2}\frac{\sqrt{2}-1}{2}\right)}$ V. T. 398. N°. 3.

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17) $\int Cos.(lx)\frac{dx}{\sqrt{l\frac{1}{x}}} = \sqrt{\left(\frac{\pi}{2}\frac{\sqrt{2}+1}{2}\right)}$ V. T. 398. N°. 4.

18) $\int Cos.\left\{2\sqrt{\left(q\,l\frac{1}{x}\right)}\right\}\frac{dx}{\sqrt{l\frac{1}{x}}} = e^{-q}\sqrt{\pi}$ V. T. 396. N°. 7.

19) $\int\frac{Cos.(p\,lx) - Cos.(q\,lx)}{lx}dx = \frac{1}{2}l\frac{1+p^2}{1+q^2}$ Euler, N. C. Petr. 20. 59.

20) $\int Cos.\left\{q\sqrt{l\frac{1}{x}}\right\}dx = \sum_0^\infty(-1)^n\frac{q^{2n}}{(n+1)^{n/1}}$ V. T. 388. N°. 13.

21) $\int Tang.\left\{q\sqrt{l\frac{1}{x}}\right\}dx = 2a\sqrt{\pi}\sum_1^\infty(-1)^n n e^{-n^2a^2}$ V. T. 388. N°. 20.

22) $\int Cot.\left\{q\sqrt{l\frac{1}{x}}\right\}dx = 2a\sqrt{\pi}\sum_1^\infty n e^{-n^2a^2}$ V. T. 388. N°. 21.

23) $\int Cosec.\left\{q\sqrt{l\frac{1}{x}}\right\}dx = 4a\sqrt{\pi}\sum_1^\infty(2n-1)e^{-(2n-1)^2a^2}$ V. T. 388. N°. 22.

24) $\int Sin.\left\{q\sqrt{l\frac{1}{x}}\right\}\frac{dx}{lx} = \frac{1}{2}q\sqrt{\pi}\sum_0^\infty\frac{(-1)^{n-1}}{(2n+1)1^{n/1}}\left(\frac{q}{2}\right)^{2n}$ V. T. 392. N°. 15.

25) $\int l\,Sin.(-q\,lx)\,dx = -\frac{1}{2}l2 - \frac{1}{2}\sum_1^\infty\frac{1}{n}\frac{1}{1+n^2q^2}$ V. T. 439. N°. 6.

26) $\int l\,Cos.(-q\,lx)\,dx = -\frac{1}{2}l2 - \frac{1}{2}\sum_1^\infty\frac{(-1)^n}{n}\frac{1}{1+n^2q^2}$ V. T. 439. N°. 7.

27) $\int l\,Tang.(-q\,lx)dx = -\sum_1^\infty\frac{1}{2n-1}\frac{1}{1+(2n-1)^2q^2}$ V. T. 439. N°. 8.

F. Log. Circ. Dir. TABLE 302. Lim. 0 et ∞.

1) $\int l\,Sin.^2x\,dx = \infty$

2) $\int l\,Cos.^2x\,dx = \infty$

3) $\int l(1+Cos.x)\,dx = \infty$

4) $\int l(1-Cos.x)dx = \infty$

(1–4) Raabe, Int. 188.

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5) $\int l(1+p^2+2p\,Cos.\,x)\,dx = \infty \quad , p > 1;$ Raabe, Int. 188.

6) $\qquad = 0 \quad , p \leqq 1;$

7) $\int l(1+p^2+2p\,Sin.\,x)\,dx = \sum_0^\infty \frac{1}{2n+1}\frac{2^{n/2}}{3^{n/2}}\left(\frac{2p}{1+p^2}\right)^{2n+1} \quad , p \leqq 1;$

(6, 7) Raabe, Cr. 15. 355.

8) $\int l\frac{p^2+x^2}{q^2+x^2}\,Cos.\,rx\,dx = \frac{\pi}{r}(e^{-qr}-e^{-pr})$

9) $\int l\frac{x^2}{q^2+x^2}\,Cos.\,rx\,dx = \frac{\pi}{r}(e^{-qr}-1)$

10) $\int l\left(1+\frac{p^2}{x^2}\right)Cos.\,rx\,dx = \frac{\pi}{r}(1-e^{-pr})$

(8–10) Raabe, Int. 170.

F. Log. de Circ. Dir. en num. $(l Sin.\,ax)^b$. Circ. Dir. ent. TABLE 303. Lim. 0 et $\frac{\pi}{4}$.

1) $\int l\,Sin.\,x\,dx = -\frac{1}{4}\pi l2 - \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 238. N°. 4.

2) $\int l\,Sin.\,x.\,Cos.^a 2x.\,Sin.\,2x\,dx = \frac{-1}{4(a+1)}\left\{l2 + \sum_0^a \frac{1}{n+1}\right\}$ V. T. 47. N°. 8.

3) $\int l(2\,Sin.^2 x).\,Tang.\,2x\,dx = -\frac{1}{12}\pi^2$ V. T. 160. N°. 5.

4) $\int l\,Sin.\,2x.\,Tang.\left(\frac{\pi}{4}+x\right)dx = -\frac{1}{12}\pi^2$ V. T. 316. N°. 8.

5) $\int l\,Sin.\,2x.\,Tang.\left(\frac{\pi}{4}-x\right)dx = -\frac{1}{24}\pi^2$ V. T. 316. N°. 4.

6) $\int l\,Sin.\,2x.\,Tang.\left(\frac{\pi}{4}+x\right).\,Sin.\,2x\,dx = \frac{6-\pi^2}{12}$ V. T. 152. N°. 8.

7) $\int l\,Sin.\,2x.\,Tang.^2\left(\frac{\pi}{4}+x\right).\,Cos.\,2x\,dx = \frac{3-\pi^2}{6}$ V. T. 152. N°. 9.

8) $\int (l\,Sin.\,2x)^3\,Tang.\left(\frac{\pi}{4}+x\right)dx = -\frac{1}{30}\pi^4$ V. T. 154. N°. 11.

9) $\int (l\,Sin.\,2\,x)^3\ Tang.\left(\frac{\pi}{4} - x\right) d\,x = -\frac{7}{240}\pi^4$ V. T. 154. N°. 10.

10) $\int (l\,Sin.\,2\,x)^5\ Tang.\left(\frac{\pi}{4} + x\right) d\,x = -\frac{4}{63}\pi^6$ V. T. 155. N°. 3.

11) $\int (l\,Sin.\,2\,x)^5\ Tang.\left(\frac{\pi}{4} - x\right) d\,x = -\frac{31}{504}\pi^6$ V. T. 155. N°. 2.

12) $\int (l\,Sin.\,2\,x)^{2a}\ Tang.\left(\frac{\pi}{4} + x\right) d\,x = \frac{1^{2a/1}}{2}\sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 157. N°. 3.

13) $\int (l\,Sin.\,2\,x)^{a-1}\ Tang.\left(\frac{\pi}{4} - x\right) d\,x = (-1)^{a-1}\,1^{a-1/1}\frac{1}{2}\sum_0^\infty \frac{1}{(1+n)^a}$ V. T. 157. N°. 9.

14) $\int (l\,Sin.\,2\,x)^{a-1}\ Tang.\left(\frac{\pi}{4} - x\right) d\,x = (-1)^{a-1}\,1^{a-1/1}\frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(1+n)^a}$ V. T. 157. N°. 8.

15) $\int (l\,Sin.\,2\,x)^{2a-1}\ Tang.\left(\frac{\pi}{4} + x\right) d\,x = -\frac{1}{8\,a}(2\,\pi)^{2a}\,B_{2a-1}$ V. T. 157. N°. 6.

16) $\int (l\,Sin.\,2\,x)^{2a-1}\ Tang.\left(\frac{\pi}{4} - x\right) d\,x = \frac{1-2^{2a-1}}{4\,a}\pi^{2a}\,B_{2a-1}$ V. T. 157. N°. 5.

17) $\int (l\,Sin.\,2\,x)^{b-1}\ Tang.\left(\frac{\pi}{4} + x\right).\,Sin.^a\,2\,x\,d\,x = \frac{1}{2}(-1)^{b-1}\,1^{b-1/1}\sum_0^\infty \frac{1}{(a+n+1)^b}$ V. T. 157. N°. 12.

18) $\int (l\,Sin.\,2\,x)^{b-1}\ Tang.\left(\frac{\pi}{4} - x\right).\,Sin.^a\,2\,x\,d\,x = \frac{1}{2}(-1)^{b-1}\,1^{b-1/1}\sum_0^\infty \frac{(-1)^n}{(a+n+1)^b}$ V. T. 157. N°. 11.

F.Log. de Circ. Dir. en num. $(l\,Cos.\,a\,x)^b$. Circ. Dir. ent. TABLE 304. Lim. 0 et $\frac{\pi}{4}$.

1) $\int l\,Cos.\,x\,d\,x = -\frac{1}{4}\pi\,l\,2 + \frac{1}{2}\sum_0^\infty \frac{(-1)^n}{(2\,n+1)^2}$ V. T. 303. N°. 1 et T. 305. N°. 1.

2) $\int l\,Cos.\,x.\,Cos.^{p-1}\,2\,x.\,Tang.\,2\,x\,d\,x = \frac{-1}{8\,(p-1)}\left\{Z'\left(\frac{p+1}{2}\right) - Z'\left(\frac{p}{2}\right)\right\}$ V. T. 46. N°. 11.

3) $\int l\,(2\,Cos.^2\,x).\,Tang.\,2\,x\,d\,x = \frac{1}{24}\pi^2$ V. T. 160. N°. 1.

4) $\int l\,Cos.\,2\,x.\,Tang.\,x\,d\,x = -\frac{1}{48}\pi^2$ V. T. 304. N°. 3.

5) $\int (l\,Cos.\,2\,x)^3\;Tang.\,x\,d\,x = -\frac{7}{240}\pi^4$ V. T. 154. N°. 10.

6) $\int (l\,Cos.\,2\,x)^5\;Tang.\,x\,d\,x = -\frac{31}{504}\pi^6$ V. T. 154. N°. 2.

7) $\int (l\,Cos.\,2\,x)^{a-1}\;Tang.\,x\,d\,x = (-1)^{a-1}\,1^{a-1/1}\sum_0^\infty \frac{(-1)^n}{(1+n)^a}$ V. T. 157. N°. 8.

8) $\int (l\,Cos.\,2\,x)^{2a-1}\;Tang.\,x\,d\,x = \frac{1-2^{2a-1}}{4\,a}\pi^{2a}\,B_{2a-1}$ V. T. 157. N°. 5.

9) $\int (l\,Cos.\,2\,x)^{2a}\;Tang.\,x\,d\,x = \frac{2^{2a}-1}{2^{2a+1}}\,1^{2a/1}\sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 157. N°. 2.

10) $\int (l\,Cos.\,2\,x)^{b-1}\;Tang.\,x.\;Cos.^a\,2\,x\,d\,x = \frac{1}{2}(-1)^{b-1}\,1^{b-1/1}\sum_0^\infty \frac{(-1)^n}{(a+n+1)^b}$ V. T. 157. N°. 11.

1) $\int l\,Tang.\,x\,d\,x = -\sum_0^\infty (-1)^n \frac{1}{(2\,n+1)^2}$ V. T. 237. N°. 4.

2) $\int l\,Tang.\,x.\;Tang.\,x\,d\,x = -\frac{1}{48}\pi^2$ V. T. 152. N°. 12.

3) $\int l\,Tang.\,x.\;Tang.\,2\,x\,d\,x = -\frac{1}{16}\pi^2$ V. T. 160. N°. 15.

4) $\int l\,Tang.\,x.\;Cos.\,2\,x.\;Sin.^{2p-1}\,2\,x\,d\,x = -2^{2p-4}\frac{\{\Gamma(p)\}^2}{p\,\Gamma(2\,p)}$ V. T. 153. N°. 20.

5) $\int (l\,Tang.\,x)^2\;d\,x = \frac{1}{16}\pi^3$ V. T. 154. N°. 1.

6) $\int (l\,Tang.\,x)^3\;Tang.\,x\,d\,x = -\frac{7}{1920}\pi^4$ V. T. 154. N°. 13.

7) $\int (l\,Tang.\,x)^3\;Tang.\,2\,x\,d\,x = -\frac{1}{128}\pi^4$ V. T. 154. N°. 16.

8) $\int (l\,Tang.\,x)^4\;d\,x = \frac{5}{64}\pi^5$ V. T. 155. N°. 1.

9) $\int (l\,Tang.\,x)^6\,dx = \frac{61}{256}\pi^7$ V. T. 155. N°. 8.

10) $\int (l\,Tang.\,x)^{q-1}\,dx = \frac{\Gamma(q)}{(-1)^{q-1}}\sum_0^\infty \frac{(-1)^n}{(2n+1)^q}$ } Arndt, Gr. 6. 434.

11) $\int (l\,Tang.\,x)^{2a}\,dx = \frac{1}{2^{2a+2}} 1^{2a/1}\,\pi^{2a+1}\,B_{2a+2}$ } Arndt, Gr. 6. 434.

12) $\int (l\,Tg.\,x)^{2a}(Tg.\,qx + Cot.\,qx)\,dx = \frac{(-1)^{a+1}}{b}\left(2b\pi\right)^{2a+1}\sum_1^b (-1)^{n+1} B''\left(\frac{2n-1}{4b}\right) Cos.\left(\frac{2n-1}{2}q\pi\right)$ V. T. 158. N°. 8.

où $q^2 = \alpha^2 b^2 + 1$, α arbitraire.

13) $\int (l\,Tang.\,x)^{b-1}\,Tang.^a x\,dx = \frac{1^{b-1/1}}{(-1)^{b-1}}\sum_0^\infty \frac{(-1)^n}{(a+1+2n)^b}$ Arndt, Gr. 6. 434.

F. Log. en num. } Circ. Dir. ent. } Autre forme. TABLE 306. Lim. 0 et $\frac{\pi}{4}$.

1) $\int l(1 + Tang.\,x)\,dx = \frac{1}{8}\pi\,l2$ Serret, L. 9. 436. — Grunnert, Gr. 6. 448.

2) $\int l\,Tang.\left(\frac{\pi}{4} + x\right).\,Sin.\,2x\,dx = \frac{1}{4}\pi$ } V. T. 47. N°. 5, 6.

3) $\int l\,Tang.\left(\frac{\pi}{4} + x\right).\,Tang..\,x.\,(Sin.\,2x + Tang.\,x)\,dx = \frac{3}{2}l2$ } V. T. 47. N°. 5, 6.

4) $\int l\,Tang.\,x\,(l\,Cos.\,2x)^2\,Tang.\,2x\,dx = -\frac{1}{96}\pi^4$ V. T. 337. N°. 15.

5) $\int l\,Tang.\,x\,(l\,Cos.\,2x)^4\,Tang.\,2x\,dx = -\frac{1}{80}\pi^6$ V. T. 337. N°. 17.

6) $\int l\,Tang.\,x\,(l\,Cos.\,2x)^6\,Tang.\,2x\,dx = -\frac{17}{448}\pi^8$ V. T. 337. N°. 19.

7) $\int l\,Tang.\,x\,(l\,Cos.\,2x)^{2a}\,Tang.\,2x\,dx = -\frac{2^{2a+1}-1}{(2a+1)(2a+2)}\pi^{2a+2}\,B_{2a+1}$ V. T. 337. N°. 20.

8) $\int l\,Tang.\,x\,(l\,Cos.\,2x)^{2a-1}\,Tang.\,2x\,dx = \frac{2^{2a+1}-1}{2^{2a+2}} 1^{2a-1/1}\sum_0^\infty \frac{n^{2a+1}}{1}$ V. T. 337. N°. 22.

9) $\int l\,Tang.\,x\,(l\,Cos.\,2x)^a\,Tang.\,2x\,dx = (-1)^{a+1}\frac{1^{a/1}}{2a}\sum_0^\infty \frac{1}{(1+2n)^{a+1}}$ V. T. 317. N°. 7.

1) $\int l\,Sin.\,x \frac{Sin.^{2a}x}{Cos.^{2a+2}x}\,dx = -\frac{1}{2a+1}\left\{\frac{1}{2}l\,2+(-1)^a\frac{\pi}{4}+(-1)^a\sum_0^a(-1)^n\frac{1}{2n-1}\right\}$ V. T. 46. N°. 4.

2) $\int l\,Sin.\,x \frac{Sin.^{2a-1}x}{Cos.^{2a+1}x}\,dx = \frac{1}{2a}\left\{-\frac{1}{2}l\,2+(-1)^a\frac{1}{2}l\,2+(-1)^a\sum_1^{a-1}\frac{(-1)^n}{2n}\right\}$ V. T. 46. N°. 5.

3) $\int l\,Sin.\,x \frac{Sin.\,2x}{Cos^{p+1}2x}\,dx = \frac{1}{4p}\{A+Z'(1-p)\}$, $0>p>-1$; V. T. 47. N°. 11.

4) $\int l\,Sin.\,x(Cos.^p 2x+Sec.^p 2x)Tang.\,2x\,dx = \frac{1}{4p}\left\{\pi\,Cot.\,p\pi-\frac{1}{p}\right\}$, $0>p>-1$; V. T. 47. N°. 12.

5) $\int l\,Sin.\,x \frac{(p-1)Cos.^{2p-1}2x+p}{Cos.^p 2x}Tang.\,2x\,dx = \frac{1}{4}\pi\,Cot.\,p\pi$, $0>p>-1$; V. T. 47. N°. 10.

6) $\int l\,Cos.\,x \frac{dx}{Sin.\,2x} = -\frac{1}{96}\pi^2$ V. T. 305. N°. 2.

7) $\int l\,Cos.\,2x \frac{dx}{Tang.\,x} = -\frac{1}{12}\pi^2$ V. T. 303. N°. 3.

8) $\int l\,Cos.\,2x \frac{Sin.^2 x}{Tang.\,x}\,dx = -\frac{1}{4}$ V. T. 307. N°. 9, 10.

9) $\int l\,Cos.\,2x \frac{Cos.^2 x}{Tang.\,x}\,dx = \frac{1}{4}-\frac{1}{12}\pi^2$ V. T. 152. N°. 9.

10) $\int l\,Cos.\,2x \frac{Cos.\,2x}{Tang.\,x} = \frac{1}{12}(6-\pi^2)$ V. T. 152. N°. 8.

11) $\int l\,Cos.\,2x \frac{Sin.\,2x}{Tang^2 x}\,dx = \frac{1}{6}(3-\pi^2)$ V. T. 152. N°. 9.

12) $\int l\,Cos.\,x \frac{Sin.^{2a}x}{Cos.^{2a+2}x}\,dx = \frac{1}{2a+1}\left\{-\frac{1}{2}l\,2+(-1)^{a+1}\frac{\pi}{4}+(-1)^{a+1}\sum_1^{a+1}\frac{(-1)^n}{2n-1}\right\}$ V. T. 46. N°. 4.

13) $\int l\,Cos.\,x \frac{Sin.^{2a-1}x}{Cos.^{2a+1}x}\,dx = \frac{1}{2a}\left\{-\frac{1}{2}l\,2+(-1)^a\frac{1}{2}l\,2+(-1)^a\sum_1^a\frac{(-1)^n}{2n}\right\}$ V. T. 46. N°. 5.

14) $\int l\,Cos.\,x \frac{Tang^p x}{Sin.\,2x}\,dx = \frac{1}{8p}\left\{2l\frac{1}{2}+Z'\left(\frac{p+4}{4}\right)-Z'\left(\frac{p+2}{4}\right)\right\}$ V. T. 151. N°. 13.

15) $\int l\,Cos.\,2x \frac{Cos.^{p-1}2x}{Tang.\,x}\,dx = -\frac{1}{2}\sum_0^\infty\frac{1}{(p+n)^2}$ V. T. 152. N°. 10.

F. Log. en num. $(l\,Sin.\,a\,x)^b$, $(l\,Cos.\,a\,x)^b$. Circ. Dir. rat. en dén. monôme.	TABLE 308.	Lim. 0 et $\frac{\pi}{4}$.

1) $\int (l\,Sin.\,2\,x)^{q-1} \frac{Sin.^a\,2\,x}{Tang.\left(\frac{\pi}{4} - x\right)}\,d\,x = \frac{1}{2}\,\frac{\Gamma(q)}{(-1)^{q-1}} \sum_0^\infty \frac{1}{(a+n+1)^q}$ V. T. 157. N°. 12.

2) $\int (l\,Sin.\,2\,x)^{2a-1}\ Tang.^2 \left(\frac{\pi}{4} + x\right) \frac{d\,x}{Tang.\,2\,x} = \frac{1}{2\,a}(-2)^{2a-1}\,\pi^{2a}\,B_{2a-1}$ V. T. 158. N°. 14.

3) $\int (l\,Cos.\,2\,x)^3 \frac{d\,x}{Tang.\,x} = -\frac{1}{30}\,\pi^4$ V. T. 154. N°. 11.

4) $\int (l\,Cos.\,2\,x)^5 \frac{d\,x}{Tang.\,x} = -\frac{4}{63}\,\pi^6$ V. T. 155. N°. 3.

5) $\int (l\,Cos.\,2\,x)^{a-1} \frac{d\,x}{Tang.\,x} = \frac{1}{2}(-1)^{a-1}\,1^{a-1/1} \sum_0^\infty \frac{1}{(1+n)^a}$ V. T. 157. N°. 9.

6) $\int (l\,Cos.\,2\,x)^{2a-1} \frac{Tang.\,2\,x}{Tang.^2\,x}\,d\,x = \frac{1}{2\,a}(-2)^{2a-1}\,\pi^{2a}\,B_{2a-1}$ V. T. 158. N°. 14.

7) $\int (l\,Cos.\,2\,x)^{2a-1} \frac{d\,x}{Tang.\,x} = -\frac{1}{a}\,2^{2a-3}\,\pi^{2a}\,B_{2a-1}$ V. T. 157. N°. 6.

8) $\int (l\,Cos.\,2\,x)^{q-1} \frac{Cos.^a\,2\,x}{Tang.\,x}\,d\,x = \frac{1}{2}\,\frac{\Gamma(q)}{(-1)^{q-1}} \sum_0^\infty \frac{1}{(a+n+1)^q}$ V. T. 157. N°. 12.

F. Log. en num. $l\,Tang.\,a\,x$. Circ. Dir. rat. en dén. monôme.	TABLE 309.	Lim. 0 et $\frac{\pi}{4}$.

1) $\int l\,Tang.\,x \frac{dx}{Cos.\,2\,x} = -\frac{1}{8}\,\pi^2$ V. T. 152. N°. 13.

2) $\int l\,Tang.\,x \frac{dx}{Sin.\,4\,x} = -\infty$ V. T. 153. N°. 11.

3) $\int l\,Tang.\,x \frac{d\,x}{Tang.\,2\,x} = -\infty$ V. T. 153. N°. 10.

4) $\int l\,Tang.\,x \frac{Tang.\,2\,x}{Cos.^2\,x}\,d\,x = -\frac{1}{12}\,\pi^2$ V. T. 340. N°. 4.

5) $\int l\,Tang.\,x \frac{Tang.\,x}{Cos.\,2\,x}\,d\,x = -\frac{1}{24}\,\pi^2$ V. T. 152. N°. 14.

6) $\int l\,Tang.\,x \frac{Sin.^{2a}\,x}{Cos.^{2a+2}\,x}\,d\,x = -\left(\frac{1}{2\,a+1}\right)^2$ V. T. 307. N°. 1, 12.

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F. Log. en num. $l\,Tang.\,a\,x$. Circ. Dir. rat. en dén. monôme.	TABLE 309 suite.	Lim. 0 et $\frac{\pi}{4}$.

7) $\int l\,Tang.\,x \frac{Sin.^{2a-1} x}{Cos.^{2a+1} x}\,dx = -\frac{1}{4a}$ V. T. 307. N°. 8, 13.

8) $\int l\,Tang.\,x.\,Sin.(p\,Cot.\,x)\frac{dx}{Sin.^2 x} = \infty$ V. T. 47. N°. 23.

9) $\int l\,Tang.\,x.\,Cos.(p\,Tang.\,x)\frac{dx}{Cos.^2 x} = -\frac{1}{p}Si.(p)$ V. T. 47. N°. 22.

10) $\int l\,Tang.\,x.\,Tang.\left(\frac{\pi}{4}+x\right)\frac{dx}{Cos.^2 x} = \frac{1}{2}(3-\pi^2)$ V. T. 152. N°. 9.

11) $\int l\,Tang.\,x\frac{Sin.^3 x}{Cos.\,2x.\,Cos.\,x}\,dx = -\frac{1}{96}\pi^2$ V. T. 152. N°. 17.

12) $\int l\,Tang.\,x\left(\frac{Cos.\,x - Sin.\,x}{Sin.\,x}\right)^p\frac{dx}{Sin.^2 x} = -\frac{\pi}{p}Cosec.\,p\,\pi, 0 > p > -1;$ V. T. 47. N°. 18.

13) $\int l\,Tang.\,x\frac{dx}{Tang.^a x.\,Sin.\,2x} = \frac{-1}{2(a+2)^2}$ V. T. 152. N°. 1.

F. Log. en num. $(l\,Tang.\,a\,x)^b$. Circ. Dir. rat. en dén. monôme.	TABLE 310.	Lim. 0 et $\frac{\pi}{4}$.

1) $\int (l\,Tang.\,x)^3\frac{dx}{Cos.\,2x} = -\frac{1}{16}\pi^4$ V. T. 154. N°. 14.

2) $\int (l\,Tang.\,x)^3\frac{Tang.\,x}{Cos.\,2x}\,dx = -\frac{1}{240}\pi^4$ V. T. 154. N°. 15.

3) $\int (l\,Tang.\,x)^3\frac{Sin.\,x.\,Cos.\,x}{Cos.\,2x}\,dx = -\frac{1}{256}\pi^4$ V. T. 154. N°. 16.

4) $\int (l\,Tang.\,x)^3\frac{Sin.^3 x}{Cos.\,2x.\,Cos.\,x}\,dx = -\frac{1}{3840}\pi^4$ V. T. 154. N°. 17.

5) $\int (l\,Tang.\,x)^5\frac{dx}{Cos.\,2x} = -\frac{1}{8}\pi^6$ V. T. 155. N°. 4.

6) $\int (l\,Tang.\,x)^5\frac{Tang.\,x}{Cos.\,2x}\,dx = -\frac{1}{504}\pi^6$ }

7) $\int (l\,Tang.\,x)^5\frac{Sin.\,x.\,Cos.\,x}{Cos.\,2x}\,dx = -\frac{1}{512}\pi^6$ } V. T. 155. N°. 5.

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8) $\int (l\,Tang.\,x)^7 \frac{dx}{Cos.\,2\,x} = -\frac{17}{32}\pi^8$ V. T. 155. N°. 10.

9) $\int (l\,Tang.\,x)^{2a-1} \frac{dx}{Cos.\,2\,x} = \frac{1-2^{2a}}{4\,a}\pi^{2a}\,B_{2a-1}$ V. T. 158. N°. 5.

10) $\int (l\,Tang.\,x)^{2a} \frac{dx}{Cos.\,2\,x} = \frac{2^{2a+1}-1}{2^{2a+1}} 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 158. N°. 4.

11) $\int (l\,Tang.\,x)^{2a-1} \frac{Tang.\,x}{Cos.\,2\,x} dx = -\frac{1}{4\,a}\pi^{2a}\,B_{2a-1}$ V. T. 158. N°. 6.

12) $\int (l\,Tang.\,x)^{2a} \frac{Tang.^q\,x + Cot.^q\,x}{Cos.\,2\,x} dx = \frac{(-1)^a}{b}(2b\pi)^{2a+1} \sum_0^b (-1)^{n-1} B' \left(\frac{n}{2b}\right) Sin.\,nq\pi$ V. T. 158. N°. 9.

où $q^2 = \alpha^2 b^2 < 1$, α arbitraire.

13) $\int (l\,Tang\,x)^a\,Tang^{p-1}\,x \frac{dx}{Sin.\,2\,x} = \frac{(-1)^a}{2\,p^{a+1}} 1^{a/1}$ V. T. 151. N°. 2.

14) $\int l\,(Tang.\,x)^{2a-1}\,Tang.\left(\frac{\pi}{4}+x\right) \frac{dx}{Sin.\,2\,x} = -\frac{2^{2a-2}}{a}\pi^{2a}\,B_{2a-1}$ V. T. 158. N°. 14.

15) $\int (l\,Tang.\,x)^{2a} \frac{dx}{Cos.^2\left(\frac{\pi}{4}+x\right)} = (2\,\pi)^{2a}\,B_{2a-1}$ V. T. 310. N°. 14.

F. Log. en num. $l\ Tang.\ a\ x$. Circ. Dir. rat. en dén. binôme. TABLE 311. Lim. 0 et $\frac{\pi}{4}$.

1) $\int l\,Tang.\,x \frac{dx}{2-Sin.\,2\,x} = -\frac{2}{27}\pi^2$ V. T. 153. N°. 3.

2) $\int l\,Tang.\,x \frac{Cos.\,2\,x}{1+p\,Sin.\,2\,x} dx = \frac{1}{16\,p}\left\{4\,(Arccos.\,p)^2 - \pi^2\right\}, p \leqq 1$; V. T. 339. N°. 30.

3) $\int l\,Tang.\,x \frac{Cos.\,2\,x}{1-p\,Sin.\,2\,x} dx = -\frac{1}{4\,p}\left\{\pi + Arcsin.\,p\right\} Arcsin.\,p$, $p < 1$; V. T. 311. N°. 2, 10.

4) $\int l\,Tang.\,x \frac{Tang.\,x}{1-Sin.\,x.\,Cos.\,x} dx = -\frac{5}{108}\pi^2$ V. T. 153. N°. 4.

5) $\int l\,Tang.\,x \frac{Cos.\,\lambda - Tang.\,x}{1-Sin.\,2\,x.\,Cos.\,\lambda} dx = \frac{1}{6}\pi^2 - \frac{1}{2}\pi\lambda + \frac{1}{4}\lambda^2$ V. T. 153. N°. 6.

6) $\int l\,Tang.\,x \frac{Sin.\,2\,x}{4-3\,Sin.^2\,2x} dx = -\frac{1}{54}\pi^2$ V. T. 153. N°. 7.

7) $\int l\,Tang.\,x \frac{Cos.\,2\,x}{1-Sin.^2\,\lambda.\,Sin.^2\,2x} dx = -\frac{1}{4}\pi\,\lambda\,Cosec.\,\lambda$ V. T. 153. N°. 9.

8) $\int l\,Tang.\,x \frac{Cos.\,2\,x}{1-Sin.^2\,2\,x.Sin.^2\,\lambda}\,dx = -\frac{1}{4}\pi\lambda\,Cosec.\,\lambda$ V. T. 340. N°. 10.

9) $\int l\,Tang.\,x \frac{Sin.\,4\,x}{1-p^2\,Sin.^2\,2\,x}\,dx = \frac{-1}{4\,p^2}(Arcsin.\,p)^2\,,\ p<1$; V. T. 311. N°. 2, 10.

10) $\int l\,Tang.\,x \frac{Cos.\,2\,x}{1-p^2\,Sin.^2\,2\,x}\,dx = -\frac{\pi}{4\,p}Arcsin.\,p\,,\ p\leqq 1$; V. T. 340. N°. 3.

11) $\int l\,Tang.\,x \frac{Cos.\,2\,x}{1+p^2\,Sin.^2\,2\,x}\,dx = -\frac{\pi}{4\,p}\,l\,\{p+\sqrt{(1+p^2)}\}$ V. T. 369. N°. 3.

12) $\int l\,Tang.\,x \frac{Cos.\,2\,x}{Cos.^2\,2\,x+p^2\,Sin.^2\,2\,x}\,dx = -\frac{\pi}{4\sqrt{(1-p^2)}}Arccos.\,p\,,\ p<1$; V. T. 340. N°. 5.

13) $\int l\,Tang.\,x \frac{Cos.\,2\,x}{1+\left(\frac{e^p-e^{-p}}{2}\right)^2 Sin.^2\,2\,x}\,dx = -\frac{1}{2}p\frac{\pi}{e^p-e^{-p}}$ V. T. 153. N°. 5.

F. Log. en num. $(l\,Tang.\,a\,x)^b$. Circ. Dir. rat. en dén. binôme. TABLE 312. Lim. 0 et $\frac{\pi}{4}$.

1) $\int (l\,Tang.\,x)^2 \frac{dx}{2+Sin.\,2\,x} = \frac{4}{243}\pi^3\sqrt{3}$ V. T. 156. N°. 1.

2) $\int (l\,Tang.\,x)^2 \frac{dx}{1+Sin.\,2\,x.Cos.\,\lambda} = \frac{\pi^2-\lambda^2}{6}\lambda\,Cosec.\,\lambda$ V. T. 156. N°. 3.

3) $\int (l\,Tang.\,x)^2 \frac{dx}{2-Sin.\,2\,x} = \frac{5}{243}\pi^3\sqrt{3}$ V. T. 156. N°. 2.

4) $\int (l\,Tang.\,x)^2 \frac{dx}{1-Sin.\,2\,x.Cos.\,\lambda} = 2\,\lambda\,Cosec.\,\lambda\left\{\frac{1}{6}\pi^2-\frac{1}{4}\pi\lambda+\frac{1}{12}\lambda^2\right\}$ V. T. 156. N°. 4.

5) $\int (l\,Tang.\,x)^2 \frac{dx}{Sin.^4\,x+Cos.^4\,x} = \frac{3}{64}\pi^3\sqrt{2}$ V. T. 154. N°. 2.

6) $\int (l\,Tang.\,x)^2 \frac{dx}{1-Sin.^2\,x.\,Cos.^2\,x} = \frac{1}{27}\pi^3\sqrt{3}$ V. T. 154. N°. 3.

7) $\int (l\,Tang.\,x)^2 \frac{Sin.\,2\,x}{1-Sin.^2\,x.Cos.^2\,x}\,dx = \frac{2}{243}\pi^3\sqrt{3}$ V. T. 154. N°. 4.

8) $\int (l\,Tang.\,x)^4 \frac{dx}{1+Sin.\,2\,x.Cos.\,\lambda} = \frac{\pi^2-\lambda^2}{Sin.\,\lambda}\,\frac{7\,\pi^2-3\,\lambda^2}{5}\,\lambda$ V. T. 156. N°. 5.

9) $\int (l\,Tang.\,x)^{2a} \frac{dx}{2 + Sin.\,2x} = \frac{(-1)^{a+1}}{2\sqrt{3}} (2\pi)^{2a+1} B''\left(\frac{1}{3}\right)$ V. T. 159. N°. 1.

10) $\int (l\,Tang.\,x)^{2a} \frac{dx}{2 - Sin.\,2x} = \frac{(-1)^{a+1}}{2\sqrt{3}} (2\pi)^{2a+2} B''\left(\frac{1}{6}\right)$ V. T. 159. N°. 2.

11) $\int (l\,Tang\;x)^{2a} \frac{dx}{1 - Sin.\,2x.\,Cos.\,2p\pi} = \frac{1}{2} Cosec.\,2p\pi.\,(-1)^{a+1} (2\pi)^{2a-1} B''(p)$ V. T. 159. N°. 5.

1) $\int l\,Tang.\,x \frac{dx}{Cos.\,x\,(Sin.\,x + Cos.\,x)} = -\frac{1}{12}\pi^2$ V. T. 316. N°. 6.

2) $\int l\,Tang.\,x \frac{dx}{Cos.\,x\,(Cos.\,x - Sin.\,x)} = -\frac{1}{6}\pi^2$ V. T. 316. N°. 7.

3) $\int l\,Tang.\,x \frac{Tang.^p\,x}{Cos.\,x - Sin.\,x} \frac{dx}{Sin.\,2x} = -\frac{1}{2}\sum_0^\infty \frac{1}{(p+n)^2}$ V. T. 152. N°. 10.

4) $\int l\,Tang.\,x \frac{Sin.^a\,2x}{Cos.^{2a}x - Sin.^{2a}x} \frac{dx}{Sin.\,2x} = -2^{a-4}\frac{\pi^2}{a^2}$ V. T. 152. N°. 18.

5) $\int l\,Tang.\,x \frac{Sin.^2\,2x}{Sin.^4\,x + Cos.^4\,x} \frac{dx}{Cos.\,2x} = -\frac{\pi}{4(2+\sqrt{2})}$ V. T. 153. N°. 16.

6) $\int l\,Tang.\,x \frac{Cos.\,2x}{Tang.^p\,x + Cot.^p\,x} \frac{dx}{Sin.^2 2x} = -\frac{\pi^2}{16p^2} Sin.\frac{\pi}{2p}.\,Sec.^2\frac{\pi}{2p}$ V. T. 152. N°. 19.

7) $\int l\,Tang.\,x \frac{dx}{Sin.^2\,2x\,(Tang.^p\,x - Cot.^p\,x)} = \frac{\pi^2}{16p^2} Sec.^2\frac{\pi}{2p}$ V. T. 152. N°. 20.

8) $\int l\,Tang.\,x \frac{Tang.^q\,x - Cot.^q\,x}{Tang.^p\,x + Cot.^p\,x} \frac{dx}{Sin.\,2x} = \frac{\pi^2}{8p^2} Sin.\frac{q\pi}{2p}.\,Sec.^2\frac{q\pi}{2p}$ V. T. 153. N°. 12.

9) $\int l\,Tang.\,x \frac{Tang.^q\,x + Cot.^q\,x}{Tang.^p\,x - Cot.^p\,x} \frac{dx}{Sin.\,2x} = \frac{\pi^2}{8p^2} Sec.^2\frac{q\pi}{2p}$ V. T. 153. N°. 13.

10) $\int l\,Tang.\,x \frac{dx}{(Cos.\,x + Sin.\,x)^2} = -l\,2$ V. T. 153. N°. 14.

11) $\int l\,Tang.\,x \frac{Sin.^{p-1}\,x}{(Cos.\,x - Sin.\,x)^{p+1}}\,dx = -\frac{1}{p}\pi\,Cosec.\,p\pi\,,\; p < 1;$ V. T. 49. N°. 24.

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12) $\int l\,Tang.\,x \frac{Sin.^{p-1}2x.\,Cos.\,2x}{(1+Sin.\,2x)^{p+1}}\,dx = -\frac{1}{p\,2^{p+1}}\frac{\Gamma(p)\sqrt{\pi}}{\Gamma(p+\frac{1}{2})}$, $p \leqq 1$; V. T. 49. N°. 5.

13) $\int l\,Tang.\,x \frac{Tang.^p x - Cot.^p x}{(Tang.^p x + Cot.^p x)^2}\frac{dx}{Sin.\,2x} = \frac{\pi}{8p^2}$ V. T. 49. N°. 19.

14) $\int l\,Tang.\,x \frac{p\,Tang.^p x(1-Tang.^{2q}x) + q\,Tang.^q x(1-Tang.^{2p}x)}{(Tang.^{p+q}x+1)^2}\frac{dx}{Sin.\,2x} = -\frac{1}{2}\frac{\pi}{p+q}\,Sec\left\{\frac{q-p}{p+q}\frac{\pi}{2}\right\}$, $p < q$; V. T. 49. N°. 13.

15) $\int l\,Tang.\,x \frac{p\,Tang.^p x(1-Tang.^{2q}x) - q\,Tang.^q x(1-Tang.^{2p}x)}{(Tang.^{p+q}x-1)^2}\frac{dx}{Sin.\,2x} = -\frac{1}{2}\frac{\pi}{p+q}\,Tang.\left\{\frac{q-p}{p+q}\frac{\pi}{2}\right\}$, $p < q$; V. T. 49. N°. 14.

16) $\int l\,Tang.\,x \frac{(q-p)(Tang.^{p+q}x - Cot.^{p+q}x) - (p+q)(Tang.^{p-q}x - Cot.^{p-q}x)}{(Tang.^p x + Cot.^p x)^2}\frac{dx}{Sin.\,2x} = -\frac{\pi}{4p}\,Sec.\,\frac{q\pi}{2p}$, $p > q$; V. T. 49. N°. 18.

17) $\int l\,Tang.\,x \frac{(q-p)(Tang.^{p+q}x - Cot.^{p+q}x) - (p+q)(Tang.^{p-q}x - Cot.^{p-q}x)}{(Tang.^p x - Cot.^p x)^2}\frac{dx}{Sin.\,2x} = -\frac{\pi}{4p}\,Tang.\,\frac{q\pi}{2p}$, $p > q$; V. T. 49. N°. 17.

18) $\int l\,Tang.\,x \frac{dx}{(Tang.\,x + Cot.\,x)^{2p+1}.\,Tang.\,2x.\,Sin.\,2x} = -\frac{\{\Gamma(p)\}^2}{32p\,\Gamma(2p)}$ V. T. 49. N°. 21.

19) $\int l\,Tang.\,2x \frac{Sin.\,(p\,l\,Tang.\,x)}{Sin.\,2x}\,dx = \infty$ V. T. 329. N°. 13.

20) $\int l\,Tang.\left(\frac{\pi}{4} \pm x\right)\frac{Cos.\,(p\,l\,Tang.\,x)}{Sin.\,2x}\,dx = \infty$ V. T. 329. N°. 7.

F. Log. en num. $(l\,Tang.\,a\,x)^b$. Circ. Dir. rat. en dén. à fact. mon. et bin. TABLE 314. Lim. 0 et $\frac{\pi}{4}$.

1) $\int (l\,Tang.\,x)^2 \frac{Tang.^q x + Cot.^q x}{Tang.^p x + Cot.^p x}\frac{dx}{Sin.\,2x} = \frac{\pi^3}{16p^3}\left\{2\,Sec.^3\frac{q\pi}{2p} - Sec.\frac{q\pi}{2p}\right\}$ V. T. 154. N°. 5.

2) $\int (l\,Tang.\,x)^2 \frac{Tang.^q x - Cot.^q x}{Tang.^p x - Cot.^p x}\frac{dx}{Sin.\,2x} = \frac{\pi^3}{16p^3}\,Sin.\,\frac{q\pi}{2p}.\,Sec.^3\frac{q\pi}{2p}$ V. T. 154. N°. 6.

3) $\int (l\,Tang.\,x)^2 \frac{Tang.^a x + Cot.^a x}{(Tang.^a x - Cot.^a x)^2} \frac{dx}{Sin.\,2\,x} = \frac{\pi}{8\,a^3}$ V. T. 313. N°. 4.

4) $\int (l\,Tang.\,x)^3 \frac{dx}{Cos.\,x\,(Cos.\,x + Sin.\,x)} = -\frac{7}{120}\pi^4$ V. T. 154. N°. 10.

5) $\int (l\,Tang.\,x)^3 \frac{dx}{Cos.\,x\,(Cos.\,x - Sin.\,x)} = -\frac{1}{15}\pi^4$ V. T. 154. N°. 11.

6) $\int (l\,Tang.\,x)^5 \frac{dx}{Cos.\,x\,(Cos.\,x - Sin.\,x)} = -\frac{8}{63}\pi^6$ V. T. 155. N°. 3.

7) $\int (l\,Tang.\,x)^5 \frac{dx}{Cos.\,x\,(Cos.\,x + Sin.\,x)} = -\frac{31}{252}\pi^6$ V. T. 155. N°. 2.

8) $\int (l\,Tang.\,x)^7 \frac{dx}{Cos.\,x\,(Cos.\,x + Sin.\,x)} = -\frac{127}{240}\pi^8$ V. T. 155. N°. 9.

9) $\int (l\,Tang.\,x)^a \frac{dx}{Cos.\,x\,(Cos.\,x + Sin.\,x)} = \frac{2^a - 1}{2^a}(-1)^a 1^{a/1} \sum_1^\infty \frac{1}{n^{a+1}}$ V. T. 157. N°. 2.

10) $\int (l\,Tang.\,x)^a \frac{dx}{Cos.\,x\,(Cos.\,x - Sin.\,x)} = (-1)^a 1^{a/1} \sum_1^\infty \frac{1}{n^{a+1}}$ V. T. 157. N°. 3.

11) $\int (l\,Tang.\,x)^{b-1} \frac{Tang.^a x}{Cos.\,x + Sin.\,x} \frac{dx}{Cos.\,x} = \frac{1^{b-1/1}}{(-1)^{b-1}} \sum_0^\infty \frac{(-1)^n}{(a+n+1)^b}$ V. T. 157. N°. 11.

12) $\int (l\,Tang.\,x)^{b-1} \frac{Tang.^a x}{Cos.\,x - Sin.\,x} \frac{dx}{Cos.\,x} = (-1)^{b-1} 1^{b-1/1} \sum_0^\infty \frac{1}{(a+n+1)^b}$ V. T. 157. N°. 12.

13) $\int (l\,Tang.\,x)^{2a-1} \frac{dx}{Cos.\,x\,(Cos.\,x + Sin.\,x)} = \frac{1 - 2^{2a-1}}{2\,a}\pi^{2a} B_{2a-1}$ V. T. 157. N°. 5.

14) $\int (l\,Tang.\,x)^{2a-1} \frac{dx}{Cos.\,x\,(Cos.\,x - Sin.\,x)} = -\frac{1}{a} 2^{2a-2} \pi^{2a} B_{2a-1}$ V. T. 157. N°. 6.

15) $\int (l\,Tang.\,x)^{2a} \frac{1}{Tang.^q x + Cot.^q x} \frac{dx}{Sin.\,2\,x} = \frac{1}{4}(-1)^{a+1}\left(\frac{2\,\pi}{q}\right)^{2a+1} B''\left(\frac{1}{4}\right)$ V. T. 158. N°. 15.

16) $\int (l\,Tang.\,x)^{2a-1} \frac{1}{Tang.^q x - Cot.^q x} \frac{dx}{Sin.\,2\,x} = \frac{1}{8}(-1)^a \left(\frac{2\,\pi}{q}\right)^{2a} B'\left(\frac{1}{2}\right)$ V. T. 158. N°. 16.

17) $\int (l\,Tang.\,x)^{p-1} \frac{Cos.\,\lambda - Tang.\,x}{1 - Cos.\,\lambda.\,Sin.\,2\,x} \frac{Tang.^q x}{Sin.\,2\,x} dx = \frac{1}{2}(-1)^{p-1}\Gamma(p) \sum_1^\infty \frac{Cos.\,n\,\lambda}{(q+n-1)^p}$ V. T. 159. N°. 7.

1) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{dx}{Sin.\, 2x} = \pm \frac{1}{8}\pi^2$ V. T. 309. N°. 6.

2) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{dx}{Tang.\, 2x} = \pm \frac{1}{16}\pi^2$ V. T. 336. N°. 2.

3) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{Tang.^{p-1} x + Cot.^{p-1} x}{Sin.\, 2x} dx = \mp \frac{\pi}{2(p-1)} Cot. \frac{1}{2} p\pi$, $p < 1$; V. T. 47. N°. 7.

4) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{(p+q)(Tang.^{p+q} x - Cot.^{p+q} x) + (p-q)(Tang.^{p-q} x - Cot.^{p-q} x)}{Sin.\, 2x} dx =$

$= \pm \frac{\pi\, Sin.\, p\pi}{Cos.\, p\pi + Cos.\, q\pi}$, $p + q < 1$; V. T. 47. N°. 20.

5) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{(Cos.^{2p-1} 2x - 1)(p\, Sin.^2 2x + Cos.\, 2x) - Sin.^2 2x}{Sin.^2 x.\, Cos.^p 2x} dx = \pm \pi\, Cot.\, p\pi$ V. T. 47. N°. 10.

6) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{Sin.\, 2x}{1 + p\, Cos. 2x} dx = \pm \frac{1}{16p} \{\pi^2 - 4 (Arccos.\, p)^2\}$, $p^2 \leqq 1$; V. T. 339. N°. 28.

7) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{Sin.\, 2x}{1 - p\, Cos.\, 2x} dx = \pm \frac{1}{4p} Arcsin.\, p\, (\pi + Arcsin.\, p)$, $p^2 \leqq 1$; V.T. 315. N°. 6, 8.

8) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{Sin.\, 2x}{1 - p^2\, Cos.^2 2x} dx = \pm \frac{\pi}{4p} Arcsin.\, p$, $p^2 \leqq 1$; V. T. 340. N°. 9.

9) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{Sin.\, 4x}{1 - p^2\, Cos.^2 2x} dx = \pm \frac{1}{4p^2} (Arcsin.\, p)^2$, $p^2 \leqq 1$, V. T. 315. N°. 6, 8.

10) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{Sin.\, 2x}{1 - Cos.^2 2x.\, Sin.^2 \lambda} dx = \pm \frac{1}{4} \pi \lambda\, Cosec.\, \lambda$ V. T. 345. N°. 11.

11) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{Sin.\, 2x}{1 + p^2\, Cos.^2 2x} dx = \pm \frac{\pi}{4p} l \{p + \sqrt{(1 + p^2)}\}$ V. T. 368. N°. 6.

12) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{Sin.\, 2x}{Sin.^2 2x + p^2\, Cos.^2 2x} dx = \pm \frac{\pi}{4\sqrt{(1 - p^2)}} Arccos.\, p$, $p < 1$; V. T. 340. N°. 12.

13) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{Sin.\, 2x}{1 - p^4\, Cos.^4 2x} dx = \pm \frac{\pi}{8p} \left[l\{p + \sqrt{(1 + p^2)}\} + Arcsin\, p\right]$, $p < 1$;

14) $\int l\, Tang. \left(\frac{\pi}{4} \pm x\right) \frac{Sin.\, 4x.\, Cos.\, 2x}{1 - p^4\, Cos.^4 2x} dx = \pm \frac{\pi}{4p^3} \left[Arcsin.\, p - l\{p + \sqrt{(1 + p^2)}\}\right]$, $p < 1$;

(13, 14) V. T. 315. N°. 8, 11.

F. Log. en num. Autre forme: un fact. log. Circ. Dir. rat. en dén. **TABLE 316.** Lim. 0 et $\frac{\pi}{4}$.

1) $\int l\,Cos.\,x \frac{dx}{(Cos.\,x + p\,Sin.\,x)^2} = \frac{1}{1+p^2}\left\{-\frac{\pi}{4} + \frac{1}{p}l(1+p) - \frac{1-p^3}{2(p+1)p}l2\right\},\ p<1;$ V. T. 48. N°. 1.

2) $\int l\,Cos.\,x \frac{Cos.\,2x\,dx}{(1-p\,Sin.\,2x)^2} = \frac{1}{4p}l(1-p) - \frac{1}{4(1-p)}l2 + \frac{1}{4}Arccos.(-p).Cosec.\{Arccos.(-p)\},\ p<1;$ V. T. 48. N°. 10.

3) $\int l\,Cos.\,x \frac{Cos.\,2x}{(1-Cos.\,\lambda.\,Sin.\,2x)^2}dx = \frac{\pi-\lambda}{4\,Sin.\,\lambda} + \frac{l\,Sin.\frac{1}{2}\lambda}{2\,Cos.\,\lambda} - \frac{1}{4}\frac{1+Cos.\,\lambda}{1-Cos.\,\lambda}Sec.\,\lambda\,l2$ V. T. 48. N°. 8.

4) $\int l\left\{2\,Sin.^2\left(\frac{\pi}{4}+x\right)\right\}\frac{dx}{Tang.\,2x} = \frac{1}{24}\pi^2$ V. T. 160. N°. 1.

5) $\int l\left\{2\,Sin.^2\left(\frac{\pi}{4}-x\right)\right\}\frac{dx}{Tang.\,2x} = -\frac{1}{12}\pi^2$ V. T. 160. N°. 5.

6) $\int l(1+Tang.\,x)\frac{dx}{Sin.\,2x} = \frac{1}{24}\pi^2$ V. T. 160. N°. 1.

7) $\int l(1-Tang.\,x)\frac{dx}{Sin.\,2x} = -\frac{1}{12}\pi^2$ V. T. 160. N°. 5.

8) $\int l(Sin.x.Cos.x)\frac{Sin.^{2a}x}{Cos.^{2a+2}x}dx = \frac{1}{2a+1}\left\{(-1)^{a+1}\frac{\pi}{2} - l2 + \frac{1}{2a+1} + 2(-1)^{a+1}\sum_1^a\frac{(-1)^n}{2n-1}\right\}$ V. T. 307. N°. 1, 12.

9) $\int l(Sin.x\,Cos.x)\frac{Sin.^{2a-1}x}{Cos.^{2a+1}x}dx = \frac{1}{2a}\left\{\frac{\pi}{2}(-1)^a l2 - l2 + \frac{1}{2a} + (-1)^a\sum_1^{a-1}\frac{(-1)^n}{n}\right\}$ V. T. 307. N°. 2, 13.

10) $\int l\left\{\frac{Cos.\,2x}{Cos.^2\,x}\right\}\frac{dx}{Sin.\,2x} = -\frac{1}{24}\pi^2$ V. T. 160. N°. 10.

11) $\int l\left\{\frac{1-Sin.\,2x.\,Cos.\,\lambda}{Cos.^2\,x}\right\}\frac{dx}{Sin.\,2x} = \frac{1}{6}\pi^2 - \frac{1}{2}\pi\lambda + \frac{1}{4}\lambda^2$ V. T. 160. N°. 14.

F. Log. en num. Autre forme: deux fact. log. Circ. Dir. rat. en dén. **TABLE 317.** Lim. 0 et $\frac{\pi}{4}$.

1) $\int l\,Cos.\,x.\,(l\,Tang.\,x)^2\frac{dx}{Sin.\,2x} = -\frac{7}{11520}\pi^4$ V. T. 305. N°. 6.

2) $\int l\,Cos.\,x.\,(l\,Tang.\,x)^{2a}(2a+1+Sin.2x.Tang.2x\,l.Tang.x)\frac{dx}{Sin.4x} = \frac{1}{16(a+1)}\pi^{2a+2}B_{2a+1}$ V. T. 310. N°. 11.

3) $\int l\,Cos.\,2x.\,(l\,Tang.\,x)^2\frac{dx}{Sin.\,2x} = -\frac{1}{384}\pi^4$ V. T. 305. N°. 7.

4) $\int l\,Tang.\left(\frac{\pi}{4}\pm x\right).(l\,Sin.\,2x)^2\frac{dx}{Tang.\,2x} = \pm\frac{1}{96}\pi^4$ V. T. 336. N°. 12.

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5) $\int l\,Tang.\left(\frac{\pi}{4} \pm x\right).(l\,Sin.\,2\,x)^4 \frac{dx}{Tang.\,2\,x} = \pm \frac{1}{80}\pi^6$ V. T. 336. N°. 13.

6) $\int l\,Tang.\left(\frac{\pi}{4} \pm x\right).(l\,Sin.\,2\,x)^6 \frac{dx}{Tang.\,2\,x} = \pm \frac{17}{448}\pi^8$ V. T. 336. N°. 14.

7) $\int l\,Tang.\left(\frac{\pi}{4} \pm x\right).(l\,Sin.\,2\,x)^{a-1} \frac{dx}{Tang.\,2\,x} = \pm \frac{1}{2a}(-1)^a 1^{a/1} \sum_0^\infty \frac{1}{(2n+1)^{a+1}}$ V. T. 336. N°. 17.

8) $\int l\,Tang.\left(\frac{\pi}{4} \pm x\right).(l\,Sin.2x)^{2a} \frac{dx}{Tang.\,2\,x} = \pm \frac{2^{2a+2}-1}{8.a+1.2a+1}\pi^{2a+2} B_{2a+1}$ V. T. 336. N°. 16

9) $\int l\,Tang.\left(\frac{\pi}{4} \pm x\right).(l\,Sin.\,2\,x)^{2a-1} \frac{dx}{Tang.\,2\,x} = \pm \frac{1-2^{2a+1}}{a\,2^{2a+3}} 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 336. N°. 15.

10) $\int l\,Tang.\left(\frac{\pi}{4} \pm x\right).(l\,Tang.\,x)^2 \frac{dx}{Sin.\,2\,x} = \pm \frac{1}{48}\pi^4$ V. T. 310. N°. 1.

11) $\int l\,Tang.\left(\frac{\pi}{4} \pm x\right).(l\,Tang.\,x)^4 \frac{dx}{Sin.\,2\,x} = \pm \frac{1}{40}\pi^6$ V. T. 310. N°. 5.

12) $\int l\,Tang.\left(\frac{\pi}{4} \pm x\right).(l\,Tang.\,x)^6 \frac{dx}{Sin.\,2\,x} = \pm \frac{17}{224}\pi^8$ V. T. 310. N°. 8.

13) $\int l\,Tang.\left(\frac{\pi}{4} \pm x\right).(l\,Tang\,x)^{2a} \frac{dx}{Sin.\,2\,x} = \pm \frac{2^{2a+2}-1}{4.a+1.2a+1}\pi^{2a+2} B_{2a+1}$ V. T. 310. N°. 9.

14) $\int l\,Tang\left(\frac{\pi}{4} \pm x\right).(l\,Tang\,x)^{2a-1} \frac{dx}{Sin.\,2\,x} = \pm \frac{1-2^{2a+1}}{2^{2a+2}\,a} 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 310. N°. 10.

15) $\int (l\,Tang.\,x)^{2b}.\,l\,\frac{1-Sin.2x.Cos.2p\pi}{Cos.^2\,x}\frac{dx}{Sin.2x} = \frac{(2\pi)^{2b+1}}{2(2b+1)}\left\{(-1)^{b+1}B'(p) - \frac{1}{2b+2}B_{2+2b}\right\}$ V. T. 161. N°. 12.

F. Log. en num. Log. de Log. Circ. Dir. rat. en dén. — TABLE 318. — Lim. 0 et $\frac{\pi}{4}$.

1) $\int l\,l\,Cot.\,x\,\frac{Tang.^q\,x}{Sin.\,2\,x}\,dx = -\frac{1}{2q}(A + l\,q)$ V. T. 190. N°. 1.

2) $\int l\,l\,Cot.\,x\,\frac{dx}{2 - Sin.\,2\,x} = \frac{\pi}{\sqrt{3}}\left\{\frac{5}{6}l\,2\pi - l\,\Gamma\left(\frac{1}{6}\right)\right\}$ V. T. 191. N°. 7.

3) $\int l\,l\,Cot.\,x\,\frac{dx}{1 + Sin.\,2\,x.\,Cos.\,\lambda} = \frac{1}{2}\pi\,Cosec.\lambda.\,l\,\frac{(2\pi)^{\frac{\lambda}{\pi}}\Gamma\left(\frac{\pi+\lambda}{2\pi}\right)}{\Gamma\left(\frac{\pi-\lambda}{2\pi}\right)}$ V. T. 190. N°. 9.

4) $\int l\,l\,Cot.x \frac{dx}{(Sin.x + Cos.x)^2} = \frac{1}{2} Z'\left(\frac{1}{2}\right) + \frac{1}{2} l\,2\pi$ V. T. 190. N°. 7.

5) $\int l\,l\,Cot.x \frac{Tang.^a x + Cot.^a x}{Tang.^b x + Cot.^b x} \frac{dx}{Sin.2x} = \frac{\pi}{4b} Sec.\frac{a\pi}{2b} l\,2\pi + \frac{\pi}{2b} \sum_1^b (-1)^{n-1} Cos.\left(\frac{n-\frac{1}{2}}{b} a\pi\right) l \frac{\Gamma\left(\frac{b+n-\frac{1}{2}}{2b}\right)}{\Gamma\left(\frac{n-\frac{1}{2}}{2b}\right)}$, $a+b$ impair;

6) $= \frac{\pi}{4b} Sec.\frac{a\pi}{2b} l\pi + \frac{\pi}{2b} \sum_1^{\frac{b-1}{2}} (-1)^{n-1} Cos.\left(\frac{n-\frac{1}{2}}{b} a\pi\right) l \frac{\Gamma\left(\frac{b-n+\frac{1}{2}}{b}\right)}{\Gamma\left(\frac{n-\frac{1}{2}}{b}\right)}$, $a+b$ pair;

V. T. 191. N°. 8, 9.

7) $\int l(p + l\,Tang.x) \frac{Tang.^q x}{Sin.2x} dx = \frac{1}{2q} \left\{ l\,p - e^{-pq} Ei.(pq) \right\}$ V. T. 325. N°. 6.

8) $\int l(p - l\,Tang.x) \frac{Tang.^q x}{Sin.2x} dx = \frac{1}{2q} \left\{ l\,p + e^{pq} Ei.(-pq) \right\}$ V. T. 325. N°. 7.

9) $\int l\left\{a^2 + (l\,Tang.x)^2\right\} dx = \pi l \frac{2\Gamma\left(\frac{2a+3\pi}{4\pi}\right)}{\Gamma\left(\frac{2a+\pi}{4\pi}\right)} - \frac{1}{2}\pi l \frac{\pi}{2}$ V. T. 190. N°. 10.

10) $\int l\,l\,Cot.x \frac{Tg^p x - Cot^p x}{Cos.2x} dx = -\frac{\pi}{2}(l\pi - A)\,Tg.\frac{p\pi}{2} + \pi \sum^{\infty} \left[\frac{l\{(2n+1)\pi - p\pi\}}{(2n+1)\pi - p\pi} - \frac{l\{(2n+1)\pi + p\pi\}}{(2n+1)\pi + p\pi} \right]$ V. T. 190. N°. 6.

11) $\int l\,l\,Cot.x\,(l\,Cot.x)^{p-1} \frac{Tang.^q x}{Sin.2x} dx = \frac{\Gamma(p)}{2q^p} \left\{ l\,q - Z'(p) \right\}$ V. T. 190. N°. 2.

F. Logar. en num. $(l\,Tang.\,x)^a$. Circ. Dir. en dén. irrat.

TABLE 319.

Lim. 0 et $\frac{\pi}{4}$.

1) $\int l\,Tang.x \frac{dx\sqrt{Cos.2x}}{Cos.^3 x} = -\frac{1}{4}\pi\left(\frac{1}{2} + l\,2\right)$ V. T. 162. N°. 1.

2) $\int l\,Tang.x \frac{Sin.x\,dx\sqrt{Cos.2x}}{Cos.^4 x} = \frac{1}{3}\left(l\,2 - \frac{4}{3}\right)$ V. T. 162. N°. 2.

3) $\int l\,Tang.x \frac{(Cos.2x)^{a-\frac{1}{2}}}{Cos.^{2a+1} x} dx = -\frac{1^{a/2}\pi}{2^{a+2}\,1^{a/1}} \left\{ A + Z'(a+1) + 2\,l\,2 \right\}$ V. T. 162. N°. 3.

4) $\int l\,Tang.x \frac{Tang.x}{\sqrt{Cos.2x}} dx = -\frac{1}{8}\pi\,l\,2$ V. T. 163. N°. 12.

5) $\int l\, Tang.\, x \frac{Tang.^3\, x}{\sqrt{Cos.\, 2x}} dx = \frac{1}{4}(l2 - 1)$ V. T. 163. N°. 13.

6) $\int l\, Tang.\, x \frac{dx}{Cos.\, x \sqrt{Cos.\, 2x}} = -\frac{1}{2}\pi\, l2$ V. T. 163. N°. 2.

7) $\int l\, Tang.\, x \frac{Sin.\, x}{Cos.^2\, x \sqrt{Cos.\, 2x}} dx = l2 - 1$ V. T. 163. N°. 3.

8) $\int l\, Tang.\, x \frac{Sin.^2\, x}{Cos.^3\, x \sqrt{Cos.\, 2x}} dx = \frac{1}{4}\pi\left(\frac{1}{2} - l2\right)$ V. T. 163. N°. 4.

9) $\int l\, Tang.\, x \frac{Sin.\, x}{Cos.^4\, x \sqrt{Cos.\, 2x}} dx = \frac{2}{3}\left(l2 - \frac{5}{6}\right)$ V. T. 163. N°. 5.

10) $\int l\, Tang.\, x \frac{Sin.^4\, x}{Cos.^5\, x \sqrt{Cos.\, 2x}} dx = \frac{3}{16}\pi\left(\frac{7}{12} - l2\right)$ V. T. 163. N°. 6.

11) $\int l\, Tang.\, x \frac{Sin.^5\, x}{Cos.^6\, x \sqrt{Cos.\, 2x}} dx = \frac{8}{15}\left(l2 - \frac{47}{60}\right)$ V. T. 163. N°. 7.

12) $\int l\, Tang.\, x \frac{dx}{Cos.\, x \sqrt[3]{(Cos.^3\, x - Sin^3.\, x)}} = -\frac{1}{27}\pi^2 - \frac{\pi\, l3}{3\sqrt{3}}$ V. T. 163. N°. 10.

13) $\int l\, Tang.\, x \frac{Sin.\, x}{Cos.^2\, x \sqrt[3]{(Cos.^3\, x - Sin.^3\, x)}} dx = \frac{1}{27}\pi^2 - \frac{\pi\, l3}{3\sqrt{3}}$ V. T. 163. N°. 11.

14) $\int l\, Tang.\, x \frac{(Cot.\, x - 1)^{p-1}}{Sin.^2\, x} dx = -\frac{2}{2p+1}\pi\, Sec.\, p\pi\, ,\ p < \frac{1}{2}$; V. T. 52. N°. 15.

15) $\int l\, Tang.\, x \frac{1}{(Cot.\, x - 1)^{1+p}} \frac{dx}{Sin.^2\, x} = \frac{2}{2p-1}\pi\, Sec.\, p\pi\, ,\ p < \frac{1}{2}$; V. T. 50. N°. 12.

16) $\int l\, Tang.\, x \frac{dx}{\sqrt{\{Cos.\, x\,(Cos.\, x - Sin.\, x)^3\}}} = -4\, l2$ V. T. 52. N°. 9.

17) $\int l\, Tang.\, x \frac{\sqrt[3]{Tang.\, x} + \sqrt[3]{Cot.\, x}}{Cos.\, x - Sin.\, x} \frac{dx}{\sqrt{Sin.\, 2x}} = -\pi^2 \sqrt{2}$ V. T. 163. N°. 18.

18) $\int (l\, Tang.\, x)^2 \frac{dx}{Cos.\, x \sqrt{Cos.\, 2x}} = \frac{\pi}{2}\left\{(l2)^2 + \frac{1}{12}\pi^2\right\}$ V. T. 164. N°. 1.

19) $\int (l\, Tang.\, x)^{2a-1} \frac{1}{Cos.\, x - Sin.\, x} \frac{dx}{\sqrt{Sin.\, 2x}} = \frac{1 - 2^{2a}}{4a\sqrt{2}}(2\pi)^{2a}\, B_{2a-1}$ V. T. 164. N°. 3.

20) $\int (l\, Tang.\, x)^{2a} \frac{Sin.\, x + Cos.\, x}{(Sin.\, x - Cos.\, x)^2 \sqrt{Sin.\, 2x}} dx = \frac{2^{2a} - 1}{\sqrt{2}}(2\pi)^{2a}\, B_{2a-1}$ V. T. 319. N°. 19.

1) $\int l\,Sin.\,x \frac{3\sqrt{Tang.\,x}+\sqrt{Cot.\,x}}{Cos.^2\,x}\,dx = -\pi\sqrt{2}-2\,l\,2$ V. T. 50. N°. 15.

2) $\int l\,Sin.\,2x \frac{1+\sqrt{Sin.\,2x}}{\sqrt{Sin.^3\,2x}}\,Tang.\left(\frac{\pi}{4}+x\right)dx = -\pi^2$ V. T. 163. N°. 18.

3) $\int (l\,Sin.\,2x)^{2a-1} \frac{Tang.\left(\frac{\pi}{4}+x\right)}{\sqrt{Sin.\,2x}}\,dx = \frac{1-2^{2a}}{8a}\,2\pi^{2a}\,B_{2a-1}$ V. T. 164. N°. 3.

4) $\int l\,Cos.\,x \frac{\sqrt{Tang.\,x}+3\sqrt{Cot.\,x}}{Sin.^2\,x}\,dx = -\pi\sqrt{2}+2\,l\,2$ V. T. 50. N°. 15.

5) $\int l\,Cos.\,x \frac{Cos.\,3x}{Sin.^2\,x.\,Cos.\,2x\sqrt{Cos.\,2x}} = \frac{1}{\sqrt{2}}\,l\,2-\frac{1}{2}\pi$ V. T. 51. N°. 2.

6) $\int l\,Cos.\,x \frac{1+Cos.^2\,2x}{Sin.^2\,2x}\,\frac{dx}{\sqrt{Cos.\,2x}} = \frac{\pi\sqrt{2\pi}}{2\{\Gamma(\frac{1}{4})\}^2} - \frac{\{\Gamma(\frac{1}{4})\}^2}{8\sqrt{2\pi}}$ V. T. 50. N°. 1.

7) $\int l\,Cos.\,x \frac{\sqrt{Cos.\,2x}}{Sin.\,2x.\,Cos.^2\,x}\,dx = \frac{\{\Gamma(\frac{1}{4})\}^2}{8\sqrt{2\pi}} - \frac{\pi\sqrt{2\pi}}{2\{\Gamma(\frac{1}{4})\}^2}$ V. T. 165. N°. 5.

8) $\int l\,Cos.\,x \frac{2\,Cot.\,2x+Sin.\,2x\,l\,Tang.\,x}{Cos.\,2x\sqrt{Cos.\,2x}}\,dx = -\frac{1}{8}\pi\,l\,2$ V. T. 319. N°. 4.

9) $\int l\,Cos.\,2x \frac{1+\sqrt{Cos.\,2x}}{Tang.\,x}\,\frac{dx}{\sqrt{Cos.^3\,2x}} = -\pi^2$ V. T. 163. N°. 18.

10) $\int (l\,Cos.\,2x)^{2a-1} \frac{dx}{Tang.\,x\sqrt{Cos.\,2x}} = \frac{1-2^{2a}}{8a}\,2\pi^{2a}\,B_{2a-1}$ V. T. 164. N°. 3.

11) $\int (l\,Cot.\,x)^{a-1} \frac{Tang.^p\,x}{Sin.\,2x}\,dx = \frac{1^{a|2}}{(2p)^{a+1}}\sqrt{p\pi}$ V. T. 162. N°. 4.

12) $\int l\,Tang.\left(\frac{\pi}{4}\pm x\right) \frac{Sin.\,x}{Cos.^2\,x\sqrt{Cos.\,2x}}\,dx = \pm\pi$ V. T. 51. N°. 2.

13) $\int l\,Tang.\left(\frac{\pi}{4}\pm x\right) \frac{1-2\,Tang.^2\,x}{Cos.\,x\sqrt{Cos.\,2x}}\,dx = \mp 2$ V. T. 51. N°. 3.

14) $\int l\,Tang.\left(\frac{\pi}{4}\pm x\right) \frac{2\,Sin.^2\,x.\,Cos.^2\,x-3\,Cos.\,2x}{Cos.^4\,x\sqrt{Cos.\,2x}}\,Sin.^2\,x\,dx = \pm 1$ V. T. 50. N°. 2.

15) $\int l\,Tang.\left(\frac{\pi}{4}\pm x\right) \frac{Sin.\,x}{Cos.^2\,x+p^2\,Cos.\,2x}\,\frac{dx}{\sqrt{Cos.\,2x}} = \pm\frac{\pi}{p}\,l\{p+\sqrt{(1+p^2)}\}$ V.T. 374. N°. 5.

16) $\int dx \sqrt{l\,Cot.x} = \frac{1}{2}\sqrt{\pi}\sum_0^\infty (-1)^n \frac{1}{\sqrt{(2n+1)^3}}$ V. T. 187. N°. 17.

17) $\int l\frac{Cos.2x}{Cos.^2x}\frac{dx}{Cos.x\sqrt{Cos.2x}} = -\pi l2$ V. T. 165. N°. 11.

18) $\int dx\, l\frac{Cos.x+p\sqrt{Cos.2x}}{Cos.x-p\sqrt{Cos.2x}} = \pi\, Arcsin.p\,,\ p \leqq 1;$ V. T. 160. N°. 17.

19) $\int l\frac{Cos.x+Sin.x.Sin.\lambda}{Cos.x-Sin.x.Sin.\lambda}\frac{dx}{Sin.x\sqrt{Cos.2x}} = \pi\lambda$ V. T. 166. N°. 7.

20) $\int l l Cot.x(Tg.^{\frac{b}{c}}x+Cot.^{\frac{b}{c}}x)dx = \frac{1}{4}\pi(l\pi-A)Sec.\frac{b\pi}{2c}-\pi\sum_0^\infty(-1)^n\left\{\frac{l\left\{(2n+1)\pi-\frac{b\pi}{c}\right\}}{(2n+1)\pi-\frac{b\pi}{c}}+\frac{l\left\{(2n+1)\pi+\frac{b\pi}{c}\right\}}{(2n+1)\pi+\frac{b\pi}{c}}\right\}$ V. T. 190. N°. 5.

F. Log. en dén.. Fonct. monôme. Circ. Dir. ent. **TABLE 321.** Lim. 0 et $\frac{\pi}{4}$.

1) $\int Sin.^2\left(\frac{\pi}{4}-x\right).Tang.\left(\frac{\pi}{4}-x\right)\frac{dx}{l\,Sin.2x} = \frac{1}{4}l\frac{2}{\pi}$ V. T. 171. N°. 1.

2) $\int Sin.^4\left(\frac{\pi}{4}-x\right).Tang.\left(\frac{\pi}{4}-x\right)\frac{dx}{l\,Sin.2x} = \frac{1}{8}l\frac{8}{\pi^2}$

3) $\int Sin.^2\left(\frac{\pi}{4}-x\right).Sin.2x.Tang.\left(\frac{\pi}{4}-x\right)\frac{dx}{l\,Sin.2x} = \frac{1}{4}l\frac{\pi}{4}$ } V.T. 321. N°. 1, 4.

4) $\int Sin.^2\left(\frac{\pi}{4}-x\right).Cos.2x\frac{dx}{l\,Sin.2x} = -\frac{1}{4}l2$ V. T. 167. N°. 2.

5) $\int (1-Sin.^{q-1}2x)\,Tang.\left(\frac{\pi}{4}-x\right)\frac{dx}{l\,Sin.2x} = \frac{1}{2}l\frac{\Gamma\left(\frac{q}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}$ V. T. 171. N°. 2.

6) $\int (1-Sin.^p2x)(1-Sin.^q2x)\,Tang.\left(\frac{\pi}{4}+x\right)\frac{dx}{l\,Sin.2x} = \frac{1}{2}l\frac{\Gamma(p+1)\Gamma(q+1)}{\Gamma(p+q+1)}$ V. T. 171. N°. 17.

7) $\int Sin.^2x.Tang.x\frac{dx}{l\,Cos.2x} = \frac{1}{4}l\frac{2}{\pi}$ V. T. 171. N°. 1.

8) $\int Sin.^2x.Sin.2x\frac{dx}{l\,Cos.2x} = -\frac{1}{4}l2$ V. T. 167. N°. 2.

9) $\int Sin.^2x.Cos.2x.Tang.x\frac{dx}{l\,Cos.2x} = \frac{1}{4}l\frac{\pi}{4}$ V. T. 321. N°. 7, 8.

10) $\int Sin.^4 x . Tang. x \frac{dx}{l Cos. 2x} = \frac{1}{8} l \frac{8}{\pi^2}$ V. T. 321. N°. 7, 8.

11) $\int Cos.^q 2x . Sin.^4 x . Tang. 2x \frac{dx}{(l Cos. 2x)^2} = \frac{1}{8} \{(q+2) l (q+2) - 2(q+1) l (q+1) + q l q\}$ V. T. 168. N°. 3.

12) $\int Tang. \left(\frac{\pi}{4} - x\right) \frac{dx}{Cos.^2 x \, l Tang. x} = l \frac{2}{\pi}$ V. T. 171. N°. 1.

13) $\int (1 - Tang. x)^2 \frac{dx}{l Tang. x} = l \frac{\pi}{4}$ V. T. 172. N°. 1.

14) $\int Tang. \left(\frac{\pi}{4} - x\right) \frac{Tang.^2 x}{l Tang. x} dx = l \frac{2\sqrt{2}}{\pi}$ V. T. 175. N°. 17.

15) $\int Tang. \left(\frac{\pi}{4} - x\right) \frac{dx}{l Tang. x} = -\frac{1}{2} l 2$ V. T. 175. N°. 18.

16) $\int l Tang. \left(\frac{\pi}{4} \pm x\right) \frac{dx}{Cos.^2 x \, l Tang. x} = \pm l \frac{\pi}{2}$ V. T. 321. N°. 14, 15.

F. Log. en dén.. Fonct. monôme. Circ. Dir. fract. à dén. monôme.

TABLE 322.

Lim. 0 et $\frac{\pi}{4}$.

1) $\int (Sin.^{q-1} 2x - Cosec.^{q-1} 2x) \, Tang. \left(\frac{\pi}{4} - x\right) \frac{dx}{l Sin. 2x} = \frac{1}{2} l Tang. \frac{1}{2} q \pi$ V. T. 175. N°. 2.

2) $\int (Sin.^q 2x - Cosec.^q 2x)^2 \, Tang. \left(\frac{\pi}{4} + x\right) \frac{dx}{l Sin. 2x} = \frac{1}{2} l \frac{Sin. 2q\pi}{2q\pi}$ V. T. 175. N°. 4.

3) $\int (Sin.^q 2x - Cosec.^q 2x)^2 \, Tang. \left(\frac{\pi}{4} - x\right) \frac{dx}{l Sin. 2x} = \frac{1}{2} l q\pi \, Cot. q\pi$ V. T. 175. N°. 3.

4) $\int \frac{Sin.^q 2x . Sin.^2 \left(\frac{\pi}{4} - x\right)}{Tang. 2x} \frac{dx}{(l Sin. 2x)^2} = \frac{1}{8} \{(q+2) l (q+2) - 2(q+1) l (q+1) + q l q\}$ V. T. 168. N°. 3.

5) $\int \frac{1 - Cos.^{q-1} 2x}{Cot. x} \frac{dx}{l Cos. 2x} = \frac{1}{2} l \frac{\Gamma\left(\frac{q}{2}\right)}{\Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{q+1}{2}\right)}$ V. T. 171. N°. 2.

6) $\int (Cos.^{q-1} 2x - Sec.^q 2x) \, Tang. x \frac{dx}{l Cos. 2x} = \frac{1}{2} l Tang. \frac{1}{2} q \pi$ V. T. 175. N°. 2.

7) $\int \frac{(1 - Cos.^p 2x)(1 - Cos.^q 2x)}{Tang. x} \frac{dx}{l Cos. 2x} = \frac{1}{2} l \frac{\Gamma(p+1) \Gamma(q+1)}{\Gamma(p+q+1)}$ V. T. 171. N°. 17.

8) $\int (Cos.^q 2x - Sec.^q 2x)^2 \, Tang.\, x \frac{dx}{l\, Cos.\, 2x} = \frac{1}{2} l (q\pi \, Cot.\, q\pi)$ V. T. 175. N°. 3.

9) $\int \frac{Cos.^q 2x + Sec.^q 2x - 2}{Tang.\, x} \frac{dx}{l\, Cos.\, 2x} = \frac{1}{2} l \frac{Sin.\, q\pi}{q\pi}$ V. T. 175. N°. 4.

10) $\int \frac{1 - Sin. 2x + (1 - Tang.\, x) Cos. 2x}{Cos.^2 x . Cos.\, 2x} \frac{dx}{l\, Tang.\, x} = - l\pi$ V. T. 172. N°. 6.

11) $\int \frac{Tang.\left(\frac{\pi}{4} - x\right)}{Cos.^2 x} \frac{dx}{l\, Tang.\, x} = l \frac{2}{\pi}$ V. T. 171. N°. 1.

12) $\int (Tang.^p x - Cot.^p x) \frac{dx}{l\, Tang.\, x} = l\, Tang.\left\{\frac{1+p}{4}\pi\right\}$ V. T. 175. N°. 5.

13) $\int \frac{Cos.\, x - Sin.\, x}{Cos.^3 x} \frac{dx}{l\, Tang.\, x} = - l 2$ V. T. 167. N°. 2.

14) $\int \frac{Tang.^q x - Tang.^p x}{Sin.\, 2x} \frac{dx}{l\, Tang.\, x} = \frac{1}{2} l \frac{q}{p}$ V. T. 170. N°. 2.

15) $\int \frac{(Tang.^q x - Cot.^q x)^2}{Cos.\, 2x} \frac{dx}{l\, Tang.\, x} = l\, Cos.\, q\pi$ V. T. 175. N°. 6.

16) $\int \frac{(Tang.^q x - Cot.^q x)^2}{Cos.\, 2x} Tang.\, x \frac{dx}{l\, Tang.\, x} = l \frac{Sin.\, q\pi}{q\pi}$ V. T. 175. N°. 7.

17) $\int \frac{(1 - Tang.^q x)(1 - Tang.^{q+1} x)}{Cos.\, 2x} \frac{dx}{l\, Tang.\, x} = - q l 2$ V. T. 172. N°. 3.

18) $\int \left(\frac{Cos.\, x - Sin.\, x}{Cos.^2 x}\right)^2 \frac{dx}{(l\, Tang.\, x)^2} = l \frac{27}{16}$ V. T. 168. N°. 1.

19) $\int \left(\frac{Cos.\, x - Sin.\, x}{Cos.^3 x}\right)^2 \frac{Sin.\, 2x}{(l\, Tang.\, x)^2} dx = 4 l \frac{32}{27}$ V. T. 168. N°. 2.

20) $\int (Tang.^q x + Cot.^q x) \frac{dx}{(l\, Tang.\, x)^p} = (-1)^p \Gamma(1-p) \sum_0^\infty (-1)^n \left\{\frac{1}{(2n+1-q)^{1-p}} - \frac{1}{(2n+1+q)^{1-p}}\right\}$ V.T. 176. N°. 4.

21) $\int \frac{Tang.^q x - Cot.^q x}{Cos.\, 2x} \frac{dx}{(l\, Tang.\, x)^p} = (-1)^{p-1} \Gamma(1-p) \sum_0^\infty \left\{\frac{1}{(2n+1-q)^{1-p}} - \frac{1}{(2n+1+q)^{1-p}}\right\}$ V.T. 176. N°. 5.

22) $\int \frac{l\, Tang.\left(\frac{\pi}{4} \pm x\right)}{(l\, Sin.\, 2x)^2} \frac{dx}{Tang.\, 2x} = \mp \frac{1}{4} l 2$ V. T. 350. N°. 5.

1) $\int \frac{Sin.^2 x . Tang. x}{1 + Cos.^2 2x} \frac{dx}{l Cos. 2x} = -\frac{1}{4} l 2$ V. T. 175. N°. 18.

2) $\int \frac{Sin.^2 x . Tang. x}{1 + Sec.^2 2x} \frac{dx}{l Cos. 2x} = \frac{1}{2} l \frac{2\sqrt{2}}{\pi}$ V. T. 175. N°. 17.

3) $\int \frac{(1 - Tang.^q x)(1 - Tang.^p x) - (1 - Tang. x)^2}{Cos. x - Sin. x} \frac{dx}{Sin. x \, l Tang. x} = l \mathrm{B}(p,q)$ V. T. 175. N°. 1.

4) $\int \frac{Cos. 2x}{1 - 2 Sin.^2 x . Cos.^2 x} \frac{dx}{l Tang. x} = l Cot. \frac{3\pi}{8}$ V. T. 172. N°. 4.

5) $\int \frac{1 - Tang.^q x}{Sin. x + Cos. x} \frac{Tang.^p x \, dx}{Cos. x \, l Tang. x} = l \frac{\Gamma\left(\frac{1}{2}p + 1\right)\Gamma\left(\frac{p+q+1}{2}\right)}{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{p+q}{2} + 1\right)}$ V. T. 171. N°. 3.

6) $\int \frac{Tang.^p x - Tang.^q x}{Sin. x + Cos. x} \frac{dx}{Sin. x \, l Tang. x} = l \frac{\Gamma\left(\frac{q}{2}\right)\Gamma\left(\frac{p+1}{2}\right)}{\Gamma\left(\frac{q+1}{2}\right)\Gamma\left(\frac{p}{2}\right)}$ V. T. 171. N°. 4.

7) $\int \frac{Tang.^q x - Cot.^q x}{Tang.^p x + Cot.^p x} \frac{dx}{Sin. 2x \, l Tang. x} = \frac{1}{2} l Tang. \left\{\frac{p+q}{4p}\pi\right\}$ V. T. 172. N°. 6.

8) $\int \frac{Tang.^q x + Cot.^q x - 2}{Tang.^p x - Cot.^p x} \frac{dx}{Sin. 2x \, l Tang. x} = \frac{1}{2} l Cos. \frac{q\pi}{2p}$ V. T. 172. N°. 7. corr.

9) $\int \frac{Tang.^{q-1} x - Cot.^q x}{Sin. x + Cos. x} \frac{dx}{Cos. x \, l Tang. x} = l Tang. \frac{1}{2} q\pi$ V. T. 175. N°. 2.

10) $\int \frac{(Tang.^p x - Cot.^p x)^2}{Sin. x + Cos. x} \frac{dx}{Cos. x \, l Tang. x} = l(q\pi \, Cot. q\pi)$ V. T. 175. N°. 3.

11) $\int \frac{(Tang.^p x - Cot.^p x)^2}{Sin. x - Cos. x} \frac{dx}{Cos. x \, l Tang. x} = l(2q\pi \, Cosec. 2q\pi)$ V. T. 175. N°. 4.

12) $\int \frac{1 - Tang.^q x}{Cos. x - Sin. x} \frac{1 - Tang.^p x}{Sin. x} \frac{Tang.^r x}{l Tang. x} dx = l \frac{\Gamma(p+r)\Gamma(q+r)}{\Gamma(p+q+r)\Gamma(r)}$ V. T. 171. N°. 18.

13) $\int \frac{1}{1 + Sin. 2x . Cos. \lambda} \frac{dx}{(l Cot. x)^{1-q}} = Cosec. \lambda \, \Gamma(q) \sum_1^\infty (-1)^{n-1} \frac{Sin. n\lambda}{n^q}$ V. T. 174. N°. 4.

14) $\int \frac{Cos. 2x}{1 - Sin.^2 2x . Sin.^2 \lambda} \frac{dx}{(l Cot. x)^{1-q}} = Sec. \lambda \, \Gamma(q) \sum_1^\infty (-1)^n \frac{Cos.\{(2n-1)\lambda\}}{(2n+1)^q}$ V. T. 174. N°. 13.

15) $\int \left\{\frac{1}{Cos. 2x(1 + Cos. 2x)} + \frac{1}{2 Cos.^2 x \, l Tang x}\right\} \frac{dx}{l Tang. x} = \frac{1}{2}(l 2 - 1)$ V. T. 172. N°. 5.

1) $\int \frac{dx}{\pi^2 + (l\, Tang.\, x)^2} = \frac{4 - \pi}{4\pi}$ V. T. 173. N°. 7.

2) $\int \frac{dx}{\pi^2 + (l\, Tang.^2\, x)^2} = \frac{1}{4\pi}\, l2$ V. T. 173. N°. 8.

3) $\int \frac{dx}{q^2 + (l\, Tang.\, x)^2} = \frac{1}{4q}\left\{Z'\left(\frac{2q + 3\pi}{4\pi}\right) - Z'\left(\frac{2q + \pi}{4\pi}\right)\right\}$ V. T. 173. N°. 9.

4) $\int Tang.\left(\frac{\pi}{4} + x\right) \frac{l\, Sin.\, 2x\, dx}{q^2 + (l\, Sin.\, 2x)^2} = \frac{1}{4}\left\{l\frac{2\pi}{q} + \frac{\pi}{q} + Z'\left(\frac{q}{2\pi}\right)\right\}$ V. T. 173. N°. 2.

5) $\int Tang.\left(\frac{\pi}{4} + x\right) \frac{l\, Sin.\, 2x}{4\pi^2 + (l\, Sin.\, 2x)^2}\, dx = \frac{1}{8}(1 - 2A)$ V. T. 173. N°. 3.

6) $\int Tang.\left(\frac{\pi}{4} + x\right) \frac{l\, Sin.\, 2x}{q^2 - (l\, Sin.\, 2x)^2}\, dx = \frac{2\pi^4}{q^4} \sum_0^\infty (-1)^{n+1} \left(\frac{2\pi}{q}\right)^{2n} \frac{B_{2n+1}}{n+1}$ V. T. 173. N°. 4.

7) $\int Tang.\left(\frac{\pi}{4} + x\right) \frac{l\, Sin.\, 2x}{\{q^2 + (l\, Sin.\, 2x)^2\}^2}\, dx = \frac{-\pi^2}{2q^4} \sum_0^\infty B_{2n+1} \left(\frac{2\pi}{q}\right)^{2n}$ V. T. 173. N°. 5.

8) $\int Tang.\left(\frac{\pi}{4} + x\right) \frac{l\, Sin.\, 2x}{\{q^2 - (l\, Sin.\, 2x)^2\}^2}\, dx = \frac{\pi^2}{2q^4} \sum_0^\infty (-1)^{n+1} \left(\frac{2\pi}{q}\right)^{2n} B_{2n+1}$ V. T. 173. N°. 6.

9) $\int Tang.\ 2x\, l\, Sin.\, x\, \frac{q^2 - (l\, Cos.\, 2x)^2}{\{q^2 + (l\, Cos.\, 2x)^2\}^2}\, dx = \frac{1}{8}\left\{l\frac{2\pi}{q} + \frac{\pi}{q} + Z'\left(\frac{q}{2\pi}\right)\right\}$ V. T. 325. N°. 1.

10) $\int Tang.\ 2x\, l\, Sin.\, x\, \frac{4\pi^2 - (l\, Cos.\, 2x)^2}{\{4\pi^2 + (l\, Cos.\, 2x)^2\}^2}\, dx = \frac{1}{16}(1 - 2A)$ V. T. 325 N°. 2.

11) $\int Tang.\, 2x\, l\, Sin.\, x\, \frac{q^2 + (l\, Cos.\, 2x)^2}{\{q^2 - (l\, Cos.\, 2x)^2\}^2}\, dx = \frac{\pi^4}{q^4} \sum_0^\infty (-1)^{n+1} \left(\frac{2\pi}{q}\right)^{2n} \frac{B_{2n+1}}{n+1}$ V. T. 325. N°. 3.

12) $\int Tang.\, 2x\, l\, Sin.\, x\, \frac{q^2 - 3(l\, Cos.\, 2x)^2}{\{q^2 + (l\, Cos.\, 2x)^2\}^3}\, dx = \frac{-\pi^2}{4q^4} \sum_0^\infty \left(\frac{2\pi}{q}\right)^{2n} B_{2n+1}$ V. T. 325. N°. 4.

13) $\int Tang.\, 2x\, l\, Sin.\, x\, \frac{q^2 + 3(l\, Cos.\, 2x)^2}{\{q^2 - (l\, Cos.\, 2x)^2\}^3}\, dx = \frac{\pi^2}{4q^4} \sum_0^\infty (-1)^{n-1} \left(\frac{2\pi}{q}\right)^{2n} B_{2n+1}$ V. T. 325. N°. 5.

1) $\int \frac{1}{Tang.\,x} \frac{l\,Cos.\,2\,x}{q^2 + (l\,Cos.\,2\,x)^2}\,dx = \frac{1}{8}\left\{l\frac{2\pi}{q} + \frac{\pi}{q} + Z'\left(\frac{q}{2\pi}\right)\right\}$ V. T. 173. N°. 2.

2) $\int \frac{1}{Tang.\,x} \frac{l\,Cos.\,2\,x}{4\pi^2 + (l\,Cos.\,2\,x)^2}\,dx = \frac{1}{8}(1 - 2\,A)$ V. T. 173. N°. 3.

3) $\int \frac{1}{Tang.\,x} \frac{l\,Cos.\,2\,x}{q^2 - (l\,Cos.\,2\,x)^2}\,dx = \frac{2\pi^4}{q^4}\sum_0^\infty (-1)^{n+1}\left(\frac{2\pi}{q}\right)^{2n}\frac{B_{2n+1}}{n+1}$ V. T. 173. N°. 4.

4) $\int \frac{1}{Tang.\,x} \frac{l\,Cos.\,2\,x}{\{q^2 + (l\,Cos.\,2\,x)^2\}^2}\,dx = \frac{-\pi^2}{2\,q^4}\sum_0^\infty \left(\frac{2\pi}{q}\right)^{2n} B_{2n+1}$ V. T. 173. N°. 5.

5) $\int \frac{1}{Tang.\,x} \frac{l\,Cos.\,2\,x}{\{q^2 - (l\,Cos.\,2\,x)^2\}^2}\,dx = \frac{\pi^2}{2\,q^4}\sum_0^\infty (-1)^{n+1}\left(\frac{2\pi}{q}\right)^{2n} B_{2n+1}$ V. T. 173. N°. 6.

6) $\int \frac{Tang.^q\,x}{Sin.\,2\,x} \frac{dx}{p + l\,Tang.\,x} = \frac{1}{2} e^{-pq}\,Ei.(p\,q)$ V. T. 169. N°. 1.

7) $\int \frac{Tang.^q\,x}{Sin.\,2\,x} \frac{dx}{p - l\,Tang.\,x} = -\frac{1}{2} e^{pq}\,Ei.(-p\,q)$ V. T. 169. N°. 2.

8) $\int \frac{Tang.\,x}{Cos.\,2\,x} \frac{l\,Tang.\,x}{q^2 + (l\,Tang.\,x)^2}\,dx = \frac{\pi}{4\,q} + \frac{1}{2} l\frac{\pi}{q} + \frac{1}{2} Z'\left(\frac{q}{\pi}\right)$ V. T. 173. N°. 12.

9) $\int \frac{Tang.\,x}{Cos.\,2\,x} \frac{l\,Tang.\,x}{q^2 - (l\,Tang.\,x)^2}\,dx = \frac{\pi^4}{4\,q^4}\sum_0^\infty (-1)^{n-1}\frac{B_{2n+1}}{n+1}\left(\frac{\pi}{q}\right)^2$ V. T. 173. N°. 14.

10) $\int \frac{l\,Tang.\,x}{\pi^2 + (l\,Tang.\,x)^2} \frac{dx}{Cos.\,2\,x} = \frac{1}{2}\left\{\frac{1}{2} - l\,2\right\}$ V. T. 173. N°. 11.

11) $\int \frac{l\,Tang.\,x}{\pi^2 + (l\,Tang.\,x)^2} \frac{Tang.\,x}{Cos.\,2\,x}\,dx = \frac{1}{4} - \frac{1}{2} A$ V. T. 173. N°. 13.

12) $\int \frac{l\,Tang.\,x}{\pi^2 + (l\,Tang.^2\,x)^2} \frac{dx}{Cos.\,2\,x} = \frac{2 - \pi}{16}$ V. T. 173. N°. 10.

13) $\int \frac{l\,Tang.\,x}{q^2 + (l\,Tang.\,x)^2} \frac{Tang.\,x}{Cos.\,2\,x}\,dx = \frac{1}{2} l\frac{q}{2\pi} - \frac{\pi}{2\,q} + \frac{1}{2} Z'\left(\frac{2\pi + q}{2\pi}\right)$ V. T. 173. N°. 1.

14) $\int \frac{l\,Tang.\,x}{\pi^2 + (l\,Tang.^4\,x)^2} \frac{dx}{Cos.\,2\,x} = -\frac{\pi\sqrt{2}}{64} + \frac{1}{16} + \frac{1}{32\sqrt{2}} l\frac{\sqrt{2}-1}{\sqrt{2}+1}$ V. T. 173. N°. 18.

15) $\int \frac{Tang.^p\,x - Cot.^p\,x}{\pi^2 + (l\,Tang.\,x)^2} \frac{dx}{Cos.\,2\,x} = \frac{1}{2\pi}\left[p\pi Cos.\,p\pi - Sin.\,p\pi.\,l\{2(1 + Cos.\,p\pi)\}\right],\ p < 1;$ V. T. 176. N°. 8.

16) $\int \frac{Tang.^p\,x + Cot.^p\,x}{\pi^2 + (l\,Tang.\,x)^2} \frac{l\,Tang.\,x}{Cos.\,2\,x}\,dx = \frac{1}{2}\left[1 - p\pi Sin.\,p\pi - Cos.\,p\pi.\,l\{2(1 + Cos\,p\pi)\}\right],\ p < 1;$ V.T. 176. N°. 9.

17) $\int \frac{Tang.^p x - Cot.^p x}{\pi^2 + (l\,Tang.^2 x)^2} \frac{dx}{Cos.\,2x} = -\frac{1}{4} Sin. \frac{1}{2} p\pi + \frac{\pi}{4} Cos. \frac{1}{2} p\pi . l \frac{1 + Sin. \frac{1}{2} p\pi}{1 - Sin. \frac{1}{2} p\pi}, p < 1;$ V. T. 176. N°. 6.

18) $\int \frac{Tang.^p x + Cot.^p x}{\pi^2 + (l\,Tang.^2 x)^2} \frac{l\,Tang.\,x}{Cos.\,2x} dx = \frac{1}{4} - \frac{\pi}{8} Cos. \frac{1}{2} p\pi + \frac{1}{8} Sin. \frac{1}{2} p\pi . l \frac{1 - Sin. \frac{1}{2} p\pi}{1 + Sin. \frac{1}{2} p\pi}, p < 1;$ V. T. 176. N°. 7.

19) $\int \frac{l\,Tang.\,x}{\{q^2 + l\,Tang.\,x)^2\}^2} \frac{Tang.\,x}{Cos.\,2x} dx = -\frac{\pi^2}{4q^4} \sum_0^\infty \left(\frac{\pi}{q}\right)^{2n} B_{2n+1}$ V. T. 173. N°. 15.

20) $\int \frac{l\,Tang.\,x}{\{q^2 - (l\,Tang.\,x)^2\}^2} \frac{Tang.\,x}{Cos.\,2x} dx = \frac{\pi^2}{4q^4} \sum_0^\infty (-1)^{n-1} \left(\frac{\pi}{q}\right)^{2n} B_{2n+1}$ V. T. 173. N°. 16.

21) $\int l\,Tang. \left(\frac{\pi}{4} \pm x\right) \frac{\pi^2 - (l\,Tang.\,x)^2}{\{\pi^2 + (l\,Tang.\,x)^2\}^2} \frac{dx}{Sin.\,2x} = \pm \frac{1}{2} \left\{ l\,2 - \frac{1}{2} \right\}$ V. T. 325. N°. 10.

22) $\int l\,Tang. \left(\frac{\pi}{4} \pm x\right) \frac{\pi^2 - (l\,Tang.^2 x)^2}{\{\pi^2 + (l\,Tang.^2 x)^2\}^2} \frac{dx}{Sin.\,2x} = \pm \frac{\pi - 2}{16}$ V. T. 325. N°. 12.

23) $\int l\,Tang. \left(\frac{\pi}{4} \pm x\right) \frac{\pi^2 - (l\,Tang.^4 x)^2}{\{\pi^2 + (l\,Tang.^4 x)^2\}^2} \frac{dx}{Sin.\,2x} = \pm \left\{ \frac{\pi\sqrt{2}}{64} - \frac{1}{16} + \frac{1}{32\sqrt{2}} l \frac{\sqrt{2}+1}{\sqrt{2}-1} \right\}$ V. T. 325. N°. 14.

24) $\int \frac{l\,Tang.\,x}{4\pi^2 + (l\,Tang.\,x)^2} \frac{dx}{Cos.\,x\,(Cos.\,x - Sin.\,x)} = \frac{1}{4}(1 - 2\,A)$ V. T. 173. N°. 3.

25) $\int \frac{l\,Tang.\,x}{q^2 - (l\,Tang.\,x)^2} \frac{dx}{Cos.\,x\,(Cos.\,x - Sin.\,x)} = \frac{4\pi^4}{q^4} \sum_0^\infty (-1)^{n+1} \left(\frac{\pi}{q}\right)^{2n} \frac{B_{2n+1}}{n+1}$ V. T. 173. N°. 4.

F. Log. en dén.. Fonct. binôme. Circ. Dir. en dén. irrat. monôme. **TABLE 326.** Lim. 0 et $\frac{\pi}{4}$.

1) $\int \frac{Tang.\left(\frac{\pi}{4} - x\right)}{\pi^2 + (l\,Sin.\,2x)^2} \frac{dx}{\sqrt{Sin.\,2x}} = \frac{1}{4\pi} l\,2$ V. T. 177. N°. 1.

2) $\int \frac{Tang.\left(\frac{\pi}{4} - x\right)}{\pi^2 + 4(l\,Sin.\,2x)^2} \frac{dx}{\sqrt{Sin.\,2x}} = \frac{1}{8\pi\sqrt{2}} \left\{ \pi - l \frac{\sqrt{2}+1}{\sqrt{2}-1} \right\}$ V. T. 177. N°. 4.

3) $\int \frac{Tang.\left(\frac{\pi}{4} - x\right)}{q^2 + (l\,Sin.\,2x)^2} \frac{dx}{\sqrt{Sin.\,2x}} = \frac{1}{8q} \left\{ Z'\left(\frac{q + 3\pi}{4\pi}\right) - Z'\left(\frac{q + \pi}{4\pi}\right) \right\}$ V. T. 177. N°. 3.

4) $\int \frac{Tang.\left(\frac{\pi}{4} + x\right)}{\pi^2 + (l\,Sin.\,2x)^2} \frac{Sin^p\,2x - Cosec^p\,2x}{\sqrt{Sin.\,2x}} dx = \frac{1}{4} \left\{ \frac{1}{\pi} Cos.\,p\pi . l \frac{1 + Sin\,p\pi}{1 - Sin.\,p\pi} - Sin.\,p\pi \right\}$ V. T. 177. N°. 9.

5) $\int \frac{Tang.\left(\frac{\pi}{4}+x\right)}{q^2+(l\,Sin.2x)^2}\frac{Sin.^p 2x - Cosec.^p 2x}{\sqrt{Sin.2x}}dx = \frac{\pi}{q}\sum_1^\infty \frac{Sin.\{(p+1)n\pi\}}{q+2n\pi}$ V. T. 177. N°. 13.

6) $\int \frac{Tang.\left(\frac{\pi}{4}+x\right)}{\pi^2+(l\,Sin.2x)^2}\frac{l\,Sin.2x}{\sqrt{Sin.2x}}dx = \frac{2-\pi}{8}$ V. T. 177. N°. 5.

7) $\int \frac{Tang.\left(\frac{\pi}{4}+x\right)}{\pi^2+4(l\,Sin.2x)^2}\frac{l\,Sin.2x}{\sqrt{Sin.2x}}dx = \frac{1}{4\sqrt{2}}\left\{\pi+2\sqrt{2}-l\frac{\sqrt{2}+1}{\sqrt{2}-1}\right\}$ V. T. 177. N°. 6.

8) $\int \frac{Tang.\left(\frac{\pi}{4}+x\right)}{\pi^2+(l\,Sin.2x)^2}\frac{Sin.^p 2x + Cosec.^p 2x}{\sqrt{Sin.2x}}l\,Sin.2x\,dx = \frac{1}{2}-\frac{\pi}{4}Cos.p\pi - \frac{1}{4}Sin.p\pi.l\frac{1+Sin.p\pi}{1-Sin.p\pi}$ V. T. 177. N°. 10.

9) $\int \frac{Tang.\left(\frac{\pi}{4}+x\right)}{q^2+(l\,Sin.2x)^2}\frac{Sin.^p 2x + Cosec.^p 2x}{\sqrt{Sin.2x}}l\,Sin.2x\,dx = -\frac{\pi}{2q}-\pi\sum_1^\infty \frac{Cos.\{(p+1)n\pi\}}{q+2n\pi}$ V. T. 177. N°. 14.

10) $\int \frac{Tang.x}{\pi^2+(l\,Cos.2x)^2}\frac{dx}{\sqrt{Cos.2x}} = \frac{1}{4\pi}l\,2$ V. T. 177. N°. 1.

11) $\int \frac{Tang.x}{\pi^2+4(l\,Cos.2x)^2}\frac{dx}{\sqrt{Cos.2x}} = \frac{1}{8\pi\sqrt{2}}\left\{\pi-l\frac{\sqrt{2}+1}{\sqrt{2}-1}\right\}$ V. T. 177. N°. 4.

12) $\int \frac{Tang.x}{q^2+(l\,Cos.2x)^2}\frac{dx}{\sqrt{Cos.2x}} = \frac{1}{8q}\left\{Z'\left(\frac{q+3\pi}{4\pi}\right)-Z'\left(\frac{q+\pi}{4\pi}\right)\right\}$ V. T. 177. N°. 3.

13) $\int \frac{Cos.^p 2x - Sec.^p 2x}{\pi^2+(l\,Cos.2x)^2}\frac{dx}{Tang.x\sqrt{Cos.2x}} = \frac{1}{4}\left\{\frac{1}{\pi}Cos.p\pi.l\frac{1+Sin.p\pi}{1-Sin.p\pi}-Sin.p\pi\right\}$ V. T. 177. N°. 9.

14) $\int \frac{Cos.^p 2x - Sec.^p 2x}{4\pi^2+(l\,Cos.2x)^2}\frac{dx}{Tang.x\sqrt{Cos.2x}} = \frac{-1}{8\pi}\left[2p\pi Cos.2p\pi + Sin.2p\pi.l\{2(1+Cos.2p\pi)\}\right]$ V. T. 177. N°. 11.

15) $\int \frac{Cos.^p 2x - Sec.^p 2x}{q^2+(l\,Cos.2x)^2}\frac{dx}{Tang.x\sqrt{Cos.2x}} = \frac{\pi}{q}\sum_1^\infty \frac{Sin.\{(p+1)n\pi\}}{q+2n\pi}$ V. T. 177. N°. 13.

16) $\int \frac{l\,Cos.2x}{\pi^2+(l\,Cos.2x)^2}\frac{dx}{Tang.x\sqrt{Cos.2x}} = \frac{2-\pi}{8}$ V. T. 177. N°. 5.

17) $\int \frac{l\,Cos.2x}{\pi^2+4(l\,Cos.2x)^2}\frac{dx}{Tang.x\sqrt{Cos.2x}} = \frac{1}{4\sqrt{2}}\left\{\pi+2\sqrt{2}-l\frac{\sqrt{2}+1}{\sqrt{2}-1}\right\}$ V. T. 177. N°. 6.

F. Log. en dén.. Fonct. binôme. Circ. Dir. en dén. irrat. monôme.	TABLE 326 suite.	Lim. 0 et $\frac{\pi}{4}$.

18) $\int \frac{Cos.^p 2x + Sec.^p 2x}{\pi^2 + (l\,Cos.\,2x)^2} \frac{l\,Cos.\,2x}{Tang.x \sqrt{Cos.\,2x}} dx = \frac{1}{2} - \frac{\pi}{4} Cos.\,p\pi - \frac{1}{4} Sin.\,p\pi.\,l \frac{1 + Sin.\,p\pi}{1 - Sin.\,p\pi}$ V. T. 177. N°. 10.

19) $\int \frac{Cos.^p 2x + Sec.^p 2x}{4\pi^2 + (l\,Cos.\,2x)^2} \frac{l\,Cos.\,2x}{Tang.x \sqrt{Cos.2x}} dx = \frac{1}{4}\left[1 - 2p\pi Sin.2p\pi - Cos.2p\pi.\,l\{2(1 + Cos.2p\pi)\}\right]$ V. T. 177. N°. 12.

20) $\int \frac{Cos.^p 2x + Sec.^p 2x}{q^2 + (l\,Cos.\,2x)^2} \frac{l\,Cos.\,2x}{Tang.\,x \sqrt{Cos.\,2x}} dx = -\frac{\pi}{2q} - \pi \sum_1^\infty \frac{Cos.\,\{(p+1)n\pi\}}{q + 2n\pi}$ V. T. 177. N°. 14.

F. Log. en dén.. Fonct. binôme. Circ. Dir. en dén. irrat. composée.	TABLE 327.	Lim. 0 et $\frac{\pi}{4}$.

1) $\int \frac{1}{q^2 + (l\,Tang.x)^2} \frac{1}{Sin.\,x + Cos.\,x} \frac{dx}{\sqrt{Sin.\,2x}} = \frac{1}{4q\sqrt{2}}\left\{Z'\left(\frac{q + 3\pi}{4\pi}\right) - Z'\left(\frac{q+\pi}{4\pi}\right)\right\}$ V. T. 177. N°. 3.

2) $\int \frac{1}{q^2 + (l\,Tang.\,x)^2} \frac{1}{Sin.\,x - Cos.\,x} \frac{dx}{\sqrt{(1 + Cos.\,2x)}} = \frac{1}{2\sqrt{2}}\left\{\frac{\pi}{q} + l\frac{2\pi}{q} + Z'\left(\frac{q}{2\pi}\right)\right\}$ V. T. 173. N°. 2.

3) $\int \frac{Tang.^{\frac{p-1}{2}} x - Cot.^{\frac{p-1}{2}} x}{q^2 + (l\,Tang.\,x)^2} \frac{1}{Sin.\,x - Cos.\,x} \frac{dx}{\sqrt{Sin.\,2x}} = -\frac{\pi\sqrt{2}}{q} \sum_1^\infty \frac{Sin.\,np\pi}{q + 2n\pi}, p < 1;$ V. T. 177. N°. 13.

4) $\int \frac{Tang.^{\frac{p-1}{2}} x + Cot.^{\frac{p-1}{2}} x}{q^2 + (l\,Tang.\,x)^2} \frac{l\,Tang.\,x}{Sin.x - Cos.x} \frac{dx}{\sqrt{Sin.\,2x}} = \frac{\pi}{q\sqrt{2}} + \pi\sqrt{2} \sum_1^\infty \frac{Cos.\,np\pi}{q + 2n\pi}, p < 1;$ V. T. 177. N°. 14.

5) $\int \frac{1 - Tang.^{q-1} x}{Sin.\,x - Cos.\,x} \frac{1 - Tang.^{q-\frac{1}{2}} x}{\sqrt{Sin.\,2x}} \frac{dx}{l\,Tang.\,x} = \frac{2q - 2}{\sqrt{2}} l\,2$ V. T. 177. N°. 15.

F. Log. sous forme irrat. Circ. Dir.	TABLE 328.	Lim. 0 et $\frac{\pi}{4}$.

1) $\int \frac{dx}{\sqrt{l\,Cot.\,x}} = \sqrt{\pi} \sum_0^\infty \frac{(-1)^n}{\sqrt{(2n+1)}}$ V. T. 178. N°. 4.

2) $\int \frac{Sin.\,x}{\sqrt{l\,Sec.\,x}} dx = \sqrt{\pi}$ V. T. 349. N°. 6.

3) $\int \frac{Cos.\,x}{\sqrt{l\,Cosec.\,x}} dx = \sqrt{\pi}$ V. T. 349. N°. 1.

4) $\int \frac{Cos.\,x}{\sqrt{(l\,Cosec.\,x)^{2a+1}}} dx = \frac{(-2)^a \sqrt{\pi}}{1^{a/2}}$ V. T. 44. N°. 6.

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5) $\int \frac{l\, l\, Cot.\, x}{\sqrt{l\, Cot.\, x}}\, dx = \sqrt{\pi} \sum_0^\infty (-1)^{n-1} \frac{l(2n+1) + 2\, l\, 2 + A}{\sqrt{(2n+1)}}$ V. T. 190. N°. 4.

6) $\int \frac{dx}{Cos.^2 x \sqrt{l\, Cot.\, x}} = \sqrt{\pi}$ V. T. 187. N°. 18.

7) $\int \frac{Tang.^p x}{Sin.\, 2x \sqrt{l\, Cot.\, x}}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{p}}$ V. T. 178. N°. 1.

8) $\int \frac{Tang.^q x .\, l\, l\, Cot.\, x}{Sin.\, 2x \sqrt{l\, Cot.\, x}}\, dx = -\frac{1}{2}\sqrt{\frac{\pi}{a}} \{A + 2\, l\, 2 + l a\}$ V. T. 190. N°. 3.

9) $\int \frac{1}{2 + Sin.\, 2x} \frac{dx}{\sqrt{l\, Cot.\, x}} = \frac{\sqrt{\pi}}{2\, Sin.\, \frac{\pi}{3}} \sum_1^\infty (-1)^{n-1} \frac{Sin.\, \frac{n\pi}{3}}{\sqrt{n}}$ V. T. 178. N°. 5.

10) $\int \frac{l\, l\, Cot.\, x}{2 + Sin.\, 2x} \frac{dx}{\sqrt{l\, Cot.\, x}} = \frac{\sqrt{\pi}}{2\, Sin.\, \frac{\pi}{3}} \sum_1^\infty (-1)^n Sin.\, \frac{n\pi}{3} \frac{l n + 2\, l\, 2 + A}{\sqrt{(2n+1)}}$ V. T. 190. N°. 8.

F. Log. Circ. Dir. } Autre forme. — TABLE 329. — Lim. 0 et $\frac{\pi}{4}$.

1) $\int Sin.\, (2\, p\, l\, Tang.\, x) .\, l\, Tang.\, x\, dx = \frac{\pi^2}{4} \frac{e^{p\pi} - e^{-p\pi}}{(e^{p\pi} + e^{-p\pi})^2}$ V. T. 404. N°. 6.

2) $\int Sin.\, (p\, l\, Tang.\, x) .\, (Tang.^q x - Cot.^q x)\, dx = \pi\, Sin.\, \tfrac{1}{2} q \pi \frac{e^{\frac{1}{2}p\pi} - e^{-\frac{1}{2}p\pi}}{e^{p\pi} + 2\, Cos.\, q\pi + e^{-p\pi}}, p^2 < 1, q^2 < 1;$ V. T. 404. N°. 8.

3) $\int Sin.^2 (p\, l\, Tang.\, x)\, dx = \frac{\pi}{8} \frac{(e^{p\pi} - 1)^2}{e^{2p\pi} + 1}$ V. T. 404. N°. 17.

4) $\int Cos.\, (p\, l\, Tang.\, x)\, dx = \frac{\pi}{2} \frac{e^{\frac{1}{2}p\pi}}{e^{p\pi} + 1}$ V. T. 404. N°. 2.

5) $\int Cos.^2 (p\, l\, Tang.\, x)\, dx = \frac{\pi}{8} \frac{(e^{p\pi} + 1)^2}{e^{2p\pi} + 1}$ V. T. 404. N°. 8.

6) $\int Cos.\, (p\, l\, Tang.\, x) .\, (Tang.^q x + Cot.^q x)\, dx = \pi\, Cos.\, \tfrac{1}{2} q \pi \frac{e^{\frac{1}{2}p\pi} + e^{-\frac{1}{2}p\pi}}{e^{p\pi} + 2\, Cos.\, q\pi + e^{-p\pi}}, p^2 < 1, q^2 < 1;$ V. T. 404. N°. 9.

7) $\int Sin.\, (p\, l\, Tang.\, x) \frac{dx}{Cos.\, 2x} = \frac{\pi}{4} \frac{1 - e^{p\pi}}{1 + e^{p\pi}}$ V. T. 404. N°. 10.

8) $\int Sin.\, (p\, l\, Tang.\, x) \frac{dx}{Sin.\, 4x} = \frac{\pi}{8} \frac{1 + e^{p\pi}}{1 - e^{p\pi}}$ V. T. 405. N°. 4.

9) $\int Sin.(p\,l\,Tang.x)\frac{Tang.^{q-1}x}{Cos.2x}dx = -\sum_1^\infty \frac{p}{(2n+q)^2+p^2}$ V. T. 404. N°. 12.

10) $\int Sin.(p\,l\,Tang.x).Tang.\left(\frac{\pi}{4}+x\right)\frac{dx}{Sin.2x} = \frac{\pi}{4}\frac{1+e^{2p\pi}}{1-e^{2p\pi}}$ V. T. 405. N°. 2.

11) $\int Sin.(p\,l\,Tang.x).Tang.\left(\frac{\pi}{4}-x\right)\frac{dx}{Sin.2x} = \frac{-\pi}{e^{p\pi}-e^{-p\pi}}$ V. T. 405. N°. 1.

12) $\int Sin.(p\,l\,Tang.x)\frac{Tang.^q x+Cot.^q x}{Cos.2x}dx = -\frac{\pi}{2}\frac{e^{p\pi}-e^{-p\pi}}{e^{p\pi}+2\,Cos.q\pi+e^{-p\pi}}$ V. T. 404. N°. 13.

13) $\int Cos.(p\,l\,Tang.x)\frac{dx}{Sin.4x} = \frac{1}{4}l\left(e^{\frac{1}{2}p\pi}-e^{-\frac{1}{2}p\pi}\right)$ V. T. 406. N°. 18.

14) $\int Cos.(p\,l\,Tang.x)\frac{l\,Tang.x}{Sin.4x}dx = \frac{1}{4}\pi^2\frac{e^{p\pi}}{(1-e^{p\pi})^2}$ V. T. 405. N°. 5.

15) $\int Cos.(p\,l\,Tang.x)\frac{l\,Tang.x}{Cos.2x}dx = \frac{1}{2}\pi^2\frac{e^{p\pi}}{(e^{p\pi}+1)^2}$ V. T. 404. N°. 14.

16) $\int Cos.(p\,l\,Tang.x).Tang.\left(\frac{\pi}{4}-x\right)\frac{l\,Tang.x}{Sin.2x}dx = \pi^2 e^{-p\pi}\frac{1+e^{-2p\pi}}{(1-e^{-2p\pi})^2}$ V. T. 405. N°. 3.

17) $\int Cos.(p\,l\,Tang.x)\frac{Tang.^q x-Cot.^q x}{Cos.2x}dx = \frac{\pi}{2}\frac{Sin.q\pi}{e^{p\pi}+2\,Cos.q\pi+e^{-p\pi}}$ V. T. 404. N°. 15.

18) $\int Sin.(p\,l\,Tang.x)\frac{1}{1-Sin.2x.Cos.\lambda}\frac{dx}{Tang.2x} = -\frac{\pi}{2}\frac{e^{p\lambda}+e^{-p\lambda}}{e^{p\pi}-e^{-p\pi}}$ V. T. 405. N°. 12.

19) $\int Cos.(p\,l\,Tang.x)\frac{dx}{1+Sin.2x.Cos.\lambda} = \frac{\pi}{2}Cosec.\lambda\frac{e^{p\lambda}-e^{-p\lambda}}{e^{p\pi}-e^{-p\pi}}$ V. T. 405. N°. 7.

20) $\int Cos.(p\,l\,Tang.x)\frac{1}{1+Sin.2x.Cos.\lambda}\frac{dx}{Sin.2x} = -\frac{\pi}{2}Cot.\lambda\frac{e^{p\lambda}-e^{-p\lambda}}{e^{p\pi}-e^{-p\pi}}$ V. T. 405. N°. 11.

21) $\int Sin.(p\,l\,Tang.x)\frac{dx}{l\,Tang.x} = Arctang.\left(e^{\frac{1}{2}p\pi}\right)$ V. T. 406. N°. 15.

22) $\int Sin.(2p\,l\,Tang.x)\frac{dx}{Tang.2x.l\,Tang.x} = \frac{1}{2}l\frac{1+e^{-p\pi}}{1-e^{-p\pi}}$ V. T. 406. N°. 17.

23) $\int Cos.(2p\,l\,Tang.x)\frac{dx}{Cos.2x.l\,Tang.x} = -\frac{1}{2}l\left(e^{p\pi}+e^{-p\pi}\right)$ V. T. 406. N°. 16.

Page 430.

1) $\int l\,Sin.\,x\,dx = -\frac{1}{2}\pi\, l\,2$ Euler, Calc. Int. IV. S. 3. 123. — Id., N. C. Petr. 14. 129. — Cauchy, Exerc. 1826. p. 205. — Id., Lim. Imag. 149. — Serret, L. 8. 1. — Roberts, L. 11. 471. — Grunert, Gr. 4. 113. — Lindmann, Stockh. Handl. 1850. III.

2) $\int l((Sin.\,x))\,dx = -\frac{1}{2}\pi(l\,2 - 2\alpha\pi i)$ Arndt, Gr. 6. 187.

3) $= -\frac{1}{2}\pi\, l\,2 + \alpha\pi^2 i$ } Lindmann, Gr. 16. 94.

4) $\int l((-Sin.\,x))\,dx = -\frac{1}{2}\pi\, l\,2 + \frac{2\alpha+1}{2}\pi^2 i$ } Lindmann, Gr. 16. 94.

5) $\int l\,Sin.\,x.\,Sin.\,x\,dx = l\,2 - 1$ V. T. 163. N°. 3.

6) $\int l\,Sin.\,x.\,Sin.^2\,x\,dx = \frac{1}{8}\pi(1 - 2\,l\,2)$ V. T. 163. N°. 4.

7) $\int l\,Sin.\,x.\,Sin.^3\,x\,dx = \frac{2}{3}\left(l\,2 - \frac{5}{6}\right)$ V. T. 163. N°. 5.

8) $\int l\,Sin.\,x.\,Sin.^4\,x\,dx = \frac{3}{16}\pi\left(\frac{7}{12} - l\,2\right)$ V. T. 163. N°. 6.

9) $\int l\,Sin.\,x.\,Sin.^5\,x\,dx = \frac{8}{15}\left(l\,2 - \frac{47}{60}\right)$ V. T. 163. N°. 7.

10) $\int l\,Sin.\,x.\,Cos.^2\,x\,dx = -\frac{1}{8}\pi(1 + 2\,l\,2)$ V. T. 162. N°. 1.

11) $\int l\,Sin.\,x.\,Sin.\,x.\,Cos.^2\,x\,dx = -\frac{1}{9}(4 - 3\,l\,2)$ V. T. 162. N°. 2.

12) $\int l\,Sin.\,x.\,Cos.\,2\,x\,dx = -\frac{1}{4}\pi$ V. T. 330. N°. 6, 10.

13) $\int l\,Sin.\,x.\,Tang.\,x\,dx = -\frac{1}{24}\pi^2$ V. T. 152. N°. 14.

14) $\int l\,Sin.\,x.\,Cos.^{2a}\,x\,dx = -\frac{\pi\,1^{a/2}}{2^{a+2}\,1^{a/1}}\{A + Z'(a+1) + 2\,l\,2\}$ V. T. 162. N°. 3.

15) $\int l\,Sin.\,x.\,Sin.^{2a}\,x.\,Cos.\,x\,dx = -\frac{1}{(2a+1)^2}$ V. T. 151. N°. 1.

16) $\int l\,Sin.\,x.\,Cos.(p\,Sin.\,x).\,Cos.\,x\,dx = -\frac{1}{p}\sum_1^\infty \frac{1}{2n-1}\,\frac{p^{2n-1}}{1^{2n-1/1}}$ V. T. 71. N°. 16.

1) $\int l\,Cos.\,x\,dx = -\frac{1}{2}\pi l 2$ Euler, Calc. Int. IV. S. 3. 126. — Id., N. C. Petr. 14. 129. — Cauchy, Exerc. 1826. p. 205. — Id., Lim. Imag. 146. — Serret, L. 8. 1. — Roberts, L. 11. 471. — Grunert, Gr. 4. 113. — Lindmann, Stockh. Handl. 1850. III.

2) $\int l\,Cos.\,x.\,Cos.\,x\,dx = l2 - 1$ V. T. 163. N°. 3.

3) $\int l\,Cos.\,x.\,Cos.^2 x\,dx = \frac{1}{8}\pi(1 - 2l2)$ V. T. 163. N°. 4.

4) $\int l\,Cos.\,x.\,Cos.^3 x\,dx = \frac{2}{3}\left(l2 - \frac{5}{6}\right)$ V. T. 163. N°. 5.

5) $\int l\,Cos.\,x.\,Cos.^4 x\,dx = \frac{3}{16}\pi\left(\frac{7}{12} - l2\right)$ V. T. 163. N°. 6.

6) $\int l\,Cos.\,x.\,Cos.^5 x\,dx = \frac{8}{15}\left(l2 - \frac{47}{60}\right)$ V. T. 163. N°. 7.

7) $\int l\,Cos.\,x.\,Sin.^2 x\,dx = -\frac{1}{8}\pi(1 + 2l2)$ V. T. 162. N°. 1.

8) $\int l\,Cos.\,x.\,Sin.^2 x.\,Cos.\,x\,dx = -\frac{1}{9}(4 - 3l2)$ V. T. 162. N°. 2.

9) $\int l\,Cos.\,x.\,Cos.\,2x\,dx = \frac{1}{4}\pi$ V. T. 331. N°. 3, 7.

10) $\int l\,Cos.\,x.\,Sin.^{2a} x\,dx = -\frac{1^{a/2}}{2^{a+1}\,1^{a/1}}\frac{\pi}{2}\left\{1 + 2l2 + \sum_0^a \frac{1}{2}\right\}$ Lindmann, Stockh. Handl. 1850. III.

11) $\int l\,Cos.\,x.\,Cos.^{2a} x.\,Sin.\,x\,dx = -\frac{1}{(2a+1)^2}$ V. T. 151. N°. 1.

12) $\int l\,Cos.\,x.\,Cos^{p-1} x.\,Sin.\,p.\,Sin.\,p\,x\,dx = \frac{\pi}{2^{p+2}}\left\{A + Z'(p) - \frac{1}{p} - 2l2\right\}$ Lindmann, Stockh. Handl. 1850. III.

13) $\int l\,Cos.\,x.\,Cos.\,(p\,Cos.\,x).\,Sin.\,x\,dx = -\frac{1}{p}\sum_1^\infty \frac{1}{2n-1}\frac{p^{2n-1}}{1^{2n-1/1}}$ V. T. 60. N°. 4.

14) $\int l\,Cos.\,x.\,Cos.\,(p\,l\,Sin.\,x).\,Tang.\,x\,dx = \frac{1}{2p^2} + \frac{\pi}{4p}\frac{1 + e^{p\pi}}{1 - e^{p\pi}}$ V. T. 335. N°. 13.

F. Log. en num.. Prod. de $l\,Sin.\,x$ et $l\,Cos.\,x$. Circ. Dir. ent. **TABLE 332.** Lim. 0 et $\frac{\pi}{2}$.

1) $\int (l\,Sin.\,x)^2\,dx = \frac{\pi}{2}\left\{(l\,2)^2 + \frac{1}{12}\pi^2\right\}$ V. T. 164. N°. 1.

2) $\int (l\,Sin.\,x)^3\,Tang.\,x\,dx = -\frac{1}{240}\pi^4$ V. T. 154. N°. 15.

3) $\int (l\,Sin.\,x)^5\,Tang.\,x\,dx = -\frac{1}{504}\pi^6$ V. T. 155. N°. 5.

4) $\int (l\,Sin.\,x)^p\,Cos.\,x\,dx = (-1)^p\,\Gamma(p+1)$ V. T. 42. N°. 2.

5) $\int (l\,Sin.\,x)^{2a-1}\,Tang.\,x\,dx = -\frac{1}{4a}\pi^{2a}B_{2a-1}$ V. T. 158. N°. 6.

6) $\int (l\,Sin.\,x)^a\,Sin.^{p-1}x.\,Cos.\,x\,dx = \frac{(-1)^a\,1^{a/1}}{p^{a+1}}$ V. T. 151. N°. 2.

7) $\int (l\,Cos.\,x)^2\,dx = \frac{1}{2}\pi\left\{(l\,2)^2 + \frac{1}{12}\pi^2\right\}$ V. T. 164. N°. 1.

8) $\int (l\,Cos.\,x)^p\,Sin.\,x\,dx = (-1)^p\,\Gamma(1+p)$ V. T. 42. N°. 2.

9) $\int (l\,Cos.\,x)^a\,Cos.^{p-1}x.\,Sin.\,x\,dx = \frac{(-1)^a\,1^{a/1}}{p^{a+1}}$ V. T. 151. N°. 2.

10) $\int l\,Sin.\,x.\,(l\,Cos.\,x)^2\,Tang.\,x\,dx = -\frac{1}{720}\pi^4$ V. T. 337. N°. 12.

11) $\int l\,Sin.\,x.\,(l\,Cos.\,x)^4\,Tang.\,x\,dx = -\frac{1}{2520}\pi^6$ V. T. 337. N°. 14.

12) $\int l\,Sin.\,x.\,(l\,Cos.\,x)^{2a}\,Tang.\,x\,dx = \frac{-1}{4}\,\frac{\pi^{2a+2}}{(a+1)\,(2a+1)}\,B_{2a+1}$ V. T. 337. N°. 17.

F. Log. en num. $(l\,Tang.\,x)^a$. Circ. Dir. ent. **TABLE 333.** Lim. 0 et $\frac{\pi}{2}$.

1) $\int l\,Tang.\,x\,dx = 0$ Euler, Calc. Int. T. 4. S. 2. 127. — Cauchy, Exerc. 1826. p. 205.

2) $\int l\,Tang.\frac{1}{2}x.\,Sin.\,x\,dx = -l\,2$ Lobatschewsky, Mém. Kasan. 1836. 1. I. 107.

3) $\int l\,Tang.\,x.\,Sin.^2 x\,dx = \frac{1}{4}\pi$ V. T. 333. N°. 1, 5.

4) $\int l\,Tang.\,x.\,Cos.^2\,x\,dx = -\frac{1}{4}\pi$ V. T. 333. N°. 1, 5.

5) $\int l\,Tang.\,x.\,Cos.\,2\,x\,dx = -\frac{1}{2}\pi$ V. T. 330. N°. 12 et T. 331. N°. 9.

6) $\int l\,Tang.\,x.\,Sin^p\,2\,x\,dx = 0$ V. T. 183. N°. 8, 9.

7) $\int l\,Tang.\,x.\,Cos.^{2(q-1)}\,x\,dx = -\frac{\Gamma(q-\frac{1}{2})}{\Gamma(q)}\frac{1}{4}\sqrt{\pi}\left\{A + 2\,l2 + Z'\left(\frac{2q-1}{2}\right)\right\}$ Lindmann, Gr. 16. 94.

8) $\int l\,Tang.\,x.\,Cos.^{q-1}\,x.\,Cos.\{(q-1)x\}\,dx = -\frac{\pi}{2q}$

9) $\int l\,Tang.\,x.\,Cos.^{q-1}\,x.\,Cot.\,x.\,Sin.\{(q+1)\,x\}\,dx = -\frac{1}{2}\pi\,\{A+Z'(q+1)\}$

(8, 9) Lindmann, Stockh. Handl. 1850. II.

10) $\int l\,Tang.\,x.\,Sin.^{2a-1}\,2\,x.\,Cos.\,2\,x\,dx = -\frac{(1^{a-1/1})^2}{1^{2a/1}}\,2^{2a-3}$ V. T. 53. N°. 24.

11) $\int (l\,Tang.\,x)^2\,dx = \frac{1}{8}\pi^3$ V. T. 305. N°. 4 et T. 358. N°. 1.

12) $\int (l\,Tang.\,x)^4\,dx = \frac{5}{32}\pi^5$ V. T. 305. N°. 7 et T. 358. N°. 12.

13) $\int (l\,Tang.\,x)^6\,dx = \frac{61}{128}\pi^7$ V. T. 305. N°. 8 et T. 358. N°. 15.

14) $\int (l\,Tang.\,x)^{2a-1}\,dx = 0$ V. T. 180. N°. 3.

15) $\int (l\,Tang.\,x)^{2a}\,dx = 2.\,1^{2a/1}\sum_0^\infty \frac{(-1)^n}{(2n+1)^{2a+1}}$ V. T. 180. N°. 4.

16) $\int (l\,Tang.\,x)^a.\,Tang.^q\,x\,dx = \frac{1}{2}\pi\frac{d^a}{dq^a}.\,Sec.\frac{1}{2}q\pi$ V. T. 180. N°. 6.

F. Log. Circ. Dir. } Log. de Circ. Dir. d'autre forme. Sans fact. Circ. Dir. TABLE 334. Lim. 0 et $\frac{\pi}{2}$

1) $\int l(p\,Tang.\,x)\,dx = \frac{\pi}{2p}\,l\,p$ V. T. 180. N°. 8.

2) $\int l\,Sin.(p\,Tang.\,x)\,dx = \frac{1}{2}\pi\,l\frac{e^{2p}-1}{2\,e^{2p}}$ V. T. 415. N°. 1.

3) $\int l\, Cos.(p\, Tang.x)\, dx = \frac{1}{2}\pi l \frac{e^{2p}+1}{2e^{2p}}$ V. T. 415. N°. 2.

4) $\int l\, Tang.(p\, Tang.x)\, dx = \frac{1}{2}\pi l \frac{e^{2p}-1}{e^{2p}+1}$ V. T. 415. N°. 3.

5) $\int l\, Cot.(p\, Tang.x)\, dx = \frac{1}{2}\pi l \frac{e^{p}+e^{-p}}{e^{p}-e^{-p}}$ V. T. 415. N°. 12.

6) $\int l(1+Cos.x)\, dx = -\frac{1}{2}\pi l 2 + 2\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 304. N°. 12.

7) $\int l(1-Cos.x)\, dx = -\frac{1}{2}\pi l 2 - 2\sum_0^\infty \frac{(-1)^n}{(2n+1)^2}$ V. T. 238. N°. 4.

8) $\int l(1+q\, Cos.^2 x)\, dx = \pi l \frac{1+\sqrt{(1+q)}}{2}$

9) $\int l(1+Tang.^2\lambda . Cos.^2 x)\, dx = \pi l \left(Cos.^2 \frac{1}{2}\lambda . Sec.\lambda\right)$

(8, 9) Lobatschewsky, Mém. Kasan. 1835. 1.

10) $\int l(1-Sin\, hp.^2\lambda . Sin.^2 x)\, dx = 2\pi l\, Cos\, hp. \frac{1}{2}\lambda$

11) $\int l(1+q^2\, Sin.x. Cos.x)\, dx = \pi l \frac{1+\sqrt{(1+q^2)}}{2}$

12) $\int l(1-Cos\, hp.^2\lambda . Cos.^2 x)\, dx = \pi l \frac{1+Sin\, hp.\lambda}{2}$

(10–12) Lobatschewsky, Mém. Kasan. 1836. 1. I. 186, 74.

13) $\int l(1-Sin.^2\lambda . Cos.^2 x)\, dx = 2\pi l\, Cos. \frac{1}{2}\lambda$ Lobatschewsky, Mém. Kasan. 1835. 1.

14) $\int l(1+p^2\, Tang.^2 x)\, dx = \pi l(1+p)$ V. T. 181. N°. 3.

15) $\int l(q^2+Tang.^2 x)\, dx = \pi l(1+q)$ V. T. 181. N°. 2.

16) $\int l(1+p^2\, Cot.^2 x)\, dx = \pi l(1+p)$ V. T. 181. N°. 3.

17) $\int l\{1+p^2\, Tang.^2(q\, Tang.x)\}\, dx = \pi l \left[1+p\frac{e^q-e^{-q}}{e^q+e^{-q}}\right]$ V. T. 416. N°. 1.

18) $\int l\{1+p^2\, Cot.^2(q\, Tang.x)\}\, dx = \pi l \left[1+p\frac{e^q+e^{-q}}{e^q-e^{-q}}\right]$ V. T. 416. N°. 2.

19) $\int l(1+2p\,Sin.\,x+p^2)\,dx = \sum_0^\infty \frac{1}{2n+1}\,\frac{2^{n/2}}{3^{n/2}}\left(\frac{2p}{1+p^2}\right)^{2n+1}$, $p \leqq 1$; Raabe, Cr. 16. 355.

20) $\int l\{1+2p\,Cos.\,(q\,Tang.\,x)+p^2\}\,dx = \pi\,l(1+pe^{-q})$, $p^2 \leqq 1$; V. T. 416. N°. 5, 6.

21) $= \pi\,l(p+e^{-q})$, $p^2 \geqq 1$;

22) $\int l\frac{1+Sin.\,\lambda.\,Cos.^2\,x}{1-Sin.\,\lambda.\,Cos.^2\,x}\,dx = \pi\,l\frac{1+Sin.\,\frac{1}{2}\lambda}{Cos.\,\frac{1}{2}\lambda}$ Lobatschewsky, Mém. Kasan. 1836. 1. II. 15.

23) $\int l\frac{1-Sin.^2\,\lambda.\,Cos.^2\,x}{1-Sin.^2\,\mu.\,Cos.^2\,x}\,dx = 2\pi\,l(Cos.\,\frac{1}{2}\lambda.\,Sec.\,\frac{1}{2}\mu)$ Lobatschewsky, Mém. Kasan. 1835. 1.

24) $\int l\frac{Sin.^2\,\lambda-Sin.^2\,\mu.\,Cos.^2\,x}{1-Sin.^2\,\mu.\,Cos.^2\,x}\,dx = \pi\,l\left\{Tang.\frac{1}{2}\mu.\,Tang.\left(\frac{1}{2}\,Arcsin.\frac{Sin.\,\mu}{Sin.\,\lambda}\right)\right\}$

25) $\int l\left\{\frac{1+Sin.\,\lambda.\,Cos.\,x}{1-Sin.\,\lambda.\,Cos.\,x}.\frac{1+Sin.\,\mu.\,Cos.\,x}{1-Sin.\,\mu.\,Cos.\,x}\cdots\right\}dx = -2\pi\,l(Cos.\,\frac{1}{2}\lambda.\,Cos.\,\frac{1}{2}\mu\ldots)$ Lobatschewsky, Mém. Kasan. 1836. 1. II. 23.

26) $\int l\left\{\frac{1+Sin.\,\lambda.\,Cos.\,x}{1-Sin.\,\lambda.\,Cos.\,x}\cdots\frac{1-Sin.\,\mu.\,Cos.\,x}{1+Sin.\,\mu.\,Cos.\,x}\cdots\right\}dx = 2\pi\,l(Cos.\,\frac{1}{2}\mu\ldots Sec.\,\frac{1}{2}\lambda\ldots)$

27) $\int l\,l\,Tang.\,x\,dx = \frac{\pi}{2}\,l\left\{\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}\sqrt{2\pi}\right\}$ V. T. 191. N°. 1.

F. Log. Circ. Dir. } Log. de Circ. Dir. d'autre forme. Avec fact. Circ. Dir. TABLE 335. Lim. 0 et $\frac{\pi}{2}$.

1) $\int l\,Tang.\frac{1}{2}x.\,Sin.\,x\,dx = -\,l\,2$ Lobatschewsky, Mém. Kasan. 1836. II. 28.

2) $\int l\,Tang.\left(\frac{\pi}{4}\pm x\right)Sin.\,2x\,dx = \pm\frac{\pi}{2}$ V. T. 63. N°. 12.

3) $\int l(1+Cos.\,x)\,Tang.\,x\,dx = \frac{1}{12}\pi^2$ V. T. 160. N°. 1.

4) $\int l(1-Cos.\,x)\,Tang.\,x\,dx = -\frac{1}{6}\pi^2$ V. T. 160. N°. 5.

5) $\int l(1-p^2\,Sin.^2\,x)\,Sin.^2\,x\,dx = \frac{1}{2}\pi\,l\frac{1+\sqrt{(1-p^2)}}{2} - \frac{\pi}{4}\,\frac{1-\sqrt{(1-p^2)}}{1+\sqrt{(1-p^2)}}$ Roberts, L. 11. 201.

6) $\int l(1+p^2\,Cos.^2\,x)\,Sin.^2\,x\,dx = \frac{\pi}{2}\,l\frac{1+\sqrt{(1+p^2)}}{2} + \pi\,\frac{\sqrt{(1+p^2)}-1}{2p^2} - \frac{1}{4}\pi$

F. Log. Circ. Dir.	Log. de Circ. Dir. d'autre forme. Avec fact. Circ. Dir.	TABLE 335 suite.	Lim. 0 et $\frac{\pi}{2}$.

7) $\int l(1+p^2\,Cos.^2\,x)\,Cos.^2\,x\,dx = \frac{1}{2}\pi\,l\frac{1+\sqrt{(1+p^2)}}{2} + \pi\frac{1-\sqrt{(1+p^2)}}{2p^2} + \frac{1}{4}\pi$ V. T. 334. N°. 8 et T. 335. N°. 6.

8) $\int l(1+p^2\,Cos.^2\,x)\,Cos.\,2x\,dx = \pi\frac{1-\sqrt{(1+p^2)}}{p^2} + \frac{1}{2}\pi$ V. T. 334. N°. 8 et T. 335. N°. 6.

9) $\int l(1-Sin.^4\,x)\,Tang.\,x\,dx = -\frac{1}{24}\pi^2$ V. T. 160. N°. 11.

10) $\int l(1+Cos.^2\,x)\,Tang.\,x\,dx = \frac{1}{24}\pi^2$ V. T. 160. N°. 6.

11) $\int l\,Tang.\left(\frac{\pi}{4}\pm x\right)Tang.\,x\,dx = \pm\frac{1}{4}\pi^2$ V. T. 183. N°. 3.

12) $\int l(p\,Tang.\,x)\,Sin.(q\,Tang.\,x).\,Tang.\,x\,dx = \frac{\pi}{4}e^{-q}\{2\,l p - Ei.(q)\} - \frac{\pi}{4}e^{q}\,Ei.(-q)$ V. T. 417. N°. 5.

13) $\int l(p\,Tang.\,x)\,Cos.(q\,Tang.\,x)\,dx = \frac{\pi}{4q}e^{-q}\{2\,l p - Ei.(q)\} + \frac{\pi}{4q}e^{q}\,Ei.(-q)$ V. T. 417. N°. 6.

14) $\int Sin.(p\,l\,Sin.\,x)\,Tang.\,x\,dx = \frac{\pi}{4}\frac{1+e^{p\pi}}{1-e^{p\pi}} + \frac{1}{2p}$ V. T. 404. N°. 11.

15) $\int Sin.(p\,l\,Sin.\,x)\,Sin.^q\,x.\,Tang.\,x\,dx = -\sum_1^\infty\frac{p}{(2n+q)^2+p^2}$ V. T. 404. N°. 12.

16) $\int l\,Tang.\,x.\,Cos.(q\,Tang.\,x)\,dx = \frac{\pi}{4q}\{e^{q}\,Ei.(-q) - e^{-q}\,Ei.(q)\}$ V. T. 417. N°. 2.

17) $\int l\,Cot.\,x.\,Sin.(q\,Tang.\,x)\,dx = \frac{\pi}{4q}\{e^{-q}\,Ei.(q) + e^{q}\,Ei.(-q)\}$ V. T. 417. N°. 7.

18) $\int l\,Cot.\,x.\,Cos.(q\,Tang.\,x)\,dx = \frac{\pi}{4q}\{e^{-q}\,Ei.(q) - e^{q}\,Ei.(-q)\}$ V. T. 417. N°. 8.

F. Log. en num. $(l\,Sin.\,x)^a$. Circ. Dir. rat. en dén. monôme.	TABLE 336.	Lim. 0 et $\frac{\pi}{2}$.

1) $\int l\,Sin.\,x\,\frac{1+Sin.^2\,x}{Sin.\,2x}\,dx = -\infty$ V. T. 153. N°. 11.

2) $\int l\,Sin.\,x\,\frac{dx}{Cos.\,x} = -\frac{1}{8}\pi^2$ V. T. 152. N°. 13.

3) $\int l\,Sin.\,x\,\frac{dx}{Cos.\,2x} = \frac{1}{8}\pi^2$ V. T. 340. N°. 14.

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4) $\int l\,Sin.\,x \frac{dx}{Tang.^p x} = \frac{1}{2}\frac{\pi}{p-1} Sec.\frac{1}{2}p\pi, p<1;$ V. T. 63. N°. 4.

5) $\int l\,Sin.\,x \frac{Sin.^{2p-1} x}{Cos.^{p+1} x} dx = -\frac{\pi}{4p} Cosec.\frac{1}{2}p\pi, 0<p<\frac{1}{2}$ V. T. 58. N°. 25.

6) $\int l\,Sin.\,x \frac{p-2\,Cos.^2 x}{Tang.^{p-1} x} dx = \frac{1}{4}p\pi\,Cosec.\frac{1}{2}p\pi, p<2;$ V. T. 63. N°. 5.

7) $\int l\,Sin.\,x \frac{p\,Cos.\,2x + Sin.^2\,2x}{Tang.^p x}\frac{dx}{Sin.\,2x} = -\frac{p+1}{4} l\,Sec.\left\{\frac{p+1}{2}\pi\right\}, 0<p<1;$ V. T. 63. N°. 6.

8) $\int l\,Sin.\,x \frac{(1-Sin.\,x)^{p-1}}{Sin^p x}\frac{dx}{Tang.\,x} = \infty$ V. T. 64. N°. 4.

9) $\int l\,Sin.\,x.\,Cos.\,(q\,Tang.\,x)\frac{dx}{Cos.^2 x} = \pi\frac{e^{-q}-1}{2q}$ V. T. 70. N°. 2.

10) $\int l\,Sin.\,x.\,Cos.\,(p\,l\,Cos.\,x)\frac{dx}{Tang.\,x} = \frac{1}{2p^2} + \frac{\pi}{4p}\frac{1+e^{p\pi}}{1-e^{p\pi}}$ V. T. 335. N°. 14.

11) $\int (l\,Sin.\,x)^2.\,Sin.\,(pl\,Cos.\,x)\frac{dx}{Cot.\,x} = \infty$ V. T. 336. N°. 10.

12) $\int (l\,Sin.\,x)^3 \frac{dx}{Cos.\,x} = -\frac{1}{16}\pi^4$ V. T. 154. N°. 14.

13) $\int (l\,Sin.\,x)^5 \frac{dx}{Cos.\,x} = -\frac{1}{8}\pi^6$ V. T. 155. N°. 4.

14) $\int (l\,Sin.\,x)^7 \frac{dx}{Cos.\,x} = -\frac{17}{32}\pi^8$ V. T. 155. N°. 10.

15) $\int (l\,Sin.\,x)^{2a} \frac{dx}{Cos.\,x} = \frac{2^{2a+1}-1}{2^{2a+1}} 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 158. N°. 4.

16) $\int (l\,Sin.\,x)^{2a-1} \frac{dx}{Cos.\,x} = \frac{2^{2a}-1}{4a}\pi^{2a} B_{2a-1}$

17) $\int (l\,Sin.\,x)^{a-1} \frac{dx}{Cos.\,x} = \frac{1^{a-1/1}}{(-1)^{a-1}} \sum_0^\infty \frac{1}{(2n+1)^a}$

18) $\int (l\,Sin.\,x)^{a-1} \frac{Sin.^q x}{Cos.\,x} dx = \frac{1^{a-1/1}}{(-1)^{a-1}} \sum_0^\infty \frac{1}{(2n+q+1)^a}$

(16–18) Arndt, Gr. 6. 434.

19) $\int (l\,Sin.\,x)^{2a} \frac{Sin.^q x - Cosec.^q x}{Cos.\,x} dx = \frac{1}{b}(-1)^a(2b\pi)^{2a+1}\sum_1^b (-1)^{n-1} B''\left(\frac{n}{2b}\right) Sin.\,nq\pi, q^2 = \alpha^2 b^2 < 1;$ V. T. 158. N°. 9.

1) $\int l\ Cos.\ x \frac{dx}{Sin.\ x} = -\frac{1}{8}\pi^2$ V. T. 152. N°. 13.

2) $\int (l Cos. x)^{p-1} \frac{Cos.^q x}{Tang.\ x} dx = \frac{\Gamma(p)}{(-1)^{p-1}} \sum_0^\infty \frac{1}{(q+1+2n)^p}$ V. T. 158. N°. 8.

3) $\int l\ Cos.\ x \frac{dx}{Tang.\ x} = -\frac{1}{24}\pi^2$ V. T. 330. N°. 13.

4) $\int l\ Cos.\ x \frac{Tang.^{p-1} x}{Sin.\ 2x} dx = \frac{\pi}{4(p-1)} Sec.\ \frac{1}{2} p\pi,\ p < 1;$ V. T. 179. N°. 5.

5) $\int l\ Cos.\ x \frac{dx}{Tang.^{p+1} x . Sin. 2x} = -\frac{\pi}{2(p+1)} Sec. \frac{1}{2} p\pi,\ p < 1;$ V. T. 63. N°. 4.

6) $\int l\ Cos.\ x \frac{Cos.\ 2x - p}{Tang.^p x} dx = \frac{1-p}{4}\pi\ Cosec. \left(\frac{p-1}{2}\pi\right),\ p^2 < 1;$ V. T. 63. N°. 5.

7) $\int l\ Cos.\ x \frac{(1 - Cos.\ x)^{p-1}}{Cos.^p x} Tang.\ x\, dx = \infty,\ p < 1;$ V. T. 64. N°. 5.

8) $\int l\ Cos.\ x . Sin. (p\ Tang.\ x) \frac{dx}{Sin.\ 2x} = -\frac{1}{4}\pi\, Ei. (-p)$ V. T. 414. N°. 3.

9) $\int l\ Cos.\ x . Sin. (p\ Tang.\ x) \frac{dx}{Cos.^2 x} = \infty$ V. T. 59. N°. 6.

10) $\int l\ Cos.\ x . Cos. (p\ Tang.\ x) \frac{dx}{Cos.^2 x} = \infty$ V. T. 59. N°. 5.

11) $\int l\ Cos.\ x . Cos. (p\ Cot.\ x) \frac{dx}{Sin.^2 x} = -\pi \frac{1 - e^{-q}}{2p}$ V. T. 60. N°. 5.

12) $\int l\ Cos.\ x . Cosec.^2 (q\ Tang.\ x) \frac{dx}{Cos.^2 x} = \frac{1}{2} \frac{\pi}{1 - e^{2q}}$ V. T. 70. N°. 12.

13) $\int l\ Cos.\ x . Sec.^2 (q\ Tang.\ x) \frac{dx}{Cos.^2 x} = \infty$ V. T. 59. N°. 7.

14) $\int l\ Cos.\ x . Sec.^2 (q\ Cot.\ x) \frac{dx}{Sin.^2 x} = \frac{\pi}{2p} \frac{1 - e^{2p}}{1 + e^{2p}}$ V. T. 60. N°. 6.

15) $\int (l\ Cos.\ x)^3 \frac{dx}{Sin.\ x} = -\frac{1}{16}\pi^4$ V. T. 154. N°. 14.

16) $\int (l\ Cos.\ x)^3 \frac{dx}{Tang.\ x} = -\frac{1}{240}\pi^4$ V. T. 154. N°. 15.

17) $\int (l\, Cos.\, x)^5 \frac{dx}{Sin.\, x} = -\frac{1}{8}\pi^6$ V. T. 155. N°. 4.

18) $\int (l\, Cos.\, x)^5 \frac{dx}{Tang.\, x} = -\frac{1}{504}\pi^6$ V. T. 155. N°. 5.

19) $\int (l\, Cos.\, x)^7 \frac{dx}{Sin.\, x} = -\frac{17}{32}\pi^8$ V. T. 155. N°. 10.

20) $\int (l\, Cos.\, x)^{2a-1} \frac{dx}{Sin.\, x} = \frac{1-2^{2a}}{4a}\pi^{2a} B_{2a-1}$ V. T. 158. N°. 5.

21) $\int (l\, Cos.\, x)^{2a-1} \frac{dx}{Tang.\, x} = -\frac{\pi^{2a}}{4a} B_{2a-1}$ V. T. 158. N°. 6.

22) $\int (l\, Cos.\, x)^{2a} \frac{dx}{Sin.\, x} = \frac{2^{2a+1}-1}{2^{2a+1}} 1^{2a}1 \sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 158. N°. 4.

23) $\int (l\, Cos.\, x)^{2a} \frac{Cos.^q x - Sec.^q x}{Sin.\, x} dx = \frac{1}{b}(-1)^a(2b\pi)^{2a+1} \sum_1^b (-1)^{n-1} B''\left(\frac{n}{2b}\right) Sin.\, nq\pi,\ q^2 = \alpha^2 b^2 < 1;$ V. T. 158. N°. 9.

F. Log. en num. $(l\, Tang.\, x)^a$. Circ. Dir. rat. en dén. monôme. TABLE 338. Lim. 0 et $\frac{\pi}{2}$.

1) $\int l\, Tang.\, x \frac{dx}{Sin.\, 4x} = -\infty$ V. T. 309. N°. 2 et T. 357. N°. 2.

2) $\int l\, Tang.\, x \frac{dx}{Cos.\, 2x} = -\frac{1}{4}\pi^2$ V. T. 180. N°. 11.

3) $\int l\, Tang.\, x \frac{dx}{Tang.\, 2x} = -\infty$ V. T. 309. N°. 3 et T. 357. N°. 4.

4) $\int l\, Tang.\, x \frac{Tang.^p x}{Cos.\, 2x} dx = -\left\{\frac{1}{2}\pi\, Cosec.\left(\frac{p+1}{2}\pi\right)\right\}^2,\ p^2 < 1;$ V. T. 180. N°. 12.

5) $\int l\, Tang.\, x \frac{dx}{Cos.\, 2x.\, Tang.^p x} = -\left\{\frac{1}{2}\pi\, Cosec.\left(\frac{p+1}{2}\pi\right)\right\}^2,\ p^2 < 1;$ V. T. 180. N°. 12.

6) $\int l(b\, Tang.\, x).\, Sin.\,(a\, Tang.\, x) \frac{dx}{Sin.\, 2x} = -\frac{1}{2}\pi\left\{A + l\frac{a}{b}\right\}$ V. T. 414. N°. 2.

7) $\int l\, Tang.\, x \frac{1 - Tang.^p x}{Cos.\, 2x} dx = \left(\frac{1}{2}\pi\, Tang.\, \frac{1}{2}p\pi\right)^2,\ p^2 < 1;$ V. T. 185. N°. 7.

8) $\int (l\,Tang.\,x)^3 \frac{dx}{Cos.\,2\,x} = -\frac{1}{8}\pi^4$ V. T. 310. N°. 1 et T. 358. N°. 11.

9) $\int (l\,Tang.\,x)^5 \frac{dx}{Cos.\,2\,x} = -\frac{1}{4}\pi^6$ V. T. 310. N°. 5 et T. 358. N°. 14.

10) $\int (l\,Tang.\,x)^7 \frac{dx}{Cos.\,2\,x} = -\frac{17}{16}\pi^8$ V. T. 310. N°. 8 et T. 358. N°. 15.

11) $\int (l\,Tang.\,x)^{2a-1} \frac{dx}{Cos.\,2\,x} = \frac{1-2^{2a}}{2\,a} B_{2a-1}\,\pi^{2a}$ V. T. 310. N°. 9 et T. 359. N°. 3.

12) $\int (l\,Tang.\,x)^{2a} \frac{dx}{Cos.\,2\,x} = 0$ V. T. 310. N°. 10 et T. 359. N°. 4.

13) $\int (l\,Tg.\,x)^{2a} (Tg.^q x + Cot.^q x)\,dx = \frac{2}{b}(-1)^{a+1}(2b\pi)^{2a+1} \sum_1^b (-1)^{n-1} B''\left(\frac{2n-1}{4\,b}\right) Cos.\left(\frac{2\,n-1}{2} q\pi\right)$ V. T. 184. N°. 5, 6.

14) $\int (l\,Tang.\,x)^{2a} \frac{(Tang.^q x - Cot.^q x)}{Cos.\,2\,x}\,dx = \frac{2}{b}(-1)^{a+1}(2b\pi)^{2a+1} \sum_1^b (-1)^{n-1} B''\left(\frac{n}{2\,b}\right) Sin.\,q\,n\,\pi$ $\Bigg\}$, $q^2 = \alpha^2 b^2 < 1$;

F. Log. en num. d'autre fonct. ent. Circ. Dir. rat. en dén. monôme. TABLE 339. Lim. 0 et $\frac{\pi}{2}$.

1) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{dx}{Sin.\,2\,x} = \pm \frac{1}{2}\pi^2$ V. T. 338. N°. 2.

2) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{Tang.^{p-1} x}{Sin.\,2\,x}\,dx = \pm \frac{\pi}{1-p} Cot.\,\frac{1}{2}\,p\,\pi$ V. T. 63. N°. 14.

3) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{dx}{Sin.\,2\,x.\,Tang.^{p-1} x} = \pm \frac{\pi}{1-p} Cot.\,\frac{1}{2}\,p\,\pi$ V. T. 63. N°. 15.

4) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{(p+q)\,(Tang.^{p+q} x + Cot.^{p+q} x) + (p-q)\,(Tang.^{p-q} x + Cot.^{p-q} x)}{Sin.\,2\,x}\,dx =$

$= \pm \frac{4\,\pi\,Sin.\,p\,\pi}{Cos.\,p\,\pi + Cos.\,q\,\pi}$, $p + q < 1$; V. T. 64. N°. 15.

5) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{dx}{Tang.\,x} = \pm \frac{1}{2}\pi^2$ V. T. 183. N°. 3.

6) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{Tang.^p x + Cot.^p x}{Sin.\,2\,x}\,dx = \pm \frac{2\,\pi}{p+1} Tang.\,\frac{1}{2}\,p\,\pi$ V. T. 64. N°. 10.

7) $\int l\,Sin.\,(q\,Tang.\,x) \frac{dx}{Cos.\,2\,x} = \frac{1}{2}\,q\,\pi - \frac{1}{4}\pi^2$ V. T. 415. N°. 13.

8) $\int l\, Cos.(q\, Tang.\, x) \frac{dx}{Cos.\, 2x} = \frac{1}{2} q\pi$ V. T. 415. N°. 14.

9) $\int l\, Tang.(q\, Tang.\, x) \frac{dx}{Cos.\, 2x} = -\frac{1}{4}\pi^2$ V. T. 415. N°. 15.

10) $\int l(q\, Tang.\, x).\, Sin.^{2a}\, 2x\, dx = \frac{1^{a/2}}{2^{a/2}} \pi\, l\, q$ V. T. 183. N°. 8.

11) $\int l(q\, Tang.\, x).\, Sin.^{2a-1}\, 2x\, dx = \frac{1^{a-1/1}}{a^{a/1}} 2^{2a-2}\, l\, q$ V. T. 183. N°. 9.

12) $\int l(q\, Tang.\, x).\, Sin.^{p-1}\, 2x\, dx = 2^{2p-2}\, l\, q \frac{\{\Gamma(\frac{1}{2}p)\}^2}{\Gamma(p)}$ V. T. 183. N°. 10.

13) $\int l\, Tang.\, x.\, Sin.(q\, Cot.\, x) \frac{dx}{Tang.\, x} = \frac{\pi}{4}\{e^{-q} Ei.(q) + e^{q} Ei.(-q)\}$ V. T. 417. N°. 7.

14) $\int l\, Tang.\, x.\, Sin.(q\, Cot.\, x) \frac{dx}{Cos.\, 2x} = \frac{\pi}{2}\left\{Ci.(q).\, Cos.\, q + Si.(q).\, Sin.\, q - \frac{1}{2} Sin.\, q\right\}$ V. T. 417. N°. 9.

15) $\int l(p\, Cot.\, x).\, Sin.(q\, Cot.\, x) \frac{dx}{Tang.\, x} = \frac{\pi}{4} e^{-q}\{2\, l\, p - Ei.(q)\} - \frac{\pi}{4} e^{q} Ei.(-q)$ V. T. 417. N°. 5.

16) $\int l\, Tang.\, x.\, Cos.(q\, Tang.\, x) \frac{dx}{Cos.\, 2x} = -\frac{\pi}{2}\left\{Ci.(q).\, Sin.\, q - Si.(q).\, Cos.\, q + \frac{1}{2} Cos.\, q\right\}$ V. T. 417. N°. 10.

17) $\int l\, Tang.^2\left(\frac{\pi}{4} \pm x\right).\, Sin.(q\, Tang.\, x) \frac{dx}{Cos.^2\, x} = \pm \frac{2\pi}{q} Sin.\, q$ V. T. 70. N°. 6.

18) $\int l\, Tang.^2\left(\frac{\pi}{4} \pm x\right).\, Sin.(q\, Cot.\, x) \frac{dx}{Sin.^2\, x} = \pm \frac{2\pi}{q} Sin.\, q$ V. T. 71. N°. 15.

19) $\int l\, Tang.^2\left(\frac{\pi}{4} \pm x\right).\, Cos.(q\, Tang.\, x) \frac{dx}{Cos.^2\, x} = \pm \frac{2}{q}\{Si.(q).\, Cos.\, q - Ci.(q).\, Sin.\, q\}$ V. T. 70. N°. 7.

20) $\int l\, Tang.^2\left(\frac{\pi}{4} \pm x\right).\, Tang.(q\, Tang.\, x) \frac{dx}{Cos.^2\, x} = \pm 2\pi$ V. T. 339. N°. 8.

21) $\int l\, Tang.^2\left(\frac{\pi}{4} \pm x\right).\, Cot.(q\, Tang.\, x) \frac{dx}{Cos.^2\, x} = \pm \frac{\pi - 2q}{q}\pi$ V. T. 339. N°. 7.

22) $\int l\, Tang.^2\left(\frac{\pi}{4} \pm x\right).\, Cosec.(q\, Tang.\, x) \frac{dx}{Cos.^2\, x} = \pm \frac{\pi^2}{q}$ V. T. 339. N°. 9.

23) $\int l(1+Sin.x)\frac{dx}{Tang.x} = \frac{1}{12}\pi^2$ V. T. 160. N°. 1.

24) $\int l(1-Sin.x)\frac{dx}{Tang.x} = -\frac{1}{6}\pi^2$ V. T. 160. N°. 7.

25) $\int l(1+p\,Sin.x)\frac{Cos.^3 x}{(3-Cos.2x)^2}dx = \frac{1}{8}\left\{(1+p)^2 l(1+p) - p\,l2 - \frac{1}{2}p^2\pi\right\}\frac{1}{1+p^2}$ V. T. 160. N°. 4.

26) $\int l(1+Sin.^2 x)\frac{dx}{Tang.x} = \frac{1}{24}\pi^2$ V. T. 160. N°. 6.

27) $\int l(1-Sin.^4 x)\frac{dx}{Tang.x} = -\frac{1}{24}\pi^2$ V. T. 160. N°. 11.

28) $\int l(1+p\,Cos.x)\frac{dx}{Cos.x} = \frac{1}{8}\pi^2 - \frac{1}{2}(Arccos.p)^2 \;,\; p^2 < 1;$ Winckler, Cr. 45. 102.

29) $\int l(1+Cos.x)\frac{Sin.x}{3+Cos.2x}dx = \frac{\pi}{16}l2$ V. T. 160. N°. 2.

30) $\int l(1+p^2\,Tang.^2 x)\frac{dx}{Cos.2x} = -\pi\,Arctang.p$ ⎫

31) $\int l(1+p^2\,Cot.^2 x)\frac{dx}{Cos.2x} = \pi\,Arctang.p$ ⎭ V. T. 181. N°. 11.

32) $\int Sin.(p\,l\,Sin.x)\frac{dx}{Cos.x} = \frac{\pi}{4}\frac{1-e^{p\pi}}{1+e^{p\pi}}$ V. T. 404. N°. 10.

33) $\int Sin.(p\,l\,Sin.x)\frac{Tang.x}{Sin.^q x}dx = -\sum_1^\infty \frac{p}{(2n-q)^2+p^2}$ V. T. 404. N°. 16.

34) $\int Sin.(p\,l\,Cos.x)\frac{dx}{Tang.x} = \frac{\pi}{4}\frac{1+e^{p\pi}}{1-e^{p\pi}} + \frac{1}{2p}$ V. T. 404. N°. 11.

35) $\int Sin.(p\,l\,Cos.x)\frac{Cos.^q x}{Tang.x}dx = -\sum_1^\infty \frac{p}{(2n+q)^2+p^2}$ V. T. 404. N°. 12.

36) $\int Cos.(p\,l\,Sin.x)\frac{l\,Sin.x}{Cos.x}dx = \frac{\pi^2}{2}\frac{e^{p\pi}}{(e^{p\pi}+1)^2}$ V. T. 404. N°. 14.

1) $\int l\frac{Cos. 2x}{Cos.^2 x}\frac{Tang^{p-2} x}{Sin. 2x}dx = \frac{\pi}{2(p-2)}Cot.\frac{1}{2}p\pi$ V. T. 179. N°. 6.

2) $\int l\frac{1+Sin. x}{1-Sin. x}\frac{dx}{Sin. x} = \frac{1}{2}\pi^2$ Legendre, Exerc. Suppl. 34. — Schlömilch, Gr. 4. 316.

3) $\int l\frac{1+p\,Sin. x}{1-p\,Sin. x}\frac{dx}{Sin. x} = \pi\,Arcsin. p\;,\; p \leqq 1;$ Raabe, Int. 421.

4) $\int l\frac{2\,Cos. x}{1+Cos. x}\frac{dx}{Sin. x} = -\frac{1}{12}\pi^2$ V. T. 160. N°. 10.

5) $\int l\frac{1+Sin. x\sqrt{(1-p^2)}}{1-Sin. x\sqrt{(1-p^2)}}\frac{dx}{Sin. x} = \pi\,Arccos. p\;,\; p^2 \leqq 1;$ V. T. 186. N°. 1.

6) $\int\left\{l\frac{1+Sin. x}{1-Sin. x} - 2\,Sin. x\right\}\frac{dx}{Sin.^3 x} = \frac{1}{4}\pi^2$ Legendre, Exerc. Suppl. 34.

7) $\int l\frac{1+Sin. 2x}{1+Cos. \lambda. Sin. 2x}\frac{Tang^p x + Cot^p x}{Sin. 2x}dx = \frac{2\pi}{p}Cosec. p\pi. (1-Cos. p\lambda)$ V. T. 179. N°. 17.

8) $\int l\frac{(Sin. x + Cos. x)^2}{1+Cos. \lambda. Sin. 2x}\frac{dx}{Sin. 2x} = \frac{1}{2}\lambda^2\;,\; 0 < \lambda < \pi;$ V. T. 179. N°. 13.

9) $\int l\frac{1+q\,Cos. x}{1-q\,Cos. x}\frac{dx}{Cos. x} = \pi\,Arcsin. q\;,\; q \leqq 1;$ Raabe, Cr. 25. 169. — Id., Int. II. 421.

10) $\int l\frac{1+Sin. \lambda. Cos. x}{1-Sin. \lambda. Cos. x}\frac{dx}{Cos. x} = \pi\lambda$ V. T. 166. N°. 7.

11) $\int l\frac{1+q\,Cos. ax}{1-q\,Cos. ax}\frac{dx}{Cos. ax} = \pi\,Arcsin. q\;,\; q \leqq 1;$ Raabe, Cr. 25. 169.

12) $\int l\frac{Sec. x + \sqrt{(1-p^2)}}{Sec. x - \sqrt{(1-p^2)}}\frac{dx}{Cos. x} = \pi\,Arccos. p\;,\; p^2 < 1;$ V. T. 186. N°. 1.

13) $\int l\frac{1+Sin. 2x}{1+Cos. \lambda. Sin. 2x}\frac{Sin^{p-1} x}{Cos^{p+1} x}dx = \frac{2\pi}{p}Cosec. p\pi\,(1-Cos. p\lambda)\;,\; p < 1;$ V. T. 179. N°. 16.

14) $\int l\frac{1+Tang. x}{1-Tang. x}\frac{dx}{Tang. x} = \frac{1}{4}\pi^2$ Schlömilch, Gr. 4. 316. — Id., Gr. 7. 100.

15) $\int l\frac{1+Sin. (Tang. x)}{1-Sin. (Tang. x)}\frac{dx}{Sin. 2x} = \frac{1}{4}\pi^2$ V. T. 414. N°. 4.

16) $\int l\left\{\frac{1+p\,Tang. (Tang. x)}{1-p\,Tang. (Tang. x)}\right\}^2\frac{dx}{Sin. 2x} = \frac{1}{4}\pi^2$ V. T. 414. N°. 5.

1) $\int l\,Cos.\,x.\,(l\,Sin.\,x)^2\,\frac{dx}{Tang.\,x} = -\frac{1}{720}\pi^4$ V. T. 332. N°. 2.

2) $\int l\,Cos.\,x.\,(l\,Sin.\,x)^4\,\frac{dx}{Tang.\,x} = -\frac{1}{2520}\pi^6$ V. T. 332. N°. 3.

3) $\int l\,Cos.\,x.\,(l\,Sin.\,x)^{2a}\,\frac{dx}{Tang.\,x} = -\frac{\pi^{2a+2}}{4(a+1)(2a+1)}B_{2a+1}$ V. T. 332. N°. 5.

4) $\int l\,Tang.^2\left(\frac{\pi}{4}\pm x\right).(l\,Tang.\,x)^2\,\frac{dx}{Sin.\,2x} = \pm\frac{1}{12}\pi^4$ V. T. 338. N°. 8.

5) $\int l\,Tang.^2\left(\frac{\pi}{4}\pm x\right).(l\,Tang.\,x)^4\,\frac{dx}{Sin.\,2x} = \pm\frac{1}{10}\pi^6$ V. T. 338. N°. 9.

6) $\int l\,Tang.^2\left(\frac{\pi}{4}\pm x\right).(l\,Tang.\,x)^6\,\frac{dx}{Sin.\,2x} = \pm\frac{17}{56}\pi^8$ V. T. 338. N°. 10.

7) $\int l\,Tang.^2\left(\frac{\pi}{4}\pm x\right).(l\,Tang.\,x)^{2a}\,\frac{dx}{Sin.\,2x} = \pm\frac{1-2^{2a+2}}{(a+1)(2a+1)}\pi^{2a+2}B_{2a+1}$ V. T. 338. N°. 11.

8) $\int l\,Tang.^2\left(\frac{\pi}{4}\pm x\right).(l\,Tang\,x)^{2a-1}\,\frac{dx}{Sin.\,2x} = 0$ V. T. 338. N°. 12.

9) $\int l\,Tang.^2\left(\frac{\pi}{4}\pm x\right)\frac{p\,l\,Tang.\,x+1}{Sin.\,2x}Tang.^p\,x\,dx = \pm\frac{1}{2}\pi^2\,Cosec.^2\left\{\frac{p+1}{2}\pi\right\}$ V. T. 338. N°. 4.

10) $\int l\,Tang.^2\left(\frac{\pi}{4}\pm x\right)\frac{p\,l\,Tang.\,x-1}{Tang.^p\,x.\,Sin.\,2x}dx = \mp\frac{1}{2}\pi^2\,Cosec.^2\left\{\frac{p+1}{2}\pi\right\}$ V. T. 338. N°. 5.

11) $\int l\,Tang.\,x.\,l\frac{1+b\,Sin.\,2x}{1-b\,Sin.\,2x}\frac{dx}{Sin.\,2x} = 0\ ,\ b \leqq 1;$ V. T. 179. N°. 19.

12) $\int l(1+p^2\,Tang.^2\,x).\,l(1+q^2\,Cot.^2\,x)\frac{dx}{Sin.^2\,x} = 2\pi\frac{pq+1}{q}l(1+pq)-2p\pi$ V. T. 179. N°. 21.

13) $\int l(1+p^2\,Tang.^2\,x).\,l(1+q^2\,Cot.^2\,x)\frac{dx}{Cos.^2\,x} = 2\pi\frac{pq+1}{p}l(1+pq)-2q\pi$ V. T. 179. N°. 21.

14) $\int l(1+p^2\,Tang.^2\,x).\,l(1+q^2\,Cot.^2\,x)\frac{dx}{Sin.^2\,2x} = \frac{p+q}{2}\pi\left\{\frac{pq+1}{pq}l(1+pq)-1\right\}$ V. T. 341. N°. 12, 13.

15) $\int l(1+p^2\,Tang.^2\,x).\,l(1+q^2\,Cot.^2\,x)\frac{Cos.\,2x}{Sin.^2\,2x}dx = \frac{p-q}{2}\pi\left\{\frac{pq+1}{pq}l(1+pq)-1\right\}$ V. T. 341. N°. 12, 13.

1) $\int l\,Sin.x \frac{Cos.x}{1+Sin.x}\,dx = -\frac{1}{12}\pi^2$ V. T. 339. N°. 23.

2) $\int l\,Sin.x \frac{Cos.x}{1-Sin.x}\,dx = -\frac{1}{6}\pi^2$ V. T. 339. N°. 24.

3) $\int l\,Sin.x \frac{Sin.2x}{1+Sin.^2x}\,dx = -\frac{1}{24}\pi^2$ V. T. 339. N°. 26.

4) $\int l\,Sin.x \frac{Tang.^2x.Sin.2x}{1+Sin.^2x}\,dx = -\frac{1}{48}\pi^2$ V. T. 339. N°. 27.

5) $\int l\,Sin.x \frac{dx}{q^2\,Sin.^2x+Cos.^2x} = -\frac{\pi}{2q}\,l(1+q)$ V. T. 369. N°. 10.

6) $\int l\,Sin.x \frac{1+Cos.^2\lambda.Sin.^2x}{\{Sin.^2\lambda.Sec.x+Cos.^2\lambda.Cos\,x\}^2}\,\frac{dx}{Cos.x} = Sec.\lambda.\,l\,Tang.\frac{1}{2}\lambda$ V. T. 66. N°. 17.

7) $\int l\,Cos.x \frac{Sin.x}{1+Cos.x}\,dx = -\frac{1}{12}\pi^2$ V. T. 335. N°. 3.

8) $\int l\,Cos.x \frac{Sin.x}{1-Cos.x}\,dx = -\frac{1}{6}\pi^2$ V. T. 335. N°. 4.

9) $\int l\,Cos.x \frac{dx}{Sin.^2x+q^2\,Cos.^2x} = -\frac{\pi}{2q}\,l(1+q)$ V. T. 368. N°. 8.

10) $\int l\,Tang.x \frac{dx}{2-Sin.2x} = 0$ V. T. 311. N°. 1 et T. 357. N°. 5.

11) $\int l\,Tang.x \frac{Sin.2x}{4\,Cos.^2 2x+Sin.^2 2x}\,dx = 0$ V. T. 311. N°. 6 et T. 357. N°. 6.

12) $\int l\,Tang.x \frac{dx}{p^2\,Cos.^2x+q^2\,Sin.^2x} = \frac{\pi}{2pq}\,l\frac{p}{q}$ V. T. 187. N°. 10.

13) $\int l\,Tang.\tfrac{1}{2}x \frac{Sin.x}{1-Cos.^2\lambda.Sin.^2x}\,dx = Cosec.2\lambda\{2L(\tfrac{1}{2}\pi-\lambda)-(\pi-2\lambda)l2\}$ Lobatschewsky, Mém. Kasan. 1836. I. I. 106. — Id., ib. II. 38.

14) $\int (l\,Tang.x)^2 \frac{dx}{2+Sin.2x} = \frac{8}{243}\pi^3\sqrt{3}$ V. T. 312. N°. 1 et T. 358. N°. 2.

15) $\int (l\,Tang.x)^2 \frac{dx}{2-Sin.2x} = \frac{10}{243}\pi^3\sqrt{3}$ V. T. 312. N°. 3 et T. 358. N°. 3.

16) $\int (l\,Tang.x)^2 \frac{dx}{1+Cos.\lambda.Sin.2x} = \frac{\pi^2-\lambda^2}{3}\lambda\,Cosec.\lambda$ V. T. 184. N°. 9.

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17) $\int (l\,Tang.x)^2 \frac{dx}{1 - Cos.\lambda.\,Sin.2x} = 4\lambda\, Cosec.\lambda \left(\frac{1}{6}\pi^2 - \frac{1}{4}\pi\lambda + \frac{1}{12}\lambda^2\right)$ V. T. 312. N°. 4 T. 358. N°. 5.

18) $\int (l\,Tang.x)^2 \frac{dx}{1 - Sin.^2 x.\,Cos.^2 x} = \frac{2}{27}\pi^3 \sqrt{3}$ V. T. 312. N°. 6 et T. 358 N°. 6.

19) $\int (l\,Tang.x)^2 \frac{Sin.2x}{1 - Sin.^2 x.\,Cos.^2 x} dx = \frac{4}{243}\pi^3 \sqrt{3}$ V. T. 312. N°. 7 et T. 358. N°. 7.

20) $\int (l\,Tang.x)^2 \frac{dx}{Sin.^4 x + Cos.^4 x} = \frac{3}{32}\pi^3 \sqrt{2}$ V. T. 312. N°. 5 et T. 358. N°. 8.

21) $\int (l\,Tang.x)^4 \frac{dx}{1 + Cos.\lambda.\,Sin.2x} = \frac{\pi^2 - \lambda^2}{Sin\,\lambda} \frac{7\pi^2 - 3\lambda^2}{5}\lambda$ V. T. 312. N°. 8 et T. 358. N°. 13.

22) $\int (l\,Tang.x)^{2a} \frac{dx}{2 + Sin.2x} = \frac{(-1)^{a+1}}{\sqrt{3}} (2\pi)^{2a+1} B''\left(\frac{1}{3}\right)$ V. T. 184. N°. 11.

23) $\int (l\,Tang.x)^{2a} \frac{dx}{2 - Sin.2x} = \frac{(-1)^{a+1}}{\sqrt{3}} (2\pi)^{2a+1} B''\left(\frac{1}{6}\right)$ V. T. 184. N°. 12.

24) $\int (l\,Tang.x)^{2a} \frac{dx}{1 - Cos.2p\pi.\,Sin.2x} = \frac{1}{2}(-1)^{a+1}(2\pi)^{2a+1} Cosec.2p\pi.B''(p)$ V. T. 184. N°. 13.

25) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{Sin.2x}{1 + p\,Cos.2x} dx = \pm \frac{\pi}{p} Arcsin.p$, $p < 1$; V. T. 355. N°. 1.

26) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{Tang.x}{p^2 Sin.^2 x + Cos.^2 x} dx = \pm \frac{\pi}{p^2} Arctang.p$ V. T. 339. N°. 30.

27) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{Sin.2x}{(1-p)^2 + 4p\,Sin.^2 x} dx = \pm \frac{\pi}{p} Arctang.p$, $p^2 \leqq 1$; V. T. 355. N°. 3.

28) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{Sin.4x}{(1-p)^2 + 4p\,Sin.^2 2x} dx = 0$, $p < 1$; V. T. 355. N°. 4.

29) $\int l\,l\,Tang.x \frac{dx}{2 + Sin.2x} = \frac{\pi}{2\sqrt{3}} l\left(\frac{\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{3})} \sqrt[3]{2\pi}\right)$ V. T. 191. N°. 2.

F. Log. en num. de Circ. binôme. Circ. Dir. rat. en dén. binôme. TABLE 343. Lim. 0 et $\frac{\pi}{2}$.

1) $\int l(p\,Sin.x - r) \frac{Sin.x}{1 - q\,Sin.^2 x} dx = \frac{\pi}{\sqrt{q(1-q)}} l \frac{p\sqrt{q} - \{1 - \sqrt{(1-q)}\}\{r + \sqrt{(r^2 - p^2)}\}}{p\sqrt{q} + \{1 - \sqrt{(1-q)}\}\{r + \sqrt{(r^2 - p^2)}\}}$ V. T. 165. N°. 4.

2) $\int l(1 + Cothp.^2\lambda.\,Sin.^2 x) \frac{Cos.x}{1 - Coshp.^2\lambda.\,Cos.^2 x} dx = \frac{2}{Sinhp.\lambda.\,Coshp.\lambda} L\{\pi - \lambda\}$ Lobatschewsky, Mém. Kasan. 1836. 1. I. 121.

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3) $\int l(1-Cot\,hp.^2\lambda.Sin.^2x)\frac{Cos.x}{1-Cos\,hp.^2\lambda.Cos.^2x}dx = \frac{2}{Sin\,hp.\lambda.Cos\,hp.\lambda}\{\lambda\,l\,Sin\,hp.\lambda+L(\lambda)\}$

4) $\int l(1+Cos\,hp.\lambda.Cos.x)\frac{Cos.x}{1-Cos\,hp.^2\lambda.Cos.^2x}dx = \frac{1}{Sin\,hp.\lambda.Cos\,hp.\lambda}\left\{L(\lambda)+\left(\lambda-\frac{1}{2}\pi\right)l\,Sin\,hp.\lambda\right\}$

5) $\int l(1-Cos\,hp.\lambda.Cos.x)\frac{Cos.x}{1-Cos\,hp.^2\lambda.Cos.^2x}dx = \frac{1}{Sin\,hp.\lambda.Cos\,hp.\lambda}\{L(\lambda)+(\lambda+\tfrac{1}{2}\pi)l\,Sin\,hp.\lambda\}$

Lobatschewsky, Mém. Kasan. 1836. 1. I. 122, 123, 124.

6) $\int l(1+Cos.\lambda.Cos.x)\frac{Cos.x}{1-Cos.^2\lambda.Cos.^2x}dx = \frac{2}{Sin.2\lambda}\left\{L\left(\frac{\pi}{2}-\lambda\right)-\lambda\,l\,Sin.\lambda\right\}$

7) $\int l(1-Cos.\lambda.Cos.x)\frac{Cos.x}{1-Cos.^2\lambda.Cos.^2x}dx = \frac{2}{Sin.2\lambda}\left\{L\left(\frac{\pi}{2}-\lambda\right)+(\pi-\lambda)l\,Sin.\lambda\right\}$

Lobatschewsky, Mém. Kasan. 1836. 1. II. 40. 41.

8) $\int l(pCos.x-r)\frac{Cos.x}{1-qCos.^2x}dx = \frac{\pi}{\sqrt{q(1-q)}}\,l\frac{p\sqrt{q}-\{1-\sqrt{(1-q)}\}\{r+\sqrt{(r^2-p^2)}\}}{p\sqrt{q}+\{1-\sqrt{(1-q)}\}\{r+\sqrt{(r^2-p^2)}\}}$ V. T. 165. N°. 4.

9) $\int l\frac{1+p\,Sin.x}{1-p\,Sin.x}\,\frac{Sin.x}{1-Cos.^2x}dx = \pi\,Arcsin.p\ ,\ p\leqq 1$; V. T. 160. N°. 17.

10) $\int l\frac{1-Cos.\mu.Sin.x}{1+Cos.\mu.Sin.x}\,\frac{Sin.x}{1-Cos.^2\lambda.Sin.^2x}dx = 2\pi Cosec.2\lambda.l\{Sin.\tfrac{1}{2}(\mu+\lambda).Sec.\tfrac{1}{2}(\mu-\lambda)\}$

11) $\int l\frac{1+qSin.x}{1-qSin.x}\,\frac{Sin.x}{1-pSin.^2x}dx = \frac{\pi}{\sqrt{p(1-p)}}\,l\frac{q\sqrt{p}+\{1-\sqrt{(1-p)}\}\{1-\sqrt{(1-q^2)}\}}{q\sqrt{p}-\{1-\sqrt{(1-p)}\}\{1-\sqrt{(1-q^2)}\}}$

Lobatschewsky, Mém. Kasan. 1835. 1.

12) $\int l\frac{1-Cot\,hp.^2\lambda.Sin.^2x}{1+Cot\,hp.^2\lambda.Sin.^2x}\,\frac{Cos.x}{1-Cos^2hp.^2\lambda.Cos.^2x}dx = \frac{2\lambda\,l\,Sin\,hp.\lambda}{Sin\,hp.\lambda.Cos\,hp.\lambda}$ Lobatschewsky, Mém. Kasan. 1836. 1. I. 123.

13) $\int l\frac{1+Cos.\mu.Cos.x}{1-Cos.\mu.Cos.x}\,\frac{dx}{1+Cos.\lambda.Cos.x} = 2\pi\,Cosec.\lambda\,l\left\{Cos.\left(\frac{\pi}{4}-\frac{1}{2}\lambda\right)Sec.\frac{1}{2}(\lambda-\mu)\right\}$

14) $\int l\frac{1-Cos.\lambda.Cos.x}{1+Cos.\lambda.Cos.x}\,\frac{Cos.x}{1-Cos.^2\lambda\,Cos.^2x}dx = 2\pi\,Cosec.2\lambda.l\,Sin.\lambda$

Lobatschewsky, Mém. Kasan. 1836. II. 23, 41.

15) $\int l\frac{1+p\,Cos.x}{1-p\,Cos.x}\,\frac{Cos.x}{1+Sin.^2x}dx = \pi\ Arcsin.p\ ,\ p\leqq 1$; V. T. 160. N°. 17.

16) $\int l\frac{1+p\,Cos.x}{1-p\,Cos.x}\,\frac{Cos.x}{1-q\,Cos.^2x}dx = \frac{\pi}{\sqrt{q(1-q)}}\,l\frac{p\sqrt{q}-\{1+\sqrt{(1-q)}\}\{1-\sqrt{(1-p^2)}\}}{p\sqrt{q}-\{1+\sqrt{(1-q)}\}\{1-\sqrt{(1-p^2)}\}}$ V. T. 170. N°. 15.

17) $\int l \frac{1+q\,Cos.\lambda.Cos.x}{1-q\,Cos.\lambda.Cos.x} \frac{dx}{1-Cos.^2\lambda.Cos.^2x} = \pi\,Cosec.\lambda\, l \frac{1+Sin.\lambda}{Sin.\lambda+\sqrt{(1-q^2\,Cos.^2\lambda)}}$

18) $\int l \frac{1+Cos\,hp.\lambda.Cos.x}{1-Cos\,hp.\lambda.Cos.x} \frac{Cos.x}{1-Cos\,hp.^2\lambda.Cos.^2x} dx = \frac{-\pi\, l\, Sin\,hp.\lambda}{Sin\,hp.\lambda.Cos\,hp.\lambda}$

Lobatschewsky, Mém. Kasan. 1836. I. 71, 119.

19) $\int l \frac{1+Cos.\mu.Cos.x}{1-Cos.\mu.Cos.x} \frac{dx}{1-Cos.^2\lambda.Cos.^2x} = \pi\,Cosec.\lambda\, l \frac{1+Sin.\lambda}{Sin.\lambda+Sin.\mu}$

20) $\int l \frac{1+Cos.\mu.Cos.x}{1-Cos.\mu.Cos\,x} \frac{Cos.x}{1-Cos.^2\lambda.Cos.^2x} dx = 2\pi\,Cosec.2\lambda\, l\left[Cos.\left\{\tfrac{1}{2}(\lambda-\mu)\right\}.Cosec.\left\{\tfrac{1}{2}(\lambda+\mu)\right\}\right]$

Lobatschewsky, Mém. Kasan. 1836. II. 22, 16.

21) $\int l \frac{1+Cos\,hp.\mu.Cos.x}{1-Cos\,hp.\mu.Cos.x} \frac{Cos.x}{1-Cos.^2\lambda.Cos.^2x} dx =$

$= 2\pi\,Cosec.2\lambda\, l\left\{Cothp.\left(\frac{1}{2}Arccos\,hp.\frac{Cos\,hp.\mu}{Cos.\lambda}\right)Tang\,hp.\left(\frac{1}{2}Arccos\,hp.\frac{Tang.\lambda}{Tang\,hp.\mu}\right)\right\}$

Lobatschewsky, Mém. Kasan. 1836. I. 78.

F. Log. en num.
Circ. Dir. rat. en dén.: puissance de bin. TABLE 344. Lim. 0 et $\frac{\pi}{2}$.

1) $\int l\,Sin.x \frac{dx}{(Sin.x \pm q\,Cos.x)^2} = \frac{1}{q(1+q^2)}\left\{\pm lq - \frac{1}{2}q\pi\right\}$ V. T. 65. N°. 8.

2) $\int l\,Sin.x \frac{Sin.^2x - p^2\,Cos.^2x}{(p^2\,Cos.^2x + Sin.^2x)^2} dx = \frac{\pi}{2p(p+1)}$ V. T. 66. N°. 11.

3) $\int l\left(\frac{1}{2}Sin.2x\right) \frac{dx}{(Sin.x \pm q\,Cos.x)^2} = \mp \frac{\pi}{1+q^2} \pm \frac{1-q^2}{1+q^2}\frac{1}{q} lq$ V. T. 65. N°. 7, 8.

4) $\int l\left(\frac{1}{2}Sin.2x\right) \frac{Sin.2x}{(q^2\,Sin.^2x + Cos.^2x)^2} dx = \frac{2}{q^2}\frac{1+q^2}{1-q^2} lq$ V. T. 66. N°. 21, 22.

5) $\int l\,Cos.x \frac{Sin.2x}{(q^2\,Sin.^2x + Cos.^2x)^2} dx = \frac{2}{q^2(1-q^2)} lq$ V. T. 66. N°. 21.

6) $\int l\,Cos.x \frac{Sin.2x}{(Sin.^2x + q^2\,Cos.^2x)^2} dx = \frac{2}{1-q^2} lq$ V. T. 66. N°. 22.

7) $\int l\,Cos.x \frac{dx}{(Sin.x \pm q\,Cos.x)^2} = \frac{-q}{1+q^2}\left\{\frac{\pi}{2q} \pm lq\right\}$ V. T. 65 N°. 7.

8) $\int l\,Cos.x \frac{p^2\,Sin.x - Cos.^2x}{(p^2\,Sin.^2x + Cos.^2x)^2} dx = -\frac{\pi}{2p(p+1)}$ V. T. 66. N°. 3.

9) $\int l\,Cos.x \frac{Cos.^p x - Sec.^p x}{(Cos.^p x + Sec.^p x)^2} Tang.x\,dx = \frac{\pi}{4p^2}$ V. T. 66. N°. 14.

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10) $\int l\,Cos.\,x \frac{Cos.^p x}{(1-Cos.\,x)^{p+1}} Tang.\,x\,dx = -\frac{\pi}{p} Cosec.\,p\pi$ V. T. 67. N°. 1.

11) $\int l\,Tang.\,x \frac{dx}{(Cos.\,x + Sin.\,x)^2} = 0$ V. T. 182. N°. 1.

12) $\int l\,Tang.\,x \frac{dx}{(Sin.\,x \pm q\,Cos.\,x)^2} = \pm \frac{1}{q} l q$ V. T. 67. N°. 7, 8.

13) $\int l\,Tang.\,x \frac{dx}{(q\,Sin.\,x + Cos.\,x)^2} = \frac{1}{q} l \frac{1}{q}$ V. T. 182. N°. 1.

14) $\int l\,Tang.\,x \frac{Sin.\,2x}{(Cos.^2 x + q^2 Sin.^2 x)^2} dx = -\frac{2}{q} l q$ V. T. 66. N°. 21, 22.

15) $\int (l\,Tang.\,x)^2 \frac{dx}{(Sin.\,x - Cos.\,x)^2} = \frac{2}{3}\pi^2$ V. T. 182. N°. 3.

16) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right) \frac{Sin.\,2x}{(q^2\,Cos.^2 x + Sin.^2 x)^2} dx = \pm \frac{4}{1+q^2} \frac{\pi}{q}$ V. T. 182. N°. 22.

F. Log. en num.
Circ. Dir. en dén. à fact. bin. et autre. TABLE 345. Lim. 0 et $\frac{\pi}{2}$.

1) $\int l\,Sin.\,x \frac{Sin.^2 x}{1 + Sin.^4 x} \frac{dx}{Cos.\,x} = \frac{-\pi^2}{16(2+\sqrt{2})}$ V. T. 158. N°. 16.

2) $\int l\,Sin.\,x \frac{p\,Sin.^p x(Sin.^{2q} x - 1) + q\,Sin.^q x(Sin.^{2p} x - 1)}{(Sin.^{p+q} x + 1)^2} \frac{dx}{Tang.\,x} = \frac{\pi}{p+q} Sec.\left(\frac{q-p}{q+p}\frac{\pi}{2}\right)$ V. T. 68. N°. 15.

3) $\int l\,Sin.\,x \frac{p\,Sin.^p x(Sin.^{2q} x - 1) - q\,Sin.^q x(Sin.^{2p} x - 1)}{(Sin.^{p+q} x - 1)^2} \frac{dx}{Tang.\,x} = \frac{\pi}{p+q} Tang.\left(\frac{q-p}{q+p}\frac{\pi}{2}\right)$ V. T. 68. N°. 16.

4) $\int l\,Sin.\,x \frac{Sin.^p x - Cosec.^p x}{(Sin.^p x + Cosec.^p x)^2} \frac{dx}{Tang.\,x} = \frac{\pi}{4p^2}$ V. T. 68. N°. 14.

5) $\int l\,Sin.\,x \frac{Sin.^p x}{(1 - Sin.\,x)^{p+1}} \frac{dx}{Tang.\,x} = -\frac{\pi}{p} Cosec.\,p\pi$ V. T. 68. N°. 23.

6) $\int l\,Tang.\,x \frac{Tang.^p x}{Sin.\,x + Cos.\,x} \frac{dx}{Sin.\,x} = -\pi^2\,Cos.\,p\pi.\,Cosec.^2 p\pi,\ p < 1;$ V. T. 338. N°. 4 et T. 345. N°. 7.

7) $\int l\,Tang.\,x \frac{Tang.^p x}{Sin.\,x - Cos.\,x} \frac{dx}{Sin.\,x} = \pi^2\,Cosec.^2 p\pi,\ p < 1;$ V. T. 183. N°. 1.

8) $\int l\,Tang.\,x \frac{1}{Sin.\,x + Cos.\,x} \frac{dx}{Sin.\,x.\,Tang.^p x} = -\pi^2\,Cos.\,p\pi.\,Cosec.^2 p\pi,\ p<1;$ V. T. 338. N°. 5. et T. 345. N°. 7.

9) $\int l\,Tang.\,x\,\frac{1}{Sin.\,x - Cos.\,x}\,\frac{dx}{Cos.\,x.\,Tang.^p\,x} = \pi^2\,Cosec.^2\,p\pi\,,\ p<1;$ V. T. 183 N°. 2.

10) $\int l\,Tang.\,x\,\frac{Tang.^q\,x - Cot.^q\,x}{Tang.^p\,x + Cot.^p\,x}\,\frac{dx}{Sin.\,2\,x} = 0$ V. T. 313. N°. 8 et T. 357. N°. 10.

11) $\int l\,Tang.\,x\,\frac{Tang.^q\,x + Cot.^q\,x}{Tang.^p\,x - Cot.^p\,x}\,\frac{dx}{Sin.\,2\,x} = 0$ V. T. 313. N°. 9 et T. 357. N°. 11.

12) $\int l\,Tang.\,x\,\frac{Cos.\,2\,x}{Tang.^p\,x + Cot.^p\,x}\,\frac{dx}{Sin.^2\,2\,x} = -\frac{\pi^2}{8\,p^2}\,Sin.\,\frac{\pi}{2\,p}\,Sec.^2\,\frac{\pi}{2\,p}$ V. T. 313. N°. 6 et T. 357. N°. 9.

13) $\int l\,Tang.\,x\,\frac{1}{Tang.^p\,x - Cot.^p\,x}\,\frac{dx}{Sin.^2\,2\,x} = \frac{\pi^2}{8\,p^2}\,Sec.^2\,\frac{\pi}{2\,p}$ V. T. 313. N°. 7 et T. 357. N°. 8.

14) $\int l\,Tang.\,x\,\frac{Sin.^2\,2\,x}{Sin.^4\,x + Cos.^4\,x}\,\frac{dx}{Cos.\,2\,x} = -\frac{\pi}{2\,(2+\sqrt{2})}$ V. T. 313. N°. 5 et T. 357. N°. 7.

15) $\int l\,Tang.\,x\,\frac{Tang.^p\,x - Cot.^p\,x}{(Tang.^p\,x + Cot.^p\,x)^2}\,\frac{dx}{Sin.\,2\,x} = \frac{\pi}{4\,p^2}$ V. T. 68. N°. 19.

16) $\int l\,Tang.\,x\,\frac{p\,Tang.^p\,x\,(Tang.^{2q}\,x - 1) + q\,Tang.^q\,x\,(Tang.^{2p}\,x - 1)}{(Tang.^{p+q}\,x + 1)^2}\,\frac{dx}{Sin.\,2x} = \frac{\pi}{p+q}\,Sec.\left\{\frac{q-p}{q+p}\,\frac{\pi}{2}\right\}$ V. T. 68. N°. 20.

17) $\int l\,Tang.\,x\,\frac{p\,Tg.^p\,x\,(Tg.^{2q}\,x - 1) - q\,Tg.^q\,x\,(Tg.^{2p}\,x - 1)}{(Tang.^{p+q}\,x - 1)^2}\,\frac{dx}{Sin.\,2\,x} = \frac{\pi}{p+q}\,Tg.\left\{\frac{q-p}{q+p}\,\frac{\pi}{2}\right\},\ p<q;$ V. T. 68. N°. 21.

18) $\int l\,Tang.\,x\,\frac{Cos.\,2\,x}{1 + Sin\,2\,x}\,\frac{dx}{1 + Cos.\,\lambda.\,Sin.\,2\,x} = \frac{\lambda^2}{Cos.\,\lambda - 1}$ V. T. 355. N°. 2.

19) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right)\frac{dx}{(Sin.^2\,x + q^2\,Cos.^2\,x)\,Tang.\,x} = \pm\,\frac{2\pi}{p}\,Arctang.\,p$ V. T. 339. N°. 31.

20) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right)\frac{Sin.\,2\,x}{(q^2\,Tang.^2\,x + 1)^2}\,\frac{dx}{Cos.^4\,x} = \pm\,\frac{\pi}{q}\,\frac{2}{1+q^2}$ V. T. 68. N°. 7.

21) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right)\frac{Sin.\,2\,x}{(q^2 + Tang.^2\,x)^2}\,\frac{dx}{Cos.^4\,x} = \pm\,\frac{\pi}{q}\,\frac{2}{1+q^2}$ V. T. 68. N°. 6.

22) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right)\frac{Sin.\,2\,x}{(q^2\,Cot.^2\,x + 1)^2}\,\frac{dx}{Sin.^4\,x} = \pm\,\frac{\pi}{q}\,\frac{2}{1+q^2}$ V. T. 68. N°. 5.

23) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right)\frac{Sin.\,2\,x}{(Cot.^2\,x + q^2)^2}\,\frac{dx}{Sin.^4\,x} = \pm\,\frac{\pi}{q}\,\frac{2}{1+q^2}$ V. T. 68. N°. 8.

24) $\int (l\, Tang.\, x)^2 \frac{Tang.^q x + Cot.^q x}{Tang.^p x + Cot.^p x} \frac{dx}{Sin.\, 2x} = \frac{\pi^3}{8p^3}\left(2\, Sec.^3 \frac{q\pi}{2p} - Sec. \frac{q\pi}{2p}\right)$ V. T. 314. N°. 1 et T. 358. N°. 9.

25) $\int (l\, Tang.\, x)^2 \frac{Tang.^q x - Cot.^q x}{Tang.^p x - Cot.^p x} \frac{dx}{Sin.\, 2x} = \frac{\pi^3}{4p^3} Sin. \frac{q\pi}{2p} . Sec.^3 \frac{q\pi}{2p}$ V. T. 314. N°. 2 et T. 358. N°. 10.

F. Log. en num.
Circ. Dir. rat. en dén. trinôme. **TABLE 346.** Lim. 0 et $\frac{\pi}{2}$.

1) $\int l\, Sin.\, x \frac{1 - p\, Cos.\, 2x}{1 - 2p\, Cos.\, 2x + p^2} dx = \frac{1}{4}\pi\, l \frac{1-p}{4}$, $p < 1$; Cauchy, Lim. Imag. 119.

2) $\int l\, Sin.\, x \frac{Cos.\, 2x - p}{1 - 2p\, Cos.\, 2x + p^2} dx = -\frac{\pi}{4p} l \frac{1-p}{4}$, $p^2 \leqq 1$; V. T. 371. N°. 2.

3) $\int l\, Sin.\, x \frac{(1+q^2)(Sin.\, 2x - p\, Sin.\, 2px) + 2q(Sin.\, 2x. Cos.\, 2px - p\, Cos.\, 2x. Sin.\, 2px)}{(1 + 2q\, Cos.\, x + q^2)(1 + 2q\, Cos.\, 2px + q^2)} dx = 0$ V. T. 355. N°. 5.

4) $\int l\, Cos.\, x \frac{dx}{1 - 2p\, Cos.\, 2x + p^2} = \frac{\pi}{2(1-p^2)} l \frac{1+p}{2}$, $p^2 < 1$; Poisson, P. 19. 404. N°. 76.

5) $\int l\, Cos.\, x \frac{1 - p\, Cos.\, 2x}{1 - 2p\, Cos.\, 2x + p^2} dx = \frac{1}{4}\pi\, l \frac{1+p}{4}$, $p < 1$; Cauchy, Lim. Imag. 118.

6) $\int l\, Cos.\, x \frac{Cos.\, 2x - p}{1 - 2p\, Cos.\, 2x + p^2} dx = \frac{\pi}{4p} l \frac{1+p}{4}$, $p^2 \leqq 1$; V. T. 370. N°. 23.

7) $\int l\, Tang.\, x \frac{1 - p\, Cos.\, 2x}{1 - 2p\, Cos.\, 2x + p^2} dx = \frac{\pi}{4} l \frac{1-p}{1+p}$, $p < 1$; Cauchy, Lim. Imag. 120.

8) $\int l\, Tang.\, x \frac{Cos.\, 2x - p}{1 - 2p\, Cos.\, 2x + p^2} dx = \frac{\pi}{4p} l \frac{1-p}{1+p}$, $p^2 \leqq 1$; V. T. 371. N°. 1.

9) $\int l\, Tang.\, x \frac{Cos.\, 2x}{1 - 2p^2\, Cos.\, 4x + p^4} dx = \frac{\pi}{4p(1-p^2)} l \frac{1-p}{1+p}$, $p^2 \leqq 1$; V. T. 371. N°. 5.

10) $\int l\, Tang.\, x \frac{Cos.\, 4x - p}{1 - 2p\, Cos.\, 4x + p^2} dx = 0$, $p^2 \geqq 1$; V. T. 371. N°. 8.

11) $\int l\, Sin.\, x \frac{(1+p^2)\, Cos.\, 2x - 2p}{(1 - 2p\, Cos.\, 2x + p^2)^2} dx = -\frac{\pi}{4(1-p)}$, $p^2 < 1$;

12) $= -\frac{\pi}{4(p-1)}$, $p^2 > 1$; V. T. 85. N°. 1.

13) $\int l\, Cos.\, x \frac{(1+p^2)\, Cos.\, 2x - 2p}{(1 - 2p\, Cos.\, 2x + p^2)^2} dx = \frac{\pi}{4(1+p)}$ V. T. 85. N°. 2.

1) $\int l\,Sin.x \frac{Sin.x}{\sqrt{(1+Sin.^2 x)}}dx = -\frac{1}{8}\pi l 2$ V. T. 163. N°. 12.

2) $\int l\,Sin.x \frac{Sin.^3 x}{\sqrt{(1+Sin.^2 x)}}dx = \frac{1}{4}(l2-1)$ V. T. 163. N°. 13.

3) $\int l\,Sin.x \frac{Tang.^{\frac{2}{q}} x}{Sin.2x}dx = \frac{1}{8}q\pi\,Cosec.\frac{\pi}{q}$ V. T. 73. N°. 11.

4) $\int l\,Sin.x \frac{dx}{\sqrt{(1-p^2 Sin.^2 x)}} = -\frac{1}{2}F'(p)\,lp - \frac{1}{4}\pi F\{\sqrt{(1-p^2)}\}$, $p^2<1$; Roberts, L. 11. 471.

5) $\int l\,Sin.x \frac{(1-Sin.x)^{p-\frac{1}{2}}}{Sin.^{p-\frac{1}{2}}x.\,Tang.x}dx = -\frac{2\pi}{2p-1}Sec.p\pi$ V. T. 73. N°. 18.

6) $\int l\,Sin.x \frac{Sin.^{p-\frac{1}{2}}x}{(1-Sin.x)^{p+\frac{1}{2}}}\frac{dx}{Tang.x} = -\frac{2\pi}{1-2p}Sec.p\pi$, $p<\frac{1}{2}$; V. T. 76. N°. 14.

7) $\int l\,Cos.x \frac{Cos.x}{\sqrt{(1+Cos.^2 x)}}dx = -\frac{1}{8}\pi l 2$ V. T. 163. N°. 12.

8) $\int l\,Cos.x \frac{Cos.^3 x}{\sqrt{(1+Cos.^2 x)}}dx = \frac{1}{4}(l2-1)$ V. T. 163. N°. 13.

9) $\int l\,Cos.x \frac{x}{Tang.^{\frac{2}{q}}x.\,Sin.2x}dx = \frac{1}{8}q\pi\,Cosec.\frac{\pi}{q}$ V. T. 73. N°. 12.

10) $\int l\,Cos.x \frac{(1-Cos.x)^{p-\frac{1}{2}}}{Cos^{p-\frac{1}{2}}x}Tang.x\,dx = -\frac{2\pi}{2p-1}Sec.p\pi$ V. T. 73. N°. 13.

11) $\int l\,Cos.x \frac{dx}{\sqrt{(1-p^2 Sin.^2 x)}} = \frac{1}{4}F'(p)\,l\frac{1-p^2}{p^2} - \frac{1}{4}\pi F\{\sqrt{(1-p^2)}\}$, $p^2<1$; Roberts, L. 11. 471.

12) $\int l\,Cos.x \frac{Cos^{p-\frac{1}{2}}x}{(1-Cos.x)^{p+\frac{1}{2}}}Tang.x\,dx = -\frac{2\pi}{2p-1}Sec.p\pi$ V. T. 74. N°. 10.

13) $\int l\,Tang.x \frac{dx}{\sqrt{(1-p^2 Sin.^2 x)}} = -\frac{1}{2}l(1-p^2).F'(p)$, $p^2<1$; Roberts, L. 11. 471.

14) $\int l\,Cot.\frac{1}{2}x \frac{Sin.x}{Sin.^2\lambda + Tang.^2\mu.\,Sin.^2 x}\frac{Cos.x}{\sqrt{(Sin.^2\lambda - Sin.^2 x)}}dx =$
$= \frac{1}{2}\pi\frac{Cos.^2\mu}{Sin.\lambda.\,Sin.\mu}\,l\frac{Sin.\mu + \sqrt{(1-Cos.^2\lambda.\,Cos.^2\mu)}}{Sin.\mu(1+Sin.\lambda)}$ Lobatschewsky, Mém. Kasan. 1836. 1. II. 25.

15) $\int l\,Sin.x \frac{Cos.x}{1-\sqrt{Sin.x}}\frac{dx}{\sqrt[4]{Sin.^3 x}} = -2\pi^2$ V. T. 163. N°. 18.

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F. Log. en num. de Circ. monôme. Circ. Dir. de forme irrat.	TABLE 347 suite.	Lim. 0 et $\frac{\pi}{2}$.

16) $\int l\, Cos.x \frac{1+\sqrt{Cos.x}}{Sin.x}\frac{1+Cos.x}{\sqrt{Cos.^3 x}}dx = -2\pi^2$ V. T. 168. N°. 18.

17) $\int (l\, Tang.x)^{2a-1}\frac{dx}{(Sin.x - Cos.x)\sqrt{Sin.2x}} = -\frac{2^{2a}-1}{2a\sqrt{2}}(2\pi)^{2a}B_{2a-1}$ V. T. 319. N°. 19 et T. 359. N°. 8.

F. Log. en num. de Circ. polynôme. Circ. Dir. de forme irrat.	TABLE 348.	Lim. 0 et $\frac{\pi}{2}$.

1) $\int dx\, l\left\{Sin.\lambda.Sin.x + \sqrt{(1 - Cos.^2\lambda.Sin.^2x)}\right\} = \frac{1}{2}\pi l2 - \frac{1}{2}L\left(\frac{1}{2}\lambda\right) - 2L\left(\frac{\pi-\lambda}{2}\right)$

2) $\int dx\, l\frac{1+Cos.x\sqrt{(Sin.^2\lambda - Sin.^2\mu.Sin.^2x)}}{1-Cos.x\sqrt{(Sin.^2\lambda - Sin.^2\mu.Sin.^2x)}} = \pi l\frac{1}{2}\left\{Cos.^2\frac{1}{2}\lambda + \sqrt{\left(Cos.^4\frac{1}{2}\lambda + Sin.^2\frac{1}{2}\mu.Cos.^2\frac{1}{2}\mu\right)}\right\}$

Lobatschewsky, Mém. Kasan. 1836. 1. II. 36. 13.

3) $\int dx\, l\frac{1 - Cos\,h\,p.\lambda.Cos\,h\,p.\mu.Cos.x.\sqrt{(1 - Cot\,h\,p^2.\lambda.Tang\,h\,p^2.\mu.Cos.^2x)}}{1 + Cos\,h\,p.\lambda.Cos\,h\,p.\mu.Cos.x.\sqrt{(1 - Cot\,h\,p^2.\lambda.Tang\,h\,p^2.\mu.Cos.^2x)}} =$

$= \pi l\frac{4\,Sin\,h\,p.\lambda}{(1 + Sin\,h\,p.\lambda)\left\{Sin\,h\,p.\lambda + \sqrt{(1 - Cos\,h\,p^2.\lambda.Cos\,h\,p^2.\mu)}\right\}}$

4) $\int\frac{Cos.x\,dx}{1 - Cos\,h\,p^2.\lambda.Cos.^2x}\,l\frac{1 + \sqrt{(Cos\,h\,p^2.\mu - Sin\,h\,p^2.\mu.Cot\,h\,p^2.\lambda.Sin.^2x)}}{1 - Cos\,h\,p.\lambda.Cos.x} =$

$= \frac{1}{2\,Sin\,h\,p.\lambda.Cos\,h\,p.\lambda}\left\{L(\lambda+\varphi) - L(\lambda-\varphi) - 2L(\varphi) - (\pi - 2\lambda - 2\varphi)l\,Sin\,h\,p.\mu - 2\lambda l\,Sin\,h\,p.\lambda\right\}$

5) $\int\frac{Cos.x\,dx}{1 - Cos\,h\,p^2.\lambda.Cos.^2x}\,l\frac{1 + \sqrt{(Cos\,h\,p^2.\mu - Sin\,h\,p^2.\mu.Cot\,h\,p^2.\lambda.Sin.^2x)}}{\sqrt{(1 + Cot\,h\,p^2.\lambda.Sin.^2x)}} =$

$= \frac{1}{2\,Sin\,h\,p.\lambda.Cos\,h\,p.\lambda}\left\{(\pi + 2\lambda + 2\varphi)\,l\,Sin\,h\,p.\mu + L(\lambda+\varphi) - L(\lambda-\varphi) - 2L(\varphi)\right\}$

6) $\int\frac{Cos.x\,dx}{1 - Cos\,h\,p^2.\lambda.Cos.^2x}\,l\left\{1 + \sqrt{(Cos\,h\,p^2.\mu - Sin\,h\,p^2.\mu.Cot\,h\,p^2.\lambda.Sin.^2x)}\right. =$

$= \frac{1}{Sin\,h\,p.\lambda.Cos\,h\,p.\lambda}\left\{\left(\lambda + \varphi - \frac{1}{2}\pi\right)l\,Sin\,h\,p.\mu + L(\lambda) - L(\varphi) + \frac{1}{2}L(\lambda+\varphi) - \frac{1}{2}L(\lambda-\varphi)\right\}$

7) $\int\frac{Cos.x\,dx}{1 - Cos\,h\,p^2.\lambda.Cos.^2x}\,l\left\{1 - \sqrt{(Cos\,h\,p^2.\mu - Sin\,h\,p^2.\mu.Cot\,h\,p^2.\lambda.Sin.^2x)}\right. =$

$= \frac{1}{Sin\,h\,p.\lambda.Cos\,h\,p.\lambda}\left\{\left(\lambda - \varphi + \frac{1}{2}\pi\right)l\,Sin\,h\,p.\mu + L(\lambda) + L(\varphi) - \frac{1}{2}L(\lambda+\varphi) + \frac{1}{2}L(\lambda-\varphi)\right\}$

Dans les formules (3) à (7), trouvées par Lobatschewsky, Mém. Kasan. 1836. 1. I. 73. 114. 120, 125, 127, on a $Cos.\varphi = \frac{Cos\,h\,p.\lambda}{Cos\,h\,p.\mu}$.

8) $\int \frac{Cos. x\, dx}{1 - Sin.^2 \lambda . Cos.^2 x} l \{1 + \sqrt{(Sin^2 \mu - Cos.^2 \mu . Tang.^2 \lambda . Sin.^2 x)}\} =$

$= \frac{2}{Sin. 2\lambda} \{-(\varphi + \lambda - \frac{1}{2}\pi) l\, Sin. \mu + L(\lambda) - L(\varphi) + \frac{1}{2} L(\lambda + \varphi) - \frac{1}{2} L(\lambda - \varphi)\}$

9) $\int \frac{Cos. x\, dx}{1 - Sin.^2 \lambda . Cos.^2 x} l \{1 - \sqrt{(Sin.^2 \mu - Cos.^2 \mu . Tang.^2 \lambda . Sin.^2 x)}\} =$

$= \frac{2}{Sin. 2\lambda} \{\lambda - \varphi + \frac{1}{2}\pi) l\, Cos. \mu + L(\lambda) + L(\varphi) - \frac{1}{2} L(\lambda + \varphi) + \frac{1}{2} L(\lambda - \varphi)\}$

Dans les formules (8), (9), trouvées par Lobatschewsky, Mém. Kasan. 1836. 1. II, 42, 43, on a $Cos. \varphi = \frac{Sin. \lambda}{Sin. \mu}$.

10) $\int \frac{dx}{\sqrt{(1 - p^2 Sin.^2 x)}} l(1 + p\, Sin.^2 x) = \frac{1}{2} l \left\{\frac{2 + 2p}{\sqrt{p}}\right\} F'(p) - \frac{1}{8}\pi F'\{\sqrt{(1-p^2)}\}$

11) $\int \frac{dx}{\sqrt{(1 - p^2 Sin.^2 x)}} l(1 - p\, Sin.^2 x) = \frac{1}{2} l \left\{\frac{2 - 2p}{\sqrt{p}}\right\} F'(p) - \frac{1}{8}\pi F'\{\sqrt{(1-p^2)}\}$

(10), (11): $p^2 < 1$; Roberts, L. 12. 449.

12) $\int \frac{dx}{\sqrt{(1 - p^2 Sin.^2 x)}} l \{\sqrt{(1 - p^2 Sin.^2 x)}\} = \frac{1}{2} l \{\sqrt{(1 - p^2)}\} F'(p)$

13) $\int \frac{dx}{\sqrt{(1 - p^2 Sin.^2 x)}} l \{Cos.^2 x + (1-p^2)^{\frac{1}{2}} Sin.^2 x\} = \frac{1}{2} l \left\{\frac{2\sqrt[4]{(1-p^2)^3}}{1 + \sqrt{(1-p^2)}}\right\} F'(p)$

(12), (13): $p^2 < 1$; Roberts, L. 11. 157. — Id., L. 11. 174.

14) $\int \frac{dx}{\sqrt{(1 - p^2 Sin.^2 x)}} l \{1 + Cot.^2 \lambda . Sin^2 x\} = \pi F\{\sqrt{(1-p^2)}, \lambda\} - 2F'(p) Y\{\sqrt{(1-p^2)}, \lambda\} -$
$- 2F'(p) l\, Sin. \lambda - \frac{1}{2}\pi F'\{\sqrt{(1-p^2)}\} - F'(p) l p - \{E'(p) - F'(p)\} [F\{\sqrt{(1-p^2)}, \lambda\}]^2$

15) $\int \frac{dx}{\sqrt{(1 - p^2 Sin.^2 x)}} l [1 - \{1 - (1-p^2) Sin.^2 \lambda\} Sin.^2 x] = \pi F\{\sqrt{(1-p^2)}, \lambda\} - 2F'(p) Y\{\sqrt{(1-p^2)}, \lambda\} +$
$+ F'(p) l \frac{\sqrt{(1-p^2)}}{p} - \frac{1}{2}\pi F'\{\sqrt{(1-p^2)}\} - \{E'(p) - F'(p)\} [F\{\sqrt{(1-p^2)}, \lambda\}]^2$

(14), (15): $p^2 < 1$; Roberts, L. 11. 471.

16) $\int \frac{dx}{\sqrt{(1 - p^2 Sin.^2 x)}} l \{1 - p^2 Sin.^4 x\} = \frac{1}{2} l \left\{\frac{4(1-p^2)}{p^2}\right\} F'(p) - \frac{1}{4}\pi F'\{\sqrt{(1-p^2)}\}, p^2 < 1;$ Roberts, L. 12. 449.

17) $\int \frac{dx}{\sqrt{(1 - p^2 Sin.^2 x)}} l(1 - p^2 Sin.^2 \lambda . Sin.^2 x) = E'(p)\{F(p, \lambda)\}^2 - 2F'(p) Y(p, \lambda), p^2 < 1;$ Roberts, L. 11. 471.

18) $\int \frac{dx}{\sqrt{(1 - p^2 Sin.^2 x)^3}} l(1 - p^2 Sin.^2 x) = \frac{1}{2(1-p^2)} [2(p^2 - 2)F'(p) + \{4 + l(1-p^2)\} E'(p)]$ V. T. 348. N°. 12, 19.

19) $\int \frac{Sin.^2 x\,d x}{\sqrt{(1-p^2 Sin.^2 x)^3}}\, l\{\sqrt{(1-p^2 Sin.^2 x)}\} = \frac{-1}{2p^2(1-p^2)}\Big[\{2-p^2+-(1-p^2)l[\sqrt{(1-p^2)}]\}F'(p)-$
$-\{2+l[\sqrt{(1-p^2)}]\}E'(p)\Big]$ Roberts, L. 11. 157.

20) $\int \frac{Cos.^2 x\,d x}{\sqrt{(1-p^2 Sin.^2 x)^3}}\, l(1-p^2 Sin.^2 x) = \frac{1}{2p^2}\Big[\{2(2-p^2)+l(1-p^2)\}F'(p)-\{4+l(1-p^2)\}E'(p)\Big]$ V. T. 348. N°. 18, 19.

21) $\int \frac{Cos. 2x\,d x}{\sqrt{(1-p^2 Sin.^2 x)^3}}\, l(1-p^2 Sin.^2 x) = \frac{1}{2p^2(1-p^2)}\Big[\{(2-p^2)+(1-p^2)l(1-p^2)\}F'(p)-$
$-(2-p^2)\{4+l(1-p^2)\}E'(p)\Big]$ V. T. 348. N°. 18, 19.

22) $\int \frac{d x}{\sqrt{(1-p^2 Sin.^2 x)}}\, l\frac{1+q\sqrt{(1-p^2 Sin.^2 x)}}{1-q\sqrt{(1-p^2 Sin.^2 x)}} = \pi F\{\sqrt{(1-p^2)}, Arcsin. q\}$ Roberts, L. 11. 157.

F. Log. sous forme irrat.
Circ. Dir.

TABLE 349.

Lim. 0 et $\frac{\pi}{2}$.

1) $\int Cos. x\,d x \sqrt{l\,Cosec. x} = \frac{1}{2}\sqrt{\pi}$ V. T. 44. N°. 1.

2) $\int Cos. x\,(l\,Cosec. x)^{a+\frac{1}{2}}\,d x = \frac{3^{a/2}}{2^a}\sqrt{\pi}$ V. T. 44. N°. 2.

3) $\int \frac{Sin.^p x}{Tang. x}(l\,Cosec. x)^{a-\frac{1}{2}}\,d x = \frac{1^{a/2}}{(2p)^a}\sqrt{\frac{\pi}{p}}$ V. T. 162. N°. 4.

4) $\int \frac{Cos. x}{\sqrt{l\,Cosec. x}}\,d x = \sqrt{\pi}$ V. T. 44. N°. 4.

5) $\int \frac{Sin.^p x}{Tang. x.\sqrt{l\,Cosec. x}}\,d x = \sqrt{\frac{\pi}{p}}$ V. T. 178. N°. 1.

6) $\int Sin. x\,d x\sqrt{l\,Sec. x} = \frac{1}{2}\sqrt{\pi}$ V. T. 44. N°. 1.

7) $\int Sin. x\,(l\,Sec. x)^{a+\frac{1}{2}}\,d x = \frac{3^{a/2}}{2^a}\sqrt{\pi}$ V. T. 44. N°. 2.

8) $\int Cos.^p x. Tang. x\,(l\,Sec. x)^{a-\frac{1}{2}}\,d x = \frac{1^{a/2}}{(2p)^a}\sqrt{\frac{\pi}{p}}$ V. T. 162. N°. 4.

9) $\int \frac{Sin. x}{\sqrt{l\,Sec\,x}}\,d x = \sqrt{\pi}$ V. T. 44. N°. 4.

10) $\int \frac{Cos.^{p-2} x. Sin. 2x}{\sqrt{l\,Sec. x}}\,d x = 2\sqrt{\frac{\pi}{p}}$ V. T. 178. N°. 1.

1) $\int \frac{(Sin.^q x - Cosec.^q x)^2}{l\,Sin.x} Tang.x\,dx = l\frac{Sin.q\pi}{q\pi}$ V. T. 175. N°. 7.

2) $\int \frac{1 + Sin.x}{l\,Sin.x} Sin.2x.Sin.(l\,Sin.x)\,dx = \frac{1}{2}\pi$ V. T. 406. N°. 3.

3) $\int \frac{Sin.^q x - Sin.^p x}{l\,Sin.x} Sin.2x\,dx = 2l\frac{q+2}{p+2}$ V. T. 167. N°. 4.

4) $\int \frac{(Sin.^p x - Sin.^q x)(Sin.^r x - Sin.^s x)}{l\,Sin.x} Sin.2x\,dx = 2l\frac{(p+r+2)(q+s+2)}{(p+s+2)(q+r+2)}$ V. T. 167. N°. 8.

5) $\int \frac{Sin.^q x + Cosec.^q x}{Sin.^p x + Cosec.^p x} \frac{dx}{Tang.x.l\,Sin.x} = l\,Tang.\left\{\frac{p+q}{4p}\pi\right\}$ V. T. 172. N°. 6.

6) $\int \frac{Cos.(2pl\,Sin.x)}{l\,Sin.x}\frac{dx}{Cos.x} = \frac{1}{2}l\frac{1}{e^{p\pi}+e^{-p\pi}}$ V. T. 406. N°. 16.

7) $\int \frac{(1 - Sin.^{1-q}x)^2}{l\,Sin.x}\frac{Sin.^{q-1}x}{Cos.x}dx = l\,Sin.\frac{q\pi}{2}$ V. T. 172. N°. 10.

8) $\int \frac{1 - Sin.^q x}{l\,Sin.x}\frac{1 - Sin.^{q+1}x}{Cos.x}dx = -ql2, q > -1;$ V. T. 172. N°. 3.

9) $\int \frac{(Sin.^q x - Cosec.^q x)^2}{l\,Sin.x}\frac{dx}{Cos.x} = l\,Cos.q\pi$ V. T. 175. N°. 6.

10) $\int \frac{Cos.^3 x}{l\,Sin.x}\frac{dx}{1 + Sin.^4 x} = l\,Cot.\frac{3\pi}{8}$ V. T. 172. N°. 4.

11) $\int \left\{\frac{1 + Sin.^2 x}{Cos.x} + \frac{Cos.x}{l\,Sin.x}\right\}\frac{dx}{l\,Sin.x} = l2 - 1$ V. T. 172. N°. 5.

12) $\int \frac{Cosec.^q x - Sin.^q x}{(l\,Sin.x)^p}\frac{dx}{Cos.x} = (-1)^p\Gamma(1-p)\sum_1^\infty\left\{\frac{1}{(2n-1-q)^{1-p}} - \frac{1}{(2n-1+q)^{1-p}}\right\}$ V. T. 176. N°. 5.

13) $\int \frac{(Cos.^q x - Sec.^q x)^2}{l\,Cos.x}\frac{dx}{Tang.x} = l\frac{Sin.q\pi}{q\pi}$ V. T. 175. N°. 7.

14) $\int \frac{(Cos.^q x - Sec.^q x)^2}{l\,Cos.x}\frac{dx}{Sin.x} = l\,Cos.q\pi$ V. T. 175. N°. 6.

15) $\int \frac{1 + Cos.x}{l\,Cos.x} Sin.2x.Sin.(l\,Cos.x)\,dx = \frac{1}{2}\pi$ V. T. 406. N°. 3.

16) $\int \frac{Cos.^q x - Cos.^p x}{l\,Cos.x} Sin.2x\,dx = 2l\frac{q+2}{p+2}$ V. T. 167. N°. 4.

17) $\int \frac{1 - Cos.^q x}{l\,Cos.\,x} \frac{1 - Cos.^{q+1} x}{Sin.\,x} dx = -q\,l\,2\ ,\ q > -1$; V. T. 172. N°. 3.

18) $\int \frac{(Cos^p x - Cos.^q x)(Cos.^r x - Cos.^s x)}{l\,Cos.\,x} Sin.\,2x\,dx = 2\,l\frac{(p+r+2)(q+s+2)}{(p+s+2)(q+r+2)}$ V. T. 167. N°. 8.

19) $\int \frac{Tang.^{p-1} x - Tang.^{q-1} x}{l\,Tang.\,x} dx = \pi\left(l\,Tang.\frac{p\pi}{4} - l\,Tang.\frac{q\pi}{4}\right)$ V. T. 180. N°. 7.

20) $\int \frac{Tang^{p-1}x - Tang.^{q-1}x}{l\,Tang.\,x} \frac{dx}{Cos.\,2x} = \pi\,(l\,Sin.\,\tfrac{1}{2}p\pi - l\,Sin.\,\tfrac{1}{2}q\pi)$ V. T. 180. N°. 13.

21) $\int \frac{Tang.^q x - Cot.^q x}{l\,Tang.\,x} dx = 2\,l\,Tang.\left\{\frac{1+q}{4}\pi\right\}$ V. T. 322. N°. 12 et T. 360. N°. 1.

22) $\int \frac{(Tang.^q x - Cot.^q x)^2}{l\,Tang.\,x} \frac{dx}{Cos.\,2x} = 0$ V. T. 322. N°. 15 et T. 360. N°. 2.

23) $\int \frac{Tang^p x - Tang.^q x}{Sin.\,x + Cos.\,x} \frac{dx}{Sin.\,x.\,l\,Tang.\,x} = l\,(Tang.\,\tfrac{1}{2}p\pi.\,Cot.\,\tfrac{1}{2}q\pi)$ V. T. 183. N°. 18.

24) $\int \frac{Tang.^q x - Cot.^q x}{Tang^p x + Cot.^p x} \frac{dx}{Sin.\,2x.\,l\,Tang.\,x} = l\,Tang.\left\{\frac{p+q}{4p}\pi\right\}$ V. T. 323. N°. 7 et T. 360. N°. 4.

25) $\int \frac{Tang.^q x + Cot.^q x - 2}{Tang.^p x - Cot^p x} \frac{dx}{Sin.\,2x.\,l\,Tang.\,x} = l\,Cos.\frac{q\pi}{2p}$ V. T. 323. N°. 8 et T. 360. N°. 5.

F. Log. en dén. binôme: $q^2 + (l\,Sin\,x)^2$. Circ. Dir. **TABLE 351.** Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{l\,Cos.\,x}{q^2 + (l\,Sin.\,x)^2} \frac{dx}{Tang.\,x} = \frac{\pi}{2q} l\,\Gamma\left(\frac{q+\pi}{\pi}\right) + \frac{\pi}{4q} l\,2q + \frac{1}{2}\left(l\frac{q}{\pi} - 1\right)$ V. T. 170. N°. 8.

2) $\int \frac{l\,Sin.\,x}{\pi^2 + (l\,Sin.\,x)^2} \frac{dx}{Cos.\,x} = \frac{1}{2}\left\{\frac{1}{2} - l\,2\right\}$ V. T. 173. N°. 11.

3) $\int \frac{Tang.\,x.\,l\,Sin.\,x}{q^2 + (l\,Sin.\,x)^2} dx = -\frac{1}{2}\left\{l\frac{q}{\pi} - \frac{\pi}{2q} - Z'\left(\frac{q}{\pi}\right)\right\}$ V. T. 173. N°. 12.

4) $\int \frac{Tang.\,x.\,l\,Sin.\,x}{\pi^2 + (l\,Sin.\,x)^2} dx = \frac{1}{4} - \frac{1}{2}A$ V. T. 173. N°. 13.

5) $\int \frac{Tang.\,x.\,l\,Sin.\,x}{q^2 - (l\,Sin.\,x)^2} dx = \frac{\pi^4}{4q^4}\sum_0^\infty (-1)^{n+1}\frac{B_{2n+1}}{n+1}\left(\frac{\pi}{q}\right)^{2n}$ V. T. 173. N°. 14.

6) $\int \frac{l\,Sin.\,x}{\pi^2+4(l\,Sin.\,x)^2}\frac{dx}{Cos.\,x}=\frac{1}{16}(2-\pi)$ V. T. 173. N°. 10.

7) $\int \frac{Sin.^p x - Cosec.^p x}{\pi^2+(l\,Sin.\,x)^2}\frac{dx}{Cos.\,x}=\frac{1}{2\pi}\left[p\pi\,Cos.\,p\pi - Sin.\,p\pi.\,l\{2(1+Cos.\,p\pi)\}\right]$ V. T. 176. N°. 8.

8) $\int \frac{Sin.^{1-p} x - Sin.^{p-1} x}{q^2+(l\,Sin.\,x)^2}\frac{dx}{Cos.\,x}=-\frac{\pi}{q}\sum_1^\infty \frac{Sin.\,np\pi}{q+n\pi}$, $p^2<1$; V. T. 176. N°. 10.

9) $\int \frac{Sin.^p x + Cosec.^p x}{\pi^2+(l\,Sin.\,x)^2}\frac{l\,Sin.\,x}{Cos.\,x}dx=\frac{1}{2}\left[1-p\pi\,Sin.\,p\pi - Cos.\,p\pi.\,l\{2(1+Cos.\,p\pi)\}\right]$ V.T. 176. N°. 9.

10) $\int \frac{Sin.^{1-p} x + Sin.^{p-1} x}{q^2+(l\,Sin.\,x)^2}\frac{l\,Sin.\,x}{Cos.\,x}dx=-\frac{\pi}{2q}-\pi\sum_1^\infty \frac{Cos.\,np\pi}{q+n\pi}$, $p^2<1$; V. T. 176. N°. 11.

11) $\int \frac{Tang.\,x.\,l\,Sin.\,x}{\{q^2+(l\,Sin.\,x)^2\}^2}dx=-\frac{\pi^2}{4q^4}\sum_0^\infty B_{2n+1}\left(\frac{\pi}{q}\right)^{2n}$ V. T. 173. N°. 15.

12) $\int \frac{\pi^2-(l\,Sin.\,x)^2}{\{\pi^2+(l\,Sin.\,x)^2\}^2}\frac{l\,Cos.\,x}{Tang.\,x}dx=\frac{1}{4}(1-2\,A)$ V. T. 351. N°. 4.

13) $\int \frac{Tang.\,x.\,l\,Sin.\,x}{\{q^2-(l\,Sin.\,x)^2\}^2}dx=\frac{\pi^2}{4q^4}\sum_0^\infty (-1)^{n+1}B_{2n+1}\left(\frac{\pi}{q}\right)^{2n}$ V. T. 173. N°. 16.

14) $\int \frac{q^2-3(l\,Sin.\,x)^2}{\{q^2+(l\,Sin.\,x)^2\}^3}\frac{l\,Cos.\,x}{Tang.\,x}dx=-\frac{\pi^2}{4q^4}\sum_0^\infty B_{2n+1}\left(\frac{\pi}{q}\right)^{2n}$ V. T. 351. N°. 11.

15) $\int \frac{q^2+(l\,Sin.\,x)^2}{\{q^2-(l\,Sin.\,x)^2\}^2}\frac{l\,Cos.\,x}{Tang.\,x}dx=\frac{\pi^4}{4q^4}\sum_0^\infty (-1)^{n+1}\frac{B_{2n+1}}{n+1}\left(\frac{\pi}{q}\right)^{2n}$ V. T. 351. N°. 5.

16) $\int \frac{q^2+3(l\,Sin.\,x)^2}{\{q^2-(l\,Sin.\,x)^2\}^3}\frac{l\,Cos.\,x}{Tang.\,x}dx=\frac{\pi^2}{4q^4}\sum_0^\infty (-1)^{n+1}B_{2n+1}\left(\frac{\pi}{q}\right)^{2n}$ V. T. 351. N°. 13.

F. Log. en dén. à autre forme binôme. Circ. Dir. TABLE 352. Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{l\,Cos.\,x}{\pi^2+(l\,Cos.\,x)^2}\frac{dx}{Sin.\,x}=\frac{1}{2}\left\{\frac{1}{2}-l\,2\right\}$ V. T. 173. N°. 11.

2) $\int \frac{l\,Cos.\,x}{q^2+(l\,Cos.\,x)^2}\frac{dx}{Cot.\,x}=\frac{\pi}{2q}l\Gamma\left(\frac{q+\pi}{\pi}\right)+\frac{\pi}{4q}l\,2q+\frac{1}{2}\left(l\frac{q}{\pi}-1\right)$ V. T. 170. N°. 8.

3) $\int \frac{Cos.^p x - Sec.^p x}{\pi^2+(l\,Cos.^2 x)^2}\frac{dx}{Sin.\,x}=-\frac{1}{4}Sin.\frac{1}{2}p\pi+\frac{1}{4\pi}Cos.\frac{1}{2}p\pi.\,l\frac{1+Sin.\frac{1}{2}p\pi}{1-Sin.\frac{1}{2}p\pi}$, $p<1$; V. T. 176. N°. 6.

4) $\int \frac{Cos.^p x + Sec.^p x}{\pi^2 + (l\, Cos.^2 x)^2} \frac{l\, Cos. x}{Sin. x} dx = \frac{1}{4} - \frac{\pi}{8} Cos. \frac{1}{2} p\pi + \frac{1}{8} Sin. \frac{1}{2} p\pi . l \frac{1 - Sin. \frac{1}{2} p\pi}{1 + Sin. \frac{1}{2} p\pi}, p < 1;$ V. T. 176. N°. 7.

5) $\int \frac{l\, Cos. x}{q^2 + (l\, Cos. x)^2} \frac{dx}{Tang. x} = -\frac{1}{2}\left\{ l\frac{q}{\pi} - \frac{\pi}{2q} - Z'\left(\frac{q}{\pi}\right)\right\}$ V. T. 173. N°. 12.

6) $\int \frac{l\, Cos. x}{\pi^2 + (l\, Cos. x)^2} \frac{dx}{Tang. x} = \frac{1}{4}(1 - 2\, A)$ V. T. 173 N°. 13.

7) $\int \frac{l\, Cos. x}{q^2 - (l\, Cos. x)^2} \frac{dx}{Tang. x} = \frac{\pi^4}{4q^4} \sum_0^\infty (-1)^{n+1} B_{2n+1} \left(\frac{\pi}{q}\right)^n$ V. T. 173. N°. 14.

8) $\int \frac{l\, Cos. x}{\pi^2 + 4(l\, Cos. x)^2} \frac{dx}{Sin. x} = \frac{1}{16}(2 - \pi)$ V. T. 173. N°. 10.

9) $\int \frac{l\, Cos. x}{\{q^2 + (l\, Cos. x)^2\}^2} \frac{dx}{Tang. x} = -\frac{\pi^2}{4q^4} \sum_0^\infty B_{2n+1} \left(\frac{\pi}{q}\right)^{2n}$. V. T. 173. N°. 15.

10) $\int \frac{\pi^2 - (l\, Cos. x)^2}{\{\pi^2 + (l\, Cos. x)^2\}^2} Tang. x . l\, Sin. x\, dx = \frac{1}{4}(1 - 2\, A)$ V. T. 352. N°. 6.

11) $\int \frac{q^2 + (l\, Cos. x)^2}{\{q^2 - (l\, Cos. x)^2\}^2} Tang. x . l\, Sin. x\, dx = \frac{\pi^4}{4q^4} \sum_0^\infty (-1)^{n+1} \frac{B_{2n+1}}{n+1} \left(\frac{\pi}{q}\right)^{2n}$ V. T. 352. N°. 7.

12) $\int \frac{q^2 - 3(l\, Cos. x)^2}{\{q^2 + (l\, Cos. x)^2\}^3} Tang. x . l\, Sin. x\, dx = -\frac{\pi^2}{4q^4} \sum_0^\infty B_{2n+1} \left(\frac{\pi}{q}\right)^{2n}$ V. T. 352. N°. 9.

13) $\int \frac{l\, Cos. x}{\{q^2 - (l\, Cos. x)^2\}^2} \frac{dx}{Tang. x} = \frac{\pi^2}{4q^4} \sum_0^\infty (-1)^{n+1} B_{2n+1} \left(\frac{\pi}{q}\right)^{2n}$ V. T. 173. N°. 16.

14) $\int \frac{q^2 + 3(l\, Cos. x)^2}{\{q^2 - (l\, Cos. x)^2\}^3} Tang. x . l\, Sin. x\, dx = \frac{\pi^2}{4q^4} \sum_0^\infty (-1)^{n+1} B_{2n+1} \left(\frac{\pi}{q}\right)^{2n}$ V. T. 352. N°. 13.

15) $\int \frac{dx}{q^2 + (l\, Tang. x)^2} = \frac{1}{2q}\left\{Z'\left(\frac{2q + 3\pi}{4\pi}\right) - Z'\left(\frac{2q + \pi}{4\pi}\right)\right\}$ V. T. 324. N°. 3 et T. 363. N°. 3.

16) $\int \frac{1}{q^2 + (l\, Tang. x)^2} \frac{1}{Sin. x + Cos. x} \frac{dx}{\sqrt{Sin. 2x}} = \frac{1}{2q\sqrt{2}}\left\{Z'\left(\frac{q + 3\pi}{4\pi}\right) - Z'\left(\frac{q + \pi}{4\pi}\right)\right\}$ V. T. 327. N°. 1 et T. 360. N°. 6.

F. Log. Circ. Dir. } Log. de Circ. Dir. sans fact. Circ. TABLE 353. Lim. 0 et π.

1) $\int l\, Sin. x\, dx = -\pi l 2$ Grunert, Gr. 4. 113.

2) $= -\pi(l 2 - 2\alpha\pi i)$ Arndt, Gr. 6. 187.

3) $\int l((Sin. x))\, dx = -\pi l 2 \pm 2\alpha\pi^2 i$ Lindmann, Gr. 16. 94.

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4) $\int l((-Sin. x))\,dx = -\pi l 2 \pm (2\alpha+1)\pi^2 i$ Lindmann, Gr. 16. 94.

5) $\int l\,Tang. x\,dx = 0$ Ohm, Ausw. 18.

6) $\int l\,Cos.^2 x\,dx = -2\pi l 2$ V. T. 353. N°. 1, 5.

7) $\int l(1+Cos. x)\,dx = -\pi l 2$

8) $\int l(1-Cos. x)\,dx = -\pi l 2$ } Raabe, Int. 161. — Ohm, Ausw. 18.

9) $\int l(1\pm p\,Cos. x)\,dx = \pi l \frac{1+\sqrt{(1-p^2)}}{2}$, $p<1$; Ohm, Ausw. 18.

10) $\int l(p+Cos. x)\,dx = -\pi l 2$, $p<1$;

11) $= -2\pi l\{\sqrt{(p+1)}-\sqrt{(p-1)}\}, p>1$; } V. T. 245. N°. 4, 5. Lobatto, Cr. 9. 260 trouve fautivement $\pi l\{p+\sqrt{(p^2-1)}\}$

12) $\int l(p-Cos. x)\,dx = -\pi l 2$, $p^2<1$;

13) $= -2\pi l\{\sqrt{(p+1)}-\sqrt{(p-1)}\}, p^2>1$; } V. T. 124. N°. 6, 7.

14) $\int l(p^2-Cos.^2 x)^2\,dx = -4\pi l 2$, $p^2<1$;

15) $= -8\pi l\{\sqrt{(p+1)}-\sqrt{(p-1)}\}, p^2>1$; } V. T. 246. N°. 20, 21.

16) $\int l(1+q\,Cot.^2 \tfrac{1}{2} x)\,dx = \pi l \frac{1+\sqrt{(1+q)}}{2}$ Ramus, Danske Afh. 6. 265.

17) $\int l(1-2p\,Cos. x+p^2)\,dx = 0$, $p \leqq 1$;

18) $= 2\pi l p$, $p>1$; } Poisson, P. 17. 612. N°. 15. — Delaunay, L. 3. 355. — Grunert, Gr. 4. 113. — Lobatschewsky, Mém. Kasan. 1835. 1.

19) $\int l(1+2p\,Cos. x+p^2)\,dx = 0$, $p<1$;

20) $= 2\pi l p$, $p>1$; } Grunert, Gr. 4. 113.

Les form. (17), (19) se trouvent aussi chez Schlömilch, Beitr. II. 1. — Bierens de Haan, Gr. 13. 193. Page 461.

21) $\int l\left(1+2\frac{b}{a}Cos.x+\frac{b^2}{a^2}\right)dx = 0 \quad , b \leqq a;$

22) $\qquad = 2\pi l\frac{b}{a}, \; b > a;$ } Ramus, Danske Afh. 6. 265.

23) $\int l(1-2p\,Cos.2x+p^2)\,dx = 0$ Bierens de Haan, Gr. 13. 193.

24) $\int l\frac{1+2q\,Cos.x+q^2}{1+2q\,Cos.ax+q^2}dx = 0$ Raabe, Cr. 23. 105.

1) $\int l\,Sin.x.\,Cos.x\,dx = 0$ V. T. 330. N°. 12 et T. 331. N°. 9.

2) $\int l\,Sin.x.\,Sin.^{2a}2x.\,Cos.2x\,dx = \frac{-\pi}{4a+2}\,\frac{1^{a/2}}{2^{a/2}}$ V. T. 88. N°. 1.

3) $\int l\,Sin.\frac{1}{2}x.\,Cos.qx\,dx = -\frac{1}{2}\pi q$

4) $\qquad = \frac{2\pi}{k}\sum_1^{k-1} Cos.\frac{2qn\pi}{k}.\,l\,Sin.\frac{n\pi}{2k}, \; k=\infty;$ } Raabe, Cr. 25. 160 (il trouve pour 4) faut. $\frac{2}{k}\Sigma$.

5) $\int l\,Tang.x.\,Tang.2x\,dx = -\frac{1}{4}\pi^2$ V. T. 160. N°. 15.

6) $\int l(1-2p\,Cos.x+p^2)\,Cos.ax\,dx = -\frac{\pi}{a}p^a$ Schlömilch, Beitr. II. 1.

7) $\int l(1-2p\,Cos.x+p^2)\,Sin.ax.\,Sin.x\,dx = \frac{1}{2}\pi\left(\frac{p^{a+1}}{a+1}-\frac{p^{a-1}}{a-1}\right)$

8) $\int l(1-2p\,Cos.x+p^2)\,Cos.ax.\,Cos.x\,dx = -\frac{1}{2}\pi\left(\frac{p^{a+1}}{a+1}+\frac{p^{a-1}}{a-1}\right)$

9) $\int l(1-2p\,Cos.2x+p^2)\,Cos.\{(2a-1)x\}\,dx = 0$

10) $\int l(1-2p\,Cos.2x+p^2)\,Sin.2ax.\,Sin.x\,dx = 0$

11) $\int l(1-2p\,Cos.2x+p^2)\,Cos.2ax.\,Cos.x\,dx = 0$

} Bierens de Haan, Gr. 13. 193.

12) $\int l(1-2p\,Cos.\,2x+p^2)\,Sin.\{(2a-1)x\}.\,Sin.\,x\,dx = -\frac{1}{2}\pi\left(\frac{p^{a-1}}{a-1}-\frac{p^a}{a}\right)$

13) $\int l(1-2p\,Cos.\,2x+p^2)\,Cos.\{(2a-1)x\}.\,Cos.\,x\,dx = -\frac{1}{2}\pi\left(\frac{p^{a-1}}{a-1}+\frac{p^a}{a}\right)$ } Bierens de Haan, Gr. 13. 193.

14) $\int l\,x.\left(b\,Cos.\,x.\,Cos.\,\frac{2a\pi x}{b}-2a\pi\,Sin.\,x.\,Sin.\,\frac{2a\pi x}{b}\right)dx = -\frac{1}{2}b\pi$ V. T. 249. N°. 24.

F. Log. de Circ. Dir. Circ. Dir. fract. **TABLE 355.** Lim. 0 et π.

1) $\int l(p\,Cos.\,x+1)\frac{dx}{Cos.\,x} = \pi\,Arcsin.\,p\;,\; p^2>1;$ Winckler, Cr. 45. 102.

2) $\int l\frac{1+Sin.\,x}{1+Cos.\,\lambda.\,Sin.\,x}\frac{dx}{Sin.\,x} = \lambda^2$ V. T. 179. N°. 13.

3) $\int l(1-2p\,Cos.\,x+p^2)\frac{dx}{Cos.\,x} = \infty\;,\; p^2 \leqq 1;$ Schlömilch, Beitr. II. 1 trouve fautivement $-2\pi\,Arctang.\,p$.

4) $\int l(1-2p\,Cos.\,2x+p^2)\frac{dx}{Cos.\,x} = 0\;,\; p<1;$ V. T. 348. N°. 3.

5) $\int l\frac{1+2q\,Cos.\,x+q^2}{1+2q\,Cos.\,ax+q^2}\frac{dx}{Tang.\,\frac{1}{2}x} = 0$ Raabe, Cr. 28. 105.

6) $\int l\,Sin.\,x\frac{dx}{1\pm 2p\,Cos.\,x+p^2} = \frac{\pi}{1-p^2}l\frac{1-p^2}{2}\;,\; p^2<1;$

7) $= \frac{\pi}{p^2-1}l\frac{p^2-1}{2p^2}\;,\; p^2>1;$ } Raabe, Int. 161. — Ohm, Ausw. 18.

8) $\int l\,Sin.\,x\frac{p+Cos.\,x}{1+2p\,Cos.\,x+p^2}dx = \frac{\pi}{2p}l\frac{1}{1-p^2}\;,\; p<1;$ Raabe, Int. 161.

9) $= \frac{\pi}{2p}l\frac{p^2-1}{4p^2}\;,\; p^2>1;$ V. T. 373. N°. 16.

10) $\int l\,Sin.\,x\frac{Cos.\,x-p}{1-2p\,Cos.\,x+p^2}dx = \frac{\pi}{2p}l(1-p^2)\;,\; p^2<1;$

11) $= \frac{\pi}{2p}l\frac{4p^2}{p^2-1}\;,\; p^2>1;$ } V. T. 371. N°. 11, 12.

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12) $\int l\,Sin.x\,\frac{p\,Cos.x+1}{1+2p\,Cos.x+p^2}\,dx=\frac{1}{2}\pi\,l\frac{1-p^2}{4}\,,\ p^2<1;$

13) $=\frac{1}{2}\pi\,l\frac{p^2}{p^2-1}\,,\ p^2>1;$ V. T. 371. N°. 15, 16.

14) $\int l\,Sin.x\,\frac{p\,Cos.x-1}{1-2p\,Cos.x+p^2}\,dx=\frac{1}{2}\pi\,l\frac{4}{1-p^2}\,,\ p^2<1;$

15) $=\frac{1}{2}\pi\,l\frac{p^2-1}{p^2}\,,\ p^2>1;$ V. T. 371. N°. 13, 14.

16) $\int l\,Sin.x\,\frac{Cos.x}{1-2p\,Cos.2x+p^2}\,dx=0\,,\ p>0;$ V. T. 371. N°. 6.

17) $\int l\,Sin.x\,\frac{Cos.2x-p}{1-2p\,Cos.2x+p^2}\,dx=\frac{\pi}{2p}\,l(1-p)\,,\ p<1;$ V. T. 371. N°. 9.

18) $\int l\,Cos.ax\,\frac{dx}{1-2p\,Cos.2x+p^2}=\frac{\pi}{1-p^2}\,l\frac{1+p^{2a}}{2}$ Plana, Mém. Turin. 1818. 7. II. 14.

19) $\int l\,Cos.x\,\frac{Cos.2x-p}{1-2p\,Cos.2x+p^2}\,dx=\frac{\pi}{2p}\,l(1+p)\,,\ p^2<1;$ V. T. 315. N°. 17, 20.

20) $\int l\,Tang.x\,\frac{Cos.2x-p}{1-2p\,Cos.2x+p^2}\,dx=\frac{\pi}{2p}\,l\frac{1-p}{1+p}\,,\ p^2<1;$ V. T. 373. N°. 4.

21) $\int l(1-2p\,Cos.x+p^2)\frac{dx}{1-2q\,Cos.x+q^2}=\frac{2\pi}{1-q^2}\,l(1-pq)\,,\ p^2\leqq 1;$ Schlömilch, Beitr. II. 2.

22) $\int l(1+2p\,Cos.ax+p^2)\frac{dx}{1-2q\,Cos.x+q^2}=\frac{2\pi}{1-q^2}\,l(1+pq^a)$ Plana, Mém. Turin. 1818. 7. II. 14.

23) $\int l(1-p^2\,Sin.^2x)\frac{dx}{\sqrt{(1-p^2\,Sin.^2x)}}=l(1-p^2).\,F'(p)$ Roberts, L. 12. 449.

F. Log. Circ. Dir. TABLE 356. Lim. 0 et 2π.

1) $\int l\,Sin.x\,dx=-2\pi\left(l\,2-2a\pi i-\frac{2\beta+1}{2}\pi i\right)$ Arndt, Gr. 6. 187.

2) $\int l(1-2p\,Cos.x+p^2)\,dx=0$ Bierens de Haan, Gr. 13. 193.

3) $\int l\frac{1+2p\,Cos.x+p^2}{1+2p\,Cos.ax+p^2}\,dx=0$ Raabe, Cr. 23. 105.

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4) $\int l\,Sin.\frac{1}{4}x.Cos.\,ax\,dx = \frac{2\pi}{k}\sum_1^{k-1} Cos.\frac{2an\pi}{k}\,l\,Sin.\frac{n\pi}{2k}$, $k=\infty$; Raabe, Cr. 25. 160. (trouvefaut. $\frac{2}{k}\Sigma$)

5) $\int l(2+2\,Cos.x)\,Cos.\,ax\,dx = \frac{2\pi}{a}(-1)^{a-1}$

6) $\int l\frac{1+Cos.x}{1+Cos.bx}\,Cos.\,ax\,dx = 2\pi\left\{\frac{(-1)^{a-1}}{a} - \frac{(-1)^{\frac{a}{b}-1}b}{a}\right\}$

7) $\int l(1+2p\,Cos.bx+p^2)\,Cos.\,ax\,dx = 0$, où b indivisible par a;

8) $\int l(1+2p\,Cos.x+p^2)\,Cos.\,ax\,dx = 2\pi(-1)^{a-1}\frac{1}{a}p^a$, $p^2 \leqq 1$;

9) $\qquad = 2\pi(-1)^{a-1}\frac{1}{ap^a}$, $p^2 \geqq 1$;

(5–9) Raabe, Cr. 23. 105.

10) $\int l(1-2p\,Cos.x+p^2)\,Cos.\,ax\,dx = -\frac{2\pi}{a}p^a$

11) $\int l(1-2p\,Cos.x+p^2)\,Sin.\,ax.Sin.\,x\,dx = -\pi\left(\frac{p^{a-1}}{a-1}-\frac{p^{a+1}}{a+1}\right)$

12) $\int l(1-2p\,Cos.x+p^2)\,Cos.\,ax.Cos.\,x\,dx = -\pi\left(\frac{p^{a+1}}{a+1}+\frac{p^{a-1}}{a-1}\right)$

(10–12) , $p<1$; Bierens de Haan, Gr. 13. 193.

13) $\int l\frac{1+2p\,Cos.x+p^2}{1+2p\,Cos.bx+p^2}\,Cos.\,ax\,dx = 2\pi\left\{(-1)^{a-1}\frac{p^a}{a} - (-1)^{\frac{a-b}{b}}\frac{bp^{\frac{a}{b}}}{a}\right\}$, $p^2 \leqq 1$;

14) $\qquad = 2\pi\left\{(-1)^{a-1}\frac{1}{ap^a} - (-1)^{\frac{a-b}{b}}\frac{b}{ap^{\frac{a}{b}}}\right\}$, $p^2 \geqq 1$;

(13–14) Raabe, Cr. 23. 105.

15) $\int l\frac{1+2p\,Cos.x+p^2}{1+2p\,Cos.\,ax+p^2}\,\frac{dx}{Tang.\,\frac{1}{2}x} = 0$ Raabe, Cr. 23. 105.

16) $\int l\,Sin.\,x.\frac{p-Cos.x}{1-2p\,Cos.x+p^2}\,dx = \frac{\pi}{p}l(1-p^2)$, $p^2<1$; V. T. 373. N°. 5.

F. Log. $l\,Tang\,x$. Circ. Dir. TABLE 357. Lim. $\frac{\pi}{4}$ et $\frac{\pi}{2}$.

1) $\int l\,Tang.\,x\,dx = \sum_0^\infty (-1)^n\frac{1}{(2n+1)^2}$ V. T. 152. N°. 11.

2) $\int l\,Tang.\,x\frac{dx}{Sin.\,4x} = -\infty$ V. T. 153. N°. 11.

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3) $\int l\,Tang.\,x\,\frac{dx}{Cos.\,2\,x} = -\frac{1}{8}\pi^2$ V. T. 187. N°. 13.

4) $\int l\,Tang.\,x\,\frac{dx}{Tang.\,2\,x} = -\infty$ V. T. 153. N°. 10.

5) $\int l\,Tang.\,x\,\frac{dx}{2 - Sin.\,2\,x} = \frac{2}{27}\pi^2$ V. T. 153. N°. 3.

6) $\int l\,Tang.\,x\,\frac{Sin.\,2\,x}{4\,Cos.^2\,2\,x + Sin.^2\,2\,x}\,dx = \frac{1}{54}\pi^2$ V. T. 153. N°. 7.

7) $\int l\,Tang.\,x\,\frac{Sin.^2\,2\,x}{Sin.^4\,x + Cos.^4\,x}\,\frac{dx}{Cos.\,2\,x} = -\frac{1}{4}\,\frac{\pi^2}{2 + \sqrt{2}}$ V. T. 153. N°. 19.

8) $\int l\,Tang.\,x\,\frac{1}{Tang^p\,x - Cot^p\,x}\,\frac{dx}{Sin.^2\,2\,x} = \frac{\pi^2}{16\,p^2}\,Sec.^2\,\frac{\pi}{2\,p}$ V. T. 152. N°. 20.

9) $\int l\,Tang.\,x\,\frac{Cos.\,2\,x}{Tang^p\,x + Cot^p\,x}\,\frac{dx}{Sin.^2\,2\,x} = -\frac{\pi^2}{16\,p^2}\,Sin.\,\frac{\pi}{2\,p}.\,Sec.^2\,\frac{\pi}{2\,p}$ V. T. 152. N°. 19.

10) $\int l\,Tang.\,x\,\frac{Tang^q\,x - Cot^q\,x}{Tang^p\,x + Cot^p\,x}\,\frac{dx}{Sin.\,2\,x} = \frac{\pi^2}{8\,p^2}\,Sin.\,\frac{q\,\pi}{2\,p}.\,Sec.^2\,\frac{q\,\pi}{2\,p}$ V. T. 153. N°. 12.

11) $\int l\,Tang.\,x\,\frac{Tang^q\,x + Cot^q\,x}{Tang^p\,x - Cot^p\,x}\,\frac{dx}{Sin.\,2\,x} = \frac{\pi^2}{8\,p^2}\,Sec.^2\,\frac{q\,\pi}{2\,p}$ V. T. 153. N°. 13.

12) $\int l\,Tang.^2\left(\frac{\pi}{4} \pm x\right)\frac{dx}{Sin.\,2\,x} = \mp\frac{1}{4}\pi^2$ V. T. 357. N°. 3.

F. Log. $(l\,Tang.\,x)^a$ pour a spécial. Circ. Dir. **TABLE 358.** Lim. $\frac{\pi}{4}$ et $\frac{\pi}{2}$.

1) $\int (l\,Tang.\,x)^2\,dx = \frac{1}{16}\pi^3$ V. T. 154. N°. 1.

2) $\int (l\,Tang.\,x)^2\,\frac{dx}{2 + Sin.\,2\,x} = \frac{4}{243}\pi^3\sqrt{3}$ V. T. 156. N°. 1.

3) $\int (l\,Tang.\,x)^2\,\frac{dx}{2 - Sin.\,2\,x} = \frac{5}{243}\pi^3\sqrt{3}$ V. T. 156. N°. 2.

4) $\int (l\,Tang.\,x)^2\,\frac{dx}{1 + Cos.\,\lambda.\,Sin.\,2\,x} = \lambda\,Cosec.\,\lambda\,\frac{\pi^2 - \lambda^2}{6}$ V. T. 156. N°. 3.

5) $\int (l\,Tang.\,x)^2\,\frac{dx}{1 - Cos.\,\lambda.\,Sin.\,2\,x} = 2\,\lambda\,Cosec.\,\lambda\left(\frac{1}{6}\pi^2 - \frac{1}{4}\pi\,\lambda + \frac{1}{12}\lambda^2\right)$ V. T. 156. N°. 4.

6) $\int (l\,Tang.\,x)^2 \frac{dx}{1 - Sin.^2 x.\,Cos.^2 x} = \frac{1}{27}\pi^3 \sqrt{3}$ V. T. 154. N°. 3.

7) $\int (l\,Tang.\,x)^2 \frac{Sin.\,2x}{1 - Sin.^2 x.\,Cos.^2 x} dx = \frac{2}{243}\pi^3 \sqrt{3}$ V. T. 154. N°. 4.

8) $\int (l\,Tang.\,x)^2 \frac{dx}{Sin.^4 x + Cos.^4 x} = \frac{3}{64}\pi^3 \sqrt{2}$ V. T. 154. N°. 2.

9) $\int (l\,Tang.\,x)^2 \frac{Tang.^q x + Cot.^q x}{Tang.^p x + Cot.^p x} \frac{dx}{Sin.\,2x} = \frac{\pi}{16p^3}\left\{2\,Sec.^3 \frac{q\pi}{2p} - Sec.\frac{q\pi}{2p}\right\}$ V. T. 154. N°. 5.

10) $\int (l\,Tang.\,x)^2 \frac{Tang.^q x - Cot.^q x}{Tang.^p x - Cot.^p x} \frac{dx}{Sin.\,2x} = \frac{\pi^2}{8p^3} Sin.\frac{q\pi}{2p}.\,Sec.^3 \frac{q\pi}{2p}$ V. T. 154. N°. 6.

11) $\int (l\,Tang.\,x)^3 \frac{dx}{Cos.\,2x} = -\frac{1}{16}\pi^4$ V. T. 154. N°. 14.

12) $\int (l\,Tang.\,x)^4\, dx = \frac{5}{64}\pi^5$ V. T. 154. N°. 1.

13) $\int (l\,Tang.\,x)^4 \frac{dx}{1 + Cos.\,\lambda.\,Sin.\,2x} = \lambda \frac{\pi^2 - \lambda^2}{Sin.\,\lambda} \frac{7\pi^2 - 3\lambda^2}{5}$ V. T. 156 N°. 5.

14) $\int (l\,Tang.\,x)^5 \frac{dx}{Cos.\,2x} = -\frac{1}{8}\pi^6$ V. T. 154. N°. 4.

15) $\int (l\,Tang.\,x)^6\, dx = \frac{61}{256}\pi^7$ V. T. 155. N°. 8.

16) $\int (l\,Tang.\,x)^7 \frac{dx}{Cos.\,2x} = -\frac{17}{32}\pi^8$ V. T. 155. N°. 10.

F. Log. $(l\,Tang.\,x)^a$ pour a général. Circ. Dir. TABLE 359. Lim. $\frac{\pi}{4}$ et $\frac{\pi}{2}$.

1) $\int (l\,Tang.\,x)^q\, dx = \Gamma(q+1) \sum_0^\infty \frac{(-1)^n}{(2n+1)^{q+1}}$ V. T. 187. N°. 7.

2) $\int (l\,Tang.\,x)^{2a}\, dx = \frac{1}{2}(-1)^{a+1}(2\pi)^{2a+1} B''\left(\frac{1}{4}\right)$ V. T. 158. N°. 2.

3) $\int (l\,Tang.\,x)^{2a-1} \frac{dx}{Cos.\,2x} = \frac{1-2^{2a}}{4a} B_{2a-1}\pi^{2a}$ V. T. 158. N°. 5.

4) $\int (l\,Tang.\,x)^{2a} \frac{dx}{Cos.\,2x} = \frac{1-2^{2a+1}}{2^{2a+1}} 1^{2a}/1 \sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 158. N°. 4.

5) $\int (l\,Tang.\,x)^{2a} \frac{dx}{2+Sin.\,2x} = \frac{(-1)^{a+1}}{2\sqrt{3}} (2\pi)^{2a+1} B''\left(\frac{1}{3}\right)$ V. T. 159. N°. 1.

6) $\int (l\,Tang.\,x)^{2a} \frac{dx}{2-Sin.\,2x} = \frac{(-1)^{a+1}}{2\sqrt{3}} (2\pi)^{2a+1} B''\left(\frac{1}{6}\right)$ V. T. 159. N°. 2.

7) $\int (l\,Tang.\,x)^{2a} \frac{dx}{1-Cos.\,2p\pi.\,Sin.\,2x} = \frac{1}{2}(-1)^{a+1}(2\pi)^{2a+1} Cosec.\,2p\pi\,B''(p)$ V. T. 159. N°. 5.

8) $\int (l\,Tang.\,x)^{2a-1} \frac{1}{Sin.\,x - Cos.\,x} \frac{dx}{\sqrt{Sin.\,2x}} = \frac{2^{2a}-1}{4a\sqrt{2}} (2\pi)^{2a} B_{2a-1}$ V. T. 164. N°. 3.

F. Log. en dén. Circ. Dir. TABLE 360. Lim. $\frac{\pi}{4}$ et $\frac{\pi}{2}$.

1) $\int \frac{Tang.^q x - Cot.^q x}{l\,Tang.\,x} dx = l\,Tang.\left\{\frac{1+q}{4}\pi\right\}$ V. T. 175. N°. 5.

2) $\int \frac{(Tang.^q x - Cot.^q x)^2}{l\,Tang.\,x} \frac{dx}{Cos.\,2x} = l\,Cos.\,q\pi$ V. T. 175. N°. 6.

3) $\int \frac{dx}{q^2 + (l\,Tang.\,x)^2} = \frac{1}{4q}\left\{Z'\left(\frac{2q+3\pi}{4\pi}\right) - Z'\left(\frac{2q+\pi}{4\pi}\right)\right\}$ V. T. 173. N°. 9.

4) $\int \frac{Tang.^q x - Cot.^q x}{Tang.^p x + Cot.^p x} \frac{dx}{Sin.\,2x.\,l\,Tang.\,x} = \frac{1}{2} l\,Tang.\left\{\frac{p+q}{4p}\pi\right\}$ V. T. 172. N°. 6.

5) $\int \frac{Tang.^q x + Cot.^q x - 2}{Tang.^p x - Cot.^p x} \frac{dx}{Sin.\,2x.\,l\,Tang.\,x} = \frac{1}{2} l\,Cos.\frac{q\pi}{2p}$ V. T. 172. N°. 7. corr.

6) $\int \frac{1}{q^2 + (l\,Tang.\,x)^2} \frac{1}{Sin.\,x + Cos.\,x} \frac{dx}{\sqrt{Sin.\,2x}} = \frac{1}{4q\sqrt{2}}\left\{Z'\left(\frac{q+3\pi}{4\pi}\right) - Z'\left(\frac{q+\pi}{4\pi}\right)\right\}$ V. T. 177. N°. 3.

7) $\int \frac{Cos.\,x}{\sqrt{l\,Cosec.\,x}} dx = \sqrt{\pi}$ V. T. 349. N°. 6.

8) $\int \frac{Sin.\,x}{\sqrt{l\,Sec.\,x}} dx = \sqrt{\pi}$ V. T. 349. N°. 1.

TABLE 361.

1) $\int_0^{a\pi} l\,Sin.\,x\,dx = -a\pi\,l2$ Clausen, Cr. 7. 309.

2) $\int_0^{2a\pi} l((Sin.\,x))\,dx = -2a\pi\left\{l2 - a\pi i - \frac{1}{2}(2\beta+1)\pi i\right\}$

3) $\int_0^{(2a+1)\pi} l((Sin.\,x))\,dx = -(2a+1)\pi\left\{l2 - 2a\pi i - \frac{2\beta+1}{2a+1}a\pi i\right\}$

Arndt, Gr. 6. 187. — Lindmann, Gr. 16. 94. où fautives.

4) $\int_0^{2a\pi} Cos.\frac{bx}{a}.\,l Sin.\frac{x}{4a}\,dx = \frac{2a\pi}{k}\sum_1^{k-1} Cos.\frac{2bn\pi}{k}.\,l\,Sin.\frac{n\pi}{2k}$, $k=\infty$; Raabe, Cr. 25. 160.

5) $\int_0^{\frac{1}{2}a\pi} l\,Tang.^2\left(\frac{\pi}{4}\pm x\right)\frac{Sin.\,2x}{1-q^2\,Cos.^2\,2x}\,dx = \pm\frac{a\pi}{2q}Arcsin.\,q$, $q<1$; V. T. 361. N°. 6.

6) $\int_0^{\frac{1}{2}a\pi} l\frac{1+q\,Cos.\,x}{1-q\,Cos.\,x}\frac{dx}{Cos.\,x} = a\pi\,Arcsin.\,q$, $q<1$; Raabe, Cr. 25. 169.

F. Log. Circ. Dir. TABLE 362. Lim. 0 et λ.

TABLE 362.

1) $\int l\,x.\,Cos.\,qx\,dx = \frac{1}{q}\{Sin.\,q\lambda.\,l\lambda - Si.\,(q\lambda)\}$ V. T. 251. N°. 4.

2) $\int dx\,l\{Cos.\,x + \sqrt{(Cos.^2\,x - Sinhp.^2\,\mu)}\} = \left(\lambda+\varphi-\frac{1}{2}\pi\right)l\,Sin\,hp.\,\mu +$
$+\frac{1}{2}\mathrm{L}(\lambda+\varphi) - \frac{1}{2}\mathrm{L}(\lambda-\varphi) - \mathrm{L}(\varphi)$

3) $\int dx\,l\{Cos.\,x + \sqrt{(Cos.^2\,x - Sin\,hp.^2\,\lambda)}\} = (\lambda - \frac{1}{2}\pi)\,l\,Sin\,hp.\,\lambda$

4) $\int dx\,l\frac{Cos.\,x + \sqrt{(Cos.^2\,x - Sin\,hp.^2\,\mu)}}{Cos.\,x - \sqrt{(Cos.^2\,x - Sin\,hp.^2\,\lambda)}} = \left(\varphi+\lambda-\frac{1}{2}\pi\right)l\,Sin\,hp.\,\mu -$
$-\left(\frac{1}{2}\pi+\lambda\right)l\,Sin\,hp.\,\lambda + \frac{1}{2}\mathrm{L}(\lambda+\varphi) - \frac{1}{2}\mathrm{L}(\lambda-\varphi) - \mathrm{L}(\varphi)$

$,\ \mu<\lambda,\ Cos.\,\varphi = \frac{Cos\,hp.\,\lambda}{Cos\,hp.\,\mu}$; Lobatschewsky, Mém. Kasan. 1836. 1. I. 118, 117, 114.

5) $\int dx\,l\{Cos.\,x + \sqrt{(Cos.^2\,x - Cos.^2\,\lambda)}\} = \left(\lambda - \frac{1}{2}\pi\right)l\,Cos.\,\lambda$

6) $\int dx\,l\{Cos.\,x + \sqrt{(Cos.^2\,x - Cos.^2\,\mu)}\} = \left(\lambda+\varphi-\frac{1}{2}\pi\right)l\,Cos.\,\mu +$
$+\frac{1}{2}\mathrm{L}(\lambda+\varphi) - \frac{1}{2}\mathrm{L}(\lambda-\varphi) - \mathrm{L}(\varphi)$

$,\ Cos.\,\varphi = Sin.\,\lambda.\,Cosec.\,\mu,\ \mu<\lambda$; Lobatschewsky, Mém. Kasan. 1836. 1. II. 21, 39.

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7) $\int dx\, l \frac{Cos. x + Cos. \lambda}{Cos. x - Cos. \lambda} = \pi l 2 - 2 L(\lambda) - 2 L\left(\frac{1}{2}\pi - \lambda\right)$

8) $\int dx\, l \frac{Sin. \lambda + Sin. \mu . Cos. x \sqrt{(Sin.^2 \lambda - Sin.^2 x)}}{Sin. \lambda - Sin. \mu . Cos. x \sqrt{(Sin.^2 \lambda - Sin.^2 x)}} =$

$= \pi l \left\{Tang. \frac{1}{2}\mu . Sin. \lambda + \sqrt{\left(Tang.^2 \frac{1}{2}\mu . Sin.^2 \lambda + 1\right)}\right\}$

Lobatschewsky, Mém. Kasan, 1836. 1. II. 20, 14.

9) $\int l \frac{1 + Sin. x}{1 - Sin\, x} \frac{Cos. x}{Sin. x \sqrt{(Sin.^2 \lambda - Sin. x)}} dx = \pi \lambda\ Cosec. \lambda$

10) $\int \left(l \frac{1 + Sin. x}{1 - Sin. x} - 2 Sin. x\right) \frac{Cos. x}{Sin.^3 x \sqrt{(Sin.^2 \lambda - Sin.^2 x)}} dx = \pi\, Cosec.^3 \lambda \frac{\lambda - Sin. \lambda . Cos. \lambda}{2}$

Legendre, Exerc. Suppl. 34.

11) $\int l \frac{1 + Sin. x}{1 - Sin. x} \frac{Sin. x . Cos. x}{\sqrt{(Sin.^2 \lambda - Sin.^2 x)}} dx = \pi (1 - Cos. \lambda)$ Legendre, Exerc. Suppl. 35.

12) $\int l \frac{1 + Sin. x}{1 - Sin. x} \frac{Sin. x}{Cos. x \sqrt{(Sin.^2 \lambda - Sin.^2 x)}} dx = \pi\, Sec. \lambda . l\, Sec. \lambda$

13) $\int l \frac{1 + Sin. x}{1 - Sin. x} \frac{Sin.^3 x}{Cos.^3 x \sqrt{(Sin.^2 \lambda - Sin.^2 x)}} dx = \frac{1}{4}\pi\, Sin.^2 \lambda . Sec.^3 \lambda - \frac{1}{2}\pi\, Sec.^3 \lambda . l\, Cos. \lambda$

Legendre, Exerc. Suppl. 39.

14) $\int \left\{l \frac{1 + Sin. x}{1 - Sin. x} - 2\, Sin. x\right\} \frac{Cos. x}{Sin.^2 x \sqrt{(Sin.^2 \lambda - Sin.^2 x)}} dx = 2\, Cosec. \lambda (1 - \lambda\, Cot. \lambda)$ Legendre, Exerc. Suppl. 49.

1) $\int l\, Cot. \tfrac{1}{2} x \frac{Sin. x . Cos. x}{1 - Cos.^2 \lambda . Cos.^2 x} \frac{dx}{\sqrt{(Sin.^2 x - Sin.^2 \mu)}} =$

$= \frac{\pi}{Sin. 2\lambda} Sin. \left(Arctang. \frac{Tang. \lambda}{Sin. \mu}\right) l \left\{Tang. \frac{1}{2}\lambda . Cot. \left(\frac{1}{2} Arctang. \frac{Tang. \lambda}{Sin. \mu}\right)\right\}$

Lobatschewsky, Mém. Kasan. 1835. I.

2) $\int l Cot. \tfrac{1}{2} x \frac{Sin. x . Cos. x}{Sin.^2 x - Sin.^2 \mu} \frac{dx}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)}} = \frac{1}{2}\pi\, Cosec. \lambda . Sec. \varphi\, l \left\{Cot. \frac{1}{2}\varphi . Tg. \frac{1}{2}\mu\right\}, Sin. \varphi = \frac{Sin. \mu}{Sin. \lambda};$

Lobatschewsky, Mém. Kasan. 1835. 1.

3) $\int l Cot. \tfrac{1}{2} x \frac{Sin. x . Cos. x}{Sin.^2 \lambda + Tg.^2 \mu\, Sin.^2 x} \frac{dx}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)}} = \frac{\pi\, Cos.^2 \mu}{2 Sin. \lambda . Sin. \mu} l \frac{Sin. \mu + \sqrt{(1 - Cos.^2 \lambda . Cos.^2 \mu)}}{Sin. \mu (1 + Sin. \lambda)}$

Lobatschewsky, Mém. Kasan. 1836. 1. II. 25. où elle était fautive.

4) $\int l \frac{1 + Sin. x}{1 - Sin. x} \frac{dx}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)}} = \pi\, F'(Sin. \lambda)$ Legendre, Exerc. Suppl. 34.

5) $\int l \frac{1+Sin.x}{1-Sin.x} \frac{\sqrt{(Sin.^2 x - Sin.^2\lambda)}}{Sin.^2 x} dx = -\pi Sin.\lambda + \pi E'(Sin.\lambda)$ Legendre, Exerc. Suppl. 34.

6) $\int l \frac{1+Sin.x}{1-Sin.x} \frac{Cos.^2 x}{\sqrt{(Sin.^2 x - Sin.^2\lambda)}} dx = -\pi + \pi E'(Sin.\lambda)$

7) $\int l \frac{1+Sin.x}{1-Sin\,x} \frac{Sin.^4 x - Sin.^2\lambda}{Sin.^2 x \sqrt{(Sin.^2 x - Sin.^2\lambda)}} dx = \pi(1 - Sin.\lambda)$

8) $\int l \frac{1+Sin.x}{1-Sin.x} dx \sqrt{(Sin.^2 x - Sin.^2\lambda)} = \pi + Cos.^2\lambda\, F'(Sin.\lambda) - \pi E'(Sin.\lambda)$

(6–8) Legendre, Exerc. Suppl. 35.

9) $\int l\, Cot.\tfrac{1}{2}x \frac{Sin.x.\,Cos.x}{Sin.^2\lambda.\,Cos.^2\mu + Sin.^2\mu.\,Sin.^2 x} \frac{dx}{\sqrt{(Sin.^2 x - Sin.^2\lambda)}} =$

$= \frac{1}{Sin.\lambda.\,Sin.\mu}\left\{l\,Sin.\lambda - \frac{1}{2}\pi\, l \frac{1+Sin.\mu}{Sin.\mu + \sqrt{(1 - Cos.^2\lambda.\,Cos.^2\mu)}}\right\}$

10) $\int l \frac{Sin.x + \sqrt{(Sin.^2 x - Sin.^2\lambda)}}{Sin.x - \sqrt{(Sin.^2 x - Sin.^2\lambda)}} \frac{dx}{1 - Cos.^2\mu.\,Cos.^2 x} =$

$= Cosec.\mu\left\{-\pi\, l\,Sin.\lambda - \pi\, l \frac{1+Sin.\mu}{Sin.\mu + \sqrt{(1 - Cos.^2\lambda.\,Cos.^2\mu)}}\right\}$

(9–10) Lobatschewsky, Mém. Kasan. 1836. 1. I. 193. 191.

11) $\int l\,\{Sin.x + \sqrt{(Sin.^2 x - Sin.^2\lambda)}\} \frac{dx}{1 - Cos.^2\mu.\,Cos.^2 x} =$

$= Cosec.\mu\left\{-Arctang.\frac{Tang.\lambda}{Sin.\mu}.\,l\,Sin.\lambda - \frac{1}{2}\pi\, l \frac{1+Sin.\mu}{Sin.\mu + \sqrt{(1 - Cos.^2\lambda.\,Cos.^2\mu)}}\right\}$ Lobatschewsky, Mém. Kasan. 1836. 1. I. 193. — Id., ib. II. 24.

F. Log. Circ. Dir. TABLE 364. Lim. λ et μ.

1) $\int l\,Sin.x.\,dx = L\left(\frac{1}{2}\pi - \mu\right) - L\left(\frac{1}{2}\pi - \lambda\right)$

2) $\int l\,Cos.x.\,dx = L(\lambda) - L(\mu)$

3) $\int l\,Tang.x.\,dx = L\left(\frac{1}{2}\pi - \mu\right) + L(\mu) - L\left(\frac{1}{2}\pi - \lambda\right) - L(\lambda)$

(1–3) Lobatschewsky, Mém. Kasan. 1836. 1. I. 17, 18, 19.

4) $\int l \frac{1+Sin.x}{1-Sin.x} \frac{Cos.x}{\sqrt{(Sin.^2 x - Sin.^2\lambda)(Sin.^2\mu - Sin.^2 x)}} dx = \pi\,Cosec.\mu\, F(c,\mu)$

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5) $\int l\frac{1-Sin.x}{1+Sin.x}\frac{Cos.x}{Sin.^2x\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}dx=\frac{\pi}{Sin.\lambda.Sin.\mu}+$

$+\frac{\pi}{Sin.^2\lambda.Sin.\mu}F(c,\mu)-\frac{\pi}{Sin.^2\lambda.Sin.\mu}E(c,\mu)$

6) $(2a+1)Sin.^2\lambda.Sin.^2\mu\int l\frac{1+Sin.x}{1-Sin.x}\frac{Cos.x}{Sin.^{2a+2}x\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}dx=$

$=2a(Sin.^2\lambda+Sin.^2\mu)\int_\lambda^\mu l\frac{1+Sin.x}{1-Sin.x}\frac{Cos.x\,dx}{Sin.^{2a}x\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}-$

$-(2a-1)\int l\frac{1+Sin.x}{1-Sin.x}\frac{Cos.x\,dx}{Sin.^{2a-2}x\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}-$

$-2\int_\lambda^\mu\frac{dx\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}{Cos.x.Sin.^{2a+1}x}$ Voir pour la dernière Intégrale T. 106. N°. 12.

7) $\int l\frac{1+Sin.x}{1-Sin.x}\frac{Cos.x}{Sin.^2x}dx\sqrt{\frac{Sin.^2\mu-Sin.^2x}{Sin.^2x-Sin.^2\lambda}}=\pi Sin.\mu.Cosec.\lambda+\pi\frac{Sin.^2\mu-Sin.^2\lambda}{Sin.^2\mu.Sin.^2\lambda}F(c,\mu)-\frac{\pi Sin.\mu}{Sin.^2\lambda}E(c,\mu)$

8) $\int l\frac{1+Sin.x}{1-Sin.x}\frac{Cos.x}{Sin.^2x}dx\sqrt{\frac{Sin.^2x-Sin.^2\lambda}{Sin.^2\mu-Sin.^2x}}=-\pi Sin.\lambda.Cosec.\mu+\pi Cosec.\mu E(c,\mu)$

9) $\int l\frac{1+Sin.x}{1-Sin.x}\frac{Cos.x.Sin.^2x}{\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}dx=\pi(1-Cos.\lambda.Cos.\mu)+$

$+\pi Sin.\mu F(c,\mu)-\pi Sin.\mu E(c,\mu)$

10) $(2a+1)\int l\frac{1+Sin.x}{1-Sin.x}\frac{Cos.x.Sin.^{2a+2}x}{\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}dx$

$=2a(Sin.^2\lambda+Sin.^2\mu)\int_\lambda^\mu l\frac{1+Sin.x}{1-Sin.x}\frac{Cos.x.Sin.^{2a}x\,dx}{\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}-$

$-(2a-1)Sin.^2\lambda.Sin.^2\mu\int_\lambda^\mu l\frac{1+Sin.x}{1-Sin.x}\frac{Cos.x.Sin.^{2a-2}x\,dx}{\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}+$

$+2\int_\lambda^\mu\frac{Sin.^{2a-1}x}{Cos.x}dx\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}$ Voir pour la dernière Intégrale T. 106. N°. 16.

11) $\int l\frac{1+Sin.x}{1-Sin.x}Cos.x\,dx\sqrt{\frac{Sin.^2\mu-Sin.^2x}{Sin.^2x-Sin.^2\lambda}}=\pi(Cos.\lambda.Cos.\mu-1)+\pi Sin.\mu E(c,\mu)$

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12) $\int l\frac{1+Sin.x}{1-Sin.x}\,Cos.x dx\sqrt{\frac{Sin.^2x-Sin.^2\lambda}{Sin.^2\mu-Sin.^2x}}=\pi(1-Cos.\lambda.\,Cos.\mu)+\frac{Sin.^2\mu-Sin.^2\lambda}{Sin.\,\mu}\pi\,F(c,\mu)-\pi\,Sin.\mu\,E(c,\mu)$

13) $\int l\frac{1+Sin.x}{1-Sin.x}\,\frac{dx}{Cos.x\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}=\pi\,Cosec.\mu\,\Pi(-Sin.^2\lambda,c,\mu)+$
$+\frac{1}{2}\pi\,Sec.\lambda.\,Sec.\mu\,l(1+Tang.^2\lambda+Tang.^2\mu)$

14) $\int l\frac{1+Sin.x}{1-Sin.x}\,\frac{dx}{Cos.^3x\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}=$
$=\frac{\pi}{4}\,\frac{Sin.^2\lambda.Cos.^2\mu+Sin.^2\mu.Cos.^2\lambda}{Cos.^3\lambda.\,Cos.^3\mu}-\frac{1}{2}\pi\,Cosec.\mu.\,Sec.^2\lambda\,F(c,\mu)-\frac{\pi\,Sin.\mu}{2\,Cos.^2\lambda.\,Cos.^2\mu}E(c,\mu)+$
$+\frac{Cos.^2\lambda+Cos.^2\mu+Cos.^2\lambda.Cos.^2\mu}{Cos.^2\lambda.\,Cos.^2\mu}\left\{\frac{1}{2}\pi\,Cosec.\mu\,\Pi(-Sin.^2\lambda,c,\mu)+\right.$
$\left.+\frac{1}{4}\pi\,Sec.\lambda.\,Sec.\mu\,l(1+Tang.^2\lambda+Tang.^2\mu)\right\}$

15) $2a\,Cos.^2\lambda.\,Cos.^2\mu\int l\frac{1+Sin.x}{1-Sin.x}\,\frac{dx}{Cos.^{2a+1}x\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}=$
$=(2a-1)(Cos.^2\lambda+Cos.^2\mu+Cos.^2\lambda.Cos.^2\mu)\int_\lambda^\mu l\frac{1+Sin.x}{1-Sin.x}\,\frac{dx}{Cos.^{2a-1}x\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}-$
$-(2a-2)(1+Cos.^2\lambda+Cos.^2\mu)\int_\lambda^\mu l\frac{1+Sin.x}{1-Sin.x}\,\frac{dx}{Cos.^{2a-3}x\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}+$
$+(2a-3)\int_\lambda^\mu l\frac{1+Sin.x}{1-Sin.x}\,\frac{dx}{Cos.^{2a-5}x\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}+$
$+2\int_\lambda^\mu\frac{Sin.x}{Cos.^{a+1}x}\,dx\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin^2\mu-Sin.^2x)}\Big\}$ Voir pour cette dernière Intégrale T. 106. N°. 9.

Ces formules 4—6, 7 et 8, 9—12, 13—15 se trouvent chez Legendre, Exerc. Suppl. N°. 33, 34, 35, 38; on y a partout $c=Sin.\lambda.\,Cosec.\mu$.

16) $\int l\frac{1+a\,Sin.x}{1-a\,Sin.x}\,\frac{Cos.x\,dx}{\sqrt{(Sin.^2x-Sin.^2\lambda)(Sin.^2\mu-Sin.^2x)}}=\pi\,Cosec.\mu\,F\left\{\frac{Sin.\lambda}{Sin.\mu},\,Arcsin.(a\,Sin.\mu)\right\}$ Roberts, L. 11. 157.

1) $\int_a^{a+1} l\,Sin.\pi x\,dx=a\pi i-l2$ Schaar, Mém. Cour. Brux. T. 23.

2) $\int_0^p\frac{Cos.kx}{Cos.x}\,l(1-q\,Cos.x)\,dx=0\,,\;k=\infty;$ Raabe, Int. 174.

3) $\int_0^{Arccos.(Tanghp.\lambda.\, Cothp.\mu)} dx\, l \frac{1 - Cos h p.\lambda.\, Cos h p.\mu.\, Cos.x \sqrt{(1 - Cot h p.^2 \lambda.\, Tang h p.^2 \mu.\, Cos.^2 x)}}{1 + Cos h p.\lambda.\, Cos h p.\mu.\, Cos.x \sqrt{(1 - Cot h p.^2 \lambda.\, Tang h p.^2 \mu.\, Cos.^2 x)}} =$

$= \pi l \frac{Sin h p.\mu (1 + Sin h p.\lambda)}{Sin h p.\lambda + \sqrt{(1 - Cos h p.^2 \lambda.\, Cos h p.^2 \mu)}}$ Lobatschewsky, Mém. Kasan. 1836. 1 .I. 75.

4) $\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} Cos.p x.\, Cos^p x\, dx\, l\, Cos.x = -\frac{\pi}{2^p} l 2$ V. T. 446. N°. 26.

5) $\int_0^{\frac{1}{2}\pi - \lambda} dx\, l \{Cos.x + \sqrt{(Cos.^2 x - Sin.^2 \lambda)}\} = -\lambda\, l\, Sin.\lambda$ Lobatschewsky, Mém. Kasan. 1836 1. I. 117.

F. Log. Circ. Inv. TABLE 366. Lim. 0 et 1.

1) $\int Arcsin.x.(lx + 1)\, dx = 1 - l2$ V. T. 163. N°. 3.

2) $\int Arccos.x.(1 + lx)\, dx = l2 - 1$ V. T. 163. N°. 3.

3) $\int Arctang.x.(lx + 1)\, dx = \frac{1}{48}\pi^2$ V. T. 152. N°. 1.

4) $\int Arctang.x.(lx + 3)(lx)^2\, dx = \frac{7}{1920}\pi^4$ V. T. 154. N°. 13.

5) $\int Arctang.x.(lx + 5)(lx)^4\, dx = \frac{31}{16128}\pi^6$ V. T. 155. N°. 2.

6) $\int Arctang.x.(lx + a)(lx)^{a-1}\, dx = \frac{1^{a/1}}{(-2)^{a+1}} \sum_0^\infty \frac{(-1)^n}{(n+1)^{a+1}}$ V. T. 157. N°. 8.

F. Log. Autres Fonctions. TABLE 367. Lim. diverses.

1) $\int_0^1 dx\, li\left(\frac{1}{x}\right)\left(l\frac{1}{x}\right)^{p-1} = -\pi\, Cot.p\pi\, \Gamma(p)$ V. T. 402. N°. 1.

2) $\int_0^1 l\Gamma(x)\, dx = \frac{1}{2} l 2\pi$ Raabe, Cr. 25. 146. — Id., Cr. 28. 10. — Schaar, Mém. Cour. Brux. T. 22. — Id., Mém. Cour. Brux. T. 23.

3) $\int_0^1 l\Gamma(1 + x)\, dx = -1 + \frac{1}{2} l 2\pi$ Raabe, Cr. 25. 146.

4) $\int_0^1 l\,\Gamma\,(x+q)\,dx = \frac{1}{2}\,l\,2\,\pi + q\,l\,q - q$ Raabe, Cr. 25. 146. — Id., Cr. 28. 10. — Stern, Gött. Stud. 1847.

5) $\int_a^{a+1} l\,\Gamma\,(x)\,dx = \frac{1}{2}\,l\,2\,\pi + a\,(l\,a - 1)$ Schaar, Mém. Cour. Brux. T. 22. — Id., ib. T. 23.

$= \frac{1}{2}\,l\,2\,\pi + l\,a - 1$ faute d'impression chez Raabe, Cr. 28. 10.

6) $\int_0^{\frac{1}{2}\pi} l\,\Theta\,(q, x)\,dx = \frac{\pi}{4}\,l\left(\frac{\mathrm{F}(p)}{\pi}\,(2\,p\,p')^{\frac{1}{3}}\,q^{-\frac{1}{6}}\right),\ q = e^{-\pi\frac{\mathrm{F}[\sqrt{(1-p^2)}]}{\mathrm{F}(p)}},\ p' = \sqrt{(1-p^2)}$ Roberts, L. 12. 449.

7) $\int_1^\infty dx\,l\,i\left(\frac{1}{x}\right)(l\,x)^{p-1} = -\pi\,Cosec.\,p\,\pi\,\Gamma\,(p)$ V. T. 402. N°. 2.

8) $\int_0^\infty dx\,l\,i\left(\frac{1}{x}\right)(l\,x)^{p-1} = -\,Sin.\,p\,\pi\,\Gamma\,(p)$ V. T. 367. N°. 1, 7.

F. Circ. Dir. ent. Circ. Inv. — TABLE 368. — Lim. 0 et $\frac{\pi}{2}$.

1) $\int Arctang.\,(Tang.^2\,x)\,dx = \frac{1}{8}\pi^2$ V. T. 269. N°. 1.

2) $\int Arctang.\,(Tang.^3\,x)\,dx = \frac{1}{8}\pi^2$ V. T. 269. N°. 2.

3) $\int Arctang.\,(\sqrt{Tang.\,x})\,\frac{dx}{(Sin.\,x + Cos.\,x)^2} = \frac{1}{4}\pi^2$ V. T. 269. N°. 3.

4) $\int Arccot.\,(Tang.^2\,x)\,dx = \frac{1}{8}\pi^2$ V. T. 269. N°. 4.

5) $\int Arccot.\,(Tang.^3\,x)\,dx = \frac{1}{8}\pi^2$ V. T. 269. N°. 5.

6) $\int Arccot.\,(\sqrt{Tang.\,x})\,\frac{dx}{(Sin.\,x + Cos.\,x)^2} = \frac{1}{4}\pi^2$ V. T. 269. N°. 6.

7) $\int Arcsin.\,(p\,Sin.\,x).\,Cos.\,x\,dx = Arcsin.\,p + \frac{1}{p}\sqrt{(1-p^2)} - \frac{1}{p}$ V. T. 108. N°. 4.

8) $\int Arctang.\,(p\,Cot.\,x).\,Tang.\,x\,dx = \frac{\pi}{2}\,l\,(1+p)$ V. T. 266. N°. 3.

9) $\int Arctang.(p\ Tang.x).\ Tang.2x\,dx = \frac{\pi}{4} l \frac{1+p^2}{(1+p)^2}$ V. T. 369. N°. 10, 11.

10) $\int Arctang.(p\ Cot.x).\ Tang.2x\,dx = \frac{\pi}{4} l \frac{(1+p)^2}{1+p^2}$ V. T. 368. N°. 8 et T. 369. N° 9.

11) $\int Arctang.\{Tang.\lambda.\sqrt{(1-p^2 Sin.^2 x)}\}\,dx\sqrt{(1-p^2 Sin.^2 x)} = \frac{\pi}{2}E(p,\lambda) -$

$- \frac{\pi}{2} Cot.\lambda.\{1-\sqrt{(1-p^2 Sin.^2\lambda)}\}$ Roberts, L. 11. 157.

12) $\int Arccot.(p\ Tang.x).\ Tang.x\,dx = \frac{\pi}{2} l \frac{1+p}{p}$ V. T. 265. N°. 12.

13) $\int Arccot.\{Tang.\lambda.\sqrt{(1-p^2 Sin.^2 x}\}\,dx\sqrt{(1-p^2 Sin.^2 x)} = \frac{\pi}{2}E(p,\varphi) -$

$-\frac{\pi\ Cot.\lambda}{2\sqrt{(1-p^2)}}\{\sqrt{(1-p^2 Sin.^2\varphi)}-\sqrt{(1-p^2)}\},\ Cot.\varphi = Tang.\lambda.\sqrt{(1-p^2)},\ p<1;$ Roberts, L. 11. 157.

F. Circ. Dir. fract.
Circ. Inv.

TABLE 369.

Lim. 0 et $\frac{\pi}{2}$.

1) $\int Arctang.\frac{p\ Sin.(r\ Tang.x)}{1+p\ Cos.(r\ Tang.x)}.\ Tang.x\,dx = \frac{1}{2}\pi l(1+pe^{-r})$ V. T. 431. N°. 7.

2) $\int Arctang.(Sin.x)\frac{dx}{Sin.x} = \frac{1}{2}\pi l(1+\sqrt{2})$ V. T. 261. N°. 14.

3) $\int Arctang.(p\ Sin.x)\frac{dx}{Sin.x} = \frac{1}{2}\pi l\{p+\sqrt{(1+p^2)}\}\ ,\ p \geqq 1;$ Raabe, Int. 421.

4) $\int Arctang.\left(\frac{Tang.x}{a}\right).Arctang.\left(\frac{Tang.x}{b}\right)\frac{dx}{Sin.^2 x} = \frac{1}{2}\pi\left\{\frac{1}{a} l \frac{a+b}{b} + \frac{1}{b} l \frac{a+b}{a}\right\}$ V. T. 264. N°. 14.

5) $\int Arctang.(Cos.x)\frac{dx}{Cos.x} = \frac{1}{2}\pi l(1+\sqrt{2})$ V. T. 261. N°. 14.

6) $\int Arctang.(p\ Cos.x)\frac{dx}{Cos.x} = \frac{1}{2}\pi l\{p+\sqrt{(1+p^2)}\},\ p \geqq 1;$ Raabe, Int. 421.

7) $\int \{Sin.^2 x.\ Arccot.(Sin.x) - Arctang.(Sin.x)\}\frac{dx}{Sin.2x} = \frac{1}{2}\pi l 2$ V. T. 258. N°. 28.

8) $\int Arccot.(a\ Tang.x).\ Arccot.(b\ Tang.x)\}\frac{dx}{Cos.^2 x} = \frac{\pi}{2}\left\{\frac{1}{a} l \frac{a+b}{b} + \frac{1}{b} l \frac{a+b}{a}\right\}$ V. T. 264. N°. 14.

9) $\int Arctang.(p\,Cot.\,x)\frac{Tang.\,x}{Cos.\,2\,x}dx = -\frac{1}{4}\pi\, l(1+p^2)$ V. T. 265. N°. 13.

10) $\int Arctang.(p\,Tang.\,x)\frac{dx}{Tang.\,x} = \frac{1}{2}\pi\, l(1+p)$ Mosta, Gr. 10. 449.

11) $\int Arctang.(p\,Tang.\,x)\frac{dx}{Tang.\,x.\,Cos.\,2\,x} = \frac{1}{4}\pi\, l(1+p^2)$ V. T. 265. N°. 13.

12) $\int Arctang.(p\,Cot.\,x)\frac{Sin.^3\,x}{Cos.x.\,Cos.2\,x}dx = -\frac{1}{8}\pi\, l\{(1+p^2)(1+p)^2\}$ V. T. 368. N°. 8 et T. 369. N°. 9.

13) $\int Arctang.(p\,Tang.\,x)\frac{Cos.^3\,x}{Sin.x.\,Cos.2\,x}dx = \frac{1}{8}\pi\, l\{(1+p)^2(1+p^2)\}$ V. T. 369. N°. 10, 11.

14) $\int Arctang.\{Tang.\,\lambda\sqrt{(1-p^2\,Sin.^2x)}\}\frac{dx}{\sqrt{(1-p^2\,Sin.^2x)}} = \frac{1}{2}\pi\,F(p,\lambda)$

15) $\int Arccot.\{Tang.\,\lambda\sqrt{(1-p^2\,Sin.^2x)}\}\frac{dx}{\sqrt{(1-p^2\,Sin.^2x)}} = \frac{1}{2}\pi\,F(p,\varphi)$

16) $\int Arctang.\{Tang.\,\lambda\sqrt{(1-p^2\,Sin.^2x)}\}\frac{dx}{\sqrt{(1-p^2\,Sin.^2x)^3}} =$

$= \frac{1}{2}\frac{\pi}{1-p^2}E(p,\lambda) - \frac{\pi}{2}\frac{Tg.\,\lambda}{1-p^2}\{\sqrt{(1-p^2\,Sin.^2\lambda)}-\sqrt{(1-p^2)}\}$

17) $\int Arccot.\{Tang.\,\lambda\sqrt{(1-p^2\,Sin.^2x)}\}\frac{dx}{\sqrt{(1-p^2\,Sin.^2x)^3}} =$

$= \frac{1}{2}\frac{\pi}{1-p^2}E(p,\varphi) - \frac{\pi}{2}\frac{Tang.\,\lambda}{\sqrt{(1-p^2)}}\{1-\sqrt{(1-p^2\,Sin.^2\varphi)}\}$

(14–17): $Cot.\,\varphi = Tg.\,\lambda\sqrt{(1-p^2)}$ Roberts, L. 11. 157.

F. Circ. Dir. ent. Circ. Inv. TABLE 370. Lim. 0 et π.

1) $\int Arctang.(Cos.\,x)\,dx = 0$ V. T. 245. N°. 12.

2) $\int Arctang.\left\{\frac{Sin.^2x}{\sqrt{(p^2-1)}}\right\}dx = 4\sum_0^\infty\frac{\{p-\sqrt{(p^2-1)}\}^{2n+1}}{(2n+1)^2}, p>1;$ V. T. 246. N°. 18.

3) $\int Arctang.\frac{p\,Sin.\,x}{1-p\,Cos.\,x}.Sin.\,ax\,dx = \frac{\pi}{2a}p^a,\ p^2 \leqq 1;$ Schlömilch, Beitr. II. § 1. — Bierens de Haan, Gr. 13. 193.

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4) $\int Arctang.\frac{p\, Sin.\, x}{1-p\, Cos.\, x}.\, Sin.\, x\, dx = \frac{1}{2} p\pi$

5) $\int Arctang.\frac{p\, Sin.\, x}{1-p\, Cos.\, x}.\, Sin.\, ax.\, Cos.\, x\, dx = \frac{1}{4}\pi\left(\frac{p^{a+1}}{a+1}+\frac{p^{a-1}}{a-1}\right)$

6) $\int Arctang.\frac{p\, Sin.\, x}{1-p\, Cos.\, x}.\, Cos.\, ax.\, Sin.\, x\, dx = \frac{1}{4}\pi\left(\frac{p^{a+1}}{a+1}-\frac{p^{a-1}}{a-1}\right)$

7) $\int Arctang.\frac{2p\, Sin.\, x}{1-p^2}.\, Sin.\, 2ax\, dx = 0$

8) $\int Arctang.\frac{2p\, Sin.\, x}{1-p^2}.\, Sin.\, \{(2a-1)x\}\, dx = \frac{\pi}{2a-1} p^{2a-1}$

9) $\int Arctang.\frac{2p\, Sin.\, x}{1-p^2}.\, Sin.\, 2ax.\, Cos.\, x\, dx = \frac{1}{2}\pi\left(\frac{p^{2a+1}}{2a+1}+\frac{p^{2a-1}}{2a-1}\right)$

10) $\int Arctang.\frac{2p\, Sin.\, x}{1-p^2}.\, Sin.\, \{(2a-1)x\}.\, Cos.\, x\, dx = 0$

11) $\int Arctang.\frac{2p\, Sin.\, x}{1-p^2}.\, Cos.\, 2ax.\, Sin.\, x\, dx = \frac{1}{2}\pi\left(\frac{p^{2a+1}}{2a+1}-\frac{p^{2a-1}}{2a-1}\right)$

12) $\int Arctang.\frac{2p\, Sin.\, x}{1-p^2}.\, Cos.\, \{(2a-1)x\}.\, Sin.\, x\, dx = 0$

13) $\int Arctang.\frac{q\, Sin.\, 2x}{1-q\, Cos.\, 2x}.\, Sin.\, 2ax\, dx = \frac{\pi}{a} q^a$

14) $\int Arctang.\frac{q\, Sin.\, 2x}{1-q\, Cos.\, 2x}.\, Sin.\, \{(2a-1)x\}\, dx = 0$

15) $\int Arctang.\frac{q\, Sin.\, 2x}{1-q\, Cos.\, 2x}.\, Sin.\, 2ax.\, Cos.\, x\, dx = 0$

16) $\int Arctang.\frac{q\, Sin.\, 2x}{1-q\, Cos.\, 2x}.\, Cos.\{(2a-1)x\}.\, Sin.\, x\, dx = \frac{1}{4}\pi\left(\frac{1}{a}q^a+\frac{1}{a-1}q^{a-1}\right)$

17) $\int Arctang.\frac{q\, Sin.\, 2x}{1-q\, Cos.\, 2x}.\, Cos.\, 2ax.\, Sin.\, x\, dx = 0$

18) $\int Arctang.\frac{q\, Sin.\, 2x}{1-q\, Cos.\, 2x}.\, Cos.\{(2a-1, x\}.\, Sin.\, x\, dx = \frac{1}{4}\pi\left(\frac{1}{a}q^a-\frac{1}{a-1}q^{a-1}\right)$

$, p^2 < 1, 1 > q > 0;$

Bierens de Haan, Gr. 13. 193.

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19) $\int (1+2a\,Cos.x+a^2)^{\frac{1}{2}c}(a^2+2ab\,Cos.x+b^2)^{\frac{1}{2}g} Sin.\left\{c\,Arccos.\frac{1+a\,Cos.x}{\sqrt{(1+2a\,Cos.x+a^2)}}\right\}.Sin.\left\{g\,Arccos.\frac{a+b\,Cos.x}{\sqrt{(a^2+2ab\,Cos.x+b^2)}}\right\}dx$

$$= \frac{1}{2}\pi a^g \sum_1^\infty \binom{c}{n}\binom{g}{n} b^n$$

20) $\int (1+2a\,Cos.x+a^2)^{\frac{1}{2}c}(a^2+2ab\,Cos.x+b^2)^{\frac{1}{2}g} Cos.\left\{c\,Arccos.\frac{1+a\,Cos.x}{\sqrt{(1+2a\,Cos.x+a^2)}}\right\}.Cos.\left\{g\,Arccos.\frac{a+b\,Cos.x}{\sqrt{(a^2+2ab\,Cos.x+b^2)}}\right\}dx$

$$= \frac{1}{2}\pi a^g \left\{2 + \sum_1^\infty \binom{c}{n}\binom{g}{n} b^n\right\}$$

Sur les intégrales (19), (22) voyez Smaasen Cr. 42. 222.

21) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.Tang.\frac{1}{2}x\,dx = \pi\,l(1+p)$, $p^2 \leqq 1$; Schlömilch, Beitr. II. § 1.

F. Circ. Dir. fract. à dén. monôme. Circ. Inv. TABLE 371. Lim. 0 et π.

1) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.\frac{dx}{Sin.x} = \frac{1}{2}\pi\,l\frac{1+p}{1-p}$

2) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.\frac{dx}{Tang.\frac{1}{2}x} = -\pi\,l(1-p)$

(1, 2) , $p^2 \leqq 1$; Schlömilch, Beitr. II. § 1.

3) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.\frac{dx}{Tang.x} = -\frac{1}{2}\pi\,l(1-p^2)$

4) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.\frac{Cos.^2x}{Sin.x}dx = \frac{1}{2}\pi\left\{l\frac{1+p}{1-p}-p\right\}$

5) $\int Arctang.\frac{2p\,Sin.x}{1-p^2}.\frac{dx}{Sin.x} = \pi\,l\frac{1+p}{1-p}$

6) $\int Arctang.\frac{2p\,Sin.x}{1-p^2}.\frac{dx}{Tang.x} = 0$

7) $\int Arctang.\frac{2p\,Sin.x}{1-p^2}.\frac{Cos.^2x}{Sin.x}dx = \pi\left\{l\frac{1+p}{1-p}-p\right\}$

8) $\int Arctang.\frac{q\,Sin.2x}{1-q\,Cos.2x}.\frac{dx}{Sin.x} = 0$

9) $\int Arctang.\frac{q\,Sin.2x}{1-q\,Cos.2x}.\frac{dx}{Tang.x} = -\pi\,l(1-q)$

10) $\int Arctang.\frac{q\,Sin.2x}{1-q\,Cos.2x}.\frac{Cos.^2x}{Sin.x}dx = 0$

(3–10) , $p^2 < 1$, $1 > q \geqq 0$; Bierens de Haan, Gr. 18. 193.

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11) $\int Arctang.\frac{p\,Sin.x}{1+p\,Cos.x}.\frac{dx}{Tang.x} = \frac{1}{2}\pi\,l(1-p^2)\,,\ p^2<1;$

12) $= \frac{1}{2}\pi\,l\frac{4p^2}{p^2-1}\,,\ p^2>1;$

13) $\int Arccot.\frac{p+Cos.x}{Sin.x}.\frac{dx}{Tang.x} = \frac{1}{2}\pi\,l\frac{4}{1-p^2}\,,\ p^2<1;$

14) $= \frac{1}{2}\pi\,l\frac{p^2-1}{p^2}\,,\ p^2>1;$

15) $\int Arccot.\frac{p-Cos.x}{Sin.x}.\frac{dx}{Tang.x} = \frac{1}{2}\pi\,l\frac{1-p^2}{4}\,,\ p^2<1;$

16) $= \frac{1}{2}\pi\,l\frac{p^2}{p^2-1}\,,\ p^2>1;$

(11–16) Ohm, Ausw. 18.

Pour les intégrales (13) et (15) il trouve fautivement $\pi l\frac{2}{1-p^2}$.

1) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.\frac{Sin.x}{1-2q\,Cos.x+q^2}dx = -\frac{\pi}{2q}l(1-pq)\,,\ p^2\leqq 1\,,\ q^2\leqq 1;$ Schlömilch, Beitr. II. § 2.

2) $\int Arctg.\frac{a\,Sin.x}{b+a\,Cos.x}.\frac{Sin.x}{\sqrt{(1+a^2-2a\,Cos.x)}}dx = \frac{1+a}{ab}\frac{a+b^2}{b-a}\frac{a+b}{1+b}F'\left\{\frac{4ab}{(b-a)^2}\frac{2\sqrt{b}}{1+b}\right\} -$

$- \frac{2}{b}E'(b) + \frac{1+b}{b}D$

où $D = \pi$, pour $a < -b$;

3) $= \frac{1-b}{1+b}\frac{\pi}{2}\,,\quad a = -b;$

4) $= 0\,,\quad -b < a < b;$

5) $= \frac{1}{2}\pi\,,\quad a = b;$

6) $= \pi\,,\quad a > b;$

7) $\int Arctang.\left\{\frac{Tang.\lambda}{\sqrt{(1-c^2\,Sin.^2\lambda)}}\sqrt{(1+p^2-2p\,Cos.x)}\right\}.\frac{dx}{\sqrt{(1+p^2-2p\,Cos.x)}} = \pi F(p,\lambda)$

(2–7) Ramus, Danske Afh. 6. 265.

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8) $\int Arctang.\frac{b\,Cos.x}{\sqrt{(a-b^2\,Cos.^2x)}}\cdot\frac{Cos.x}{\sqrt{(a-b^2\,Cos.^2x)}}dx=\frac{\pi}{2b}l\frac{a}{a-b^2}$ Winckler, Cr. 45. 102.

9) $\int Cos.\left\{c\,Arccos.\frac{a+b\,Cos.x}{\sqrt{(a^2+2ab\,Cos.x+b^2)}}\right\}\frac{1-a^g\,Cos.gx}{1-2a^g\,Cos\,gx+a^{2g}}(a^2+2ab\,Cos.x+b^2)^{\frac{1}{2}c}dx=$

$=\frac{1}{2}\pi a^c\left\{2+\sum_1^\infty\binom{c}{ng}b^{ng}\right\}$ Smaasen, Cr. 42. 222.

F. Circ. Dir. Circ. Inv. TABLE 373. Lim. 0 et 2 π.

1) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.Sin.ax\,dx=\frac{\pi}{a}p^a$

2) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.Sin.ax.Cos.x\,dx=\frac{1}{2}\pi\left(\frac{p^{a+1}}{a+1}+\frac{p^{a-1}}{a-1}\right)$

3) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.Cos.ax.Sin.x\,dx=\frac{1}{2}\pi\left(\frac{p^{a+1}}{a+1}-\frac{p^{a-1}}{a-1}\right)$

4) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.\frac{dx}{Sin.x}=\pi l\frac{1+p}{1-p}$

5) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.\frac{dx}{Tang.x}=-\pi l(1-p^2)$

6) $\int Arctang.\frac{p\,Sin.x}{1-p\,Cos.x}.\frac{Cos.^2x}{Sin.x}dx=\pi\left(l\frac{1+p}{1-p}-p\right)$

(1–6): $1>p\geqq 0$; Bierens de Haan, Gr. 13. 193.

F. Circ. Dir. Circ. Inv. TABLE 374. Lim. diverses.

1) $\int_0^\infty Arctang.\frac{a}{x}.Sin.bx\,dx=\frac{\pi}{2b}(1-e^{-ab})$ Cauchy, P. 28. 147. I. § 5.

2) $\int_0^\infty Arctang.px.Sin.qx\,dx=\frac{\pi}{2q}e^{-q/p}$ Raabe, Int. 170.

3) $\int_0^\infty Cos.^{a+1}\left(Arctang.\frac{x}{q}\right).Sin.\left\{(a+1)Arctang.\frac{x}{q}\right\}.Sin.x\,dx=\frac{\pi q^a e^{-q}}{2\Gamma(a+1)}$ V. T. 59. N°. 17.

4) $\int_0^\infty Cos.^{a+1}\left(Arctang.\frac{x}{q}\right).Cos.\left\{(a+1)Arctang.\frac{x}{q}\right\}.Cos.x\,dx=\frac{\pi q^a e^{-q}}{2\Gamma(a+1)}$ V. T. 59. N°. 18.

5) $\int_0^{\frac{\pi}{4}} Arctang. \left(\frac{p\sqrt{Cos. 2x}}{Cos. x}\right). \frac{dx}{Cos. 2x} = \frac{1}{2}\pi l \{p + \sqrt{(1+p^2)}\}$ V. T. 262. N°. 2.

6) $\int_0^{\frac{\pi}{4}} Arcsin. (Tang. x). \frac{dx}{Sin. 2x} = \frac{1}{4}\pi l 2$ V. T. 257. N°. 1.

7) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} Arctg.(a+b Tg. x) dx = -\pi \left\{\frac{1}{2} Arctg. \frac{2ab}{1+a^2-b^2} - Arctg. \frac{Tang. \left(\frac{1}{2} Arctang. \frac{2ab}{1+a^2-b^2}\right)}{a Tg. \left(\frac{1}{2} Arctg. \frac{2ab}{1+a^2-b^2}\right) - b}\right\}$ V. T. 271. N°. 1.

F. Circ. Dir. Autres Fonctions. TABLE 375. Lim. diverses.

1) $\int_0^{\frac{\pi}{2}} E(Cos. \lambda, Sin. x)\, dx = \frac{1}{2}\pi\, Cot. \lambda$ Lobatschewsky, Mém. Kasan. 1835. 1.

2) $\int_0^{\frac{\pi}{2}} E(x) \frac{dx}{\sqrt{(1-p^2 Sin.^2 x)}} = \frac{1}{2} E'(p). F'(p) - \frac{1}{4} l(1-p^2)$ Roberts, L. 12. 449.

3) $\int_0^{\frac{\pi}{2}} E(p, Sin. x) \frac{Sin. x}{1-p^2 Sin.^2 x} dx = \frac{\pi}{2\sqrt{(1-p^2)}}$ Roberts, L. 10. 453.

4) $\int_0^{\frac{\pi}{2}} F(p, x) \frac{Sin. x. Cos. x}{1-p^2 Sin.^2 x} dx = \frac{-1}{4p^2} l(1-p^2). F'(p)$

5) $\int_0^{\pi} F(\sqrt{(1-p^2)}, x) \frac{Sin. x. Cos. x}{Cos.^2 x + p\, Sin.^2 x} dx = \frac{1}{4(1-p)} l \frac{1}{(1+p)\sqrt{p}}. F'\{\sqrt{(1-p^2)}\}$

6) $\int_\lambda^\mu E(p, x) \frac{dx}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)}} =$

$= \frac{1}{2 Cos. \lambda. Sin. \mu} E'(p). F'\left\{\sqrt{\left(1 - \frac{Tang.^2 \lambda}{Tang.^2 \mu}\right)}\right\} + \frac{p^2 Sin. \mu}{2 Cos. \lambda} F'\left\{\sqrt{\left(1 - \frac{Sin.^2 2\lambda}{Sin.^2 2\mu}\right)}\right\}$

7) $\int_\lambda^\mu F(p, x) \frac{dx}{\sqrt{(Sin.^2 x - Sin.^2 \lambda)(Sin.^2 \mu - Sin.^2 x)}} =$

$= \frac{1}{2 Cos. \lambda. Sin. \mu} F'(p). F'\{\sqrt{(1 - Tang.^2 \lambda. Cot.^2 \mu)}\}$

où dans 6) et 7) on a $p^2 = 1 - Cot.^2 \lambda. Cot.^2 \mu$.

(4–7): $, p < 1$; Roberts, L. 11. 157.

8) $\int_0^{\frac{\pi}{2}} \mathrm{r}(p,x)\frac{dx}{\sqrt{(1-p^2 Sin.^2 x)}} = \frac{1}{12}\pi F'\{\sqrt{(1-p^2)}\} + \frac{1}{6}E'(p).\{F'(p)\}^2 + \frac{1}{6}F'(p).l\frac{p}{4(1-p^2)}$

9) $\int_0^{\pi} \mathrm{r}(p,x)\frac{dx}{\sqrt{(1-p^2 Sin.^2 x)}} = \frac{1}{6}\pi F'\{\sqrt{(1-p^2)}\} + \frac{4}{3}E'(p).\{F'(p)\}^2 + \frac{1}{3}F'(p).l\frac{p}{4(1-p^2)}$

Sur les formules (8) et (9) voyez: Roberts, L. 12. 449. où $p < 1$.

10) $\int_0^1 B'(x).Sin.2c\pi x\,dx = 0$

11) $\int_0^1 B''(x).Cos.2c\pi x\,dx = 0$

12) $\int_0^1 B'(x).Cos.2c\pi x\,dx = \frac{(-1)^a}{(2\pi)^{2a+2}}\frac{1^{2a+1/1}}{c^{2a+2}}$

13) $\int_0^1 B''(x).Sin.2c\pi x\,dx = \frac{(-1)^{a-1}}{(2\pi)^{2a+1}}\frac{1^{2a/1}}{c^{2a+1}}$

Raabe, Cr. 42. 348. (10–13)

14) $\int_0^{\lambda} E'(Sin\ x)\frac{Tang.x}{\sqrt{(Sin.^2\lambda - Sin.^2 x)}}dx = \frac{p\pi}{2\sqrt{(1-p^2)}}$ V. T. 272. N°. 3.

TABLES

D'INTÉGRALES DÉFINIES

PAR

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TROISIÈME PARTIE.

AMSTERDAM,
C. G. VAN DER POST.
1858.

PARTIE TROISIÈME.

F. Algébr.
Expon.
Logar.

TABLE 376.

Lim. 0 et 1.

1) $\int e^{-x}(1-x)\, lx\, dx = \frac{1-e}{e}$ V. T. 112. N°. 2.

2) $\int e^{ax}(ax+2)\, x\, lx\, dx = -\frac{1}{a^2}\{(a-1)e^a+1\}$ V. T. 112. N°. 1.

3) $\int e^{-x^2}(x^2-1)\, x\, lx\, dx = \frac{e-1}{4e}$ V. T. 112. N°. 2.

4) $\int e^{-(1-x)^2}(2-x)(1-x)\, x\, l(1-x)\, dx = \frac{1-e}{4e}$ V. T. 376. N°. 3.

5) $\int e^{x-1}\, x\, l(1-x)\, dx = \frac{e-1}{e}$ V. T. 112. N°. 2.

6) $\int e^x \frac{x^2+x+2}{(x+1)^3}\, x\, lx\, dx = 1-\frac{1}{2}e$ V. T. 112. N°. 5.

F. Algébr. ent.
Expon. monôme.
Logar.

TABLE 377.

Lim. 0 et ∞.

1) $\int e^{-x} x^{p-1}\, lx\, dx = \frac{d\Gamma(p)}{dp}$ Cauchy, P. 28. 147. P. 1. § 6. — Lejeune-Dirichlet, Cr. 15. 258. — Grunert, Gr. 2. 266. — Lobatschewsky, Mém. Kasan. 1835. 211.

2) $\int e^{-ax} x^{p-1}\, l\frac{1}{x}\, dx = \frac{\Gamma(p)}{a^p}\{la - Z'(p)\}$ Cauchy, P. 28. 147. I. § 6. — Schlömilch, Stud. I. 14.

3) $\int e^{-ax} x^b\, lx\, dx = \frac{1^{b/1}}{a^{b+1}}\left\{-A - la + \sum_1^b \frac{1}{n}\right\}$ Schlömilch, Gr. 4. 167.

4) $\int e^{-x}(x-p)\, x^{p-1}\, lx\, dx = \Gamma(p)$ V. T. 113. N°. 3.

5) $\int e^{-x^{2p}}(2x^{2p}-1)\, x^{p-1}\, lx\, dx = \frac{1}{2p^2}\sqrt{\pi}$ V. T. 115. N°. 5.

6) $\int e^{-px^2}(px^2-a)\, x^{2a-1}\, lx\, dx = \frac{1}{2(2p)^a}1^{a-1/1}$ V. T. 114. N°. 9

7) $\int e^{-x^2}(2x^2-2a-1)\, x^{2a}\, lx\, dx = \frac{1}{2}\Gamma\left(\frac{2a+1}{2}\right)$ V. T. 114. N°. 6.

8) $\int e^{-px^2}(2px^2-2a-1)\, x^{2a}\, lx\, dx = \frac{1}{2}\frac{1}{(2p)^a}1^{a/2}\sqrt{\frac{\pi}{p}}$ V. T. 114. N°. 8.

9) $\int e^{-px} x^a \, l(q+x)^2 \, dx = \frac{1}{p^{a+1}} \Big[1^{a/1} \{ l q^2 - 2 e^{pq} Ei.(-pq) \} +$

$+ 2 \{ 1 + pq e^{pq} \mathrm{E}(-pq) \} 2^{a-1/1} \sum_0^{a-1} 2^{n/1} (-pq)^n + 2 . 3^{a-2/1} \sum_0^{a-2} \left\{ \frac{(pq)^n}{3^{n/1}} \sum_0^n \frac{1^{m+1/1}}{(-pq)^m} \right\} \Big]$

10) $\int e^{-px} x^a \, l(q-x)^2 \, dx = \frac{1}{p^{a+1}} \Big[1^{a/1} \{ l q^2 - 2 e^{-pq} Ei.(pq) \} +$

$+ 2 \{ 1 - pq e^{-pq} Ei.(pq) \} 2^{a-1/1} \sum_0^{a-1} 2^{n/1} (pq)^n + 2 . 3^{a-2/1} \sum_0^{a-2} \left\{ \frac{(-pq)^n}{3^{n/1}} \sum_0^n \frac{1^{m+1/1}}{(pq)^m} \right\} \Big]$

11) $\int e^{-px} x^{2a} \, l(q^2 - x^2)^2 \, dx = \frac{2}{p^{2a+1}} \Big[1^{2a/1} l q^2 - 1^{2a/1} e^{pq} Ei.(-pq) \sum_0^{2a-1} \frac{(-pq)^n}{1^{n/1}} - 1^{2a/1} e^{-pq} Ei.(pq) \sum_0^{2a} \frac{(pq)^n}{1^{n/1}} +$

$+ 2^{2a-1/1} \sum_1^a \left\{ \frac{1}{1^{2n/1}} \sum_0^{n-1} 1^{2n-2m/1} (p^2 q^2)^m \right\} + 3^{2a-2/1} \sum_1^a \left\{ \frac{1}{1^{2n-1/1}} \sum_0^{n-1} 1^{2n-2m-1/1} (p^2 q^2)^m \right\} \Big]$

12) $\int e^{-px} x^{2a+1} \, l(q^2 - x^2)^2 \, dx = \frac{2}{p^{2a+2}} \Big[1^{2a+1/1} l q^2 - 1^{2a+1/1} e^{pq} Ei.(-pq) \sum_0^{2a} \frac{(-pq)^n}{1^{n/1}} - 1^{2a+1/1} e^{-pq} Ei.(pq) \sum_0^{2a} \frac{(pq)^n}{1^{n/1}} +$

$+ 2^{2a/1} \sum_1^{a+1} \left\{ \frac{1}{1^{2n+1/1}} \sum_0^{n-1} 1^{2n-2m+1/1} (p^2 q^2)^m \right\} + 3^{2a-1/1} \sum_1^a \left\{ \frac{1}{1^{2n/1}} \sum_0^{n-1} 1^{2n-2m/1} (p^2 q^2)^m \right\} \Big]$

13) $\int e^{-px} x^{2a} \, l(q^2 + x^2)^2 \, dx = \frac{2}{p^{2a+1}} \Big[1^{2a/1} l q^2 - 1^{2a/1} \{ 2 Ci.(pq).Cos.pq + 2 Si.(pq).Sin.pq - \pi Sin.pq \} \sum_0^a \frac{(-p^2 q^2)^n}{1^{2n/1}} +$

$+ 1^{2a/1} \{ 2 Ci.(pq).Sin.pq - 2 Si.(pq).Cos.pq + \pi Cos.pq \} \sum_0^a \frac{(pq)^{2n-1}}{1^{2n-1/1}} +$

$+ 2^{2a-1/1} \sum_1^a \left\{ \frac{1}{1^{2n/1}} \sum_0^{n-1} 1^{2n-2m/1} (-p^2 q^2)^m \right\} + 3^{2a-2/1} \sum_1^a \left\{ \frac{1}{1^{2n-1/1}} \sum_0^{n-1} 1^{2n-2m-1/1} (-p^2 q^2)^m \right\} \Big]$

14) $\int e^{-px} x^{2a+1} \, l(q^2 + x^2)^2 \, dx = \frac{2}{p^{2a+2}} \Big[1^{2a+1/1} l q^2 - 1^{2a+1/1} \{ 2 Ci.(pq).Cos.pq + 2 Si.(pq).Sin.pq - \pi Sin.pq \} \sum_0^a \frac{(-p^2 q^2)^n}{1^{2n/1}} +$

$+ 1^{2a+1/1} \{ 2 Ci.(pq).Sin.pq - 2 Si.(pq).Cos.pq + \pi Cos.pq \} \sum_1^{a+1} \frac{(pq)^{2n-1}}{1^{2n-1/1}} +$

$+ 2^{2a/1} \sum_1^{a+1} \left\{ \frac{1}{1^{2n+1/1}} \sum_0^{n-1} 1^{2n-2m+1/1} (-p^2 q^2)^m \right\} + 3^{2a-1/1} \sum_1^a \left\{ \frac{1}{1^{2n/1}} \sum_0^{n-1} 1^{2n-2m/1} (-p^2 q^2)^m \right\} \Big]$

15) $\int e^{-px} x \, l(q^4 - x^4)^2 \, dx = 8 + 4 l q^2 + (pq - 1) 2 e^{pq} Ei.(-pq) + (pq + 1) 2 e^{-pq} Ei.(pq) -$

$- 2pq \{ 2 Ci.(pq).Sin.pq - 2 Si.(pq).Cos.pq + \pi Cos.pq \} - 2 \{ Ci.(pq).Cos.pq + 2 Si.(pq).Sin.pq - \pi Sin.pq \}$

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16) $\int e^{-px} x^2 \, l(q^4 - x^4)^2 \, dx = 24 + 8 lq^2 - (p^2 q^2 - 2pq + 2) 2 e^{pq} Ei.(-pq) -$

$- (p^2 q^2 + 2pq + 2) 2 e^{-pq} Ei.(pq) - 4pq \{2 Ci.(pq). Sin. pq - 2 Si.(pq). Cos. pq + \pi Cos. pq\} +$

$+ (p^2 q^2 - 2) 2 \{2 Ci.(pq). Cos. pq + 2 Si.(pq). Sin. pq - \pi Sin. pq\}$

17) $\int e^{-px} x^3 \, l(q^4 - x^4)^2 \, dx = 88 + 24 lq^2 + (p^3 q^3 - 3p^2 q^2 + 6pq - 6) 2 e^{pq} Ei.(-pq) -$

$-(p^3 q^3 + 3p^2 q^2 + 6pq + 6) 2 e^{-pq} Ei.(pq) + (p^2 q^2 - 6) 2pq \{2 Ci.(pq). Sin. pq - 2 Si.(pq). Cos. pq + \pi Cos. pq\} +$

$+ (p^2 q^2 - 6) 2 \{2 Ci.(pq). Cos. pq + 2 Si.(pq). Sin. pq - \pi Sin.(pq)\}$

Sur les intégrales 9) à 17) voyez Bierens de Haan, Verh. K. Ak. v. Wet. 1854. bl. 19.

18) $\int e^{-qx^a} (q x^a - p) x^{ap-1} \, lx \, dx = \frac{1}{a^2 q^p} \Gamma p$ V. T. 115. N°. 9.

1) $\int e^{-x} \, lx \, \frac{x + p - 1}{x^p} \, dx = \frac{\pi}{\Gamma(p)} Cosec. p\pi, \; p < 1;$ V. T. 126. N°. 8.

2) $\int e^{-x^2} \, lx \frac{dx}{x^2} = \infty$ V. T. 126. N°. 3 et T. 273. N°. 3.

3) $\int e^{-px} \, lx \frac{px + a}{x^{a+1}} \, dx = p^a \Gamma(-a), \; a < 0;$ V. T. 126. N°. 10.

4) $\int l(1 - e^{-2a\pi x}) \frac{dx}{1 + x^2} = \pi \{\tfrac{1}{2} l 2a\pi - l\Gamma(a+1) + a(la - 1)\}$ Schaar, Mém. Cour. Brux. T. 22, 23.

5) $\int e^{-px} \, l(q + x)^2 \, \frac{p(x+q) \, l(q+x)^2 - 4}{x + q} = (lq^2)^2$

6) $\int e^{-px} \, l(q - x)^2 \, \frac{p(x-q) \, l(q-x)^2 - 4}{x - q} = (lq^2)^2$

7) $\int e^{-px} \, l(q^2 + x^2)^2 \, \frac{p(x^2+q^2) \, l(q^2+x^2)^2 - 8}{x^2 + q^2} = (lq^4)^2$

8) $\int e^{-px} \, l(q^2 - x^2)^2 \, \frac{p(x^2-q^2) \, l(q^2-x^2)^2 - 8}{x^2 - q^2} = (lq^4)^2$

(5–8: Bierens de Haan, Verh. K. Ak. v. Wet. 1854. bl. 19.)

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9) $\int\left\{\frac{e^{-x}}{x}-\frac{1}{(1+x)^2\, l(1+x)}\right\} dx = 0$ Cauchy, C. R. 16. 422. — Schlömilch, Stud. I. 10. — Féaux, Funct. Transc.

10) $\int\left\{(p-1)\, e^{-x}+\frac{(1+x)^{-p}-(1+x)^{-1}}{l(1+x)}\right\}\frac{dx}{x} = l\,\Gamma(p)$ Féaux, Funct. Transc.

11) $\left\{e^{-x}+\frac{(1+x)^{-p-1}}{l(1+x)}\right\}\frac{dx}{x} = l\,p$ Stern, Gött. Stud. 1847.

12) $\int e^{-q^2x^2-\frac{1}{x^2}}\, l x\, \frac{2\,q^2\,x^4+x^2-2}{x4}\, dx = \frac{1}{2}\, e^{-2q}\sqrt{\pi}$ V. T. 126. N°. 16.

F. Algébr. fract. à dén. puiss. de bin.
Expon. monôme. TABLE 379. Lim. 0 et ∞.
Logar.

1) $$\int e^{-px}\, l\,(q+x)^2\, \frac{px+pq+a-1}{(x+q)^a}\, dx = \frac{l\,q^2}{q^{a-1}} - 2\,\frac{(-p)^{a-1}}{1^{a-1/1}}\, e^{pq}\, Ei.(-pq) + \\ + \frac{2}{1^{a-1\,1}\, q^{a-1}}\sum_1^{a-1} 1^{a-n-1\,1}\,(-pq)^{n-1}$$

2) $$\int e^{-px}\, l(q-x)^2\, \frac{px-pq+a-}{(x-q)^a}\, dx = (-1)^{a-1}\left\{\frac{l\,q^2}{q^{a-1}} - 2\,\frac{p^{a-1}}{1^{a-1/1}}\, e^{-pq}\, Ei.(pq) + \right. \\ \left. + \frac{2}{1^{a-1/1}\, q^{a-1}}\sum_1^{a-1} 1^{a-n-1/1}\,(pq)^{n-1}\right\}$$

3) $$\int e^{-px}\, l(q+x)^2\, \frac{px+pq+1}{(x+q)^2}\, dx = 2\,p\, e^{pq}\, Ei.(-pq) + \frac{1}{q}(2+l\,q^2)$$

4) $$\int e^{-px}\, l(q-x)^2\, \frac{px-pq+1}{(x-q)^2}\, dx = 2\,p\, e^{-pq}\, Ei.(pq) - \frac{1}{q}(2+l\,q^2)$$

5) $$\int e^{-px}\, l(q+x)^2\, \frac{px-pq+1}{(x-q)^2}\, dx = \frac{1}{q}\left\{e^{pq}\, Ei.(-pq) - e^{-pq}\, Ei.(pq) - l\,q^2\right\}$$

6) $$\int e^{-px}\, l(q-x)^2\, \frac{px+pq+1}{(x+q)^2}\, dx = \frac{1}{q}\left\{e^{pq}\, Ei.(-pq) - e^{-pq}\, Ei.(pq) + l\,q^2\right\}$$

7) $$\int e^{-px}\, l(q^2-x^2)^2\, \frac{px+pq+1}{(x+q)^2}\, dx = \frac{1}{q}\left\{(2pq+1)\, e^{pq}\, Ei.(-pq) - e^{-pq}\, Ei.(pq) + 2\,l\,q^2+2\right\}$$

8) $$\int e^{-px}\, l(q^2-x^2)^2\, \frac{px-pq+1}{(x-q)^2}\, dx = \frac{1}{q}\left\{e^{pq}\, Ei.(-pq) + (2pq-1)\, e^{-pq}\, Ei.(pq) - 2\,l\,q^2 - 2\right\}$$

9) $\int e^{-px} l(q+x)^2 \frac{px^2-(pq+2a-1)x+2aq}{(x-q)^2} x^{2a-1}\,dx = q^{2a-1}\{e^{pq}Ei.(-pq)-e^{-pq}Ei.(pq)\}+$
$+\frac{2}{p^{2a-1}}\sum_1^a 1^{2a-2n,1}(p^2q^2)^{n-}$

10) $\int e^{-px} l(q+x)^2 \frac{px^2-(pq+2a)x+(2a+1)q}{(x-q)^2} x^{2a}\,dx = -q^{2a}\{e^{pq}Ei.(-pq)+e^{-pq}Ei.(pq)\}+$
$+\frac{2}{p^{2a}}\sum_1^a 1^{2a-2n+1/1}(p^2q^2)^{n-1}$

11) $\int e^{-px} l(q-x)^2 \frac{px^2+(pq-2a+1)x-2aq}{(x+q)^2} x^{2a-1}\,dx = q^{2a-1}\{e^{pq}Ei.(-pq)-e^{-pq}Ei.(pq)\}+$
$+\frac{2}{p^{2a-1}}\sum_1^a 1^{2a-2n,1}(p^2q^2)^{n-1}$

12) $\int e^{-px} l(q-x)^2 \frac{px^2+(pq-2a)x-(2a+1)q}{(x+q)^2} x^{2a}\,dx = -q^{2a}\{e^{pq}Ei.(-pq)+e^{-pq}Ei.(pq)\}+$
$+\frac{2}{p^{2a}}\sum_1^a 1^{2a-2n+1/1}(p^2q^2)^{n-1}$

13) $\int e^{-px} l(q+x)^2 \frac{px^2+2x-pq^2}{(x^2-q^2)^2}\,dx = \frac{1}{2q^2}\{2-4lq^2-(2pq-1)e^{pq}Ei.(-pq)-e^{-pq}Ei.(pq)\}$

14) $\int e^{-px} l(q-x)^2 \frac{px^2+2x-pq^2}{(x^2-q^2)^2}\,dx = \frac{1}{2q^2}\{2-4lq^2-e^{pq}Ei.(-pq)+(2pq+1)e^{-pq}Ei.(pq)\}$

15) $\int e^{-px} l(q^2-x^2)^2 \frac{px^2+2x-pq^2}{(x^2-q^2)^2}\,dx = \frac{1}{q^2}\{2-4lq^2-pq\,e^{pq}Ei.(-pq)+pq\,e^{-pq}Ei.(pq)\}$

Sur toutes ces intégrales voyez: Bierens de Haan, Verh. K. Ak. v. Wet. 1854. bl. 19.

F. Algébr. rat.
Expon. en dén. polynôme. TABLE 380. Lim. 0 et ∞.
Logar.

1) $\int lx \frac{(x-q)e^x-q}{(e^x+1)^2} x^{q-1}\,dx = \Gamma(q)\sum_0^\infty \frac{(-1)^n}{(1+n)^q}$ V. T. 117. N°. 16.

2) $\int lx \frac{(x-2a-1)e^x-(x+2a+1)}{(e^x+1)^3} x^{2a}e^x\,dx = (2^{2a-1}-1)\pi^{2a}B_{2a-1}$ V. T. 119. N°. 9.

3) $\int lx \frac{(ax-p)(1+e^{-x})-xe^{-x}}{(1+e^{-x})^2} e^{-ax}x^{p-1}\,dx = \Gamma(p)\sum_0^\infty \frac{(-1)^n}{(a+n)^p}$ V. T. 113. N°. 5.

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4) $\int lx \frac{(\pi x - 2a)e^{\pi x} - 2a}{(e^{\pi x}+1)^2} x^{2a-1}\, dx = \frac{2^{2a-1}-1}{2a} B_{2a-1}$ V. T. 117. N°. 21.

5) $\int lx \frac{(x-q)e^x + q}{(e^x - 1)} x^{q-1}\, dx = \Gamma(q) \sum_0^\infty \frac{1}{(1+n)^q}$ V. T. 117. N°. 17.

6) $\int lx \frac{(x - 2a - 1)e^x + (x + 2a + 1)}{(e^x - 1)^3} x^{2a} e^x\, dx = 2^{2a-1}\pi^{2a} B_{2a-1}$ V. T. 119. N°. 10.

7) $\int lx \frac{(ax - p)(e^x - 1) + xe^x}{(e^x - 1)^2} e^{-ax} x^{p-1}\, dx = \Gamma(p) \sum_0^\infty \frac{1}{(a+n)^p}$ V. T. 118. N°. 8.

8) $\int lx \frac{\pi x e^{\pi x} - 2a(e^{2\pi x} - 1)}{(e^{\pi x} - 1)^2} x^{2a-1}\, dx = \frac{2^{2a-2}}{a} B_{2a-1}$ V. T. 117. N°. 22.

9) $\int lx \frac{a e^{2\pi x} + \pi x e^{\pi x} - a}{(e^{\pi x} - 1)^2} x^{2a-1}\, dx = \frac{2^{2a-2}}{a} B_{2a-1}$ V. T. 118. N°. 15.

10) $\int lx \frac{(q+1)(e^x + e^{-x}) - x(e^x - e^{-x})}{(e^x + e^{-x})^2} x^q\, dx = \Gamma(q+1) \sum_0^\infty \frac{(-1)^{n+1}}{(2n+1)^{q+1}}$ V. T. 120. N°. 11.

11) $\int lx \frac{(4a+2)(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}) - \pi x (e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})}{(e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x})^2} x^{2a}\, dx = -B_{2a}$ V. T. 120. N°. 14.

12) $\int l(1+x^2) \frac{e^{\pi x}(1+\pi x) + e^{-\pi x}(1 - \pi x)}{(e^{\pi x} + e^{-\pi x})^2} \frac{dx}{x^2} = 2 - \frac{1}{2}\pi$ V. T. 138. N°. 2.

13) $\int l(1+4x^2) \frac{e^{\pi x}(1+\pi x) + e^{-\pi x}(1 - \pi x)}{(e^{\pi x} + e^{-\pi x})^2} \frac{dx}{x^2} = 2l2$ V. T. 138. N°. 3.

14) $\int lx \frac{e^x(x - 2a - 1) + e^{-x}(x + 2a + 1)}{(e^x - e^{-x})^2} x^{2a}\, dx = \frac{2^{2a+1}-1}{2^{2a+1}} 1^{2a/1} \sum_1^\infty \frac{1}{n^{2a+1}}$ V. T. 120. N°. 13.

15) $\int lx \frac{e^{\pi x}(\pi x - 2a) + e^{-\pi x}(\pi x + 2a)}{(e^{\pi x} - e^{-\pi x})^2} x^{2a-1}\, dx = \frac{2^{2a}-1}{4a} B_{2a-1}$ V. T. 120. N°. 18.

16) $\int lx \frac{e^{\frac{1}{2}\pi x}(\pi x - 4a) + e^{\frac{1}{2}\pi x}(\pi x + 4a)}{(e^{\frac{1}{2}\pi x} - e^{-\frac{1}{2}\pi x})^2} x^{2a-1}\, dx = \frac{2^{2a}-1}{a} 2^{2a} B_{2a-1}$ V. T. 120. N°. 20.

17) $\int lx \frac{(2a+1)(e^x + e^{-x} + 1) - x(e^x - e^{-x})}{(e^x + e^{-x} + 1)^2} x^{2a}\, dx = \frac{(-1)^a}{\sqrt{3}} (2\pi)^{2a+1} B''\left(\frac{1}{3}\right)$ V. T. 124. N°. 11.

18) $\int lx \frac{x(e^x - e^{-x}) - 3(e^{\frac{1}{2}x} - e^{-\frac{1}{2}x})^2 - 12\, Cos.^2 \frac{1}{2}\lambda}{(e^x + e^{-x} + 2\, Cos. \lambda)^2} x^2\, dx = \frac{\lambda}{2\, Sin. \lambda} \frac{\pi^2 - \lambda^2}{3}$ V. T. 124. N°. 7.

F. Algébr. rat.
Expon. en dén. polynôme. **TABLE 380 suite.** Lim. 0 et ∞.
Logar.

19) $\int lx \frac{q(e^x+e^{-x}+2\,Cos.\lambda)-x(e^x-e^{-x})}{(e^x+e^{-x}+2\,Cos.\lambda)^2} x^{q-1}\,dx = \frac{\Gamma(q)}{Sin.\lambda}\sum_1^\infty (-1)^n \frac{Sin.\,n\lambda}{n^q}$ V. T. 137. N°. 5.

20) $\int lx \frac{x(e^x-e^{-x})-2(e^{\frac{1}{2}x}-e^{-\frac{1}{2}x})^2}{(e^x+e^{-x}-1)^2}\,x\,dx = \frac{4}{27}\pi^2$ V. T. 124. N°. 1.

21) $\int lx \frac{2a(e^x+e^{-x}-1)-x(e^x-e^{-x})}{(e^x+e^{-x}-1)^2} x^{2a-1}\,dx = \frac{(-1)^a}{\sqrt{3}}(2\pi)^{2a+1}B''\left(\frac{1}{6}\right)$ V. T. 124. N°. 12.

22) $\int lx \frac{(x-2)e^{2x}+2}{\sqrt{(e^{2x}-1)^3}}\,x\,dx = \frac{\pi}{2}l2$ V. T. 141. N°. 6.

23) $\int lx \frac{2(x-1)e^x+(2-x)e^{-x}}{\sqrt{(e^{2x}-1)^3}}\,x\,dx = 1-l2$ V. T. 141. N°. 7.

24) $\int \frac{e^{px}+e^{-px}}{e^{\pi x}+e^{-\pi x}}\frac{lx}{x^q}dx = Z'(1-q)\Gamma(1-q)\sum_0^\infty(-1)^n\left[\frac{1}{\{(2n+1)\pi-p\}^{1-q}}+\frac{1}{\{(2n+1)\pi+p\}^{1-q}}\right]-$
$-\Gamma(1-q)\sum_0^\infty(-1)^n\left[\frac{l\{(2n+1)\pi-p\}}{\{(2n+1)\pi-p\}^{1-q}}+\frac{l\{(2n+1)\pi+p\}}{\{(2n+1)\pi+p\}^{1-q}}\right]$

25) $\int \frac{e^{px}-e^{-px}}{e^{\pi x}-e^{-\pi x}}\frac{lx}{x^q}dx = Z'(1-q)\Gamma(1-q)\sum_0^\infty\left[\frac{1}{\{(2n+1)\pi-p\}^{1-q}}-\frac{1}{\{(2n+1)\pi+p\}^{1-q}}\right]-$
$-\Gamma(1-q)\sum_0^\infty\left[\frac{l\{(2n+1)\pi-p\}}{\{(2n+1)\pi-p\}^{1-q}}-\frac{l\{(2n+1)\pi+p\}}{\{(2n+1)\pi+p\}^{1-q}}\right]$

(24, 25) Malmsten, Cr. 38. 1.

F. Algébr. irrat.
Expon. **TABLE 381.** Lim. 0 et ∞.
Logar.

1) $\int e^{-x}\,lx\,dx\sqrt{x} = \left(1-l2-\frac{1}{2}A\right)\sqrt{\pi}$ V. T. 140. N°. 1. et T. 381. N°. 6.

2) $\int e^{-qx}\,lx\,dx\sqrt{x} = \frac{1}{2q}(2-lq-2l2-A)\sqrt{\frac{\pi}{q}}$ V. T. 140. N°. 2. et T. 381. N°. 6.

3) $\int e^{-qx}\left(qx-a-\frac{1}{2}\right)x^{a-\frac{1}{2}}\,dx\,lx = \frac{1^{a/2}}{(2q)^a}\sqrt{\frac{\pi}{q}}$ V. T. 140. N°. 4.

4) $\int e^{-px}(2px-3)\,lx\,dx\sqrt{x} = \frac{1}{p}\sqrt{\frac{\pi}{p}}$ V. T. 139. N°. 2.

5) $\int e^{-\left(px+\frac{q}{x}\right)}\{2px^2-(2c+1)x-2q\}x^{c-\frac{1}{2}}dx\,lx = 2\left(\frac{q}{p}\right)^{\frac{1}{2}c}e^{-2\sqrt{pq}}\sqrt{\frac{\pi}{p}}\sum_0^\infty\frac{(c-n+1)^{2n/1}}{2^{n/2}(2\sqrt{pq})^n}$ V. T. 139. N°. 8.

F. Algébr. irrat.
Expon.
Logar.

TABLE 381 suite.

Lim. 0 et ∞.

6) $\int e^{-qx}\, lx \frac{dx}{\sqrt{x}} = -(lq + 2l2 + A)\sqrt{\frac{\pi}{q}}$ Malmsten, Cr. 38. 1. — Schlömilch, Gr. 4. 167.

7) $\int e^{-q^2\left(x+\frac{1}{x}\right)}\, lx \frac{2a^2x^2 - 3x - 2a^2}{\sqrt{x}}\, dx = \frac{2a^2+1}{a^3} e^{-2a^2}\sqrt{\pi}$ V. T. 139. N°. 6.

8) $\int e^{-\left(px+\frac{q}{x}\right)}\, lx \frac{2px^2 - x - 2q}{x\sqrt{x}}\, dx = 2e^{-2\sqrt{pq}}\sqrt{\frac{\pi}{p}}$ V. T. 140. N°. 9.

9) $\int e^{-\frac{1+x^2}{2qx}}\, lx \frac{1+qx-x^2}{x\sqrt{x}}\, dx = -\frac{\sqrt{2q\pi}}{\sqrt[q]{e}}\, 2q$ V. T. 140. N°. 5.

10) $\int e^{-\frac{1+x^2}{2qx}}\, lx \frac{x^2+qx-1}{x^2\sqrt{x}}\, dx = \frac{2q}{\sqrt[q]{e}}\sqrt{2q\pi}$ V. T. 140. N°. 7.

11) $\int e^{-\frac{1+x^2}{2qx}}\, lx \frac{x^2+3qx-1}{x^3\sqrt{x}}\, dx = \frac{1+q}{\sqrt[q]{e}}\, 2q\sqrt{2q\pi}$ V. T. 140. N°. 8

12) $\int e^{-px-\frac{q}{x}}\, lx \frac{2px^2+(2a-1)x-2q}{x^{a+\frac{3}{2}}}\, dx = 2\left(\frac{p}{q}\right)^{\frac{1}{2}a}\sqrt{\frac{\pi}{p}}\, e^{-2\sqrt{pq}}\sum_0^\infty \frac{(a-n)^{2n/1}}{2^{n/2}(2\sqrt{pq})^n}$ V. T. 140. N°. 12.

13) $\int lx \frac{(2x-3)e^x - (2x+3)e^{-x}}{(e^x+e^{-x})^2}\, dx\sqrt{x} = \sqrt{\pi}\sum_0^\infty (-1)^n \frac{1}{\sqrt{(2n+1)^3}}$ V. T. 139. N°. 11.

14) $\int \frac{lx}{e^x+e^{-x}}\frac{dx}{\sqrt{x}} = \sqrt{\pi}\sum_0^\infty \left\{(-1)^{n+1}\frac{l(2n+1)+2l2+A}{\sqrt{(2n+1)}}\right\}$

15) $\int \frac{lx}{e^x+1+e^{-x}}\frac{dx}{\sqrt{x}} = Cosec.\frac{1}{3}\pi.\sqrt{\pi}\sum_1^\infty \left\{(-1)^n Sin.\frac{1}{3}n\pi\frac{ln+2l2+A}{\sqrt{n}}\right\}$

(14, 15) Malmsten, Cr. 38. 1.

16) $\int lx \frac{(2x-1)e^x-(2x+1)e^{-x}}{(e^x+e^{-x})^2}\frac{dx}{\sqrt{x}} = 2\sqrt{\pi}\sum_0^\infty \frac{(-1)^n}{\sqrt{(2n+1)}}$ V. T. 140. N°. 19.

17) $\int lx \frac{(2x-1)e^x-(2x+1)e^{-x}-1}{(e^x+1+e^{-x})^2}\frac{dx}{\sqrt{x}} = 2\, Cosec.\frac{1}{3}\pi.\sqrt{\pi}\sum_1^\infty (-1)^{n-1}\frac{Sin.\frac{1}{3}n\pi}{\sqrt{n}}$ V. T. 140. N°. 20.

F. Algébr.
Expon.
Logar.

TABLE 382.

Lim. −∞ et ∞.

1) $\int e^{-x^2}(x^2-a)x^{2a-1}\, lx\, dx = 0$ V. T. 142. N°. 9.

2) $\int e^{-x^2}(2x^2-2a-1)x^{2a}\, lx\, dx = \frac{1^{a/2}}{2^a}\sqrt{\pi}$ V. T. 142. N°. 8.

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3) $\int e^{-px^2-\frac{q}{x^2}} l\,x \frac{2px^4+x^2-2q}{x^4}\,dx = e^{-2\sqrt{pq}}\sqrt{\frac{\pi}{q}}$ V. T. 146. N°. 2.

4) $\int e^{-px^2-\frac{q}{x^2}} l\,x \frac{2px^4+(2a-1)x^2-2q}{x^{2a+2}}\,dx = \left(\frac{p}{q}\right)^{\frac12 a} e^{-2\sqrt{pq}}\sqrt{\frac{\pi}{p}}\sum_0^\infty \frac{(a-n)^{2n|1}}{1^{n|1}(4\sqrt{pq})^n}$ V. T. 146. N°. 3.

5) $\int e^{bxi}(-xi)^{a-1}\,l\left(1+\frac{ci}{x}\right)\frac{dx}{p-xi} = 0$

6) $\int e^{bxi}(-xi)^{a-1}\,l\left(1+\frac{ci}{x}\right)\frac{dx}{p+xi} = 2\pi p^{a-1}e^{-bp}\,l\left(1+\frac{c}{p}\right)$

Cauchy, Lim. Imag. Add. 20, 21.

7) $\int e^{pxi}\,l(q+xi)\frac{dx}{(q+xi)^a} = \frac{2\pi}{1^{a/1}}p^{a-1}e^{-pq}\{Z'(a)-l\,p\}$

8) $\int e^{pxi}\,l(q-xi)\frac{dx}{(q-xi)^a} = 0$

Cauchy, Exerc. 1827. p. 141.

9) $\int \frac{1-e^{pxi}}{l(q-xi)}\frac{dx}{x} = \frac{2\pi i}{1-q}\{1-e^{p(q-1)}\}$, $q<1$;

10) $= 0$, $q>1$;

11) $= \pi p i$, $q=1$;

Cauchy, Lim. Imag. Add. 18.

12) $\int \frac{e^{-bxi}}{l(1+xi)}\frac{dx}{c^2+x^2} = \frac{\pi e^{-bc}}{c\,l(1+c)}-\frac{1}{c}$

13) $\int \frac{e^{-bxi}}{l(1+xi)}(xi)^a\frac{dx}{c^2+x^2} = \frac{\pi c^{a-1}e^{-bc}}{l(1+c)}$

Cauchy, P. 19. 511.

14) $\int \frac{e^{axi}}{l(1-pxi)}(-xi)\frac{dx}{1+x^2} = \frac{\pi e^{-a}}{l(1+p)}$ Cauchy, Cours. Leç. 39.

15) $\int \frac{e^{-axi}}{\{l(h+xi)\}^m}\frac{1}{(k+xi)^p(l+xi)^q\ldots}\frac{dx}{b^2+x^2} = \frac{\pi}{b}e^{-ab}\frac{1}{(b+k)^p(b+l)^q\ldots}\frac{1}{\{l(b+h)\}^m}$

16) $\int \frac{e^{-axi}}{\{l(h+xi)\}^m\{l(g+xi)\}^n\ldots}\frac{1}{(k+xi)^p(l+xi)^q\ldots}\frac{dx}{b^2+x^2} = \frac{\pi}{b}e^{-ab}\frac{1}{(b+k)^p(b+l)^q\ldots}\frac{1}{\{l(b+h)\}^m\{l(b+g)\}^n\ldots}$

Les intégrales (15) (16) se trouvent: Lejeune-Dirichlet, Cr. 4. 94. — Schlömilch, Stud. II. 17.

17) $\int e^{-(px^2+qx)}(2px^2+qx-a-1)x^a\,l\,x\,dx = \left(\frac{-q}{2p}\right)^a e^{\frac{q^2}{4p}}\sqrt{\frac{\pi}{p}}\sum_0^\infty\frac{a^{2n|-1}}{1^{n|1}}\left(\frac{p}{q^2}\right)^n$ V. T. 142. N°. 14.

18) $\int e^{-px^2+2qx}(px^2-qx-1)x\,l\,x\,dx = \frac{q}{2p}e^{\frac{q^2}{p}}\sqrt{\frac{\pi}{p}}$ V. T. 142. N°. 13.

F. Algébr. Expon. Logar.	TABLE 383.	Lim. diverses.

1) $\displaystyle\int_0^{l\frac{b^2}{a^2}} \frac{x\,e^{-x}}{\sqrt{\frac{b^2-a^2e^x}{e^x-1}}}\,\frac{dx}{(1-e^{-x})^2}\,l\frac{b^2-a^2e^x}{e^x-1} = -\frac{4\pi}{a+b}+\frac{4\pi}{a^2-b^2}\,l\frac{a^a}{b^b}$ V. T. 45. N°. 1.

2) $\displaystyle\int_1^{\infty} \frac{e^{-2ax}\,l\,x}{2x-1}\left\{a(2x-1)\,l(2x-1)-1\right\}dx = \frac{1}{4}\left\{li.(e^{-a})\right\}^2$ V. T. 383. N°. 3.

3) $\displaystyle\int_1^{\infty} e^{-2ax}\,l(2x-1)\,\frac{dx}{x} = \frac{1}{2}\left\{li.(e^{-a})\right\}^2$ ⎫ Winckler, Cr. 45. 102.

4) $\displaystyle\int_{2a}^{\infty} e^{-x}\,l\frac{x-a}{a}\,\frac{dx}{x} = \frac{1}{2}\left\{li.(e^{-a})\right\}^2$ ⎭

F. Algébr. rat. ent. Expon. Circ. Dir.	TABLE 384.	Lim. 0 et $\frac{\pi}{2}$.

1) $\displaystyle\int e^{-q\,Tang.x}\,\frac{x}{Cos.^2 x}\,dx = \frac{1}{q}\left\{Ci.(q).Sin.q+Cos.q.\left(\frac{\pi}{2}-Si.(q)\right)\right\}$ V. T. 288. N°. 4.

2) $\displaystyle\int e^{-Tang.^2x}\,\frac{x\,Sin.^3 2x}{Cos.^8 x}\,dx = 2\sqrt{\pi}$ V. T. 290. N°. 3.

3) $\displaystyle\int e^{-Tang.^2x}\,\frac{x\,Sin.4x}{Cos.^8 x}\,dx = \frac{3}{2}\sqrt{\pi}$ V. T. 290. N°. 8.

4) $\displaystyle\int e^{-q\,Cot.x}\,\frac{Sin.x+Cos.x}{Sin.^3 x}\,x\,dx = Sin.q.\left\{\frac{\pi}{2}-Si.(q)\right\}-Ci.(q).Sin.q$ V. T. 290. N°. 10.

5) $\displaystyle\int \frac{e^{\pi\,Tang.x}-e^{-\pi\,Tang.x}}{(e^{\pi\,Tang.x}+e^{-\pi\,Tang.x})^2}\,\frac{x}{Cos.^2 x}\,dx = \frac{4-\pi}{4\pi}$ V. T. 292. N°. 3.

6) $\displaystyle\int \frac{e^{\frac{1}{2}\pi\,Tang.x}-e^{-\frac{1}{2}\pi\,Tang.x}}{(e^{\frac{1}{2}\pi\,Tang.x}+e^{-\frac{1}{2}\pi\,Tang.x})^2}\,\frac{x}{Cos.^2 x}\,dx = \frac{\sqrt{2}}{\pi}\left\{\pi+l\frac{\sqrt{2}-1}{\sqrt{2}+1}\right\}$ V. T. 292. N°. 1.

7) $\displaystyle\int \frac{e^{\frac{1}{4}\pi\,Tang.x}-e^{-\frac{1}{4}\pi\,Tang.x}}{(e^{\frac{1}{4}\pi\,Tang.x}+e^{-\frac{1}{4}\pi\,Tang.x})^2}\,\frac{x}{Cos.^2 x}\,dx = \frac{1}{\pi}\,l\,2$ V. T. 292. N°. 2.

8) $\displaystyle\int \frac{4(e^{\frac{1}{4}\pi\,Tg.x}-e^{-\frac{1}{4}\pi\,Tg.x})-\pi\,Tg.x(e^{\frac{1}{4}\pi\,Tg.x}+e^{-\frac{1}{4}\pi\,Tg.x})}{(e^{\frac{1}{4}\pi\,Tang.x}-e^{-\frac{1}{4}\pi\,Tang.x})^2}\,\frac{x}{Cos.^2 x}\,dx = -\pi\sqrt{2}+4+\sqrt{2}\,l\frac{\sqrt{2}-1}{\sqrt{2}+1}$ V.T. 292. N°. 5.

9) $\displaystyle\int \frac{2(e^{\frac{1}{2}\pi\,Tang.x}-e^{-\frac{1}{2}\pi\,Tang.x})-\pi\,Tang.x\,(e^{\frac{1}{2}\pi\,Tang.x}+e^{-\frac{1}{2}\pi\,Tang.x})}{(e^{\frac{1}{2}\pi\,Tang.x}-e^{-\frac{1}{2}\pi\,Tang.x})^2}\,\frac{x}{Cos.^2 x}\,dx = \frac{2-\pi}{2}$ V. T. 292. N°. 6.

10) $\displaystyle\int \frac{(e^{\pi\,Tang.x}-e^{-\pi\,Tang.x})-\pi\,Tang.x\,(e^{\pi\,Tang.x}+e^{-\pi\,Tang.x})}{(e^{\pi\,Tang.x}-e^{-\pi\,Tang.x})^2}\,\frac{x}{Cos.^2 x}\,dx = \frac{1}{2}\left(l2-\frac{1}{2}\right)$ V. T. 292. N°. 10.

1) $\int e^{-qx} \, Sin.\, qx.\, x\, dx = \frac{1}{2q^2}$

2) $\int e^{-qx} \, Sin.\, qx.\, x^2\, dx = \frac{1}{2q^3}$

3) $\int e^{-qx} \, Sin.\, qx.\, x^3\, dx = 0$

4) $\int e^{-qx} \, Sin\, qx.\, x^4\, dx = \frac{-3}{q^5}$

5) $\int e^{-qx} \, Sin.\, qx.\, x^5\, dx = -\frac{15}{q^6}$

6) $\int e^{-px} \, Sin.\, qx.\, x\, dx = \frac{2pq}{(p^2+q^2)^2}$

7) $\int e^{-px} \, Sin.\, qx.\, x^2\, dx = 2\frac{3p^2q-q^3}{(p^2+q^2)^3}$

8) $\int e^{-px} \, Sin.\, qx.\, x^3\, dx = 24pq\frac{p^2-q^2}{(p^2+q^2)^4}$

(1–8) Oettinger, Cr. 38. 216, où les intégrales 4), 5) sont fautives.

9) $\int e^{-px} \, Sin.\, qx.\, x^4\, dx = 24\frac{5p^4q-10p^2q^3+q^5}{(p^2+q^2)^5}$ Sohnke, Samml.

10) $\int e^{-qx} \, Cos.\, qx.\, x\, dx = 0$ Poisson, Chal. 1. 159. — Oettinger, Cr. 38. 216.

11) $\int e^{-qx} \, Cos.\, qx.\, x^2\, dx = -\frac{1}{2q^3}$

12) $\int e^{-qx} \, Cos.\, qx.\, x^3\, dx = -\frac{3}{2q^4}$

13) $\int e^{-qx} \, Cos.\, qx.\, x^4\, dx = -\frac{3}{q^5}$

14) $\int e^{-px} \, Cos.\, qx.\, x\, dx = \frac{p^2-q^2}{(p^2+q^2)^2}$

15) $\int e^{-px} \, Cos.\, qx.\, x^2\, dx = 2\frac{p^3-3pq^2}{(p^2+q^2)^3}$

16) $\int e^{-px} Cos.\, qx.\, x^3\, dx = 6\frac{p^4-6p^2q^2+q^4}{(p^2+q^2)^4}$

(11–16) Oettinger, Cr. 38. 216.

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F. Algébr. rat. ent. x^a pour a spécial.
Expon. $e^{\pm px}$. TABLE 385 suite. Lim. 0 et ∞.
Circ. Dir. monôme.

17) $\int e^{-px}\, Cos.\, q\, x.\, x^3\, dx = 6\frac{p^4 - 5q^4}{(p^2+q^2)^4}$ (fautive) Sohnke, Samml.

18) $\int e^{-px}\, Cos.\, q\, x.\, x^4\, dx = 24\, p\frac{p^4 - 10\, p^2 q^2 + 5q^4}{(p^2+q^2)^5}$ Sohnke, Samml. où elle est fautive.

F. Alg. rat. ent. x^a pour a général.
Expon. $e^{\pm px}$. TABLE 386. Lim. 0 et ∞.
Circ. Dir. monôme.

1) $\int e^{-x}\, Sin.\, x.\, x^{p-1}\, dx = \frac{1}{2^{\frac{1}{2}p}} Sin. \frac{1}{4} p\pi . \Gamma(p)$
2) $\int e^{-x}\, Cos.\, x.\, x^{p-1}\, dx = \frac{1}{2^{\frac{1}{2}p}} Cos. \frac{1}{4} p\pi . \Gamma(p)$
Cauchy, Sav. Etr. 1827. 599. P. 1. § 3.

3) $\int e^{-x}\, Sin.\, (x\, Tang.\, \lambda).\, x^{p-1}\, dx = \Gamma(p)\, Cos.^p \lambda.\, Sin.\, p\, \lambda$
4) $\int e^{-x}\, Cos.\, (x\, Tang.\, \lambda).\, x^{p-1}\, dx = \Gamma(p)\, Cos.^p \lambda.\, Cos.\, p\, \lambda$
Kummer, Cr. 17. 210.

5) $\int e^{-x}\, Sin.\, \left(px + \frac{1}{2} a\pi\right).\, x^a\, dx = \frac{d^a}{dp^a}.\frac{p}{1+p^2}$
6) $\int e^{-x}\, Cos.\, \left(px + \frac{1}{2} a\pi\right).\, x^a\, dx = \frac{d^a}{dp^a}.\frac{1}{1+p^2}$
Dienger, Cr. 46. 119.

7) $\int e^{-ax}\, Cos.\, x.\, x^{p-1}\, dx = \frac{\Gamma(p)}{(1+a^2)^{\frac{1}{2}p}} Sin.\, (p\, Arccot.\, a)$
8) $\int e^{-ax}\, Cos.\, x.\, x^{p-1}\, dx = \frac{\Gamma(p)}{(1+a^2)^{\frac{1}{2}p}} Cos.\, (p\, Arccot.\, a)$
Boncompagni, Cr. 25, 74.

9) $\int e^{-kx}\, Sin.\, a\, x.\, x^{p-1}\, dx = \frac{\Gamma(p)}{a^p} Sin. \frac{1}{2} p\, \pi$, pour k très-petit;
10) $\int e^{-kx}\, Cos.\, a\, x.\, x^{p-1}\, dx = \frac{\Gamma(p)}{a^p} Cos. \frac{1}{2} p\, \pi$
Cauchy, P. 19. 511.

11) $\int e^{-px}\, Sin.\, q\, x.\, x^a\, dx = \frac{1^{a/1}}{p^{a+1}} \sum_0^\infty (-1)^n \frac{(a+1)^{2n/1}}{1^{2n+1/1}} \left(\frac{q}{p}\right)^{2n+1}, a \leqq 1;$ Oettinger, Cr. 38. 216. où elle est fautive.

F. Alg. rat. ent. x^a pour a général.
Expon. $e^{\pm px}$.
Circ. Dir. monôme.

TABLE 386 suite.

Lim. 0 et ∞.

12) $\int e^{-px} \, Sin. \, qx . x^{a-1} \, dx = \frac{1^{a-1/1}}{(p^2+q^2)^{\frac{1}{2}a}} Sin. \left(a \, Arctang. \frac{q}{p}\right)$

13) $\int e^{-px} \, Cos. \, qx . x^{a-1} \, dx = \frac{1^{a-1/1}}{(p^2+q^2)^{\frac{1}{2}a}} Cos. \left(a \, Arctang. \frac{q}{p}\right)$

Euler, Calc. Int. 4. S. 5. 134. — Lacroix, Calc. Diff. T. 3. p. 490 (démonstration de Poisson.) — Legendre, Exerc. P. 3. 54. — Cauchy, P. 28. 147. P. 1. § 2. — Id., Cours. Leç. 32. — Id., Exerc. 1826. p. 58. — Fuss, Mém. Pétersb. 1830. — Plana, Mém. Brux. 1837. — Grunert, Cr. 8. 146. — Liouville, Cr. 13. 210. — Schlömilch, Gr. 6. 200.

Chez Oettinger, Cr. 38. 216 et Schlömilch, Stud. I. 13 ces deux formules valent pour a aussi fractionnaire.

14) $\int e^{-px} \, Sin. \, qx . x^{r-1} \, dx = \frac{(p-qi)^{-r} - (p+qi)^{-r}}{2i} \Gamma(r)$ Cauchy, P. 19. 511. — Id., Sav. Etr. 1827. 124. Note 6. — Id., P. 28. 147. I. § 2. — Plana, Mém. Brux. 1837.

15) $= \frac{(p+qi)^r - (p-qi)^r}{2i(p^2+q^2)^r} \Gamma(r)$ Boncompagni, Cr. 25. 74, où faut. $e^{-qx} \, Sin. \, px$.

16) $= \frac{\Gamma(r)}{p^r} Cos.^r \left(Arctg. \frac{q}{p}\right) . Sin. \left(r \, Arctg. \frac{q}{p}\right)$ Serret, L. 8. 489. — Boncompagni, Cr. 25. 74.

17) $= \frac{\Gamma(r)}{(p^2+q^2)^{\frac{1}{2}r}} Sin. \left\{r \, Arcsin. \frac{q}{\sqrt{(p^2+q^2)}}\right\}$ Cauchy, Sav. Etr. 1827. 599. P. 1. § 3. — Fuss, Mém. Pétersb. 1830.

18) $\int e^{-px} \, Cos. \, qx . x^{r-1} \, dx = \frac{(p-qi)^{-r} + (p+qi)^{-r}}{2} \Gamma(r)$ Cauchy, P. 19. 511. — Id., Sav. Etr. 1827. 124. Note 6. — Id., P. 28. 147. I. § 2.

19) $= \frac{(p+qi)^r + (p-qi)^r}{2(p^2+q^2)^r} \Gamma(r)$ Boncompagni, Cr. 25. 74, où faut. $e^{-qx} \, Cos. \, px$.

20) $= \frac{\Gamma(r)}{p^r} Cos.^r \left(Arctg. \frac{q}{p}\right) . Cos. \left(r \, Arctg. \frac{q}{p}\right)$ Serret, L. 8. 489. — Boncompagni, Cr. 25. 74.

21) $= \frac{\Gamma(r)}{(p^2+q^2)^{\frac{1}{2}r}} Cos. \left\{r \, Arcsin. \frac{q}{\sqrt{(p^2+q^2)}}\right\}$ Cauchy, Sav. Etr. 1827. 599. P. 1. § 2. — Fuss, Mém. Pétersb. 1830.

F. Alg. rat. ent.
Expon. $e^{\pm ax}$.
Circ. Dir. polynôme.

TABLE 587.

Lim. 0 et ∞.

1) $\int e^{-cx} (b \, Sin. \, ax + a \, Cos. \, ax) \, dx = a \frac{b+c}{a^2+c^2}$ Poisson, Chal. 154.

2) $\int e^{-qx} \, Cos. (2x^2 + qx) . x \, dx = 0$

3) $\int e^{-qx} \{Sin. (2x^2 + qx) + Cos. (2x^2 + qx)\} x^2 \, dx = 0$

Poisson, Chal. Suppl. Note B.

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F. Alg. rat. ent.
Expon. $e^{\pm ax}$.
Circ. Dir. polynôme.

TABLE 387 suite.

Lim. 0 et ∞.

4) $\int e^{-qx}\left\{Sin.(2x^2-qx)-Cos.(2x^2-qx)\right\}x^2\,dx=\frac{1}{16}(2-q^2)e^{-\frac{1}{4}q^2}\sqrt{\pi}$

5) $\int e^{-qx}\,Cos.(2x^2-qx).x\,dx=\frac{1}{8}q\,e^{-\frac{1}{4}q^2}\sqrt{\pi}$

(4 et 5) Poisson, Chal. Suppl. Note B.

6) $\int e^{-qx}\left\{Cos.\,bx-i\,Sin.\,bx\right\}x^c\,dx=\frac{1^{c|1}}{(a+bi)^{c+1}}$ Cauchy, Cours. Leç. 32. — Moigno, Calc. Int. 42. où fautivement $+\,i\,Sin.\,bx$.

7) $\int e^{-\frac{1}{2}x}\,Cos.\left\{\frac{1}{2}x\,Tang.\,\lambda+b\lambda\right\}.x^{q-1}\,dx=2^q\,\Gamma(q)\,Cos.^q\lambda.\,Cos.\left\{(b+q)\lambda\right\}$ Kummer, Cr. 17. 228.

F. Alg. rat. ent.
Expon. e^{-x^2}.
Circ. Dir.

TABLE 388.

Lim. 0 et ∞.

1) $\int e^{-x^2}\,Sin.\,ax.x\,dx=\frac{1}{4}a\,e^{-\frac{1}{4}a^2}\sqrt{\pi}$ Legendre, Exerc. 3, 48. — Dienger, Cr. 46. 119. — Svanberg, Transf. 4.

2) $\int e^{-x^2}\,Cos.\,ax.x\,dx=\frac{1}{2}-\frac{1}{4}a\sum_0^\infty(-1)^n\frac{a^{2n+1}}{(n+1)^{n+1/1}}$ Legendre, Exerc. 3, 49.

3) $\int e^{-x^2}\,Sin.\,2x.\,x\,dx=\frac{1}{2e}\sqrt{\pi}$

4) $\int e^{-x^2}\,Cos.\,2x.\,x^2\,dx=-\frac{1}{4e}\sqrt{\pi}$

5) $\int e^{-x^2}\,Cos.\,2px.x^2\,dx=\frac{1-2p^2}{4}e^{-p^2}\sqrt{\pi}$

(3 à 5) Dienger, Cr. 46. 119. où dans 3) il est fautivement $x^2\,dx$.

6) $\int e^{-x^2}\,Sin.\,ax.x^2\,dx=\frac{1}{4}a+\frac{2-a^2}{8}\sum_0^\infty(-1)^n\frac{a^{2n+1}}{(n+1)^{n+1/1}}$

7) $\int e^{-x^2}\,Sin.\,ax.x^3\,dx=\frac{6a-a^3}{16}e^{-\frac{1}{4}a^2}\sqrt{\pi}$

8) $\int e^{-x^2}\,Sin.\,ax.x^4\,dx=\frac{10a-a^3}{16}+\frac{12-12a^2+a^4}{32}\sum_0^\infty(-1)^n\frac{a^{2n+1}}{(n+1)^{n+1/1}}$

9) $\int e^{-x^2}\,Sin.\,ax.x^5\,dx=\frac{60a-20a^3+a^5}{64}e^{-\frac{1}{4}a^2}\sqrt{\pi}$

10) $\int e^{-x^2}\,Cos.\,ax.x^2\,dx=\frac{2-a^2}{8}e^{-\frac{1}{4}a^2}\sqrt{\pi}$

(6 à 10) Legendre, Exerc. 3. 49.

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11) $\int e^{-x^2} Cos.ax.x^3\,dx = \frac{4-a^2}{8} - \frac{6a-a^3}{16}\sum_0^\infty (-1)^n \frac{a^{2n+1}}{(n+1)^{n+1/1}}$

12) $\int e^{-x^2} Cos.ax.x^4\,dx = \frac{12-12a^2+a^4}{32} e^{-\frac{1}{4}a^2}\sqrt{\pi}$

Legendre, Exerc. 3. 49.

13) $\int e^{-p^2x^2} Cos.qx.x\,dx = \frac{1}{2p^2}\sum_0^\infty (-1)^n \frac{1}{(n+1)^{n/1}}\left(\frac{q}{p}\right)^{2n}$ Oettinger, Cr. 38. 216.

14) $\int e^{-x^2} Sin.ax.x^{2b-1}\,dx = \frac{1}{2}\sqrt{\pi}\frac{d^{b-1}}{dy^{b-1}}.(y^{\frac{1}{2}(b-1)}e^{-y})$ (après la différentiation mettez $y=\frac{1}{4}a^2$.) Liouville, L. 5. 311.

15) $= \frac{b^{b/1}}{2^{2b}}e^{-\frac{1}{4}a^2}\sqrt{\pi}\sum_0^\infty (-1)^n \frac{(b-1)^{n/-1}}{1^{2n+1/1}}a^{2n+1}$ Cauchy, Sav. Etr. 1827. 599. P. 1. § 2.

16) $\int e^{-x^2} Cos.ax.x^{2b}\,dx = \frac{1}{2a}\sqrt{\pi}\frac{d^b}{dy^b}.(y^{b-\frac{1}{2}}e^{-y})$(après la différentiation mettez $y=\frac{1}{4}a^2$.) Liouville. L. 5. 311.

17) $= \frac{(b+1)^{b/1}}{2^{2b+1}}e^{-\frac{1}{4}a^2}\sqrt{\pi}\sum_0^\infty (-1)^n \frac{b^{n/-1}}{1^{2n/1}}a^{2n}$ Cauchy, Sav. Etr. 1827. 599. P. 1. § 2.

18) $\int e^{-x^2} Sin.2ax.x^b dx = (-1)^{\frac{1}{2}(b-1)}\frac{1}{2}e^{-a^2}a^b\sqrt{\pi}\left\{1+\sum_0^\infty (-1)^n \frac{b^{2n/-1}}{1^{n/1}}\left(\frac{1}{2a}\right)^{2n}\right\}$, b impair;

19) $\int e^{-x^2} Cos.2ax.x^b dx = (-1)^{\frac{1}{2}b}\frac{1}{2}e^{-a^2}a^b\sqrt{\pi}\left\{1+\sum_0^\infty (-1)^n \frac{b^{2n/-1}}{1^{n/1}}\left(\frac{1}{2a}\right)^{2n}\right\}$, b pair;

Cauchy, P. 28. 147. P. III. — Id., Exerc. 1826. p. 57.

20) $\int e^{-x^2} Tang.ax.x\,dx = a\sqrt{\pi}\sum_1^\infty (-1)^{n-1} n\,e^{-(na)^2}$ V. T. 439. N°. 7.

21) $\int e^{-x^2} Cot.ax.x\,dx = -a\sqrt{\pi}\sum_1^\infty n\,e^{-(na)^2}$ V. T. 439. N°. 6.

22) $\int e^{-x^2} Cosec.ax.x\,dx = -a\sqrt{\pi}\sum_1^\infty (2n-1)\,e^{-(2n-1)^2a^2}$ V. T. 439. N°. 8.

23) $\int e^{-x^2} Sin.\left(2px+\frac{1}{2}a\pi\right).x^{a+1}\,dx = \frac{\sqrt{\pi}}{2^{a+1}}\frac{d^a}{dp^a}.pe^{-p^2}$ Dienger, Cr. 46. 119.

24) $\int e^{-x^2} Cos.\left(2px+\frac{1}{2}b\pi\right).x^b\,dx = \frac{(-1)^b}{2}e^{-p^2}\sqrt{\pi}\sum_0^\infty (-1)^n p^{b-2n}\binom{b}{2n}\frac{(n+1)^{n/1}}{2^{2n}}$ Schlömilch, Cr. 33. 268.

25) $\int e^{-x^2}(a\,Sin.ax+2x\,Cos.ax)\,dx = 1$

26) $\int e^{-x^2}(4x^2+a^2-2)\,Sin.ax\,dx = a$

Legendre, Exerc. 3. 49.

F. Alg. rat. ent.
Expon. e^{-ax^2}.
Circ. Dir.

TABLE 389.

Lim. 0 et ∞.

1) $\int e^{x^2 i} Sin.\, a\, x.\, x\, d\, x = \frac{i-1}{4\, i}\, a\, e^{-\frac{a^2 i}{4}} \sqrt{\pi}$ Cauchy, Lim. Imag. 191.

2) $\int e^{-\frac{1}{4}a^2x^2} Sin.\, a\, x.\, x\, d\, x = \frac{2\sqrt{\pi}}{a^2\, e}$ Oettinger, Cr. 38. 216.

3) $\int e^{-a^2x^2} Sin.\, b\, x.\, x\, d\, x = \frac{b}{4\, a^3}\, e^{-\frac{b^2}{4a^2}} \sqrt{\pi}$ Oettinger, Cr. 38. 216. — Dienger, Cr. 46. 119.

4) $\int e^{-\frac{1}{2}bx^2} Cos.\, (a\, x \sqrt{b}).\, x^4\, d\, x = \frac{27}{b^2}\, e^{-\frac{1}{2}a^2} (1 - 6\, a^3 + 3\, a^4) \sqrt{\frac{3\,\pi}{2\, b}}$ Laplace, Mém. Inst. 1809. 353. § 3.

5) $\int e^{-a^2x^2} Sin.\, b\, x.\, x^{2c+1}\, d\, x = (-1)^{c+1} \frac{\sqrt{\pi}}{2\, a} \frac{d^{2c+1}}{d\, b^{2c+1}}.\, e^{-\frac{b^2}{4a^2}}$

6) $\int e^{-a^2x^2} Cos.\, b\, x.\, x^{2c}\, d\, x = (-1)^c \frac{\sqrt{\pi}}{2\, a} \frac{d^{2c}}{d\, b^{2c}}.\, e^{-\frac{b^2}{4a^2}}$

(5, 6) Laplace, Probab. L. 1. N°. 25.

7) $\int e^{-px^2} Sin. \left(2\, q\, x + \frac{1}{2}\, a\, \pi\right).\, x^{a+1}\, d\, x = \frac{\sqrt{\pi}}{2^{a+1}\, p \sqrt{p}} \frac{d^a}{d\, q^a}.\, q\, e^{-\frac{q^2}{p}}$ Dienger, Cr. 46. 119.

8) $\int e^{-q^2x^2} Cos. \left(p\, x + \frac{1}{2}\, a\, \pi\right).\, x^a\, d\, x = \frac{1}{2\, q} \sqrt{\pi} \frac{d^a}{d\, p^a}.\, e^{-\frac{p^2}{4q^2}}$ Schlömilch, Gr. 5. 90. — Id., Stud. I. 12.

9) $\int e^{-px^2}(e^{2qx Sin.\lambda} + e^{-2qx Sin.\lambda}) Sin. (2\, q\, x\, Cos.\lambda).\, x\, d\, x = \frac{q}{p}\, e^{-\frac{q^2}{p} Cos.\, 2\lambda} \sqrt{\frac{\pi}{p}}.\, Cos. \left(\lambda - \frac{q^2}{p} Sin.\, 2\, \lambda\right)$

10) $\int e^{-px^2}(e^{2qx Sin.\lambda} - e^{-2qx Sin.\lambda}) Cos. (2\, q\, x\, Cos.\lambda).\, x\, d\, x = \frac{q}{p}\, e^{-\frac{q^2}{p} Cos.\, 2\lambda} \sqrt{\frac{\pi}{p}}.\, Sin. \left(\lambda - \frac{q^2}{p} Sin.\, 2\, \lambda\right)$

(9, 10) Dienger, Cr. 46. 119.

F. Alg. rat. ent.
Exp. d'autre forme monôme.
Circ. Dir.

TABLE 390.

Lim. 0 et ∞.

1) $\int e^{-x Cos.\lambda} Sin.\, (x\, Sin.\, \lambda).\, x^{p-1}\, d\, x = Sin.\, p\, \lambda.\, \Gamma\, (p)$

2) $\int e^{-x Cos.\lambda} Cos.\, (x\, Sin.\, \lambda).\, x^{p-1}\, d\, x = Cos.\, p\, \lambda.\, \Gamma\, (p)$

(1, 2) Cauchy, Lim. Imag. 162.

3) $\int e^{-ax^p} Sin.\, (b\, x^p).\, x^{c-1}\, d\, x = \frac{1}{2\, p\, i} \Gamma \left(\frac{c}{p}\right) \left\{(a - b\, i)^{-\frac{c}{p}} - (a + b\, i)^{-\frac{c}{p}}\right\}$

4) $= \frac{1}{p} \Gamma \left(\frac{c}{p}\right) (a^2 + b^2)^{-\frac{c}{2p}} Sin. \left\{\frac{c}{p}\, Arctang. \frac{b}{a}\right\}$

(3, 4) Plana, Mém. Brux. 1837.

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5) $\int e^{-ax^p} Cos.(b x^p).x^{c-1}\,dx = \frac{1}{2p}\Gamma\left(\frac{c}{p}\right)\left\{(a-bi)^{-\frac{c}{p}} + (a+bi)^{-\frac{c}{p}}\right\}$

6) $= \frac{1}{p}\Gamma\left(\frac{c}{p}\right)(a^2+b^2)^{-\frac{c}{2p}} Cos.\left\{\frac{c}{p} Arctg.\frac{b}{a}\right\}$

Plana, Mém. Brux. 1837.

7) $\int e^{-ax^q} Sin.(a\,x\,Tang.\,\lambda).x^{pq-1}\,dx = \frac{\Gamma(p)\,Sin.\,p\,\lambda}{q\,a^p(1+Tang.^2\lambda)^{\frac{1}{2}p}}$

8) $\int e^{-ax^q} Cos.(a\,x\,Tang.\,\lambda).x^{pq-1}\,dx = \frac{\Gamma(p)\,Cos.\,p\,\lambda}{q\,a^p(1+Tang.^2\lambda)^{\frac{1}{2}p}}$

Boncompagni, Cr. 25. 74. ne valent que pour $q = 1$.

9) $\int e^{-px^2-\frac{q^2}{x^2}} Sin.(rx^2).x^2\,dx = \frac{1}{4}\sqrt{\pi}\left(\frac{1}{p}Cos.\varphi\right)^{\frac{3}{2}}.Sin.\left(2bq+\frac{3}{2}\varphi\right).e^{-2aq} + \frac{q}{2p}\sqrt{\pi}.Cos.\varphi.Sin.(2bq-\varphi).e^{-2aq}$

10) $\int e^{-px^2-\frac{q^2}{x^2}} Cos.(rx^2).x^2\,dx = \frac{1}{4}\sqrt{\pi}\left(\frac{1}{p}Cos.\varphi\right)^{\frac{3}{2}}.Cos.\left(2bq+\frac{3}{2}\varphi\right).e^{-2aq} + \frac{q}{2p}\sqrt{\pi}.Cos.\varphi.Cos.(2bq-\varphi).e^{-2aq}$

11) $\int e^{-px^2-\frac{q^2}{x^2}} Sin.(r x^2).x^4\,dx = \frac{1}{2}\sqrt{\pi}.e^{-2aq}\Big[\frac{3}{4}\left(\frac{1}{p}Cos.\varphi\right)^{\frac{5}{2}}.Sin.\left(2bq+\frac{5}{2}\varphi\right) +$

$+ \frac{q}{p^2}Cos.^2\varphi.(Cos.2bq + Sin.2bq) + q^2\left(\frac{1}{p}Cos.\varphi\right)^{\frac{1}{2}}.Sin.\left(2bq-\frac{5}{2}\varphi\right)\Big]$

12) $\int e^{-px^2-\frac{q^2}{x^2}} Cos.(r x^2).x^4\,dx = \frac{1}{2}\sqrt{\pi}.e^{-2aq}\Big[\frac{3}{4}\left(\frac{1}{p}Cos.\varphi\right)^{\frac{5}{2}}.Cos.\left(2bq+\frac{5}{2}\varphi\right) +$

$+ \frac{q}{p^2}Cos.^2\varphi.(Cos.2bq - Sin.2bq) + q^2\left(\frac{1}{p}Cos.\varphi\right)^{\frac{1}{2}}.Cos.\left(2bq-\frac{5}{2}\varphi\right)\Big]$

Dans les formules (8) à (12), dues à Helmling, Transf. 41—44, on a

$$Tang.\,\varphi = \frac{r}{p},\; a = \sqrt{\frac{p+\sqrt{(p^2+r^2)}}{2}},\; b = \sqrt{\frac{-p+\sqrt{(p^2+r^2)}}{2}}.$$

13) $\int e^{-a^2x^4+b^2x^2}\{2\,a\,x\,Cos.(2\,a\,b\,x^3) + b\,Sin.(2\,a\,b\,x^3)\}\,dx = \frac{1}{2}\sqrt{\pi}$

14) $\int e^{-a^2x^4+b^2x^2}\{2\,a\,x\,Sin.(2\,a\,b\,x^3) - b\,Cos.(2\,a\,b\,x^3)\}\,dx = 0$

Cauchy, Sav. Etr. 1827. 599. P. 1. § 5.

15) $\int e^{b^2x^2-(ax+c)^2} Sin.\{2bx(ax+c)\}.x^{h-1}\,dx = \frac{e^{-c^2}\Gamma(\frac{1}{2}h)}{\sqrt{\{(a^2+b^2)(1+2c)\}^h}} Sin.\left\{h\,Arcsin.\frac{b}{\sqrt{(a^2+b^2)}}\right\}$

16) $\int e^{b^2x^2-(ax+c)^2} Cos.\{2bx(ax+c)\}.x^{h-1}\,dx = \frac{e^{-c^2}\Gamma(\frac{1}{2}h)}{\sqrt{\{(a^2+b^2)(1+2c)\}^h}} Cos.\left\{h\,Arcsin.\frac{b}{\sqrt{(a^2+b^2)}}\right\}$

$, b \leqq a;$

Sur 15) et 16) voir Cauchy, Sav. Etr. 1827. 599. P. 1. § 3. — Voir T. 116. N°. 4.

17) $\int e^{(b^2-a^2)\left(x^2+\frac{c^2}{x^2}\right)} Sin.\left\{2ab\left(x^2-\frac{c^2}{x^2}\right)\right\}.x^{h-1}dx=\frac{1}{2\sqrt{(a^2+b^2)^h}}Cos.\left\{h\,Arcsin.\frac{b}{\sqrt{(a^2+b^2)}}\right\}$

$\left[\Gamma\left(\frac{1}{2}h\right)\psi\left\{1-\frac{1}{2}h, c^2(a^2+b^2)\right\}-\Gamma\left(-\frac{1}{2}h\right)c^h\sqrt{(a^2+b^2)^h}\psi\left\{1+\frac{1}{2}h, c^2(a^2+b^2)\right\}\right]$, $a<b$;

18) $\int e^{(b^2-a^2)\left(x^2+\frac{c^2}{x^2}\right)} Cos.\left\{2ab\left(x^2-\frac{c^2}{x^2}\right)\right\}.x^{h-1}dx=\frac{1}{2\sqrt{(a^2+b^2)^h}}Sin.\left\{h\,Arcsin.\frac{b}{\sqrt{(a^2+b^2)}}\right\}$

$\left[\Gamma\left(\frac{1}{2}h\right)\psi\left\{1-\frac{1}{2}h, c^2(a^2+b^2)\right\}-\Gamma\left(-\frac{1}{2}h\right)c^h\sqrt{(a^2+b^2)^h}\psi\left\{1+\frac{1}{2}h, c^2(a^2+b^2)\right\}\right]$

Sur 17) et 18) voir Cauchy, Sav. Etr. 1827. 599. P. 1. § 3. — Voir T. 116. N°. 3.

19) $\int e^{(b^2-a^2)\left(x^2+\frac{c^2}{x^2}\right)} Sin.\left\{2ab\left(x^2-\frac{c^2}{x^2}\right)\right\}.x^{2h}\,dx=$

$=\frac{1}{2c}e^{-2c\sqrt{(a^2+b^2)}}Cos.\left\{(2h+1)\,Arcsin.\frac{b}{\sqrt{(a^2+b^2)}}\right\}\frac{\sqrt{\pi}}{(a^2+b^2)^{h+\frac{1}{4}}}\sum_0^\infty\frac{(h-n)^{n/1}}{2^{n/2}(2c)^n\sqrt{(a^2+b^2)^n}}$, $a<b$;

20) $\int e^{(b^2-a^2)\left(x^2+\frac{c^2}{x^2}\right)} Cos.\left\{2ab\left(x^2-\frac{c^2}{x^2}\right)\right\}.x^{2h}\,dx=$

$=\frac{1}{2c}e^{-2c\sqrt{(a^2+b^2)}}Sin.\left\{(2h+1)\,Arcsin.\frac{b}{\sqrt{(a^2+b^2)}}\right\}\frac{\sqrt{\pi}}{(a^2+b^2)^{h+\frac{1}{4}}}\sum_0^\infty\frac{(h-n)^{n/1}}{2^{n/2}(2c)^n\sqrt{(a^2+b^2)^n}}$

Sur 19) et 20) voir Cauchy, Sav. Etr. 1827. 599. P. 1. § 3. — Voir T. 116. N°. 8.

21) $\int e^{-q\left(x^2+\frac{1}{x^2}\right)} Sin.\left\{p\left(x-\frac{1}{x}\right)^2\right\}.x^{2a}\,dx=\frac{1}{2}e^{-2(q+pi)}\sqrt{\frac{\pi}{q+pi}}\sum_0^\infty\frac{(a+n)^{2n/-1}}{2^{n/2}}\left\{\frac{1}{2(p+qi)}\right\}^n$, $a<b$;

22) $\int e^{-q\left(x^2+\frac{1}{x^2}\right)} Cos.\left\{p\left(x-\frac{1}{x}\right)^2\right\}.x^{2a}\,dx=\frac{1}{2}e^{-2(q+pi)}\sqrt{\frac{\pi}{q+pi}}\sum_0^\infty\frac{(a+n)^{2n/-1}}{2^{n/2}}\left\{\frac{1}{2(p+qi)}\right\}^n$ Cauchy, Exerc. 1826. p. 54.

23) $\int e^{-q\left(x^2+\frac{1}{x^2}\right)} Cos.pa.x^{2a}\,dx=\frac{1}{2}e^{-2q}\sqrt{\frac{\pi}{q}}\sum_0^\infty\frac{(a+n)^{2n/-1}}{1^{2n/1}}(-1)^n\frac{d^{2n}}{dp^{2n}}.e^{-\frac{p^2}{4q}}$

F. Alg. rat. ent.
Exp. en dén. binôme. TABLE 391. Lim. 0 et ∞.
Circ. Dir.

1) $\int\frac{e^x-e^{-x}}{e^x+e^{-x}}Cos.ax.x\,dx=-\frac{1}{2}\pi^2e^{-\frac{1}{2}a\pi}\frac{1+e^{-a\pi}}{(1-e^{-a\pi})^2}$ V. T. 391. N°. 2, 3.

2) $\int\frac{1}{e^x-e^{-x}}Cos.ax.x\,dx=\frac{1}{2}\pi^2\frac{e^{-a\pi}}{(1+e^{-a\pi})^2}$ Plana, Mém. Turin. 1818. 7. IV. 18.

3) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} Cos. a x . x\, d x = -\frac{\pi^2 e^{-a\pi}}{(1 - e^{-a\pi})^2}$ Plana, Mém. Turin. 1818. 7. IV. 19.

4) $\int \frac{e^x - 1}{e^x + 1} Cos. a x . x\, d x = -2\pi^2 e^{-a\pi} \frac{1 + e^{-2a\pi}}{(1 - e^{-2a\pi})^2}$ V. T. 391. N°. 2, 3.

5) $\int \frac{x\, Sin.\, a x}{e^{\pi x} + e^{-\pi x}} d x = \frac{1}{4} \frac{e^{\frac{1}{2}a} - e^{-\frac{1}{2}a}}{(e^{\frac{1}{2}a} + e^{-\frac{1}{2}a})^2}$ } Legendre, Exerc. 5. 45.

6) $\int \frac{x\, Cos.\, a x}{e^{\pi x} + e^{-\pi x}} d x = \frac{1}{2} \frac{e^a}{(e^a + 1)^2}$ } Legendre, Exerc. 5. 45.

F. Alg. rat. fract. à dén. x.
Exp. monôme. TABLE 392. Lim. 0 et ∞.
Circ. Dir. monôme.

1) $\int e^{-x} Sin.\, a x \frac{d x}{x} = Arctang.\, a$ Arndt, Gr. 11. 70. — Dienger, Cr. 46. 119.

2) $\int e^{-ax} Sin.\, a x \frac{d x}{x} = \frac{1}{4}\pi$ Oettinger, Cr. 38. 216.

3) $\int e^{-px} Sin.\, q x \frac{d x}{x} = Arctang.\, \frac{q}{p}$ Euler, Calc. Int. T. 4. S. 5. § 139. — Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 34. — Poisson, P. 16. 215. N°. 2. — Id., Chal. 158. — Legendre, Exerc. 3. 55. — Plana, Mém. Brux. 1837. — Lobatto, Cr. 11. 169. — Oettinger, Cr. 38. 216. — Hoppe, Cr. 40. 139. — Lindmann, Gr. 16. 94.

4) $\int e^{-px} Cos.\, q x \frac{d x}{x} = \infty$ Poisson, P. 16. 215. N°. 2. — Legendre, Exerc. 3. 55.

5) $= -\frac{1}{2} l \left(1 + \frac{b^2}{a^2}\right)$ Lobatto, Cr. 11. 169. (fautive).

6) $= - l\, 0 - \frac{1}{2} l (a^2 + b^2) - A$ Bidone, Mém. Turin. 1812. 231. Art. 3. 35.

7) $= \frac{1}{2} l (\infty' + b^2) - \frac{1}{2} l (a^2 + b^2)$ Plana, Mém. Brux. 1837.

8) $\int e^{-pxi} Sin.\, q x \frac{d x}{x} = \frac{i}{2} l \frac{p - q}{p + q}$ Oettinger, Cr. 38. 216.

9) $\int e^{-x} Sin.^2\, p x \frac{d x}{x} = \frac{1}{4} l (1 + 4 p^2)\, ,\, p^2 < \frac{1}{4};$ Dienger, Cr. 46. 119.

10) $\int e^{-ax} Sin.^2 \frac{1}{2} b x \frac{d x}{x} = \frac{1}{4} l \frac{a^2 + b^2}{a^2}$ Lindmann, Gr. 16. 94.

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11) $\int e^{-x} Sin.\,p\,x.\,Sin.\,q\,x \frac{dx}{x} = \frac{1}{4} l \frac{1+(p+q)^2}{1+(p-q)^2}$ V. T. 301. N°. 13.

12) $\int e^{-x} Sin.\,p\,x.\,Cos.\,q\,x \frac{dx}{x} = \frac{1}{2} Arctang. \frac{2p}{1-p^2+q^2}$ V. T. 301. N°. 14.

13) $\int e^{-ax} Sin.^2 b\,x.\,Cos.^2 c\,x \frac{dx}{x} = \frac{1}{8} l \left\{ \frac{a^2+4b^2}{a^2} \cdot \frac{\sqrt{\{a^2+4(b+c)^2\}} \cdot \sqrt{\{a^2-4(b-c)^2\}}}{a^2+4c^2} \right\}$ Bidone, Mém. Turin. 1812. 231. Art. 3. 35.

14) $\int e^{a Cos.\,b\,x} Sin.\,(a\,Sin.\,b\,x) \frac{dx}{x} = \frac{1}{2}\pi(e^a-1)$ Cauchy, Exerc. 1826. p. 95. — Id., Lim. Imag Add. N°. 24. (où faut. $= \frac{1}{2}\pi e^a$).

15) $\int e^{-p^2x^2} Sin.\,q\,x \frac{dx}{x} = \frac{q}{2p}\sqrt{\pi} \sum_0^\infty (-1)^n \frac{1}{(2n+1)1^{n/1}} \left(\frac{q}{2p}\right)^{2n}$ Oettinger, Cr. 38. 216.

F. Alg. rat. fract. à dén. x.
Exp. } Circ. Dir. } Fonct. polyn. en num. TABLE 393. Lim. 0 et ∞.

1) $\int \frac{1-e^{-qx}}{x} Sin.\,x\,dx = Arctang.\,q$
2) $\int \frac{1-e^{-qx}}{x} Cos.\,x\,dx = \frac{1}{2} l(1+q^2)$
} Sohnke, Samml. — Minding, Tafeln. I.

3) $\int \frac{e^{-qx}-e^{-rx}}{x} Sin.\,p\,x\,dx = Arctang.\frac{r}{p} - Arctang.\frac{q}{p}$ Pioch, Mém. Cour. Brux. T. 15. P. 2.

4) $= Arctang.\frac{p}{q} - Arctang.\frac{p}{r}$
5) $\int \frac{e^{-qx}-e^{-rx}}{x} Cos.\,p\,x\,dx = \frac{1}{2} l \frac{p^2+r^2}{p^2+q^2}$
} Cauchy, Cours. Leç. 33. — Lindmann, Stockh. Handl. 1850. IV.

6) $\int \frac{1-Cos.\,p\,x}{x} e^{-x} dx = \frac{1}{2} l(1+p^2)$ Dienger, Cr. 46. 119.

7) $\int \frac{1-Cos.\,p\,x}{x} e^{-qx} dx = \frac{1}{2} l(p^2+q^2) - l\,q$ Malmsten, Cr. 38. 1.

8) $\int \frac{Sin.\,p\,x - Sin.\,q\,x}{x} e^{-rx} dx = Arctang.\frac{p}{r} - Arctang.\frac{q}{r}$ Arndt, Gr. 11. 70.

9) $\int \frac{Cos.\,p\,x - Cos.\,q\,x}{x} e^{-x} dx = \frac{1}{2} l \frac{1+q^2}{1+p^2}$ V. T. 301. N°. 19.

10) $\int \frac{Cos.\,p\,x - Cos.\,q\,x}{x} e^{-rx}\,dx = \frac{1}{2} l \frac{r^2+q^2}{r^2+p^2}$ Poisson, P. 16. 215. N°. 2.

11) $\int \frac{e^{-x} - Cos.\,x}{x}\,dx = 0$ Arndt, Gr. 10. 225.

12) $\int \frac{e^{-qx} - Cos.\,x}{x}\,dx = -lq$ Arndt, Gr. 11. 70.

13) $\int \frac{e^{-x} - Cos.\,q\,x}{x}\,dx = lq$

14) $\int \frac{e^{-qx} - Cos.\,q\,x}{x}\,dx = 0$

} Malmsten, Cr. 38. 1.

15) $\int \frac{e^{-x} - e^{-px}\,Cos.\,q\,x}{x}\,dx = \frac{1}{2} l (p^2+q^2)$ Cauchy, Exerc. 1826. p. 95. — Malmsten, Cr. 38. 1.

16) $\int \frac{e^{-x} - e^{-\frac{x}{p}}\,Cos.\,x}{x}\,dx = \frac{1}{2} l \frac{1+p^2}{p^2}$

17) $\int \frac{e^{-px} - e^{-qx}\,Cos.\,r\,x}{x}\,dx = \frac{1}{2} l \frac{q^2+r^2}{p^2}$

} Schlömilch, Höh. Anal. 74.

18) $\int \frac{e^{-px}\,Sin.\,q\,x - e^{-rx}\,Sin.\,s\,x}{x}\,dx = Arctang.\frac{qr-ps}{pr-qs}$ Lindmann, Stockh. Handl. 1850. IV.

19) $\int \frac{e^{-px}\,Cos.\,q\,x - e^{-rx}\,Cos.\,s\,x}{x}\,dx = \frac{1}{2} l \frac{r^2+s^2}{p^2+q^2}$ Bidone, Mém. Turin. 1812. 231. Art. 3. 35. — Lindmann, Stockh. Handl. 1850. IV.

F. Alg. rat. fract. à dén. x^a.
Exp. en num.
Circ. Dir.

TABLE 394. Lim. 0 et ∞.

1) $\int e^{-ax}\,Sin.\,b\,x\frac{dx}{x^2} = \infty$ Plana, Mém. Brux. 1837.

2) $\qquad = b - a\,Arctang.\frac{b}{a} - b\left\{A + \frac{1}{2} l (a^2+b^2) + lo\right\}$

3) $\int e^{-ax}\,Cos.\,b\,x\frac{dx}{x^2} = a\left\{A + \frac{1}{2} l (a^2+b^2) + lo\right\} - b\,Arctang.\frac{b}{a} + \frac{1}{o}$

} Bidone, Mém. Turin. 1812. 231. Art. 3. 35.

4) $\int e^{-qx}\,Sin.^2\frac{1}{2}px\frac{dx}{x^2} = \frac{1}{2}p\,Arctang.\frac{p}{q} - \frac{1}{4}q\,l\frac{p^2+q^2}{q^2}$ Lindmann, Gr. 16. 94.

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5) $\int e^{-\frac{1}{x^2}} Sin.(2p^2x^2)\frac{dx}{x^2} = \frac{1}{2}e^{-2p} Sin.2p.\sqrt{\pi}$

6) $\int e^{-\frac{1}{x^2}} Cos.(2p^2x^2)\frac{dx}{x^2} = \frac{1}{2}e^{-2p} Cos.2p.\sqrt{\pi}$

7) $\int e^{-px^2-\frac{q^2}{x^2}} Sin.(rx^2)\frac{dx}{x^2} = \frac{\sqrt{\pi}}{2q}e^{-2aq} Cos.\varphi.\, Sin.2bq$

8) $\int e^{-px^2-\frac{q^2}{x^2}} Cos.(rx^2)\frac{dx}{x^2} = \frac{\sqrt{\pi}}{2q}e^{-2aq} Cos.\varphi.\, Cos.2bq$

, $a = \sqrt{\frac{p+\sqrt{(p^2+r^2)}}{2}}$, $b = \sqrt{\frac{-p+\sqrt{(p^2+r^2)}}{2}}$, $Tang.\varphi = \frac{r}{p}$; Helmling, Transf. 30*, 31*, 39, 40.

9) $\int e^{-cx} Sin.bx\frac{dx}{x^{a+1}} = p\frac{b\,Cos.\left(Arctang.\frac{b}{c}\right) - c\,Sin.\left(Arctang.\frac{b}{c}\right)}{a(1-a)\sqrt{(b^2+c^2)^{1-a}}}$, $p = 0{,}906402$; Laplace, P. 15. 229.

10) $= \frac{(c-bi)^a-(c+bi)^a}{2i}\Gamma(-a)$ (val. extr.) Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826. p. 58.

11) $= (b^2+c^2)^{\frac{1}{2}a} Sin.\left(a\,Arctang.\frac{b}{c}\right)\Gamma(-a)$ Cauchy, Exerc. 1826. p. 58.

12) $= 1^{-a-1/1}(b^2+c^2)^{\frac{1}{2}a} Sin.\left(a\,Arctang.\frac{b}{c}\right)$

13) $\int e^{-cx} Cos.bx\frac{dx}{x^{a+1}} = 1^{-a-1/1}(b^2+c^2)^{\frac{1}{2}a} Cos.\left(a\,Arctang.\frac{b}{c}\right)$

Oettinger, Cr. 38. 216. où 12) était fautive.

14) $= \frac{(c-bi)^a+(c+bi)^a}{2}\Gamma(-a)$ (val. extr.) Cauchy, P. 28. 147. P. III. Suppl. — Id., Exerc. 1826. p. 58.

15) $= (b^2+c^2)^{\frac{1}{2}a} Cos.\left(a\,Arctang.\frac{b}{c}\right)\Gamma(-a)$ Cauchy, Exerc. 1826.

16) $\int e^{-cx} Sin.bx\frac{dx}{x^a} = p\frac{Sin.\left(Arctang.\frac{b}{c}\right)}{(1-a)(b^2+c^2)^{\frac{1-a}{2}}}$

17) $\int e^{-cx} Cos.bx\frac{dx}{x^a} = p\frac{Cos.\left(Arctang.\frac{b}{c}\right)}{(1-a)(b^2+c^2)^{\frac{1-a}{2}}}$

, $p = 0{,}906402$; Laplace, P. 15. 229.

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18) $\int \frac{Cos. bx - Cos. cx}{x^2} e^{-ax} dx = \frac{1}{2} a l \frac{a^2+b^2}{a^2+c^2} + c\, Arctang. \frac{c}{a} - b\, Arctang. \frac{b}{a}$

19) $\int \frac{e^{-cx} - e^{-bx}}{x^2} Sin. ax\, dx = \frac{1}{2} a l \frac{a^2+b^2}{a^2+c^2} + b\, Arctang. \frac{a}{b} - c\, Arctang. \frac{a}{c}$

Bidone, Mém. Turin. 1812. 231. Art. 3. 35.

20) $\int e^{-x^2} \frac{2x\, Cos. x - Sin. x}{x^2} Sin. x\, dx = \frac{e-1}{2e} \sqrt{\pi}$ Dienger, Cr. 46. 119.

21) $\int e^{-cx} \frac{Cos. bx + i\, Sin. bx}{x^a} dx = \frac{p}{(1-a)(c-bi)^{1-a}}$, $p = 0{,}906402$; Laplace, P. 15. 229.

22) $\int \frac{e^{-px} Sin. qx - e^{-rx} Sin. sx}{x^{t+1}} dx = \frac{\Gamma(1-t)}{t} \left\{ (p^2+q^2)^{\frac{1}{2}t} Sin. \left(t\, Arctg. \frac{q}{p} \right) - (r^2+s^2)^{\frac{1}{2}t} Sin. \left(t\, Arctg. \frac{s}{r} \right) \right\}$

23) $\int \frac{e^{-px} Cos. qx - e^{-rx} Cos. sx}{x^{t+1}} dx = \frac{\Gamma(1-t)}{t} \left\{ (r^2+s^2)^{\frac{1}{2}t} Cos. \left(t\, Arctg. \frac{s}{r} \right) - (p^2+q^2)^{\frac{1}{2}t} Cos. \left(t\, Arctg. \frac{q}{p} \right) \right\}$

Lindmann, Stockh. Handl. 1850. IV.

F. Alg. rat. fract. à dén. $x^2 + a^2$.
Exp. en num. TABLE 395. Lim. 0 et ∞.
Circ. Dir.

1) $\int e^{p\, Cos. bx} Sin. (p\, Sin. bx) \frac{x}{x^2+q^2} dx = \frac{1}{2}\pi \left(e^{pe^{-bq}} - 1 \right)$

2) $\int e^{p\, Cos. bx} Cos. (p\, Sin. bx) \frac{dx}{x^2+q^2} = \frac{1}{2q} \pi e^{pe^{-bq}}$

Cauchy, Lim. Imag. Add. N°. 25 où pour 1) faut. $\frac{\pi}{2} e^{pe^{-bq}}$. — Id., Exerc. d. Math. I. 95. — Boncompagni, Cr. 25. 74.

3) $\int e^{Cos. bx} Sin. \left(\frac{1}{2} a\pi - Sin. bx \right) \frac{x^{a-1}}{x^2+q^2} dx = \frac{\pi}{2q} q^{a-1} e^{e^{-bq}}$ Cauchy, Lim. Imag. Add. N°. 26.

4) $\int \frac{e^{-p\, Sin\, bx} - e^{p\, Sin. bx}}{q^2+x^2} x\, Sin. (p\, Cos. bx)\, dx = \pi \left\{ Cos. (p e^{-bq}) - 1 \right\}$

5) $\int \frac{e^{-p\, Sin. bx} + e^{p\, Sin. bx}}{q^2+x^2} Sin. (p\, Cos. bx)\, dx = \frac{\pi}{q} Sin. (p e^{-bq})$

6) $\int \frac{e^{-p\, Sin. bx} - e^{p\, Sin. bx}}{q^2+x^2} x\, Cos. (p\, Cos\, bx)\, dx = -\pi \left\{ Sin. (pe^{-bq}) - pe^{-bq} \right\}$

7) $\int \frac{e^{-p\, Sin. bx} + e^{p\, Sin. bx}}{q^2+x^2} Cos. (p\, Cos. bx)\, dx = \frac{\pi}{q} Cos. (p e^{-bq})$

Boncompagni, Cr. 25. 74. où pour 6) fautivement $= -\pi\, Sin. (p e^{-bq})$

1) $\int \frac{Sin.\,p\,x}{e^{\pi x}+e^{-\pi x}}\frac{dx}{x} = Arctang.(e^{\frac{1}{2}p})$ V. T. 281. N°. 4.

2) $\int \frac{Cos.\,p\,x}{e^{\pi x}-e^{-\pi x}}\frac{dx}{x} = -\frac{1}{2}\,l(e^{\frac{1}{2}p}+e^{-\frac{1}{2}p})$ V. T. 281. N°. 8.

3) $\int \frac{e^{\pi x}-e^{-\pi x}}{e^{\pi x}+e^{-\pi x}}\frac{Cos.\,p\,x}{x}dx = l\frac{1+e^{-\frac{1}{2}p}}{1-e^{-\frac{1}{2}p}}$ V. T. 282. N°. 1.

4) $\int \frac{e^{\pi x}+e^{-\pi x}}{e^{\pi x}-e^{-\pi x}}\frac{Cos.\,p\,x}{x}dx = -\,l(e^{\frac{1}{2}p}-e^{-\frac{1}{2}p})$ V. T. 282. N°. 8.

5) $\int \frac{Sin.\,q\,x}{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}}\frac{x}{1+x^2}dx = -\frac{\pi}{2\sqrt{2}}e^{-q}-\frac{e^q-e^{-q}}{4\sqrt{2}}\,l\frac{e^q+\sqrt{2}+e^{-q}}{e^q-\sqrt{2}+e^{-q}}+\frac{e^q+e^{-q}}{2\sqrt{2}}Arctg.\left(\frac{\sqrt{2}}{e^q-e^{-q}}\right)$ V.T.396. N°. 16.

6) $\int \frac{Sin.\,q\,x}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}}\frac{dx}{1+x^2} = -\frac{e^{-q}}{2\sqrt{2}}+\frac{e^q-e^{-q}}{4\sqrt{2}}\,l\frac{e^q+\sqrt{2}+e^{-q}}{e^q-\sqrt{2}+e^{-q}}+\frac{e^q+e^{-q}}{2\sqrt{2}}Arctg.\left(\frac{\sqrt{2}}{e^q-e^{-q}}\right)$ Schlömilch, Beitr. II. § 7.

7) $\int \frac{Sin.\,q\,x}{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}}\frac{x}{1+x^2}dx = -\frac{1}{2}q\,e^{-q}+\frac{e^q-e^{-q}}{4}\,l(1+e^{-2q})$ V. T. 396. N°. 18.

8) $\int \frac{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}}{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}}\frac{Sin.\,q\,x}{1+x^2}dx = q\,e^{-q}-\frac{e^q-e^{-q}}{2}\,l(1-e^{-2q})$ V. T. 396. N°. 17.

9) $\int \frac{Sin.\,q\,x}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}}\frac{dx}{1+x^2} = \frac{e^q+e^{-q}}{2}Arctang.(e^{-q})-\frac{1}{4}\pi\,e^{-q}$

10) $\int \frac{e^{\frac{1}{2}\pi x}-1}{e^{\frac{1}{2}\pi x}+1}\frac{Sin.\,q\,x}{1+x^2}dx = -\frac{1}{2}\pi\,e^q+\frac{e^q-e^{-q}}{2}\,l\frac{e^q+1}{e^q-1}+(e^q+e^{-q})\,Arctang.(e^q)$

11) $\int \frac{e^{\frac{1}{2}\pi x}+1}{e^{\frac{1}{2}\pi x}-1}\frac{Sin.\,q\,x}{1+x^2}dx = -\frac{1}{2}\pi\,e^{-q}+\frac{e^q-e^{-q}}{2}\,l\frac{e^q+1}{e^q-1}+(e^q+e^{-q})\,Arctang.(e^{-q})$

(9–11) Schlömilch, Beitr. II. § 7. où 11) était fautive.

12) $\int \frac{Sin.\,q\,x}{e^{\pi x}-e^{-\pi x}}\frac{dx}{1+x^2} = -\frac{1}{4}q\,e^{-q}+\frac{e^q-e^{-q}}{4}\,l(1+e^{-q})$ V. T. 396. N°. 17.

13) $\int \frac{e^{\pi x}+e^{-\pi x}}{e^{\pi x}-e^{-\pi x}}\frac{Sin.\,q\,x}{1+x^2}dx = \frac{1}{2}q\,e^{-q}-\frac{e^q-e^{-q}}{2}\,l(1-e^{-q})$ V. T. 396. N°. 17.

14) $\int \frac{e^{\pi x}+1}{e^{\pi x}-1}\frac{Sin.\,q\,x}{1+x^2}dx = \frac{e^q-e^{-q}}{2}\,l\frac{e^q+1}{e^q-1}$ Schlömilch, Beitr. II. § 7.

15) $\int \frac{e^{px}+e^{-px}}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}}\frac{Sin.\,q\,x}{1+x^2}dx = -\frac{1}{2}\pi\,e^{-q}\,Cos.\,p+\frac{e^q-e^{-q}}{4}Sin.\,p.\,l\frac{e^q+2\,Sin.\,p+e^{-q}}{e^q-2\,Sin.\,p+e^{-q}}+$

$+\frac{e^q+e^{-q}}{2}Cos.\,p.\,Arctang.\left(\frac{2\,Cos.\,p}{e^q-e^{-q}}\right),\ p^2\leqq\frac{1}{4}\pi^2;$ Schlömilch, Beitr. II. § 7. — Id., Stud II. 19.

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16) $\int \frac{e^{px}-e^{-px}}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}} \frac{x\,Sin.\,q\,x}{1+x^2}\,dx = -\frac{1}{2}\pi e^{-q}\,Sin.\,p - \frac{e^q-e^{-q}}{4}\,Cos.\,p.\,l\frac{e^q+2\,Sin.\,p+e^{-q}}{e^q-2\,Sin.\,p+e^{-q}} +$

$+ \frac{e^q+e^{-q}}{2}\,Sin.\,p.\,Arctang.\left(\frac{2\,Cos.\,p}{e^q-e^{-q}}\right), p^2 < \frac{1}{4}\pi^2$; V. T. 396. N°. 30.

17) $\int \frac{e^{px}+e^{-px}}{e^{\pi x}-e^{-\pi x}} \frac{Sin.\,q\,x}{1+x^2}\,dx = -\frac{1}{2}e^{-q}(q\,Cos.\,p+p\,Sin.\,p) + \frac{e^q-e^{-q}}{4}\,Cos.\,p.\,l(1+2e^{-q}Cos.\,p+e^{-2q}) +$

$+ \frac{e^q+e^{-q}}{2}\,Sin.\,p.\,Arctang.\left(\frac{Sin.\,p}{e^q+Cos.\,p}\right), p^2 \leqq \pi^2$; Schlömilch, Beitr. II. § 8. — Id., Stud. II. 19.

18) $\int \frac{e^{px}-e^{-px}}{e^{\pi x}-e^{-\pi x}} \frac{x\,Sin.\,q\,x}{1+x^2}\,dx = \frac{1}{2}e^{-q}(p\,Cos.\,p-q\,Sin.\,p) + \frac{e^q-e^{-q}}{4}\,Sin.\,p.\,l(1+2e^{-q}Cos.\,p+e^{-2q}) -$

$- \frac{e^q+e^{-q}}{2}\,Cos.\,p.\,Arctang.\left(\frac{Sin.\,p}{e^q+Cos.\,p}\right), p^2 < \pi^2$; V. T. 396. N°. 32.

19) $\int \frac{Cos.\,q\,x}{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}} \frac{dx}{1+x^2} = \frac{\pi}{2\sqrt{2}}e^{-q} - \frac{e^q+e^{-q}}{4\sqrt{2}}\,l\frac{e^q+e^{-q}+\sqrt{2}}{e^q+e^{-q}-\sqrt{2}} + \frac{e^q-e^{-q}}{2\sqrt{2}}\,Arctg.\left(\frac{\sqrt{2}}{e^q-e^{-q}}\right)$ V. T. 396. N°. 30.

20) $\int \frac{Cos.\,q\,x}{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}} \frac{dx}{1+x^2} = \frac{1}{2}q\,e^{-q} + \frac{e^q+e^{-q}}{4}\,l(1+e^{-2q})$ V. T. 396. N°. 32.

21) $\int \frac{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}}{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}} \frac{x\,Cos.\,q\,x}{1+x^2}\,dx = -q\,e^{-q} - \frac{e^q-e^{-q}}{2}\,l(1-e^{-2q})$ V. T. 396. N°. 31.

22) $\int \frac{Cos.\,q\,x}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}} \frac{x}{1+x^2}\,dx = \frac{e^q-e^{-q}}{4}\,Arctang.\,(e^{-q}) + \frac{1}{4}\pi e^{-q} - \frac{1}{2}$ V. T. 396. N°. 29.

23) $\int \frac{e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x}}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}} \frac{x\,Cos.\,q\,x}{1+x^2}\,dx = -1 + \frac{e^q+e^{-q}}{2}\,l\frac{1+e^{-q}}{1-e^{-q}}$ V. T. 396. N°. 31.

24) $\int \frac{e^{\frac{1}{2}\pi x}-1}{e^{\frac{1}{2}\pi x}+1} \frac{x\,Cos.\,q\,x}{1+x^2}\,dx = -\frac{1}{2}\pi e^{q} + \frac{e^q+e^{-q}}{2}\,l\frac{e^q+1}{e^q-1} + \frac{e^q-e^{-q}}{2}\,Arctg.\,(e^q)$ V. T. 396. N°. 29.

25) $\int \frac{e^{\frac{1}{2}\pi x}+1}{e^{\frac{1}{2}\pi x}-1} \frac{x\,Cos.\,q\,x}{1+x^2}\,dx = -2 + \frac{1}{2}\pi e^{-q} + \frac{e^q+e^{-q}}{2}\,l\frac{e^q+1}{e^q-1} + \frac{e^q-e^{-q}}{2}\,Arctg.\,(e^{-q})$ V. T. 396. N°. 29.

26) $\int \frac{Cos.\,q\,x}{e^{\pi x}-e^{-\pi x}} \frac{x}{1+x^2}\,dx = -\frac{1}{4} + \frac{1}{4}q\,e^{-q} + \frac{e^q+e^{-q}}{4}\,l(1+e^{-q})$ V. T. 396. N°. 31.

27) $\int \frac{e^{\pi x}+e^{-\pi x}}{e^{\pi x}-e^{-\pi x}} \frac{x\,Cos.\,q\,x}{1+x^2}\,dx = -\frac{1}{2} - \frac{1}{2}q\,e^{-q} - \frac{e^q+e^{-q}}{2}\,l(1-e^{-q})$ V. T. 396. N°. 31.

28) $\displaystyle\int \frac{e^{\pi x}+1}{e^{\pi x}-1}\,\frac{x\, Cos.\, q x}{1+x^2}\, dx = -1 + \frac{e^q + e^{-q}}{2}\, l\, \frac{e^q + 1}{e^q - 1}$ V. T. 396. N°. 29.

29) $\displaystyle\int \frac{e^{px}+e^{-px}}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}}\,\frac{x\, Cos.\, q x}{1+x^2}\, dx = -1 + \frac{1}{2}\pi e^{-q} Cos. p + \frac{e^q + e^{-q}}{4} Sin. p.\, l\, \frac{e^q + 2\, Sin.\, p + e^{-q}}{e^q - 2\, Sin\, p + e^{-q}} +$

$\displaystyle + \frac{e^q - e^{-q}}{4} Cos. p.\, Arctang. \left(\frac{2\, Cos. p}{e^q - e^{-q}}\right),\ p^2 \leqq \frac{1}{4}\pi^2$; Schlömilch, Beitr. II. § 7.

30) $\displaystyle\int \frac{e^{px}-e^{-px}}{e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x}}\,\frac{Cos.\, q x}{1+x^2}\, dx = \frac{1}{2}\pi e^{-q} Sin. p - \frac{e^q + e^{-q}}{4} Cos. p.\, l\, \frac{e^q + 2\, Sin. p + e^{-q}}{e^q - 2\, Sin. p + e^{-q}} +$

$\displaystyle + \frac{e^q - e^{-q}}{2} Sin. p.\, Arctang. \left(\frac{2\, Cos. p}{e^q - e^{-q}}\right),\ p^2 < \frac{1}{4}\pi^2$; Schlömilch, Beitr. II. § 9. — Id., Stud. II. 19.

31) $\displaystyle\int \frac{e^{px}+e^{-px}}{e^{\pi x}-e^{-\pi x}}\,\frac{x\, Cos. q x}{1+x^2}\, dx = \frac{1}{2} e^{-q}(q Cos. p + p\, Sin. p) - \frac{1}{2} + \frac{e^q + e^{-q}}{4} Cos. p.\, l(1 + 2 e^{-q} Cos. p + e^{-2q}) +$

$\displaystyle + \frac{e^q - e^{-q}}{2} Sin. p.\, Arctg. \left(\frac{Sin. p}{e^q + Cos. p}\right),\ p^2 \leqq \pi^2$; V. T. 396 N°. 17.

32) $\displaystyle\int \frac{e^{px}-e^{-px}}{e^{\pi x}-e^{-\pi x}}\,\frac{Cos. q x}{1+x^2}\, dx = \frac{1}{2} e^{-q}(q\, Sin. p - p\, Cos. p) + \frac{e^q + e^{-q}}{4} Sin. p.\, l(1 + 2 e^{-q} Cos. p + e^{-2q}) -$

$\displaystyle - \frac{e^q - e^{-q}}{2} Cos. p.\, Arctang. \left(\frac{Sin. p}{e^q + Cos. p}\right),\ p^2 < \pi^2$; Schlömilch, Beitr. II. 9. — Id., Stud. II. 19.

33) $\displaystyle\int \frac{e^{px}+e^{-px}}{e^{\pi x}-e^{-\pi x}}\,\frac{Sin.\, q x}{r^2+x^2}\, dx = \frac{1}{2r^2} - \frac{\pi e^{-qr} Cos. pr}{2r Sin. r\pi} + \sum_{1}^{\infty}(-1)^{n-1} \frac{e^{-nq} Cos. np}{n^2 - r^2},\ \pi \geqq p \geqq 0$; Schlömilch, Beitr. II. 7.

34) $\displaystyle\int \frac{e^{px}-e^{-px}}{e^{\pi x}-e^{-\pi x}}\,\frac{Cos.\, q x}{r^2+x^2}\, dx = \frac{\pi e^{-qr} Sin. p r}{2 r\, Sin. r \pi} + \sum_{1}^{\infty}(-1)^{n} \frac{e^{-nq} Sin. n p}{n^2 - r^2},\ \pi > p > 0$; Schlömilch, Beitr. II. 9.

35) $\displaystyle\int \frac{e^{\pi x}+e^{-\pi x}}{e^{\pi x}-e^{-\pi x}}\,\frac{Cos. q x}{x}\,\frac{dx}{1+x^2} = \frac{1}{2}\,\frac{(1-q+qe^{-q})}{1-e^{-q}} + \frac{1}{2}(e^{\frac{1}{2}q} - e^{-\frac{1}{2}q})^2\, l(1-e^{-q})$ V. T. 396. N°. 4, 27.

36) $\displaystyle\int \frac{Sin.\, x}{e^{qx} + 2\, Cos.\, x + e^{-qx}}\,\frac{x}{x^2 - \pi^2}\, dx = \frac{1}{2} Arctang. \left(\frac{1}{q}\right) - \frac{1}{2q}$

37) $\displaystyle\int \frac{Sin.\, x}{e^{qx} - 2\, Cos.\, x + e^{-qx}}\,\frac{x}{x^2 - \pi^2}\, dx = \frac{1}{2}\,\frac{q}{1+q^2} - \frac{1}{2} Arctang. \left(\frac{1}{q}\right)$

(36, 37) Schlömilch, Beitr. III. § 3.

38) $\displaystyle\int \frac{Cos.\, q x - e^{-qx}}{x}\,\frac{dx}{x^4 + r^4} = \frac{\pi}{2 r^4} e^{-\frac{1}{2}qr\sqrt{2}} Sin. \left(\frac{1}{2} q r \sqrt{2}\right)$ Cauchy, Exerc. 1826. p. 95.

1) $\int e^{-x} Sin.\, x.\, dx \sqrt{x} = \sqrt{\frac{\pi}{2\sqrt{2}}}.\, Sin.\, \frac{3\pi}{8}$

2) $\int e^{-x} Cos.\, x.\, dx \sqrt{x} = \sqrt{\frac{\pi}{2\sqrt{2}}}.\, Cos.\, \frac{3\pi}{8}$

Fuss, Mém. Pétersb. 1830.

3) $\int e^{-qx} Sin.\, q\,x.\, dx \sqrt{x} = \frac{1}{4q}\sqrt{(1+\sqrt{2})}.\sqrt{\frac{\pi}{2q}}$

4) $\int e^{-qx} Sin.\, q\,x.\, x\, dx \sqrt{x} = \frac{3}{16q^2}\sqrt{(1+\sqrt{2})}.\sqrt{\frac{\pi}{q}}$

5) $\int e^{-qx} Sin\; q\,x.\, x^2\, dx \sqrt{x} = \frac{15}{32q^3}\sqrt{(-1+\sqrt{2})}.\sqrt{\frac{\pi}{2q}}$

6) $\int e^{-qx} Cos.\, q\,x.\, dx \sqrt{x} = \frac{1}{4q}\sqrt{(-1+\sqrt{2})}.\sqrt{\frac{\pi}{2q}}$

7) $\int e^{-qx} Cos.\, q\,x.\, x\, dx \sqrt{x} = \frac{3}{16q}\sqrt{(-1+\sqrt{2})}.\sqrt{\frac{\pi}{q}}$

8) $\int e^{-qx} Cos.\, q\,x.\, x^2\, dx \sqrt{x} = \frac{15}{32q^3}\sqrt{(1+\sqrt{2})}.\sqrt{\frac{\pi}{2q}}$

9) $\int e^{-qx} Sin.\, p\,x.\, dx \sqrt{x} = \frac{1}{4}\sqrt{\{-q^3+3qp^2+\sqrt{(q^2+p^2)^3}\}}.\sqrt{\frac{2\pi}{(q^2+p^2)^3}}$

10) $\int e^{-qx} Sin.\, p\,x.\, x\, dx \sqrt{x} = \frac{3}{8}\sqrt{\{-q^5+10q^3p^2-5qp^4+\sqrt{(q^2+p^2)^5}\}}.\sqrt{\frac{2\pi}{(q^2+p^2)^5}}$

11) $\int e^{-qx} Sin.\, px.\, x^2\, dx \sqrt{x} = \frac{15}{16}\sqrt{\{-q^7+21q^5p^2-35q^3p^4+7qp^6+\sqrt{(q^2+p^2)^7}\}}.\sqrt{\frac{2\pi}{(q^2+p^2)^7}}$

12) $\int e^{-qx} Cos.\, p\,x.\, dx \sqrt{x} = \frac{1}{4}\sqrt{\{q^3-3qp^2+\sqrt{(q^2+p^2)^3}\}}.\sqrt{\frac{2\pi}{(q^2+p^2)^3}}$

13) $\int e^{-qx} Cos.\, p\,x.\, x\, dx \sqrt{x} = \frac{3}{8}\sqrt{\{q^5-10q^3p^2+5qp^4+\sqrt{(q^2+p^2)^5}\}}.\sqrt{\frac{2\pi}{(q^2+p^2)^5}}$

14) $\int e^{-qx} Cos.\, px.\, x^2 px \sqrt{x} = \frac{15}{16}\sqrt{\{q^7-21q^5p^2+35q^3p^4-7qp^6+\sqrt{(q^2+p^2)^7}\}}.\sqrt{\frac{2\pi}{(q^2+p^2)^7}}$

Sur ces intégrales 3) à 14) voyez Oettinger, Cr. 38. 216. où 12) et 14) étaient fautives.

15) $\int e^{-px^a} x^{\frac{1}{2}a-1}\, Cos.\, (2q\sqrt{px})\, dx = \frac{1}{a} e^{-q^2}\sqrt{\frac{\pi}{p}}$ Boncompagni, Cr. 25. 74. ne vaut que pour $a=1$.

1) $\int e^{-qx} \operatorname{Sin.} qx \frac{dx}{\sqrt{x}} = \frac{1}{2}\sqrt{(-1+\sqrt{2})}.\sqrt{\frac{\pi}{q}}$

2) $\int e^{-qx} \operatorname{Cos.} qx \frac{dx}{\sqrt{x}} = \frac{1}{2}\sqrt{(1+\sqrt{2})}.\sqrt{\frac{\pi}{q}}$

Oettinger, Cr. 38. 216.

3) $\int e^{-x} \operatorname{Sin.} x \frac{dx}{\sqrt{x}} = \sqrt{\left(\frac{\sqrt{2}-1}{2}\right)}.\sqrt{\frac{\pi}{2}}$

4) $\int e^{-x} \operatorname{Cos.} x \frac{dx}{\sqrt{x}} = \sqrt{\left(\frac{\sqrt{2}+1}{2}\right)}.\sqrt{\frac{\pi}{2}}$

Fuss, Mém. Pétersb. 1830.

5) $\int e^{-qx} \operatorname{Sin.} px \frac{dx}{\sqrt{x}} = \sqrt{\frac{\sqrt{(p^2+q^2)}-q}{p^2+q^2}}.\sqrt{\frac{\pi}{2}}$

6) $\int e^{-qx} \operatorname{Cos.} px \frac{dx}{\sqrt{x}} = \sqrt{\frac{\sqrt{(p^2+q^2)}+q}{p^2+q^2}}.\sqrt{\frac{\pi}{2}}$

Euler, Calc. Int. IV. S. 5. § 136. — Oettinger, Cr. 38. 216.

7) $\int e^{-x} \operatorname{Cos.}(2\sqrt{qx}) \frac{dx}{\sqrt{x}} = e^{-q}\sqrt{\pi}$ Kummer, Cr. 17. 210.

8) $\int e^{-px} \operatorname{Cos.}(2q\sqrt{px}) \frac{dx}{\sqrt{x}} = e^{-q^2}\sqrt{\frac{\pi}{p}}$ Boncompagni, Cr. 25. 74.

9) $\int e^{-px+qx\operatorname{Cos.}\lambda} \operatorname{Sin.}(qx \operatorname{Sin.}\lambda) \frac{dx}{\sqrt{x}} = -\operatorname{Sin.}\frac{1}{2}\beta.\sqrt{\frac{\pi}{r}}$

10) $\int e^{-px+qx\operatorname{Cos.}\lambda} \operatorname{Cos.}(qx \operatorname{Sin.}\lambda) \frac{dx}{\sqrt{x}} = \operatorname{Cos.}\frac{1}{2}\beta.\sqrt{\frac{\pi}{r}}$

Dienger, Cr. 46. 119, où $r^2 = p^2 + q^2 - 2pq \operatorname{Cos.}\lambda$, $\operatorname{Cos.}\beta = \frac{p - q\operatorname{Cos.}\lambda}{r}$, $\operatorname{Sin.}\beta = \frac{-q\operatorname{Sin.}\lambda}{r}$, $p > q > 0$;

11) $\int e^{-a^2\left(x+\frac{1}{x}\right)} \operatorname{Sin.} bx \frac{dx}{\sqrt{x}} = e^{-2qa}\{q\operatorname{Sin.} 2pa + p\operatorname{Cos.} 2pa\}\sqrt{\frac{\pi}{b^2+a^4}}$

12) $\int e^{-a^2\left(x+\frac{1}{x}\right)} \operatorname{Cos.} bx \frac{dx}{\sqrt{x}} = \frac{1}{b} e^{-2qa}\{q\operatorname{Cos.} 2pa - p\operatorname{Sin.} 2pa\}\sqrt{\frac{\pi}{b^2+a^4}}$

Des intégrales 11) et 12) voyez Cauchy, P. 28. 147. P. I. § 4. où 12) était fautive, et où l'on a:

$$2p = \sqrt{\{\sqrt{(b^2+a^4)}+a\}} - \sqrt{\{\sqrt{(b^2+a^4)}-a\}}$$

$$2q = \sqrt{\{\sqrt{(b^2+a^4)}+a\}} + \sqrt{\{\sqrt{(b^2+a^4)}-a\}}$$

1) $\int e^{-qx} Sin. qx \frac{dx}{x\sqrt{x}} = \sqrt{(-1+\sqrt{2})}.\sqrt{2q\pi}$

2) $\int e^{-qx} Sin. qx \frac{dx}{x^2\sqrt{x}} = \frac{4}{3} q\sqrt{(1+\sqrt{2})}.\sqrt{q\pi}.$

3) $\int e^{-qx} Sin. qx \frac{dx}{x^3\sqrt{x}} = \frac{8}{15} q^2\sqrt{(1+\sqrt{2})}.\sqrt{2q\pi}$

4) $\int e^{-qx} Sin. qx \frac{dx}{x^4\sqrt{x}} = \frac{32}{105} q^3\sqrt{(-1+\sqrt{2})}.\sqrt{q\pi}$

5) $\int e^{-qx} Cos. qx \frac{dx}{x\sqrt{x}} = -\sqrt{(1+\sqrt{2})}.\sqrt{2q\pi}$

6) $\int e^{-qx} Cos. qx \frac{dx}{x^2\sqrt{x}} = \frac{4}{3} q\sqrt{(-1+\sqrt{2})}.\sqrt{q\pi}$

7) $\int e^{-qx} Cos. qx \frac{dx}{x^3\sqrt{x}} = \frac{8}{15} q^2\sqrt{(-1+\sqrt{2})}.\sqrt{2q\pi}$

8) $\int e^{-qx} Cos. qx \frac{dx}{x^4\sqrt{x}} = \frac{32}{105} q^3\sqrt{(1+\sqrt{2})}.\sqrt{q\pi}$

9) $\int e^{-qx} Sin. px \frac{dx}{x\sqrt{x}} = -\sqrt{\{-q+\sqrt{(p^2+q^2)}\}}.\sqrt{2\pi}$

10) $\int e^{-qx} Sin. px \frac{dx}{x^2\sqrt{x}} = \frac{2}{3}\sqrt{\{-q^3+3p^2q+\sqrt{(p^2+q^2)^3}\}}.\sqrt{2\pi}$

11) $\int e^{-qx} Sin. px \frac{dx}{x^3\sqrt{x}} = -\frac{4}{15}\sqrt{\{-q^5+10p^2q^3-5p^4q+\sqrt{(p^2+q^2)^5}\}}.\sqrt{2\pi}$

12) $\int e^{-qx} Sin. px \frac{dx}{x^4\sqrt{x}} = \frac{8}{105}\sqrt{\{-q^7+21p^2q^5-35p^4q^3+7p^6q+\sqrt{(p^2+q^2)^7}\}}.\sqrt{2\pi}$

13) $\int e^{-qx} Cos. px \frac{dx}{x\sqrt{x}} = -\sqrt{\{q+\sqrt{(p^2+q^2)}\}}.\sqrt{2\pi}$

14) $\int e^{-qx} Cos. px \frac{dx}{x^2\sqrt{x}} = \frac{2}{3}\sqrt{\{q^3-3p^2q+\sqrt{(p^2+q^2)^3}\}}.\sqrt{2\pi}$

15) $\int e^{-qx} Cos. px \frac{dx}{x^3\sqrt{x}} = -\frac{4}{15}\sqrt{\{q^5-10p^2q^3+5p^4q+\sqrt{(p^2+q^2)^5}\}}.\sqrt{2\pi}$

16) $\int e^{-qx} Cos. px \frac{dx}{x^4\sqrt{x}} = \frac{8}{105}\sqrt{\{q^7-21p^2q^5+35p^4q^3-7p^6q+\sqrt{(p^2+q^2)^7}\}}.\sqrt{2\pi}$

Sur ces intégrales 1) à 16) voyez Oettinger, Cr. 38. 216.

Page 515.

F. Alg. irrat. fract. à autre dén.
Exp.
Circ. Dir.

TABLE 399 suite.

Lim. 0 et ∞.

17) $\int e^{-a\sqrt{(\frac{1}{2}x)}} \frac{\{b+\sqrt{(\frac{1}{2}x)}\} Cos.\{a\sqrt{(\frac{1}{2}x)}\} - \sqrt{(\frac{1}{2}x)}. Sin.\{a\sqrt{(\frac{1}{2}x)}\}}{x+b\sqrt{2x}+b^2}\, dx = 0$

18) $\int e^{-a\sqrt{(\frac{1}{2}x)}} \frac{\{b+\sqrt{(\frac{1}{2}x)}\} Cos.\{a\sqrt{(\frac{1}{2}x)}\} - \sqrt{(\frac{1}{2}x)}. Sin.\{a\sqrt{(\frac{1}{2}x)}\}}{x+b\sqrt{2x}+b^2} \frac{dx}{c^2-x^2} =$

$= \frac{\{b+\sqrt{(\frac{1}{2}c)}\} Sin.\{a\sqrt{(\frac{1}{2}c)}\} + \sqrt{(\frac{1}{2}c)}. Cos.\{a\sqrt{(\frac{1}{2}c)}\}}{c+b\sqrt{2c}+b^2} \cdot \frac{\pi e^{-a\sqrt{(\frac{1}{2}c)}}}{2c}$

Les intégrales 17) et 18) se trouvent: Poisson, Chal. 159.

F. Alg.
Exp.
Circ. Dir.

TABLE 400.

Lim. diverses.

1) $\int_{-\infty}^{\infty} e^{-px^2+2qx Cos.\lambda} x\, Sin.(2qx\, Sin.\lambda).dx = \frac{q\pi}{p} e^{\frac{q^2}{p} Cos\, 2\lambda}. Sin.\left(\lambda+\frac{q^2}{p} Sin.\, 2\lambda\right).\sqrt{\frac{\pi}{p}}$ } Dienger, Cr. 46. 119.

2) $\int_{-\infty}^{\infty} e^{-px^2+2qx Cos.\lambda} x\, Cos.(2qx\, Sin.\lambda).dx = \frac{q\pi}{p} e^{\frac{q^2}{p} Cos.2\lambda}. Cos.\left(\lambda+\frac{q^2}{p} Sin.\, 2\lambda\right).\sqrt{\frac{\pi}{p}}$ }

3) $\int_{-\infty}^{\infty} e^{-px^2} x\, Sin. qx. dx = \frac{q}{2p} e^{-\frac{q^2}{4p}} \sqrt{\frac{\pi}{p}}$ Ohm, Ausw. 21.

4) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{(a+i)xi} (2\, Cos.x)^{a-1} x\, dx = \frac{\pi i}{2a}$ } Kummer, Cr. 20. 1.

5) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-a Sin.\frac{1}{2}x.\sqrt{(2Cos.x)}} Cos.\left(a\, Cos.\frac{1}{2}x.\sqrt{(2Cos.x)}-\frac{1}{2}x\right)\frac{dx}{\sqrt{(2\,Cos.x)}} = \pi Cos.a$ }

6) $\int_0^{\frac{a}{c}} \left\{Cos.x - \frac{e^{bcx}+e^{-bcx}}{2}\right\} \frac{dx}{x} = l\, b\, c + Ci.\left(\frac{a}{c}\right) - \frac{1}{2} Ei.(ab) - \frac{1}{2} Ei.(-ab)$ Arndt, Gr. 11. 70.

F. Algébr.
Exp.
Circ. Inv.

TABLE 401.

Lim. 0 et ∞.

1) $\int e^{-px} Arctang.\frac{x}{q}.x\, dx = \frac{1}{p^2}\Big[Ci.(pq). Sin. pq - \left\{Si.(pq)-\frac{1}{2}\pi\right\} Cos. pq - pq\Big\{Ci.(pq). Cos. pq$

$+ \left(Si.(pq)-\frac{1}{2}\pi\right) Sin. pq\Big\}\Big]$

2) $$\int e^{-px} Arctang.\frac{x}{q}.x^{2a}dx = \frac{1}{p^{2a+1}}\Big[\Big\{Ci.(pq).Sin.pq - \Big(Si.(pq) - \frac{1}{2}\pi\Big)Cos.pq\Big\} 1^{2a/1}\sum_0^a \frac{(-p^2q^2)^n}{1^{2n/1}} -$$

$$- pq\Big\{Ci.(pq).Cos.pq + \Big(Si.(pq) - \frac{1}{2}\pi\Big)Sin.pq\Big\} 1^{2a/1}\sum_0^{a-1}\frac{(-p^2q^2)^n}{1^{2n+1/1}} +$$

$$+ 3^{2a-2/1}pq\sum_1^a\Big\{\frac{1}{1^{2n/1}}\sum_0^{n-1}1^{2n-2m\,1}(-p^2q^2)^m\Big\} + 4^{2a-3/1}pq\sum_1^a\Big\{\frac{1}{1^{2n/1}}\sum_0^{n-1}1^{2n-2m-1/1}(-p^2q^2)^m\Big\}\Big]$$

3) $$\int e^{-px} Arctang.\frac{x}{q}.x^{2a+1}dx = \frac{1}{p^{2a+2}}\Big[\Big\{Ci.(pq).Sin.pq - \Big(Si.(pq) - \frac{1}{2}\pi\Big)Cos.pq\Big\} 1^{2a+1/1}\sum_0^a \frac{(-p^2q^2)^n}{1^{2n/1}} -$$

$$- pq\Big\{Ci.(pq).Cos.pq + \Big(Si.(pq) - \frac{1}{2}\pi\Big)Sin.pq\Big\} 1^{2a+1/1}\sum_0^{a}\frac{(-p^2q^2)^n}{1^{2n+1/1}} +$$

$$+ 3^{2a-1/1}pq\sum_1^{a+1}\Big\{\frac{1}{1^{2n+1/1}}\sum_0^{n-1}1^{2n-2m+1/1}(-p^2q^2)^m\Big\} + 4^{2a-2/1}pq\sum_1^a\Big\{\frac{1}{1^{2n/1}}\sum_0^{n-1}1^{2n-2m/1}(-p^2q^2)^m\Big\}\Big]$$

4) $$\int e^{-px} Arctang.\frac{x}{q}\frac{px+pq+1}{(x+q)^2}dx = \frac{1}{2q}\Big[-e^{pq}Ei.(-pq) + Ci.(pq).Sin.pq -$$

$$- \Big(Si.(pq) - \frac{1}{2}\pi\Big)Cos.pq + Ci.(pq).Cos.pq + \Big(Si.(pq) - \frac{1}{2}\pi\Big)Sin.pq\Big]$$

5) $$\int e^{-px} Arctang.\frac{x}{q}\frac{px-pq+1}{(x-q)^2}dx = \frac{1}{2q}\Big[-e^{-pq}Ei.(pq) - Ci.(pq).Sin.pq +$$

$$+ \Big(Si.(pq) - \frac{1}{2}\pi\Big)Cos.pq + Ci.(pq).Cos.pq + \Big(Si.(pq) - \frac{1}{2}\pi\Big)Sin.pq\Big]$$

6) $$\int e^{-px} Arctang.\frac{x}{q}\frac{(pq+1)x+pq^2+2q}{(x+q)^2}x\,dx = \frac{1}{2p}\Big[pq\,e^{pq}Ei.(-pq) + (pq+2)\Big\{Ci.(pq).Sin.pq -$$

$$- \Big(Si.(pq) - \frac{1}{2}\pi\Big)Cos.pq\Big\} - pq\Big\{Ci.(pq).Cos.pq + \Big(Si.(pq) - \frac{1}{2}\pi\Big)Sin.pq\Big\}\Big]$$

7) $$\int e^{-px} Arctang.\frac{x}{q}\frac{(pq-1)x-pq^2+2q}{(x-q)^2}x\,dx = \frac{1}{2p}\Big[-pq\,e^{-pq}Ei.(pq) + (pq-2)\Big\{Ci.(pq).Sin.pq -$$

$$- \Big(Si.(pq) - \frac{1}{2}\pi\Big)Cos.pq\Big\} + pq\Big\{Ci.(pq).Cos.pq + \Big(Si.(pq) - \frac{1}{2}\pi\Big)Sin.pq\Big\}\Big]$$

8) $$\int e^{-px} Arctang.\frac{x}{q}\frac{p(q^2+x^2)Arctang.\frac{x}{q} - 2q}{x^2+q^2}dx = 0$$

9) $\int e^{-px} Arctang. \frac{x}{q} \frac{px^2+2x+pq^2}{(x^2+q^2)^2} dx = \frac{1}{2q^2}\Big[Ci.(pq).Sin.pq - \left(Si.(pq)-\frac{1}{2}\pi\right)Cos.pq + $
$+ pq\left\{Ci.(pq).Cos.pq + \left(Si.(pq)-\frac{1}{2}\pi\right)Sin.pq\right\}\Big]$

10) $\int e^{-px} Arctg. \frac{x}{q} \frac{px^3+x^2+pq^2x-q^2}{(x^2+q^2)^2} dx = \frac{1}{2q}\Big[1-pq\left\{Ci.(pq).Sin.pq - \left(Si.(pq)-\frac{1}{2}\pi\right)Cos.pq\right\}\Big]$

Sur ces intégrales 1) à 10) voyez Bierens de Haan, Verh. d. K. Ak. v. Wet. 1854. bl. 19.

11) $\int \frac{(2\pi x-1)e^{2\pi x}+1}{(e^{2\pi x}-1)^2} Arctang. \frac{x}{q} dx = -\frac{1}{4} + \frac{q}{2} l q - \frac{1}{2} q\, Z'(q)$ V. T. 138. N°. 9.

12) $\int \frac{(\pi x-1)e^{\pi x}+(\pi x+1)e^{-\pi x}}{(e^{\pi x}-e^{-\pi x})^2} Arctang. x\, dx = \frac{1}{2}\left(l2-\frac{1}{2}\right)$ V. T. 138. N°. 12.

13) $\int \frac{e^{-2\pi x}+2\pi x-1}{(e^{\pi x}-e^{-\pi x})^2} Arctang. x\, dx = \frac{1}{2}A - \frac{1}{4}$ V. T. 138. N°. 10.

14) $\int \frac{e^{-2qx}+2qx-1}{(e^{qx}-e^{-qx})^2} Arctang. x\, dx = \frac{1}{2} l \frac{q}{\pi} + \frac{\pi}{4q} - \frac{1}{2} Z'\left(\frac{\pi+q}{\pi}\right)$ V. T. 138. N°. 11.

15) $\int \frac{\pi x(e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x})-4(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})}{(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})^2} Arctang. x\, dx = \pi\sqrt{2}-4+\sqrt{2}\, l\frac{\sqrt{2}+1}{\sqrt{2}-1}$ V. T. 138. N°. 18.

16) $\int \frac{\pi x(e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x})-2(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})}{(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})^2} Arctang. x\, dx = \frac{1}{2}\pi - 1$ V. T. 138. N°. 16.

17) $\int \frac{(x^2-q^2)e^{-2\pi x}+2\pi x^3-x^2+2\pi q^2x+q^2}{(e^{2\pi x}-1)^2} Arctang. \frac{x}{q} \frac{dx}{(q^2+x^2)^2} = \frac{1}{4q^3}\sum_0^\infty \frac{B_{2n+1}}{q^{2n}}$ V. T. 138. N°. 20.

18) $\int e^{-(Arctang. x)^2} (Arctang. x)^{2a} \frac{dx}{1+x^2} = \left(\frac{1}{2}\pi\right)^{2a+1} \sum_0^\infty \frac{1}{1^{n/1}(2a+2n+1)}\left(-\frac{1}{4}\pi^2\right)^n$ Helmling, Transf.

F. Algébr.
Exp.
Autre Fonct.
TABLE 402.
Lim. 0 et ∞.

1) $\int e^{-x} li.(e^x). x^{p-1} dx = -\pi\, Cot. p\pi. \Gamma(p)$
2) $\int e^{x} li.(e^{-x}). x^{p-1} dx = -\pi\, Cosec. p\pi. \Gamma(p)$

, $0 \leqq p \leqq 1$; Schlömilch, Beitr. III. § 6, 7.

3) $\int li.(e^{-x}).x^{p-1}\,dx = -\frac{1}{p}\Gamma(p)$

4) $\int e^{-qx}\, li.(e^{-x})\frac{dx}{\sqrt{x}} = -2\sqrt{\frac{\pi}{q}}\, l\{\sqrt{q}+\sqrt{(1+q)}\}$

$, \; 0 \leqq p \leqq 1 \;, \; 0 < q < 1;$

Schlömilch, Beitr. III. § 6, 7. où 4) fautive.

5) $\int e^{qx}\, li.(e^{-x})\frac{dx}{\sqrt{x}} = -2\, Arcsin.(\sqrt{q}).\sqrt{\frac{\pi}{q}}\;,\; q < 1;$ V. T. 300. N°. 5.

F. Alg. rat. ent.
Log.
Circ. Dir. de Log.

TABLE 403.

Lim. 0 et 1.

1) $\int Sin.(p\, l x).x^{q-1}\,dx = \frac{-p}{p^2+q^2}$ V. T. 278. N°. 8.

2) $\int Cos\,(p\, l x).x^{q-1}\,dx = \frac{q}{p^2+q^2}$ V. T. 278. N°. 9.

3) $\int Sin.(q\, l x).(l x)^{a-1}.x^{p-1}\,dx = (-1)^a \frac{1^{a-1/1}}{(p^2+q^2)^{\frac{1}{2}a}} Sin.\left(a\, Arctang.\frac{q}{p}\right)$ V. T. 386. N°. 12

4) $\int Cos.(q\, l x).(l x)^{a-1}.x^{p-1}\,dx = (-1)^{a-1} \frac{1^{a-1/1}}{(p^2+q^2)^{\frac{1}{2}a}} Cos.\left(a\, Arctang.\frac{q}{p}\right)$ V. T. 386. N°. 13.

5) $\int Sin.^{2a}(l x).x^{p-1}\,dx = \frac{1^{2a/1}}{p^2+(2a)^2.p^2+(2a-2)^2\ldots p^2+2^2}\frac{1}{p}$ V. T. 279. N°. 1.

6) $\int Sin.^{2a+1}(l x).x^{p-1}\,dx = \frac{-1^{2a+1/1}}{p^2+(2a+1)^2.p^2+(2a-1)^2\ldots p^2+1^2}$ V. T. 279. N°. 2.

7) $\int Cos.^{2a}(l x).x^{p-1}\,dx = \frac{1}{p}\frac{1^{2a/1}}{p^2+(2a)^2.p^2+(2a-2)^2\ldots p^2+2^2}\left\{1+\frac{p^2}{1.2}+\frac{p^2}{1,2}\frac{p^2+2^2}{3.4}+\right.$

$\left.+\ldots+\frac{p^2.p^2+2^2\ldots p^2+(2a-2)^2}{1^{2a/1}}\right\}$ V. T. 279. N°. 3.

8) $\int Cos.^{2a+1}(l x).x^{p-1}\,dx = \frac{1}{p}\frac{1^{2a+1/1}}{p^2+(2a+1)^2.p^2+(2a-1)^2\ldots p^2+1^2}\left\{\frac{p^2}{1}+\frac{p^2}{1}\frac{p^2+1^2}{2.3}+\right.$

$\left.+\ldots+\frac{p^2.p^2+1^2\ldots p^2+(2a-1)^2}{1^{2a+1/1}}\right\}$ V. T. 279. N°. 4.

9) $\int Sin.(q\, l x).\, l l\frac{1}{x}.x^{p-1}\,dx = \frac{1}{p^2+q^2}\left\{-p Arctang.\frac{q}{p}+\frac{1}{2}q\, l(p^2+q^2)+q\,A\right\}$ V. T. 439. N°. 2.

10) $\int Cos.(q\,l\,x).\,l l\frac{1}{x}.\,x^{p-1}\,d\,x = \frac{1}{p^2+q^2}\left\{q\,Arctang.\frac{q}{p}+\frac{1}{2}p\,l\,(p^2+q^2)+p\,A\right\}$ V. T. 439. N°. 3.

11) $\int Cot.(q\,l\,x).\,x^{p-1}\,d\,x = 4\,q\sum_1^\infty\frac{n}{p^2+4\,n^2\,q^2}$ V. T. 278. N°. 12.

12) $\int Sin.\{(q l x)^2\}.x^{2p-1}dx = \frac{1}{4q}\left\{Cos.\left(\frac{p^2}{q^2}\right)+Sin.\left(\frac{p^2}{q^2}\right)\right\}\sqrt{2\pi}-\frac{p}{q^2}\left\{Cos.\left(\frac{p^2}{q^2}\right).\sum_0^\infty\frac{(-1)^n}{(4n+1)\,1^{2n/1}}\left(\frac{p}{q}\right)^{4n}+\right.$

$\left.+\,Sin.\left(\frac{p^2}{q^2}\right).\sum_0^\infty\frac{(-1)^n}{(4\,n-1)\,1^{2n-1/1}}\left(\frac{p}{q}\right)^{4n-2}\right\}$ V. T. 279. N°. 18.

13) $\int Cos.\{(q l x)^2\}.x^{2p-1}dx = \frac{1}{4q}\left\{Cos.\left(\frac{p^2}{q^2}\right)-Sin.\left(\frac{p^2}{q^2}\right)\right\}\sqrt{2\pi}-\frac{p}{q^2}\left\{Sin.\left(\frac{p^2}{q^2}\right).\sum_0^\infty\frac{(-1)^n}{(4n+1)\,1^{2n/1}}\left(\frac{p}{q}\right)^{4n}-\right.$

$\left.-\,Cos.\left(\frac{p^2}{q^2}\right).\sum_0^\infty\frac{(-1)^n}{(4\,n-1)\,1^{2n-1/1}}\left(\frac{p}{q}\right)^{4n-2}\right\}$ V. T. 279. N°. 19.

14) $\int Sin.\{p^2-(lx)^2\}.\,x^{2p-1}dx = -\frac{1}{2}\sqrt{\frac{\pi}{2}}-p^3\sum_0^\infty\left\{\frac{p^2\,Cos.(2p^2)}{2n(4n+1)}+\frac{Sin.(2p^2)}{4n-1}\right\}\frac{(-p^4)^{n-1}}{1^{2n-1/1}}$ V. T. 403. N°. 12, 13.

15) $\int Cos.\{p^2-(lx)^2\}.\,x^{2p-1}dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}-\frac{1}{p}\sum_0^\infty\left\{\frac{p^2\,Sin.(2p^2)}{2n(4n+1)}-\frac{Cos.(2p^2)}{4n-1}\right\}\frac{(-p^4)^n}{1^{2n-1/1}}$ V. T. 403. N°. 12, 13.

16) $\int Sin.^a(l\,x).\,x^{b-1}\,d\,x = \frac{(-1)^a-e^{b\pi}}{\Gamma\left(\frac{a+b\,i}{2}+1\right)\Gamma\left(\frac{a-b\,i}{2}+1\right)}\frac{\pi}{2\,a}1^{a/1}\,e^{b\pi}$ V. T. 280. N°. 21.

17) $\int Cos.\left(q\sqrt{l\frac{1}{x}}\right).\,x^{p-1}\,d\,x = \frac{1}{p}\sum_0^\infty(-1)^n\frac{1}{(n+1)^{n/1}}\left(\frac{q^2}{p}\right)^n$ V. T. 388. N°. 13.

18) $\int Sin.(Sin.\lambda.\,l\,x).\,(l\,x)^{p-1}.\,x^{Cos.\lambda-1}\,d\,x = (-1)^p\,Sin.\,p\,\lambda.\,\Gamma(p)$ V. T. 390. N°. 1.

19) $\int Cos.(Sin.\lambda.\,l\,x).\,(l\,x)^{p-1}.\,x^{Cos.\lambda-1}\,d\,x = (-1)^{p-1}\,Cos\,p\,\lambda.\,\Gamma(p)$ V. T. 390. N°. 2.

20) $\int l\,Sin.\left(q\,l\frac{1}{x}\right).\,x^{p-1}\,d\,x = \frac{1}{2\,p}\,l\frac{1}{2}-\frac{p}{2}\sum_1^\infty\frac{1}{n}\frac{1}{p^2+n^2\,q^2}$ V. T. 439. N°. 6.

21) $\int l\,Cos.(q\,l\,x).\,x^{p-1}\,d\,x = -\frac{1}{2\,p}\,l\,2+\frac{p}{2}\sum_1^\infty\frac{(-1)^{n-1}}{n}\frac{1}{p^2+n^2\,q^2}$ V. T. 439. N°. 7.

22) $\int l\,Tang.\left(q\,l\frac{1}{x}\right).\,x^{p-1}\,d\,x = -\,p\sum_1^\infty\frac{1}{2\,n-1}\frac{1}{p^2+(2\,n-1)^2\,q^2}$ V. T. 439. N°. 8.

1) $\int Sin.(p\,l\,x)\frac{dx}{1+x} = \frac{\pi e^{p\pi}}{e^{2p\pi}-1} - \frac{1}{2p}$ V. T. 404. N°. 10, 11.

2) $\int Sin.(p\,l\,x)\frac{dx}{1-x} = -\frac{1}{2}\pi\frac{e^{2p\pi}+1}{e^{2p\pi}-1} + \frac{1}{2p}$ V. T. 281. N°. 9.

3) $\int Cos.(p\,l\,x)\frac{dx}{1-x} = \infty$ V. T. 281. N°. 8.

4) $\int Sin.(p\,l\,x)\frac{x^{a-1}dx}{1-x} = -\frac{1}{2}\pi + \frac{1}{2p} + \frac{\pi}{1-e^{2p\pi}} + \sum_{0}^{a-2}\frac{p}{p^2+(n+1)^2}$ V. T. 281. N°. 11.

5) $\int Sin.(p\,l\,x)\frac{x^{q-1}dx}{1-x} = \varphi - \frac{1}{2p}Sin.\,\varphi - \sum_{1}^{\infty}\frac{Sin.^{2n}\varphi.Sin.2n\varphi}{2np^{2n}}B_{2n-1}$, où $Cot.\,\varphi = \frac{q-1}{p}$; V. T. 282. N°. 15.

6) $\int Sin.(p\,l\,x)\frac{l\,x}{1+x^2}dx = \frac{1}{4}\pi^2\frac{e^{\frac{1}{2}p\pi}-e^{-\frac{1}{2}p\pi}}{(e^{\frac{1}{2}p\pi}+e^{-\frac{1}{2}p\pi})^2}$ V. T. 391. N°. 5.

7) $\int Cos.(p\,l\,x)\frac{dx}{1+x^2} = \frac{1}{2}\pi\frac{e^{\frac{1}{2}p\pi}}{e^{p\pi}+1}$ V. T. 281. N°. 4.

8) $\int Sin.(p\,l\,x)\frac{x^q-x^{-q}}{1+x^2}dx = \pi\,Sin.\frac{1}{2}\pi q\frac{e^{\frac{1}{2}p\pi}-e^{-\frac{1}{2}p\pi}}{e^{p\pi}+e^{-p\pi}+2\,Cos.\,q\pi}$, $p^2<1$, $q^2<1$; V. T. 282. N°. 6.

9) $\int Cos.(p\,l\,x)\frac{x^q+x^{-q}}{1+x^2}dx = \pi\,Cos.\frac{1}{2}\pi q\frac{e^{\frac{1}{2}p\pi}+e^{-\frac{1}{2}p\pi}}{e^{p\pi}+e^{-p\pi}+2\,Cos.\,q\pi}$, $p^2<1$, $q^2<1$; V. T. 282. N°. 4.

10) $\int Sin.(p\,l\,x)\frac{dx}{1-x^2} = \frac{1}{4}\pi\frac{1-e^{p\pi}}{1+e^{p\pi}}$ V. T. 281. N°. 8.

11) $\int Sin.(p\,l\,x)\frac{x}{1-x^2}dx = \frac{1}{4}\pi\frac{1+e^{p\pi}}{1-e^{p\pi}} + \frac{1}{2p}$ V. T. 281. N°. 9.

12) $\int Sin.(p\,l\,x)\frac{x^{q-1}}{1-x^2}dx = -\sum_{1}^{\infty}\frac{p}{(2n+q)^2+p^2}$ V. T. 282. N°. 13.

13) $\int Sin.(p\,l\,x)\frac{x^q+x^{-q}}{1-x^2}dx = -\frac{\pi}{2}\frac{e^{p\pi}-e^{-p\pi}}{e^{p\pi}+e^{-p\pi}+2\,Cos.\,q\pi}$, $q^2 \leqq 1$; V. T. 282. N°. 10.

14) $\int Cos.(p\,l\,x)\frac{l\,x}{1+x^2}dx = -\frac{1}{2}\pi^2\frac{e^{p\pi}}{(e^{p\pi}+1)^2}$ V. T. 391. N°. 6.

15) $\int Cos.(p\,l\,x)\frac{x^q-x^{-q}}{1-x^2}dx = -\frac{\pi}{2}\frac{Sin.\,\pi q}{e^{p\pi}+e^{-p\pi}+2\,Cos.\,q\pi}$ V. T. 282. N°. 16.

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16) $\int Sin.(p\,l\,x)\frac{dx}{(1-x^2)\,x^{q+1}} = \frac{\pi}{\text{[illegible]}}\sum_1^{\infty}\frac{p}{(2n-q)^2+p^2}$ V. T. 282. N°. 14.

17) $\int Sin.^2(p\,l\,x)\frac{dx}{1+x^2} = \frac{\pi}{8}\frac{(e^{p\pi}-1)^2}{e^{2p\pi}+1}$ V. T. 281. N°. 12.

18) $\int Cos.^2(p\,l\,x)\frac{dx}{1+x^2} = \frac{\pi}{8}\frac{(e^{p\pi}+1)^2}{e^{2p\pi}+1}$ V. T. 281. N°. 13.

19) $\int Cos.(p\,l\,x).\,l(1+x)\frac{dx}{x} = \frac{1}{2p^2} - \frac{\pi}{p}\frac{e^{p\pi}}{e^{2p\pi}-1}$ V. T. 404 N°. 1.

20) $\int Cos.(p\,l\,x).\,l(1-x)\frac{dx}{x} = \frac{1}{2p^2} - \frac{\pi}{2p}\frac{e^{2p\pi}+1}{e^{2p\pi}-1}$ V. T. 404. N°. 2.

21) $\int Cos.(p\,l\,x).\,l(1-x^2)\frac{dx}{x} = \frac{1}{p^2} + \frac{\pi}{2p}\frac{1+e^{p\pi}}{1-e^{p\pi}}$ V. T. 404. N°. 4.

F. Alg. fract. à autre dén,
Log.
Circ. Dir. de Log.

TABLE 405.

Lim. 0 et 1.

1) $\int Sin.(p\,l\,x)\frac{1-x}{1+x}\frac{dx}{x} = \frac{-2\pi}{e^{p\pi}-e^{-p\pi}}$ V. T. 282. N°. 2.

2) $\int Sin.(p\,l\,x)\frac{1+x}{1-x}\frac{dx}{x} = -\pi\frac{e^{2p\pi}+1}{e^{2p\pi}-1}$ V. T. 282. N°. 9.

3) $\int Cos.(p\,l\,x)\frac{1-x}{1+x}\frac{l\,x}{x}\,dx = 2\pi^2 e^{-p\pi}\frac{1+e^{-2p\pi}}{(1-e^{-2p\pi})^2}$ V. T. 391. N°. 4.

4) $\int Sin.(p\,l\,x)\frac{1+x^2}{1-x^2}\frac{dx}{x} = \frac{1}{2}\pi\frac{1+e^{p\pi}}{1-e^{p\pi}}$ V. T. 282. N°. 8.

5) $\int Cos.(p\,l\,x)\frac{1+x^2}{1-x^2}\frac{l\,x}{x}\,dx = \pi^2\frac{e^{p\pi}}{(1-e^{p\pi})^2}$ V. T. 391. N°. 3.

6) $\int\frac{Cos.(q\,l\,x)}{(1+x^p)^2}x^{p-1}\,dx = \frac{1}{p^2}\frac{\pi q}{e^{\frac{\pi q}{p}}-e^{-\frac{\pi q}{p}}}$ Euler, N. A. Petr. III. 3.

7) $\int\frac{Cos.(p\,l\,x)}{1+2x\,Cos.\lambda+x^2}\,dx = \frac{1}{2}\pi\,Cosec.\lambda\frac{e^{p\lambda}-e^{-p\lambda}}{e^{p\pi}-e^{-p\pi}}$ V. T. 284. N°. 1.

8) $\int Sin.(q\,l\,x)\frac{1-x^{2p}}{1+2x^{2p}Cos.(2q\,l\,x)+x^{4p}}x^{p-1}dx = -\frac{\pi}{4}\frac{q}{p^2+q^2}$ V. T. 284. N°. 6.

9) $\int Cos.(q\,l\,x)\frac{1+x^{2p}}{1+2x^{2p}Cos.(2q\,l\,x)+x^{4p}}x^{p-1}dx = \frac{\pi}{4}\frac{p}{p^2+q^2}$ V. T. 284. N°. 7.

10) $\int Cos.(q\,l\,x)\frac{l\,x}{x^p+2\,Cos.\lambda+x^{-p}}\frac{dx}{x} = \frac{\pi}{2p}Cosec.\lambda\frac{e^{\frac{q\lambda}{p}}-e^{-\frac{q\lambda}{p}}}{e^{\frac{q\pi}{p}}-e^{-\frac{q\pi}{p}}}, \lambda<\pi;$ Euler, N. A. Petr. 1787. 3. (où faut.) — Poisson, P. 18. 295. N°. 28.

11) $\int Cos.(p\,l\,x)\frac{1+x^2}{1+2x\,Cos.\lambda+x^2}\frac{dx}{x} = -\pi\,Cot.\lambda\frac{e^{p\lambda}-e^{-p\lambda}}{e^{p\pi}-e^{-p\pi}}$ V. T. 284. N°. 4.

12) $\int Sin.(p\,l\,x)\frac{1-x^2}{1+2x\,Cos.\lambda+x^2}\frac{dx}{x} = -\pi\frac{e^{p\lambda}+e^{-p\lambda}}{e^{p\pi}-e^{-p\pi}}$ V. T. 284. N°. 5.

13) $\int\frac{Cos.(q\,l\,x)}{x^p+\left(a+\frac{1}{a}\right)+x^{-p}}\frac{dx}{x} = \frac{2}{p}\frac{a\pi}{1-a^2}\frac{Sin.\left(\frac{q}{p}l\,a\right)}{e^{\frac{q\pi}{p}}-e^{-\frac{q\pi}{p}}}$ Euler, N. A. Petr. III. 3.

14) $\int Sin.(q\,l\,x)\frac{x^p-x^{1-p}}{1-x}\frac{dx}{x} = -\pi\frac{e^{2q\pi}-e^{-2q\pi}}{e^{2q\pi}-2\,Cos.2p\pi+e^{-2q\pi}}, p<1;$ V. T. 282. N°. 12.

15) $\int Cos.(p\,l\,x)\frac{dx}{(1+x)\sqrt{x}} = \frac{\pi}{e^{p\pi}+e^{-p\pi}}$ V. T. 281. N°. 6.

F. Alg. rat.
Log. en dén. $l\,x$.
Circ. Dir. de Log.

TABLE 406.

Lim. 0 et 1.

1) $\int Sin.(p\,l\,x)\frac{x^a}{l\,x}dx = Arctang.\left(\frac{p}{a+1}\right)$ Euler, N. C. Petr. 20. 59.

2) $\int Sin.\left(q\sqrt{l\frac{1}{x}}\right).x^{p-1}\frac{dx}{l\,x} = q\sqrt{\frac{\pi}{p}}.\sum_0^\infty\frac{(-1)^{n-1}}{(2n+1)1^{n/1}}\left(\frac{q^2}{4p}\right)^n$ V. T 392. N°. 15.

3) $\int Sin.(l\,x)\frac{1+x}{l\,x}x\,dx = \frac{1}{4}\pi$ Euler, N. C. Petr. 20. 59.

4) $\int\frac{x^{p-1}Sin.(r\,l\,x)-x^{q-1}Sin.(s\,l\,x)}{l\,x}dx = Arctang.\left(\frac{q\,r-p\,s}{p\,q-r\,s}\right)$ V. T. 393. N°. 18.

5) $\int Cos.(q\,l\,x).x^{p-1}\frac{dx}{l\,x} = \infty$ V. T. 392. N°. 4.

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6) $\int \frac{x^{p-1}\, Cos.(r\,l\,x) - x^{q-1}\, Cos.(s\,l\,x)}{l\,x}\, d\,x = \frac{1}{2}\, l\, \frac{p^2+r^2}{q^2+s^2}$ V. T. 393. N°. 19.

7) $\int Sin.^2 (q\,l\,x).x^{a-1} \frac{d\,x}{l\,x} = \frac{1}{4}\, l\, \frac{a^2}{a^2+4q^2}$ V. T. 392. N°. 10.

8) $\int Sin.(q\,l\,x).x^{a-1} \frac{d\,x}{(l\,x)^2} = -\infty$ V. T. 394. N°. 1.

9) $\int Cos.(q\,l\,x).x^{a-1} \frac{d\,x}{(l\,x)^2} = \infty$ V. T. 394. N°. 3.

10) $\int Sin.^2 (q\,l\,x).x^{p-1} \frac{d\,x}{(l\,x)^2} = \frac{1}{2}\, q\, Arctang.\frac{q}{p} - \frac{1}{4}\, p\, l\, \frac{p^2+q^2}{p^2}$ V. T. 394. N°. 4.

11) $\int Sin.(q\,l\,x).x^{b-1} \frac{d\,x}{(l\,x)^{a+1}} = (-1)^a (b^2+q^2)^{\frac{1}{2}a}\, \Gamma(-a).\, Sin.\left\{a\, Arctang.\frac{q}{b}\right\}$ V. T. 394. N°. 11.

12) $\int Cos.(q\,l\,x).x^{b-1} \frac{d\,x}{(l\,x)^{a+1}} = (-1)^{a+1} (b^2+q^2)^{\frac{1}{2}a}\, \Gamma(-a).\, Cos.\left\{a\, Arctang.\frac{q}{b}\right\}$ V. T. 394. N°. 15.

13) $\int \frac{x^{p-1}\, Sin.(r\,l\,x) - x^{q-1}\, Sin.(s\,l\,x)}{(l\,x)^{a+1}}\, d\,x =$

$= (-1)^{a+1} \frac{\Gamma(1-a)}{a} \left[(q^2+s^2)^{\frac{1}{2}a}\, Sin.\left(a\, Arctang.\frac{s}{q}\right) - (p^2+r^2)^{\frac{1}{2}a}\, Sin.\left(a\, Arctang.\frac{r}{p}\right)\right]$ V. T. 394. N°. 22.

14) $\int \frac{x^{p-1}\, Cos.(r\,l\,x) - x^{q-1}\, Cos.(s\,l\,x)}{(l\,x)^{a+1}}\, d\,x =$

$= (-1)^{a+1} \frac{\Gamma(1-a)}{a} \left[(q^2+s^2)^{\frac{1}{2}a}\, Cos.\left(a\, Arctang.\frac{s}{q}\right) - (p^2+r^2)^{\frac{1}{2}a}\, Cos.\left(a\, Arctang.\frac{r}{p}\right)\right]$ V. T. 394. N°. 23.

15) $\int \frac{Sin.(2\,p\,l\,x)}{l\,x} \frac{d\,x}{1+x^2} = Arctang.(e^{p\pi})$ V. T. 396. N°. 1.

16) $\int \frac{Cos.(2\,p\,l\,x)}{l\,x} \frac{d\,x}{1-x^2} = \frac{1}{2}\, l\,(e^{p\pi}+e^{-p\pi})$ V. T. 396. N°. 2.

17) $\int \frac{Cos.(2\,p\,l\,x)}{x\,l\,x} \frac{1-x^2}{1+x^2}\, d\,x = l\, \frac{1-e^{-p\pi}}{1+e^{-p\pi}}$ V. T. 396. N°. 3.

18) $\int \frac{Cos.(2\,p\,l\,x)}{x\,l\,x} \frac{1+x^2}{1-x^2}\, d\,x = l\,(e^{p\pi}-e^{-p\pi})$ V. T. 396. N°. 4.

F. Algébr. rat.
Log. en dén. $\sqrt{lx}$.
Circ. Dir. de Log.

TABLE 407.

Lim. 0 et 1.

1) $\int Sin.\left(\frac{2p^2}{lx}\right)\frac{dx}{\sqrt{l\frac{1}{x}}} = -e^{-2p}Sin.(2p).\sqrt{\pi}$ V. T. 280. N°. 17.

2) $\int Cos.\left(\frac{2p^2}{lx}\right)\frac{dx}{\sqrt{l\frac{1}{x}}} = e^{-2p}Cos.(2p).\sqrt{\pi}$ V. T. 280. N°. 18.

3) $\int Sin.(plx).x^{q-1}\frac{dx}{\sqrt{l\frac{1}{x}}} = -\sqrt{\left\{\frac{\pi}{2}\frac{-q+\sqrt{(p^2+q^2)}}{p^2+q^2}\right\}}$ V. T. 398. N°. 5.

4) $\int Cos.(plx).x^{q-1}\frac{dx}{\sqrt{l\frac{1}{x}}} = \sqrt{\left\{\frac{\pi}{2}\frac{q+\sqrt{(p^2+q^2)}}{p^2+q^2}\right\}}$ V. T. 398. N°. 6.

5) $\int Sin.\left(p\sqrt{l\frac{1}{x}}\right).x^{q-1}\frac{dx}{\sqrt{l\frac{1}{x}}} = \frac{2}{q}\sum_0^\infty\frac{(-1)^n}{(n+2)^{n+1/1}}\left(\frac{p}{q}\right)^{2n+1}$ V. T. 280. N°. 10.

6) $\int Cos.\left(p\sqrt{l\frac{1}{x}}\right).x^{q-1}\frac{dx}{\sqrt{l\frac{1}{x}}} = \frac{1}{q}e^{-\frac{p^2}{4q^2}}\sqrt{\pi}$ V. T. 280. N°. 4.

7) $\int Cot.\left(p\sqrt{l\frac{1}{x}}\right)\frac{dx}{\sqrt{l\frac{1}{x}}} = 2\sqrt{\pi}\sum_1^\infty e^{-n^2p^2}$ V. T. 280. N°. 22.

F. Alg. rat. fract.
Log. en dén. $q^2+(lx)^2$.
Circ. Dir. de Log.

TABLE 408.

Lim. 0 et 1.

1) $\int\frac{Sin.(2plx)}{\frac{1}{4}\pi^2+(lx)^2}\frac{dx}{1-x^2} = -\frac{e^{p\pi}+e^{-p\pi}}{\pi}Arctang.(e^{-p\pi})+\frac{1}{2}e^{-p\pi}$ V. T. 396. N°. 9.

2) $\int\frac{Sin.(plx)}{\pi^2+(lx)^2}\frac{dx}{1-x^2} = \frac{1}{4}pe^{-p\pi}-\frac{e^{p\pi}-e^{-p\pi}}{4\pi}l(1+e^{-p\pi})$ V. T. 396. N°. 12.

3) $\int\frac{Sin.(plx)}{\pi^2+(lx)^2}\frac{1+x^2}{1-x^2}dx = -\frac{1}{2}pe^{-p\pi}+\frac{e^{p\pi}-e^{-p\pi}}{2\pi}l(1-e^{-p\pi})$ V. T. 396. N°. 13.

4) $\int\frac{Sin.(plx)}{\pi^2+(lx)^2}\frac{x^q+x^{-q}}{1-x^2}dx = \frac{1}{2}e^{-p\pi}(p\,Cos.q\pi+q\,Sin.q\pi)-$

$-\frac{e^{p\pi}-e^{-p\pi}}{4\pi}Cos.q\pi.l(1+2e^{-p\pi}Cos.q\pi+e^{-2p\pi})-\frac{e^{p\pi}+e^{-p\pi}}{2\pi}Sin.q\pi.Arctg.\left(\frac{Sin.q\pi}{e^{p\pi}+Cos.q\pi}\right), q^2\leqq 1;$ V. T. 396. N°. 17.

5) $\int \frac{Sin.(p\,l\,x)}{\pi^2 + (l\,x)^2}\,\frac{x^q - x^{-q}}{1 - x^2}\, l\,x\,d\,x = \frac{1}{2}\pi\,(p\,Sin\;q\,\pi - q\;Cos.\,q\,\pi)\,e^{-p\pi} -$

$- \frac{e^{p\pi} - e^{-p\pi}}{4} Sin.q\pi.l(1 + 2e^{-p\pi}Cos.\,q\,\pi + e^{-2p\pi}) + \frac{e^{p\pi} + e^{-p\pi}}{2} Cos.q\pi.Arctg.\left(\frac{Sin.\,q\,\pi}{e^{p\pi} + Cos.\,q\pi}\right), p^2 < 1;$ V. T. 39 N°. 18.

6) $\int \frac{Sin.(p\,l\,x)}{r^2 + (lx)^2}\,\frac{x^q + x^{-q}}{1 - x^2}\,dx = -\frac{\pi}{2\,r^2} + \frac{\pi\,e^{-pr}\,Cos.\,q\,r}{2\,r\,Sin.\,r} + \pi \sum_1^\infty (-1)^n \frac{e^{-np\pi}\,Cos.\,n\,q\,\pi}{n^2\,\pi^2 - r^2}, 0 \leqq q \leqq 1;$ V. T. 39 N°. 33.

7) $\int \frac{Cos.(p\,l\,x)}{\pi^2 + (l\,x)^2}\,\frac{l\,x}{1 - x^2}\,d\,x = \frac{1}{4} - \frac{1}{4} p\,\pi\,e^{-p\pi} - \frac{e^{p\pi} + e^{-p\pi}}{4} l\,(1 + e^{-p\pi})$ V. T. 396. N°. 26.

8) $\int \frac{Cos.(p\,l\,x)}{\pi^2 + (lx)^2}\,\frac{x^q - x^{-q}}{1 - x^2}\,dx = \frac{1}{2} e^{-p\pi}(q Cos.q\pi - p Sin.q\pi) - \frac{e^{p\pi} + e^{-p\pi}}{4\,\pi} Sin.q\pi.l(1 + 2e^{-p\pi}Cos.q\pi + e^{-2p\pi}) +$

$+ \frac{e^{p\pi} - e^{-p\pi}}{2\,\pi}\;Cos.\,q\,\pi.\;Arctang.\left(\frac{Sin.\,q\,\pi}{e^{p\pi} + Cos.\,q\,\pi}\right), q^2 < 1;$ V. T. 396. N°. 32.

9) $\int \frac{Cos.(p\,l\,x)}{\pi^2 + (lx)^2}\,\frac{x^q + x^{-q}}{1 - x^2}\, l\,x\,d\,x = \frac{1}{2} - \frac{\pi}{2} e^{-p\pi}\,(p\;Cos.\,q\,\pi + q\,Sin.\,q\,\pi) -$

$- \frac{e^{p\pi} + e^{-p\pi}}{4}\;Cos.q\pi.l\left\{1 + 2e^{-p\pi}Cos.q\pi + e^{-2p\pi}\right\} - \frac{e^{p\pi} - e^{-p\pi}}{2} Sin.q\pi.Arctg.\left(\frac{Sin.\,q\,\pi}{e^{p\pi} + Cos.\,q\pi}\right), p^2 \leqq 1;$ V. T. 39 N°. 31.

10) $\int \frac{Cos.\,(p\,l\,x)}{r^2 + (l\,x)^2}\,\frac{x^q - x^{-q}}{1 - x^2}\,dx = -\frac{\pi\,e^{-pr}\,Sin.\,q\,r}{2\,r\,Sin.\,r} + \pi \sum_1^\infty (-1)^n \frac{e^{-np\pi}\;Sin.nq\pi}{r^2 - n^2\,\pi^2}, 0 < q < 1;$ V. T. 396. N°. 34.

11) $\int \frac{Sin.\,(2\,p\,l\,x)}{\frac{1}{4}\pi^2 + (l\,x)^2}\,\frac{1 - x}{1 + x}\,\frac{d\,x}{x} = e^{p\pi} + \frac{e^{p\pi} - e^{-p\pi}}{\pi}\, l\,\frac{e^{p\pi} - 1}{e^{p\pi} + 1} - 2\,\frac{e^{p\pi} + e^{-p\pi}}{\pi}\,Arctang.(e^{p\pi})$ V. T. 396. N°. 10.

12) $\int \frac{Sin.\,(2\,p\,l\,x)}{\frac{1}{4}\pi^2 + (l\,x)^2}\,\frac{1 + x}{1 - x}\,\frac{d\,x}{x} = e^{-p\pi} + \frac{e^{p\pi} - e^{-p\pi}}{\pi}\, l\,\frac{e^{p\pi} - 1}{e^{p\pi} + 1} - 2\,\frac{e^{p\pi} + e^{-p\pi}}{\pi}\,Arctang.\,(e^{-p\pi})$ V. T. 396. N°. 11.

13) $\int \frac{Sin.\,(p\,l\,x)}{\pi^2 + (l\,x)^2}\,\frac{1 + x}{1 - x}\,\frac{d\,x}{x} = \frac{e^{p\pi} - e^{-p\pi}}{2\,\pi}\, l\,\frac{e^{p\pi} - 1}{e^{p\pi} + 1}$ V. T. 396. N°. 14.

14) $\int \frac{Sin.\,(p\,l\,x)}{\pi^2 + (l\,x)^2}\,\frac{1 - x}{1 + x}\,\frac{d\,x}{x} = \frac{e^{p\pi} - e^{-p\pi}}{2\,\pi}\, l\,(1 - e^{-2p\pi}) - p\,e^{-p\pi}$ V. T. 396. N°. 8.

15) $\int \frac{Cos.\,(2\,p\,l\,x)}{\frac{1}{4}\pi^2 + (lx)^2}\,\frac{1 - x}{1 + x}\,\frac{lx}{x}\,d\,x = \frac{1}{2}\pi e^{-p\pi} + \frac{e^{p\pi} + e^{-p\pi}}{2}\, l\,\frac{e^{p\pi} - 1}{e^{p\pi} + 1} - (e^{p\pi} - e^{-p\pi})\,Arctg.(e^{p\pi})$ V. T. 396. N°. 24.

16) $\int \frac{Cos.\,(2\,p\,l\,x)}{\frac{1}{4}\pi^2 + (lx)^2}\,\frac{1 + x}{1 - x}\,\frac{lx}{x}\,dx = 2 - \frac{1}{2}\pi e^{-p\pi} + \frac{e^{p\pi} + e^{-p\pi}}{2}\, l\,\frac{e^{p\pi} - 1}{e^{p\pi} + 1} - (e^{p\pi} - e^{-p\pi})\,Arctg.(e^{-p\pi})$ V. T. 396. N°. 25.

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17) $\int \frac{Cos.(p\,l\,x)}{\pi^2 + (l\,x)^2} \frac{1+x}{1-x} \frac{l\,x}{x} dx = 1 + \frac{e^{p\pi}+e^{-p\pi}}{2} l \frac{e^{p\pi}-1}{e^{p\pi}+1}$ V. T. 396. N°. 28.

18) $\int \frac{Cos.(p\,l\,x)}{\pi^2 + (l\,x)^2} \frac{1+x^2}{1-x^2} \frac{l\,x}{x} dx = \frac{1}{2} + \frac{1}{2} p\,\pi\,e^{-p\pi} + \frac{e^{p\pi}+e^{-p\pi}}{2} l(1-e^{-p\pi})$ V. T. 396. N°. 27.

19) $\int \frac{Sin.(l\,x)}{x^a + 2\,Cos.(l\,x) + x^{-a}} \frac{l\,x}{\pi^2 - (l\,x)^2} \frac{dx}{x} = \frac{1}{2\,a} - \frac{1}{2} Arccot.\,a$ V. T. 396. N°. 36.

20) $\int \frac{Sin.(l\,x)}{x^a - 2\,Cos.(l\,x) + x^{-a}} \frac{l\,x}{\pi^2 - (l\,x)^2} \frac{dx}{x} = \frac{1}{2} Arccot.\,a - \frac{1}{2} \frac{a}{1+a^2}$ V. T. 396. N°. 37.

21) $\int \frac{Cos.(p\,l\,x)}{\pi^2 + (l\,x)^2} \frac{1+x^2}{1-x^2} \frac{dx}{x\,l\,x} = -\frac{1}{2\,\pi^2} \frac{1 - p\,\pi + p\,\pi\,e^{-p\pi}}{1-e^{-p\pi}} -$

$- \frac{(e^{\frac{1}{2}p\pi} - e^{-\frac{1}{2}p\pi})^2}{2\,\pi^2} l(1-e^{-p\pi})$ V. T. 396. N°. 33.

1) $\int \frac{Sin.(2\,p\,l\,x)}{\frac{1}{4}\pi^2 + (l\,x)^2} \frac{l\,x}{1+x} \frac{dx}{\sqrt{x}} = -\frac{\pi}{2\sqrt{2}} e^{-p\pi} - \frac{e^{p\pi}-e^{-p\pi}}{4\sqrt{2}} l \frac{e^{p\pi}+e^{-p\pi}+\sqrt{2}}{e^{p\pi}+e^{-p\pi}-\sqrt{2}} +$

$+ \frac{e^{p\pi}+e^{-p\pi}}{2\sqrt{2}} Arctang.\left(\frac{\sqrt{2}}{e^{p\pi}-e^{-p\pi}}\right)$ V. T. 396. N°. 5.

2) $\int \frac{Sin.(p\,l\,x)}{\pi^2 + (l\,x)^2} \frac{l\,x}{1+x} \frac{dx}{\sqrt{x}} = -\frac{1}{2} p\,\pi\,e^{-p\pi} + \frac{e^{p\pi}-e^{-p\pi}}{4} l(1+e^{-2p\pi})$ V. T. 396. N°. 7.

3) $\int \frac{Sin.(2\,p\,l\,x)}{\frac{1}{4}\pi^2 + (l\,x)^2} \frac{1}{1-x} \frac{dx}{\sqrt{x}} = \frac{e^{-p\pi}}{\pi\sqrt{2}} + \frac{e^{p\pi}-e^{-p\pi}}{2\,\pi\sqrt{2}} l \frac{e^{p\pi}-\sqrt{2}+e^{-p\pi}}{e^{p\pi}+\sqrt{2}+e^{-p\pi}} -$

$- \frac{e^{p\pi}+e^{-p\pi}}{\pi\sqrt{2}} Arctang.\left(\frac{\sqrt{2}}{e^{p\pi}-e^{-p\pi}}\right)$ V. T. 396. N°. 6.

4) $\int \frac{Sin.(p\,l\,x)}{\pi^2 + (l\,x)^2} \frac{1}{1-x} \frac{dx}{\sqrt{x}} = -\frac{e^{p\pi}+e^{-p\pi}}{2\,\pi} Arctang.(e^{-p\pi}) + \frac{1}{4} e^{-p\pi}$ V. T. 396. N°. 9.

5) $\int \frac{Sin.(p\,l\,x)}{\pi^2 + (l\,x)^2} \frac{x^q + x^{-q}}{1-x} \frac{dx}{\sqrt{x}} = \frac{1}{2} e^{-p\pi}\,Cos.\,q\,\pi + \frac{e^{p\pi}-e^{-p\pi}}{4\,\pi} Sin.\,q\,\pi.\,l \frac{e^{p\pi} - 2\,Sin.\,q\,\pi + e^{-p\pi}}{e^{p\pi} + 2\,Sin.\,q\,\pi + e^{-p\pi}} -$

$- \frac{e^{p\pi}+e^{-p\pi}}{2\,\pi} Cos.\,q\,\pi.\,Arctang.\left(\frac{2\,Cos.\,q\,\pi}{e^{p\pi}-e^{-p\pi}}\right), p^2 \leqq \frac{1}{4};$ V. T. 396. N°. 15.

6) $\int \frac{Sin.(plx)}{\pi^2+(lx)^2}\frac{x^q-x^{-q}}{1-x}\frac{lx}{\sqrt{x}}dx=\frac{1}{2}e^{-p\pi}Sin.q\pi+\frac{e^{p\pi}-e^{-p\pi}}{4\pi}Cos.q\pi.l\frac{1+2e^{-p\pi}Sin.q\pi+e^{-2p\pi}}{1-2e^{-p\pi}Sin.q\pi+e^{-2p\pi}}-$
$-\frac{e^{p\pi}+e^{-p\pi}}{2\pi}Sin.p\pi.Arctang.\left(\frac{2Cos.q\pi}{e^{p\pi}-e^{-p\pi}}\right), p^2<\frac{1}{4}$; V. T. 396. N°. 16.

7) $\int \frac{Cos.(2plx)}{\frac{1}{4}\pi^2+(lx)^2}\frac{1}{1+x}\frac{dx}{\sqrt{x}}=\frac{1}{2}e^{-p\pi}\sqrt{2}-\frac{e^{p\pi}+e^{-p\pi}}{2\pi\sqrt{2}}l\frac{1+e^{-p\pi}\sqrt{2}+e^{-2p\pi}}{1-e^{-p\pi}\sqrt{2}+e^{-2p\pi}}+$
$+\frac{e^{p\pi}-e^{-p\pi}}{\pi\sqrt{2}}Arctang.\left(\frac{\sqrt{2}}{e^{p\pi}-e^{-p\pi}}\right)$ V. T. 396. N°. 19.

8) $\int \frac{Cos.(plx)}{\pi^2+(lx)^2}\frac{1}{1+x}\frac{dx}{\sqrt{x}}=\frac{1}{2}pe^{-p\pi}+\frac{e^{p\pi}+e^{-p\pi}}{4\pi}l(1+e^{-2p\pi})$ V. T. 396. N°. 20.

9) $\int \frac{Cos.(plx)}{\pi^2+(lx)^2}\frac{1+\sqrt{x}}{1-\sqrt{x}}\frac{lx}{\sqrt{x}}dx=2-\frac{1}{2}\pi e^{-p\pi}+\frac{e^{-p\pi}+e^{-p\pi}}{2}l\frac{1-e^{-p\pi}}{1+e^{-p\pi}}+(e^{p\pi}-e^{-p\pi})Arctang.(e^{-p\pi})$ V. T. 396. N°. 25.

10) $\int \frac{Cos.(plx)}{\pi^2+(lx)^2}\frac{lx}{1-x}\frac{dx}{\sqrt{x}}=-\frac{e^{p\pi}-e^{-p\pi}}{2}Arctang.(e^{-p\pi})+\frac{1}{2}-\frac{1}{4}\pi e^{-p\pi}$ V. T. 396. N°. 22.

11) $\int \frac{Cos.(plx)}{\pi^2+(lx)^2}\frac{1-\sqrt{x}}{1+\sqrt{x}}\frac{lx}{\sqrt{x}}dx=\frac{1}{2}\pi e^{p\pi}+\frac{e^{p\pi}+e^{-p\pi}}{2}l\frac{1+e^{-p\pi}}{1-e^{-p\pi}}+(e^{p\pi}+e^{-p\pi})Arctang.(e^{p\pi})$ V. T. 396. N°. 24.

12) $\int \frac{Cos.(plx)}{\pi^2+(lx)^2}\frac{x^q-x^{-q}}{1-x}\frac{dx}{\sqrt{x}}=-e^{-p\pi}Sin.q\pi+\frac{e^{p\pi}+e^{-p\pi}}{2\pi}Cos.q\pi.l\frac{e^{p\pi}+2Sin.q\pi+e^{-p\pi}}{e^{p\pi}-2Sin.q\pi+e^{-p\pi}}-$
$-\frac{e^{p\pi}-e^{-p\pi}}{\pi}Sin.q\pi.Arctang.\left(\frac{2Cos.q\pi}{e^{p\pi}-e^{-p\pi}}\right), p^2<\frac{1}{4}$; V. T. 396 N°. 30.

13) $\int \frac{Cos.(plx)}{\pi^2+(lx)^2}\frac{x^q+x^{-q}}{1-x}\frac{lx}{\sqrt{x}}dx=-\frac{1}{2}\pi e^{-p\pi}Cos.q\pi+\frac{e^{p\pi}+e^{-p\pi}}{4}Sin.q\pi.l\frac{e^{p\pi}-2Sin.q\pi+e^{-p\pi}}{e^{p\pi}+2Sin.q\pi+e^{-p\pi}}-$
$-\frac{e^{p\pi}-e^{-p\pi}}{2}Cos.q\pi.Arctang.\left(\frac{2Cos.q\pi}{e^{p\pi}-e^{-p\pi}}\right), p^2\leqq\frac{1}{4}$; V. T. 396. N°. 29.

F. Alg. rat. ent.
Log. de TABLE 410. Lim. 0 et $\frac{\pi}{4}$.
Circ. Dir.

1) $\int l\,Sin.x.\,x^{p-1}dx=-\frac{1}{2p}\left(\frac{\pi}{4}\right)^p\left[l2-2+\sum_1^\infty\frac{4}{p+2m}\sum_1^\infty\frac{1}{(4n)^{2m}}\right]$ V. T. 233. N°. 19.

2) $\int l\,Tang.x\frac{x}{Sin.2x}dx=-\frac{1}{64}\pi^3$ V. T. 305. N°. 5.

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3) $\int (l\,Tang.\,x)^3 \frac{x}{Sin.\,2\,x}\,d\,x = -\frac{5}{512}\pi^5$ V. T. 305. N°. 8.

4) $\int (l\,Tang.\,x)^5 \frac{x}{Sin.\,2\,x}\,d\,x = -\frac{61}{3072}\pi^7$ V. T. 305. N°. 9.

5) $\int l\,(Tang.\,x)^a \frac{x}{Sin.\,2\,x}\,d\,x = \frac{1^{a/1}}{2\,(-1)^a} \sum_0^\infty \frac{(-1)^n}{(2\,n+1)^{a+2}}$ V. T. 305. N°. 10.

6) $\int (l\,Tang.\,x)^{2a-1} \frac{x}{Sin.\,2\,x}\,d\,x = \frac{1^{2a-1/1}}{2^{2a+3}}\pi^{2a+1}\,B_{2a+2}$ V. T. 305. N°. 11.

7) $\int Sin.(2\,p\,l\,Tang.\,x) \frac{x}{Sin.\,2\,x}\,d\,x = \frac{-\pi}{16\,p}\frac{(e^{p\pi}-1)^2}{e^{2p\pi}+1}$ V. T. 329. N°. 3.

8) $\int \frac{x}{\sqrt{l\,Cot.\,x}}\frac{d\,x}{Sin.\,2\,x} = \frac{1}{2}\sqrt{\pi}.\sum_0^\infty (-1)^n \frac{1}{\sqrt{(2\,n+1)^3}}$ V. T. 320. N°. 16.

9) $\int \frac{x}{\sqrt{(l\,Cot.\,x)^3}}\frac{d\,x}{Sin.\,2\,x} = \infty$ V. T. 328. N°. 1.

10) $\int \frac{x\,l\,Tang.\,x}{\{\pi^2+(l\,Tang.\,x)^2\}^2}\frac{d\,x}{Sin.\,2\,x} = \frac{\pi-3}{16\,\pi}$ V. T. 324. N°. 1.

11) $\int \frac{x\,l\,Tang.\,x}{\{\pi^2+(l\,Tang.^2\,x)^2\}^2}\frac{d\,x}{Sin.\,2\,x} = \frac{1}{64}(1-l\,2)$ V. T. 324. N°. 2.

12) $\int \frac{l\,Tang.\,x}{\{q^2+(l\,Tang.\,x)^2\}^2}\frac{x}{Sin.\,2\,x}\,d\,x = \frac{1}{16\,q}\left\{Z'\left(\frac{2\,q+3\,\pi}{4\,\pi}\right) - Z'\left(\frac{2\,q+\pi}{4\,\pi}\right) - \frac{\pi}{q}\right\}$ V. T. 324. N°. 3.

F. Alg. rat. ent.
Log. de
Circ. Dir.

TABLE 411. Lim. 0 et $\frac{\pi}{2}$.

1) $\int l\,Sin.\,x.\,x^{p-1}\,d\,x = -\frac{1}{p}\left(\frac{\pi}{2}\right)^p \left\{1 - \sum_1^\infty \frac{2}{p+2\,m}\sum_1^\infty \frac{1}{(4\,n^2)^m}\right\}$ V. T. 238. N°. 18.

2) $\int l\,Cos.\,x.\,x\,Tang.\,x\,d\,x = \infty$ V. T. 332. N°. 7.

3) $\int l\,(1-Cos.\,x).\,x^{p-1}\,d\,x = \frac{1}{2\,p}\left(\frac{\pi}{2}\right)^p \left[l\,2 + 2 - \sum_1^\infty \frac{4}{p+2\,m}\sum_1^\infty \frac{1}{(4\,n)^{2m}}\right]$ V. T. 238. N°. 19.

4) $\int l\,Sin.\,x \frac{x}{Tang.\,x}\,d\,x = -\frac{1}{4}\pi\left\{(l\,2)^2 + \frac{1}{12}\pi^2\right\}$ V. T. 332. N°. 1.

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5) $\int (l\,Tang.\,x)^a \frac{x}{Sin.\,2\,x}\,dx = \infty$ V. T. 333. N°. 14, 15.

6) $\int (l\,Cosec.\,x)^{a-\frac{1}{2}} \frac{Sin.^2\,x.\,l\,Cosec.\,x + (a+\frac{1}{2})\,Cos.^2 x}{Sin.\,x}\,x\,dx = \frac{3^{a/2}}{2^a}\sqrt{\pi}$ V. T. 349. N°. 2.

7) $\int (l\,Sec.\,x)^{a-\frac{1}{2}} \frac{Cos.^2\,x.\,l\,Cos.\,x + (a+\frac{1}{2})\,Sin.^2 x}{Cos.\,x}\,x\,dx = \infty$ V. T. 349. N°. 7.

8) $\int l\,Tang.\,x.\left\{1 - q^4\,Tang.^4\,x + \frac{4q^4\,x\,Sin.^3\,x}{Cos.^5\,x}\right\}\frac{dx}{(1-q^4\,Tang.^4\,x)^2} = \frac{1}{8}\pi\,l\left\{\frac{q^4}{(1+q)^2\,(1+q^2)}\right\}$ V. T. 241. N°. 6.

9) $\int\left\{\frac{1}{2}l(1-p^2 Sin.^2 x) - 1\right\}\frac{x\,Sin.\,2\,x}{\sqrt{(1-p^2\,Sin.\,2\,x)^3}}\,dx = \frac{1}{2\,p^2}\,l(1-p^2).\left\{\frac{\pi}{\sqrt{(1-p^2)}} - \mathrm{F}'(p)\right\}$ V. T. 348. N°. 12.

10) $\int\{3\,l\,(1-p^2\,Sin.^2\,x) - 2\}\frac{x\,Sin.\,2\,x}{\sqrt{(1-p^2\,Sin.^2\,x)^5}}\,dx =$

$= \frac{1}{p^2\,(1-p^2)}\left[2\,(2-p^2)\,\mathrm{F}'(p) - \{4 + l\,(1-p^2)\}\,\mathrm{E}'(p) + \frac{\pi}{1-p^2}\,l\,(1-p^2)\right]$ V. T. 348. N°. 18.

11) $\int\left\{\frac{p\,l\,(1+p\,Sin.^2\,x)}{1-p^2\,Sin.^2\,x} + \frac{2}{1+p\,Sin.^2\,x}\right\}\frac{x\,Sin.\,2\,x}{\sqrt{(1-p^2\,Sin.^2\,x)}}\,dx =$

$= \frac{1}{2\,p}\left\{2\,l\left(\frac{\sqrt{p}}{2\,(1+p)}\right).\,\mathrm{F}'(p) + \frac{\pi}{2}\mathrm{F}'\{\sqrt{(1-p^2)}\} + \frac{2\,\pi}{\sqrt{(1-p^2)}}\,l\,(1+p)\right\}$ V. T. 348. N°. 10.

12) $\int\left\{\frac{p\,l\,(1-p\,Sin.^2\,x)}{1-p^2\,Sin.^2\,x} - \frac{2}{1-p\,Sin.^2\,x}\right\}\frac{x\,Sin.\,2\,x}{\sqrt{(1-p^2\,Sin.^2\,x)}}\,dx =$

$= \frac{1}{2\,p}\left\{2\,l\left(\frac{\sqrt{p}}{2\,(1-p)}\right).\,\mathrm{F}'(p) + \frac{\pi}{2}\mathrm{F}'\{\sqrt{(1-p^2)}\} + \frac{2\,\pi}{\sqrt{(1-p^2)}}\,l\,(1-p)\right\}$ V. T. 348. N°. 11.

13) $\int\left\{\frac{l\,(1-p^2\,Sin.^2\,\lambda.\,Sin.^2\,x)}{1-p^2\,Sin.^2\,x} - \frac{2\,Sin.^2\,\lambda}{1-p^2\,Sin.^2\,\lambda.\,Sin.^2\,x}\right\}\frac{x\,Sin.\,2\,x}{\sqrt{(1-p^2\,Sin.^2\,x)}}\,dx =$

$= \frac{1}{p^2}\left[4\,\mathrm{F}'(p)\,\Upsilon\,(p,\lambda) - 2\,\mathrm{E}'(p)\{\mathrm{F}\,(p,\lambda)\}^2 + \frac{\pi}{\sqrt{(1-p^2)}}\,l\,(1-p^2\,Sin.^2\,\lambda)\right]$ V. T. 348. N°. 17.

14) $\int\left\{\frac{l\,(1-p^2\,Sin.^4\,x)}{1-p^2\,Sin.^2\,x} - \frac{4\,Sin.^2\,x}{1-p^2\,Sin.^4\,x}\right\}\frac{x\,Sin.\,2\,x}{\sqrt{(1-p^2\,Sin.^2\,x)}}\,dx =$

$= \frac{1}{p^2}\left\{l\left(\frac{p}{2\,(1-p^2)}\right)\mathrm{F}'(p) + \frac{\pi}{2}\mathrm{F}'\{\sqrt{(1-p^2)}\} + \frac{\pi}{\sqrt{(1-p^2)}}\,l\,(1-p^2)\right\}$ V. T. 348. N°. 16.

15) $\int\left\{l\frac{1-q\sqrt{(1-p^2 Sin.^2 x)}}{1+q\sqrt{(1-p^2 Sin.^2 x)}}-\frac{2q\sqrt{(1-p^2 Sin.^2 x)}}{1-q^2+p^2q^2 Sin.^2 x}\right\}\frac{x\, Sin.2x}{\sqrt{(1-p^2 Sin.^2 x)^3}}dx =$

$= \frac{\pi}{p^2}F\{\sqrt{(1-p^2)}, Arcsin.q\}+\frac{\pi}{p^2\sqrt{(1-p^2)}}l\frac{1-q\sqrt{(1-p^2)}}{1+q\sqrt{(1-p^2)}}$ V. T. 348. N°. 22.

16) $\int\{1+p^2 Sin.^2 x(lSin.x-1)\}\frac{x\, Cot.x}{\sqrt{(1-p^2 Sin.^2 x)^3}}dx=\frac{1}{2}F'(p)lp+\frac{1}{4}\pi F\{\sqrt{(1-p^2)}\}$ V. T. 347. N°. 4.

17) $\int\frac{2 Sin.^2 x.l Cosec.x - Cos.^2 x}{(l Cosec.x)^{\frac{3}{2}}}\frac{x}{Sin.x}dx = 2\sqrt{\pi}-\pi$ V. T. 349. N°. 4.

18) $\int\frac{l Tang.x}{\{q^2+(l Tang.x)^2\}^2}\frac{x}{Sin.2x}dx = \frac{1}{8q}\left\{Z'\left(\frac{2q+3\pi}{4\pi}\right)-Z'\left(\frac{2q+\pi}{4\pi}\right)\right\}$ V. T. 352. N°. 15.

F. Alg. rat. fract. Log. de Circ. Dir. } Dén. $x^2+(l Cos.x)^2$. TABLE 412. Lim. 0 et $\frac{\pi}{2}$.

1) $\int\frac{x\, Tang.x}{x^2+(l Cos.x)^2}dx = \frac{\pi}{2l2}$ Poisson, Bull. de la S. Phil. Sept. 1822. — Id., P. 19. 404. N°. 76. — Cauchy, P. 19. 511. — Id., Exerc. 1826. p. 205.

2) $\int\frac{l Cos.x}{x^2+(l Cos.x)^2}dx = \frac{1}{2}\pi\left(1-\frac{1}{l2}\right)$ Poisson, P. 19. 404. N°. 76. — Cauchy, P. 19. 511. — Id., Exerc. 1826. p. 205.

3) $\int\frac{Cos.2ax.l Cos.x + x Sin.2ax}{x^2+(l Cos.x)^2}dx = \frac{1}{2}\pi$ Poisson, P. 19. 404. N°. 76.

4) $\int\frac{Cos.(b Tang.x).l Cos.x + x Sin.(b Tang.x)}{x^2+(l Cos.x)^2}dx = -\frac{1}{2}\pi\left(\frac{e^{-b}}{l2}-1\right)$ V. T. 446. N°. 17.

5) $\int\frac{Sin.(b Tang.x).l Cos.x - x Cos.(b Tang.x)}{x^2+(l Cos.x)^2}Tang.x\, dx = -\frac{\pi e^{-b}}{2l2}$ V. T. 446. N°. 18.

6) $\int\frac{l Cos.x}{x^2+(l Cos.x)^2}\frac{dx}{1+Cos.2x} = \infty$

7) $\int\frac{l Cos.2x}{x^2+(l Cos.x)^2}\frac{dx}{1-Cos.2x} = \frac{1}{4}\pi$

8) $\int\frac{Sin.2x}{x^2+(l Cos.x)^2}\frac{x}{1-Cos.2x}dx = \infty$

9) $\int\frac{l Cos.x}{x^2+(l Cos.x)^2}\frac{dx}{1-2p Cos.2x+p^2} = \frac{1}{2}\frac{\pi}{p^2-1}\left\{\frac{1}{l2-l(1+p)}-\frac{1+p}{1-p}\right\}, p^2 \leqq 1;$

} Poisson, P. 19. 404. N°. 76. (6–9)

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F. Alg. rat. fract. Log. de Circ. Dir. } Dén. $x^2 + (l\,Cos.\,x)^2$. **TABLE 412 suite.** Lim. 0 et $\frac{\pi}{2}$.

$$10)\int \frac{x\,Sin.\,2\,x}{x^2+(l\,Cos.\,x)^2}\,\frac{d\,x}{1-2\,p\,Cos.\,2\,x+p^2} = \frac{\pi}{4p}\left\{\frac{1}{l\,2-l\,(1+p)}-\frac{1}{l\,2}\right\},\ p^2 \leqq 1;$$

$$11)\int \frac{Cos.^k\,x}{x^2+(l\,Cos.\,x)^2}\,\frac{x\,Sin.\,k\,x+Cos.\,k\,x.\,l\,Cos.\,x}{1-2\,p\,Cos.\,2\,x+p^2}\,d\,x = \frac{\pi}{2\,(1-p)^2}$$

$$12)\int \frac{Cos.^k\,x.\,Sin.\,2\,x}{x^2+(l\,Cos.\,x)^2}\,\frac{Sin.\,k\,x.\,l\,Cos.\,x-x\,Cos.\,k\,x}{1-2\,p\,Cos.\,2\,x+p^2}\,d\,x = 0$$

(11, 12) $,\ p^2 < 1,\ k = \infty;$ (10–12) Poisson, P. 19. 404. N°. 76.

$$13)\int \frac{(l\,Cos.\,x)^2+2\,x\,Tang.\,x.\,l\,Cos.\,x-x^2}{\{x^2+(l\,Cos.\,x)^2\}^2}\,l\,Cos.\,x\,d\,x = \frac{\pi}{2\,l\,2}$$ V. T 412. N°. 1.

$$14)\int \frac{(l\,Cos.\,x)^2-2\,x\,Cot.\,x.\,l\,Cos.\,x-x^2}{\{x^2+(l\,Cos.\,x)^2\}^2}\,x\,Tang.\,x\,d\,x = \pi\,\frac{1-l\,2}{2\,l\,2}$$ V. T. 412. N°. 2.

F. Alg. rat. ent. Log. de Circ. Dir. **TABLE 413.** Lim. 0 et π.

$$1)\int l\,Sin.\,x.\,x\,d\,x = \frac{1}{2}\pi^2\,l\frac{1}{2}$$ Grunert, Gr. 4. 113.

$$2)\qquad = -\frac{1}{2}\pi^2\,(l\,2-2\,\alpha\,\pi\,i)$$ Arndt, Gr. 6. 187.

$$3)\int l\,Sin.\,x.\,(\pi-2\,x)\,d\,x = 0$$ Grunert, Gr. 4. 113.

$$4)\int l\,Cos.^2\,x.\,x\,d\,x = \pi^2\,l\frac{1}{2}$$ V. T. 413. N°. 3.

$$5)\int l\,Tang.^2\,x.\,x\,d\,x = 0$$ V. T. 413. N°. 1, 4.

$$6)\int l((Sin.\,x)).\,x\,d\,x = \frac{1}{2}\pi^2\,l\frac{1}{2} \pm \alpha\,\pi^3\,i$$

$$7)\int l\,((-Sin.\,x)).\,x\,d\,x = \frac{1}{2}\pi^2\,l\frac{1}{2} \pm \frac{2\,\alpha+1}{1}\,\pi^3\,i$$

(6, 7) Lindmann, Gr. 16. 94.

$$8)\int l(1-2\,p\,Cos.\,2\,x+p^2).\,Cos.\,\{(2\,a-1)\,x\}.\,x^{\pm 2b}\,d\,x = 0$$

$$9)\int l(1-2\,p\,Cos.\,2\,x+p^2).\,Sin.\,\{(2\,a-1)\,x\}.\,x^{\pm 2b+1}\,d\,x = 0$$

(8, 9) $,\ p < 1;$ Bierens de Haan, Gr. 13. 193.

10) $\int l(1-2p\,Cos.\,2x+p^2).\,Sin.\,2ax.\,Sin.\,x.\,x^{\pm 2b}\,dx = 0$

11) $\int l(1-2p\,Cos.\,2x+p^2).\,Sin.\,2ax.\,Cos.\,x.\,x^{\pm 2b+1}\,dx = 0$

12) $\int l(1-2p\,Cos.\,2x+p^2).\,Cos.\,2ax.\,Sin.\,x.\,x^{\pm 2b+1}\,dx = 0$

13) $\int l(1-2p\,Cos.\,2x+p^2).\,Cos.\,2ax.\,Cos.\,x.\,x^{\pm 2b}\,dx = 0$

14) $\int l(1-2q\,Cos.\,x+q^2).\,Sin.\,ax.\,x^{2b+1}\,dx = \frac{(-1)^{b+1}\pi q^a}{a^{2b+2}}\,1^{2b+1/1}\sum_0^{2b+1}\frac{(-alq)^n}{1^{n/1}}$

15) $\int l(1-2q\,Cos.\,x+q^2).\,Cos.\,ax.\,x^{2b}\,dx = \frac{(-1)^{b+1}\pi q^a}{a^{2b+1}}\,1^{2b/1}\sum_0^{2b}\frac{(-alq)^n}{1^{n/1}}$

$, p<1, q^2<1;$ Bierens de Haan, Gr. 13. 193.

F. Alg. rat. fract. à dén. x^a.
Log.
Circ. Dir.

TABLE 414.

Lim. 0 et ∞.

1) $\int l(ax).\,Sin.\,ax\frac{dx}{x} = -\frac{1}{2}\pi A$

2) $\int l(bx).\,Sin.\,ax\frac{dx}{x} = -\frac{1}{2}\pi\left(A+l\frac{a}{b}\right)$

Arndt, Gr. 11. 70.

3) $\int l(1+x^2).\,Sin.\,ax\frac{dx}{x} = -\pi\,li.(e^{-a})$ Schlömilch, Beitr. III. § 8.

4) $\int l\left(\frac{1+Sin.\,x}{1-Sin.\,x}\right)^2\frac{dx}{x} = \pi^2$ Schlömilch, Gr. 4. 316. où la valeur est $\frac{1}{2}\pi^2$ fautivement.

5) $\int l\left(\frac{1+Tang.\,x}{1-Tang.\,x}\right)^2\frac{dx}{x} = \frac{1}{2}\pi^2$ Schlömilch, Beitr. III. § 5. — Id., Gr. 4. 316.

6) $\int l\frac{1+Tang.\,px}{1-Tang.\,px}\frac{dx}{x} = \frac{1}{4}\pi^2$ Schlömilch, Beitr. II. § 5.

7) $\int l\frac{1+2p\,Cos.\,x+p^2}{1+2p\,Cos.\,ax+p^2}\frac{dx}{x} = l(a^2).\,l(1+p),\ p^2\leqq 1;$

8) $= l(a^2).\,l\left(\frac{1+p}{p}\right),\ p^2\geqq 1;$

Raabe, Cr. 23. 105.

9) $\int l \frac{1+2p\,Cos.\,ax+p^2}{1+2p\,Cos.\,bx+p^2}\frac{dx}{x} = l(1+p).\,l\left(\frac{b^2}{a^2}\right),\ p^2 \leqq 1;$

10) $= l\left(\frac{1+p}{p}\right).\,l\left(\frac{b^2}{a^2}\right),\ p^2 \geqq 1;$ Schlömilch, Gr. 5. 152. — Raabe, Cr. 23. 105.

11) $\int l(q^2+x^2)\frac{ax\,Cot.\,ax - l\,Sin.\,ax}{x^2}dx = \frac{\pi}{q}l\frac{2e^{2aq}}{e^{2aq}-1}$ V. T. 415. N°. 4.

12) $\int l(q^2+x^2)\frac{ax\,Tang.\,ax + l\,Cos.\,ax}{x^2}dx = \frac{\pi}{q}l\frac{e^{2aq}+1}{2e^{2aq}}$ V. T. 415. N°. 5.

13) $\int l(q^2-x^2)\frac{ax\,Cot.\,ax - l\,Sin.\,ax}{x^2}dx = p\pi - \frac{\pi^2}{2q}$ V. T. 415. N°. 13.

14) $\int l(q^2-x^2)\frac{ax\,Tang.\,ax + l\,Cos.\,ax}{x^2}dx = -p\pi$ V. T. 415. N°. 14.

15) $\int l(q^2+x^2)\frac{2ax - Sin.\,2ax.\,l\,Tang.\,ax}{x^2\,Sin.\,2ax}dx = \frac{\pi}{q}l\frac{e^{2aq}+1}{e^{2aq}-1}$ V. T. 415. N°. 11.

16) $\int l(q^2-x^2)\frac{2ax - Sin.\,2ax.\,l\,Tang.\,ax}{x^2\,Sin.\,2ax}dx = -\frac{\pi^2}{2q}$ V. T. 415. N°. 17.

17) $\int l\,x.\,Sin.\,qx\frac{dx}{x^{1-p}} = \frac{1}{q^p}\left\{Sin.\,\frac{1}{2}p\pi.\,Z'(p) - Sin.\,\frac{1}{2}p\pi.\,lq + \frac{1}{2}\pi\,Cos.\,\frac{1}{2}p\pi\right\}\Gamma(p)$

18) $\int l\,x.\,Cos.\,qx\frac{dx}{x^{1-p}} = \frac{1}{q^p}\left\{Cos.\,\frac{1}{2}p\pi.\,Z'(p) - Cos.\,\frac{1}{2}p\pi.\,lq - \frac{1}{2}\pi\,Sin.\,\frac{1}{2}p\pi\right\}\Gamma(p)$ Raabe, Int. 416.

19) $\int l(q\ \ +x^2).\left\{l(1+p^2\,Tang.^2x) - \frac{2p^2\,x\,Tang.\,x}{Cos.^2x+p^2\,Sin.^2x}\right\}\frac{dx}{x^2} = \frac{2\pi}{q}\left\{1+p\frac{e^q-e^{-q}}{e^q+e^{-q}}\right\}$ V. T. 416. N°. 1.

F. Alg. rat. fract. à dén. $b^2 \pm x^2$.
Log. de
Circ. Dir. monôme.

TABLE 415.

Lim. 0 et ∞.

1) $\int l\,Sin.\,qx\frac{dx}{1+x^2} = \frac{1}{2}\pi\,l\frac{e^{2q}-1}{2e^{2q}}$

2) $\int l\,Cos.\,qx\frac{dx}{1+x^2} = \frac{1}{2}\pi\,l\frac{e^{2q}+1}{2e^{2q}}$

3) $\int l\,Tang.\,qx\frac{dx}{1+x^2} = \frac{1}{2}\pi\,l\frac{e^{2q}-1}{e^{2q}+1}$

Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 39.

4) $\int l\,Sin.\,px \frac{dx}{q^2+x^2} = \frac{\pi}{2q} l \frac{1-e^{-2pq}}{2}$

5) $\int l\,Cos.\,px \frac{dx}{q^2+x^2} = \frac{\pi}{2q} l \frac{1+e^{-2pq}}{2}$

Bidone, Mém. Turin. 1812. 231. Tableau. — Legendre, Exerc. 4. 133. — Schlömilch, Beitr. II. § 5. — Id., Gr. 10. 440.

6) $\int l\left(2\,Cos.\frac{1}{2}x\right)^2 \frac{dx}{q^2+x^2} = \frac{\pi}{q} l \frac{e^q+1}{e^q}$

7) $\int l\left(2\,Sin.\frac{1}{2}x\right)^2 \frac{dx}{q^2+x^2} = \frac{\pi}{q} l \frac{e^q-1}{e^q}$

Schlömilch, Stud. II. 18.

8) $\int l\left(Sin.\frac{1}{2}kx\right) \frac{dx}{q^2+x^2} = -\frac{\pi}{2q} l\,2$, $k = \infty$; Schlömilch, Beitr. II. 5.

9) $\int l\left(2\,Sin.\frac{1}{2}qx\right) \frac{dx}{p^2+x^2} = \frac{\pi}{2p} l(1-e^{-pq})$

10) $\int l\left(2\,Cos.\frac{1}{2}qx\right) \frac{dx}{p^2+x^2} = \frac{\pi}{2p} l(1+e^{-pq})$

Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 39. — Boncompagni, Cr. 25. 74.

11) $\int l\,Tang.\,px \frac{dx}{q^2+x^2} = \frac{\pi}{2q} l \frac{e^{2pq}-1}{e^{2pq}+1}$ Bidone, Mém. Turin. 1812. 231. Tableau. — Legendre, Exerc. 4. 133. — Schlömilch, Beitr. II. § 5. — Boncompagni, Cr. 25. 74.

12) $\int l\,Cot.\,px \frac{dx}{q^2+x^2} = \frac{\pi}{2q} l \frac{e^{pq}+e^{-pq}}{e^{pq}-e^{-pq}}$ Schlömilch, Beitr. II. § 5. — Boncompagni, Cr. 25. 74.

13) $\int l\,Sin.\,px \frac{dx}{q^2-x^2} = -\frac{1}{4q}\pi^2 + \frac{1}{2}p\pi$

14) $\int l\,Cos.\,px \frac{dx}{q^2-x^2} = \frac{1}{2}p\pi$

15) $\int l(2\,Sin.\,px) \frac{dx}{q^2-x^2} = \frac{1}{2}p\pi - \frac{1}{4q}\pi^2$

16) $\int l(2\,Cos.\,px) \frac{dx}{q^2-x^2} = \frac{1}{2}p\pi$

17) $\int l\,Tang.\,px \frac{dx}{q^2-x^2} = -\frac{1}{4q}\pi^2$

Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 39.

F. Alg. rat. fract. à dén. b^2+x^2.
Log. de
Circ. Dir. polynôme.

TABLE 416.

Lim. 0 et ∞.

1) $\int l(1+p^2\,Tang.^2\,x)\dfrac{dx}{q^2+x^2} = \dfrac{\pi}{q}\left\{1+p\dfrac{e^q-e^{-q}}{e^q+e^{-q}}\right\}$

2) $\int l(1+p^2\,Cot.^2\,x)\dfrac{dx}{q^2+x^2} = \dfrac{\pi}{q}\left\{1+p\dfrac{e^q+e^{-q}}{e^q-e^{-q}}\right\}$

3) $\int l(1+p^2\,Tang.^2\,r\,x)\dfrac{dx}{q^2+x^2} = \dfrac{\pi}{q}l\{1+p\,Tangh\,p.(q\,r)\}$

4) $\int l(1+p^2\,Cot.^2\,r\,x)\dfrac{dx}{q^2+x^2} = \dfrac{\pi}{q}l\{1+p\,Coth\,p.(q\,r)\}$

(1–4) Schlömilch, Gr. 10. 440.

5) $\int l(1+2\,p\,Cos.\,x+p^2)\dfrac{dx}{q^2+x^2} = \dfrac{\pi}{q}l(1+p\,e^{-q})\;,\;p^2\leqq 1;$

6) $= \dfrac{\pi}{q}l(p+e^{-q})\;,\;p^2\geqq 1;$

(5–6) Schlömilch, Stud. II. 18.

7) $\int l(1-2\,p\,Cos.\,x+p^2)\dfrac{dx}{q^2+x^2} = \dfrac{\pi}{q}l(1-p\,e^{-q})\;,\;p^2\leqq 1;$ Hoppe, Cr. 40. 139.

8) $\int l(1+2\,p\,Cos.\,r\,x+p^2)\dfrac{dx}{q^2+x^2} = \dfrac{\pi}{q}l(p+e^{-qr})\;,\;p>1;$ Ohm, Ausw. 26.

9) $= \dfrac{\pi}{q}l(1+p\,e^{-qr})$

10) $\int l(1-2\,p\,Cos.\,r\,x+p^2)\dfrac{dx}{q^2+x^2} = \dfrac{\pi}{q}l(1-p\,e^{-qr})$

(9–10) , $p<1$; Legendre, Exerc. 4. 133. — Plana, Mém. Turin. 1818. 7. II. 11. — Boncompagni, Cr. 25. 74.

11) $= \dfrac{\pi}{q}l(p-e^{-qr})\;,\;p>1;$ Ohm, Ausw. 26.

12) $\int l\dfrac{1+2\,p\,Cos.\,r\,x+p^2}{1-2\,p\,Cos.\,r\,x+p^2}\dfrac{dx}{q^2+x^2} = \dfrac{\pi}{q}l\dfrac{1+p\,e^{-rq}}{1-p\,e^{-rq}}$

13) $\int l\dfrac{1-2\,p\,Cos.\,r\,x+p^2}{1+2\,p\,Cos.\,r\,x+p^2}\dfrac{dx}{q^2+x^2} = \dfrac{\pi}{q}l\dfrac{1-p\,e^{-rq}}{1+p\,e^{-rq}}$

(12–13) Boncompagni, Cr. 25. 74. Elles ne valent que pour $p^2\leqq 1$.

F. Alg. rat. fract. à dén. $b^2\pm x^2$.
Log. $l(a\,x)$.
Circ. Dir.

TABLE 417.

Lim. 0 et ∞.

1) $\int l\,x.\,Sin.\,q\,x\,\dfrac{x}{1+x^2}\,dx = -\dfrac{1}{4}\pi\,\{e^q\,li.(e^{-q})+e^{-q}\,li.(e^q)\}$

2) $\int l\,x.\,Cos.\,q\,x\,\dfrac{dx}{1+x^2} = \dfrac{\pi}{4q}\{e^q\,li.(e^{-q})-e^{-q}\,li.(e^q)\}$

(1–2) Schlömilch, Gr. 5. 204.

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3) $\int l\,x.\,Sin.\,q\,x\,\frac{x\,dx}{p^2+x^2} = \frac{1}{2}\pi e^{-pq}\,l\,p - \frac{1}{4}\pi\{e^{pq}\,li.(e^{-pq}) + e^{-pq}\,li.(e^{pq})\}$

4) $\int l\,x.\,Cos.\,q\,x\,\frac{dx}{p^2+x^2} = \frac{\pi}{2p}e^{-pq}\,l\,p + \frac{\pi}{4p}\{e^{pq}\,li.(e^{-pq}) - e^{-pq}\,li.(e^{pq})\}$

Schlömilch, Gr. 5. 204. (3, 4)

5) $\int l(rx).\,Sin.qx\,\frac{x\,dx}{p^2+x^2} = \frac{1}{4}\pi e^{-pq}\{2\,l(pr) - Ei.(pq)\} - \frac{1}{4}\pi e^{pq}\,Ei.(-pq)$

6) $\int l(rx).\,Cos.qx\,\frac{dx}{p^2+x^2} = \frac{\pi}{4p}e^{-pq}\{2\,l(pr) - Ei.(pq)\} + \frac{\pi}{4p}e^{pq}\,Ei.(-pq)$

Arndt, Gr. 11. 70. (5, 6)

7) $\int l\left(\frac{p}{x}\right).\,Sin.\,q\,x\,\frac{x\,dx}{p^2+x^2} = \frac{1}{4}\pi\{e^{-pq}\,Ei.(pq) + e^{pq}\,Ei.(-pq)\}$

8) $\int l\left(\frac{p}{x}\right).\,Cos.\,q\,x\,\frac{dx}{p^2+x^2} = \frac{\pi}{4q}\{e^{-pq}\,Ei.(pq) - e^{pq}\,Ei.(-pq)\}$

9) $\int l\left(\frac{p}{x}\right).\,Sin.\,q\,x\,\frac{x\,dx}{p^2-x^2} = -\frac{1}{2}\pi\left\{Ci.(pq).Cos.pq + Si.(pq).Sin.pq - \frac{1}{2}\pi\,Sin.pq\right\}$

10) $\int l\left(\frac{p}{x}\right).\,Cos.\,q\,x\,\frac{dx}{p^2-x^2} = \frac{\pi}{2p}\left\{Ci.(pq).Sin.pq - Si.(pq).Cos.pq + \frac{1}{2}\pi\,Cos.pq\right\}$

Schlömilch, Stud. II. 21. où 9) et 10) sont faut. négatives. (7–10)

F. Alg. rat. fract. à dén. $b^n \pm x^n$.
Log.
Circ. Dir.

TABLE 418.

Lim. 0 et ∞.

1) $\int l\,Sin.\,q\,x\,\frac{dx}{p^4+x^4} = \frac{\pi}{2p^3\sqrt{2}}\,l\left[\frac{1}{2}\sqrt{\{1 - 2e^{-pq\sqrt{2}}Cos.(pq\sqrt{2}) + e^{-2pq\sqrt{2}}\}}\right] -$
$- \frac{\pi}{2p^3\sqrt{2}}\,Arcsin.\left\{\frac{e^{-pq\sqrt{2}}\,Sin.(pq\sqrt{2})}{\sqrt{\{1 - 2e^{-pq\sqrt{2}}Cos.(pq\sqrt{2}) + e^{-2pq\sqrt{2}}\}}}\right\}$

2) $\int l\,Cos.\,q\,x\,\frac{dx}{p^4+x^4} = \frac{\pi}{2p^3\sqrt{2}}\,l\left[\frac{1}{2}\sqrt{\{1 + 2e^{-pq\sqrt{2}}Cos.(pq\sqrt{2}) + e^{-2pq\sqrt{2}}\}}\right] +$
$+ \frac{\pi}{2p^3\sqrt{2}}\,Arcsin.\left\{\frac{e^{-pq\sqrt{2}}\,Sin.(pq\sqrt{2})}{\sqrt{\{1 + 2e^{-pq\sqrt{2}}Cos.(pq\sqrt{2}) + e^{-2pq\sqrt{2}}\}}}\right\}$

3) $\int l\,Tang.qx\,\frac{dx}{p^4+x^4} = \frac{\pi}{2p^3\sqrt{2}}\,l\left[\sqrt{\frac{1 - 2e^{-pq\sqrt{2}}Cos.(pq\sqrt{2}) + e^{-2pq\sqrt{2}}}{1 + 2e^{-pq\sqrt{2}}Cos.(pq\sqrt{2}) + e^{-2pq\sqrt{2}}}}\right] -$
$- \frac{\pi}{2p^3\sqrt{2}}\,Arcsin.\left\{\frac{2e^{-pq\sqrt{2}}\,Sin.(pq\sqrt{2})}{\sqrt{\{1 - 2e^{-2pq\sqrt{2}}Cos.(2pq\sqrt{2}) + e^{-4pq\sqrt{2}}\}}}\right\}$

Plana, Mém. Turin. 1818. 7. II. 9. (1–3)

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4) $$\int l(1+2p\,Cos.\,qx+p^2)\frac{dx}{1+x^{2a}}=\frac{\pi}{a}l(1+pe^{-q})-\frac{\pi}{a}\sum_{1}^{\frac{1}{2}(a-1)}\left[Cos.\frac{n\pi}{a}.\,l\left\{1+2pe^{-qCos.\frac{n\pi}{a}}Cos.\left(qSin.\frac{n\pi}{a}\right)+p^2e^{-2qCos.\frac{n\pi}{a}}\right\}\right]$$

$$-\frac{2\pi}{a}\sum_{1}^{\frac{1}{2}(a-1)}\left\{Sin.\frac{n\pi}{a}.\,Arcsin.\left[\frac{pe^{-qCos.\frac{n\pi}{a}}Sin.\left(qSin.\frac{n\pi}{a}\right)}{\sqrt{\left\{1+2pe^{-qCos.\frac{n\pi}{a}}Cos.\left(qSin.\frac{n\pi}{a}\right)+p^2e^{-2qCos.\frac{n\pi}{a}}\right\}}}\right]\right\},\ a\text{ impair};$$

5) $$=\frac{\pi}{a}\sum_{0}^{\frac{1}{2}a-1}\left[Cos.\left(\frac{2n+1}{2a}\pi\right).\,l\left\{1+2pe^{-qCos.\left(\frac{2n+1}{2a}\pi\right)}Cos.\left[qSin.\left(\frac{2n+1}{2a}\pi\right)\right]+p^2e^{-2qCos.\left(\frac{2n+1}{2a}\pi\right)}\right\}\right]$$

$$+\frac{2\pi}{a}\sum_{0}^{\frac{1}{2}a-1}\left\{Sin.\left(\frac{2n+1}{2a}\pi\right).Arcsin.\left[\frac{pe^{-qCos\left(\frac{2n+1}{2a}\pi\right)}Sin.\left[qSin.\left(\frac{2n+1}{2a}\pi\right)\right]}{\sqrt{\left\{1+2pe^{-qCos.\left(\frac{2n+1}{2a}\pi\right)}Cos\left[qSin.\left(\frac{2n+1}{2a}\pi\right)\right]+p^2e^{-2qCos.\left(\frac{2n+1}{2a}\pi\right)}\right\}}}\right]\right\},\ a\text{ pair};$$

Sur les intégrales 4), 5) voyez: Plana, Mém. Turin. 1818. 7. III. 15.

6) $\int l\,Sin\,qx\,\frac{dx}{p^4-x^4}=\frac{\pi}{8p^3}\left\{2pq-\pi+2\,l\frac{1-e^{-2pq}}{2}\right\}$ V. T. 415. N°. 4, 13.

7) $\int l\,Sin.\,qx\,\frac{x^2\,dx}{p^4-x^4}=\frac{\pi}{8p}\left\{2pq-\pi+2\,l\frac{2}{1-e^{-2pq}}\right\}$ V. T. 415. N°. 4, 13.

8) $\int l\,Cos.\,qx\,\frac{dx}{p^4-x^4}=\frac{\pi}{4p^3}\left\{pq+l\frac{1+e^{-2pq}}{2}\right\}$ V. T. 415. N°. 5, 14.

9) $\int l\,Cos.\,qx\,\frac{x^2\,dx}{p^4-x^4}=\frac{\pi}{4p}\left\{pq+l\frac{2}{1-e^{-2pq}}\right\}$ V. T. 415. N°. 5, 14.

10) $\int l(2\,Sin.\,qx)\frac{dx}{p^4-x^4}=\frac{\pi}{8p^3}\{2pq-\pi+2\,l(1-e^{-2pq})\}$ V. T. 415. N°. 9, 15.

11) $\int l(2\,Sin.\,qx)\frac{x^2\,dx}{p^4-x^4}=\frac{\pi}{8p}\{2pq-\pi-2\,l(1-e^{-2pq})\}$ V. T. 415. N°. 9, 15.

12) $\int l(2\,Cos.\,qx)\frac{dx}{p^4-x^4}=\frac{\pi}{4p^3}\{pq+l(1+e^{-2pq})\}$ V. T. 415. N°. 10, 16.

13) $\int l(2\,Cos.\,qx)\frac{x^2\,dx}{p^4-x^4}=\frac{\pi}{4p}\{pq-l(1+e^{-2pq})\}$ V. T. 415. N°. 10, 16.

14) $\int l\,Tang.\,qx\,\frac{dx}{p^4-x^4}=\frac{\pi}{8p^3}\left\{-\pi+2\,l\frac{1-e^{-2pq}}{1+e^{-2pq}}\right\}$ V. T. 415. N°. 11, 17.

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15) $\int l\,Tang.\,q\,x \dfrac{x^2\,dx}{p^4 - x^4} = \dfrac{\pi}{8p}\left\{-\pi + 2\,l\,\dfrac{1+e^{-2pq}}{1-e^{-2pq}}\right\}$ V. T. 415. N°. 11, 17.

16) $\int l\left(\dfrac{p}{x}\right).Sin\,qx \dfrac{x\,dx}{p^4-x^4} = \dfrac{\pi}{8p^2}\{\pi Sin.pq - 2Si.(pq).Sin.pq - 2Ci.(pq).Cos.pq + e^{-pq}Ei.(pq) + e^{pq}Ei.(-pq)\}$ V. T. 417. N°. 7, 9.

17) $\int l\left(\dfrac{p}{x}\right).Sin.qx \dfrac{x^3\,dx}{p^4-x^4} = \dfrac{\pi}{8}\{\pi Sin.pq - 2Si.(pq).Sin.pq - 2Ci.(pq).Cos.pq - e^{-pq}Ei.(pq) - e^{pq}Ei.(-pq)\}$ V. T. 417. N°. 7, 9.

18) $\int l\left(\dfrac{p}{x}\right).Cos\,qx \dfrac{dx}{p^4-x^4} = \dfrac{\pi}{8p^3}\{\pi Cos.pq - 2Si.(pq).Cos\,pq + 2Ci.(pq).Sin\,pq + e^{-pq}Ei.(pq) - e^{pq}Ei.(-pq)\}$ V. T. 417. N°. 8, 10.

19) $\int l\left(\dfrac{p}{x}\right).Cos\,qx \dfrac{x^2\,dx}{p^4-x^4} = \dfrac{\pi}{8p}\{\pi Cos.pq - 2Si.(pq).Cos.pq + 2Ci.(pq).Sin.pq - e^{-pq}Ei.(pq) + e^{pq}Ei.(-pq)\}$ V. T. 417. N°. 8, 10.

F. Alg. rat. fract. à autre dén.
Log.
Circ. Dir.

TABLE 419.

Lim. 0 et ∞.

1) $\int l\,Sin.qx \dfrac{dx}{x^4+2p^2x^2\,Cos.2\lambda+p^4} = \dfrac{\pi}{4p^3}Sec.\lambda.l\left[\dfrac{1}{2}\sqrt{\{1-2e^{-2pqCos.\lambda}Cos.(2pqSin.\lambda)+e^{-4pqCos.\lambda}\}}\right] -$

$- \dfrac{\pi}{4p^3}\,Cosec.\lambda.\,Arcsin.\left\{\dfrac{e^{-2pqCos.\lambda}\,Sin.(2\,p\,q\,Sin.\lambda)}{\sqrt{\{1-2\,e^{-2pqCos.\lambda}\,Cos.(2\,p\,q\,Sin.\lambda)+e^{-4pqCos.\lambda}\}}}\right\}$

2) $\int l\,Cos.qx \dfrac{dx}{x^4+2p^2x^2\,Cos.2\lambda+p^4} = \dfrac{\pi}{4p^3}Sec.\lambda.l\left[\dfrac{1}{2}\sqrt{\{1+2e^{-2pqCos.\lambda}Cos.(2pqSin.\lambda)+e^{-4pqCos.\lambda}\}}\right] +$

$+ \dfrac{\pi}{4p^3}\,Cosec.\lambda.\,Arcsin.\left\{\dfrac{e^{-2pqCos.\lambda}\,Sin.(2\,p\,q\,Sin.\lambda)}{\sqrt{\{1+2\,e^{-2pqCos.\lambda}\,Cos.(2\,p\,q\,Sin.\lambda)+e^{-4pqCos.\lambda}\}}}\right\}$

3) $\int l\,Tg.qx \dfrac{dx}{x^4+2p^2x^2\,Cos.2\lambda+p^4} = \dfrac{\pi}{4p^3}Sec.\lambda.l\left[\sqrt{\dfrac{1-2\,e^{-2pqCos.\lambda}\,Cos.(2\,p\,q\,Sin.\lambda)+e^{-4pqCos.\lambda}}{1+2\,e^{-2pqCos.\lambda}\,Cos.(2\,p\,q\,Sin.\lambda)+e^{-4pqCos.\lambda}}}\right] -$

$- \dfrac{\pi}{4p^3}\,Cosec.\lambda.\,Arcsin.\left\{\dfrac{2\,e^{-2pqCos.\lambda}\,Sin.(2\,p\,q\,Sin.\lambda)}{\sqrt{\{1-2e^{-4pqCos.\lambda}\,Cos.(4pqSin.\lambda)+e^{-8pqCos.\lambda}\}}}\right\}$

Sur les intégrales 1) à 3) voyez: Plana, Mém. Turin. 1818. 7. II. 9.

4) $\int l(1+2\,p\,Cos.r\,x+p^2)\dfrac{dx}{(x^2+q^2)^2} = \dfrac{\pi}{2q^3}l(1+pe^{-qr}) + \dfrac{\pi}{2q^3}\dfrac{p\,e^{-qr}}{(1+p\,e^{-qr})}, p<1;$ Plana, Mém. Turin. 1818. 7. II. 11. où fautive.

5) $\int l(1+2pCos.rx+p^2)\dfrac{dx}{x^4+2q^2x^2\,Cos.2\lambda+q^4} = \dfrac{\pi}{4q^3}\,Sec.\lambda.\,l\{1+2p\,e^{-qrCos.\lambda}\,Cos.(qrSin.\lambda)+p^2\,e^{-2qrCos.\lambda}\} +$

$+ \dfrac{\pi}{2q^3}\,Cosec.\lambda.\,Arcsin.\left\{\dfrac{p\,e^{-qrCos\,\lambda}\,Sin.(q\,r\,Sin.\lambda)}{\sqrt{\{1+2\,p\,e^{-qrCos.\lambda}\,Cos.(q\,r\,Sin.\lambda)+p^2\,e^{-2qrCos.\lambda}\}}}\right\}, p<1;$

F. Alg. rat. fract. à autre dén.
Log.
Circ. Dir.

TABLE 419 suite.

Lim. 0 et ∞.

6) $\int l(1+2p\,Cos.rx+p^2)\frac{dx}{x^4+2q^2x^2Cos.2\lambda+q^4}=\frac{\pi}{4q^3}Sec.\lambda.l\{p^2+2pe^{-qr\,Cos.\lambda}Cos.(qr\,Sin.\lambda)+e^{-2qr\,Cos.\lambda}\}$

$+\frac{\pi}{2q^3}Cosec.\lambda.\,Arcsin.\left\{\frac{e^{-qr\,Cos.\lambda}Sin.(qr\,Sin.\lambda)}{\sqrt{\{p^2+2pe^{-qr\,Cos.\lambda}Cos.(qr\,Sin.\lambda)+e^{-2qr\,Cos.\lambda}\}}}\right\},\ p>1;$

Les formules 5) et 6) sont trouvées par Plana, Mém. Turin. 1818. 7. II. 8.

7) $\int l\frac{1+Tang.qx}{1-Tang.qx}\frac{1}{x}\frac{dx}{p^2+x^2}=\frac{\pi}{p^2}Arctang.\left(\frac{e^{pq}-e^{-pq}}{e^{pq}+e^{-pq}}\right)$ Schlömilch, Beitr. II. § 5.

8) $\int\frac{\pi(1-Cos.qx)-2Sin.qx.lx}{\left(\frac{\pi}{2}\right)^2+(lx)^2}\frac{dx}{x}=2\pi(1-e^{-q})$ Cauchy, Lim. Imag. Add. 19.

9) $\int\frac{Cos.(qlx)}{x^p-2\,Cos.\lambda+x^{-p}}\frac{dx}{x}=\frac{\pi}{p\,Sin.\lambda}\frac{e^{-\frac{q}{p}(\pi-\lambda)}-e^{\frac{q}{p}(\pi-\lambda)}}{e^{-\frac{q\pi}{p}}-e^{\frac{q\pi}{p}}}$ Euler, N. A. Petr. III. 3.

F. Alg. rat. fract.
Log.
Circ. Dir.

TABLE 420.

Lim. $-\infty$ et ∞.

1) $\int l\,Sin\,qx\frac{r+sx}{x^2+2px\,Cos.\lambda+p^2}dx=-\frac{\pi}{p\,Sin.\lambda}\left(r-\frac{1}{2}s^2\right)l2+$

$+\frac{r-ps\,Cos.\lambda}{p\,Sin.\lambda}\pi l\left[\sqrt{\{1-2e^{-2pq\,Sin.\lambda}Cos.(2pq\,Cos.\lambda)+e^{-4pq\,Sin.\lambda}\}}\right]$

$-s\pi\,Arcsin.\left\{\frac{e^{-2pq\,Sin.\lambda}Sin.(2pq\,Cos.\lambda)}{\sqrt{\{1-2e^{-2pq\,Sin.\lambda}Cos.(2pq\,Cos.\lambda)+e^{-4pq\,Sin.\lambda}\}}}\right\}$

2) $\int l\,Cos.qx\frac{r+sx}{x^2+2px\,Cos.\lambda+p^2}dx=-\frac{\pi}{p\,Sin.\lambda}\left(r-\frac{1}{2}s^2\right)l2+$

$+\frac{r-ps\,Cos.\lambda}{p\,Sin\,\lambda}\pi l\left[\sqrt{\{1+2e^{-2pq\,Sin.\lambda}Cos.(2pq\,Cos.\lambda)+e^{-4pq\,Sin.\lambda}\}}\right]$

$+s\pi\,Arcsin.\left\{\frac{e^{-2pq\,Sin.\lambda}Sin.(2pq\,Cos.\lambda)}{\sqrt{\{1+2e^{-2pq\,Sin.\lambda}Cos.(2pq\,Cos.\lambda)+e^{-4pq\,Sin.\lambda}\}}}\right\}$

3) $\int l\,Tang.qx\frac{r+sx}{x^2+2px\,Cos.\lambda+p^2}dx=\frac{r-ps\,Cos.\lambda}{2p\,Sin.\lambda}\pi l\frac{1-2e^{-2pq\,Sin.\lambda}Cos.(2pq\,Cos.\lambda)+e^{-4pq\,Sin.\lambda}}{1+2e^{-2pq\,Sin.\lambda}Cos.(2pq\,Cos.\lambda)+e^{-4pq\,Sin.\lambda}}-$

$-s\pi\,Arcsin.\left\{\frac{2e^{-2pq\,Sin.\lambda}Sin.(2pq\,Cos.\lambda)}{\sqrt{\{1-2e^{-4pq\,Sin.\lambda}Cos.(4pq\,Cos.\lambda)+e^{-8pq\,Sin.\lambda}\}}}\right\}$

Ces intégrales se trouvent chez Plana, Mém. Turin. 1818. 7. II. 12.

1) $\int_0^{a\pi} l(Sin.x).x\,dx = -\frac{1}{2}a^2\pi^2\left\{l2-\frac{a\mp1}{a}\alpha\pi i-\frac{a\pm1}{a}\frac{2\beta+1}{2}\pi i\right\}$ ± pour a pair ou impair; Stegmann, Gr. 7. 108.

2) $\int_0^{a\pi} l(Sin.^2x).x\,dx = -a^2\pi^2 l2$ Clausen, Cr. 7. 309.

3) $= a^2\pi l2$ (fautive) Hill, Cr. 7. 102.

4) $\int_0^{2a\pi} l\,Sin.x.x\,dx = -2a^2\pi^2\left\{l2-\alpha\pi i\frac{2a-1}{2a}-\frac{2\beta+1}{2}\frac{2a+1}{2a}\pi i\right\}$ Arndt, Gr. 6. 187. — Lindmann, Gr. 16. 94.

5) $\int_0^{(2a+1)\pi} l\,Sin.x.x\,dx = -\frac{(2a+1)^2}{2}\pi^2\left\{l2-\alpha\pi i\frac{2a+2}{2a+1}-\frac{2\beta+1}{2}\frac{2a}{2a+1}\pi i\right\}$ Stegmann, Gr. 7. 108.— Lindmann, Gr. 16. 94.

6) $= -\frac{(2a+1)^2}{2}\pi^2\left\{l2-\alpha\pi i\frac{2a+2}{2a+1}-\frac{2\beta+1}{2}\frac{2a+1}{2a}\pi i\right\}$ (fautive)

7) $\int_{2a\pi}^{(2a+1)\pi} l\,Sin.x.x\,dx = -\frac{1}{2}\pi^2(4a+1)l2+\alpha\pi^3 i(4a+1)$

8) $\int_{(2a-1)\pi}^{2a\pi} l\,Sin.x.x\,dx = -\frac{1}{2}\pi^2(4a-1)l2+\frac{2\alpha+1}{2}\pi^3 i(4a-1)$

9) $\int_{(2a-2)\pi}^{2a\pi} l\,Sin.x.x\,dx = -\frac{1}{2}\pi^2(2a)^2 l2+\alpha\pi^3 i(4a-3)+\frac{2\beta+1}{2}(4a-1)\pi^3 i$ (fautive).

(6–9: Arndt, Gr. 6. 187.)

10) $= -2\pi^2(2a-1)l2+\alpha\pi^3 i(4a-3)+\frac{2\beta+1}{2}(4a-1)\pi^3 i$ Stegmann, Gr. 7. 108.

11) $\int_0^{2a\pi} l(1+2pCos.x+p^2).x^b\,dx = 2\sum_0^{b-1}\left\{1^{n/1}\binom{b}{n}(2a\pi)^{b-n}Cos.\left\{\frac{n+1}{2}\pi\right\}.\sum_1^{\infty}\frac{p^m}{m^{n+2}}\right\}$, $p^2<1$; Hoppe, Cr. 40. 139.

12) $\int_0^{\lambda}\left\{2x+l\frac{1+Sin.x}{1-Sin.x}\right\}\frac{dx}{\sqrt{(Cos.^2x-Cos.^2\lambda)(1-Cos.^2\lambda.Cos.^2x)}} = \pi\,Cosec.\varphi.\,\mathrm{F}(p,\varphi)$

13) $\int_0^{\lambda}\left\{2x\,Cos.x-l\frac{1+Sin.x}{1-Sin.x}\right\}\frac{Cos.x}{Sin.^2x.\sqrt{(Cos.^2x-Cos.^2\lambda)(1-Cos.^2\lambda.Cos.^2x)}}dx =$

$= \frac{\pi\,Cos.^2\lambda}{Sin.\lambda.Sin.\varphi}\mathrm{F}(p,\varphi)-\frac{\pi\,Sin.\varphi}{Sin.^4\lambda}\mathrm{E}(p,\varphi)+\frac{\pi\,Cos.\lambda}{Sin.^2\lambda}$

(12–13:) $Cos.\varphi = Cos.^2\lambda$, $p = Sin.\lambda.Cosec.\varphi$; Legendre, Exerc. Suppl. 49.

1) $\int Arcsin.x.\,(2\,lx+1)\,x\,dx = \frac{1}{4}\pi\left(l\,2-\frac{1}{2}\right)$ V. T. 163. N°. 4.

2) $\int Arcsin.x.\,(3\,lx+1)\,x^2\,dx = \frac{2}{3}\left(\frac{5}{6}-l\,2\right)$ V. T. 163. N°. 5.

3) $\int Arcsin.x.\,(4\,lx+1)\,x^3\,dx = \frac{3}{16}\pi\left(l\,2-\frac{7}{12}\right)$ V. T. 163. N°. 6.

4) $\int Arcsin.x.\,(5\,lx+1)\,x^4\,dx = \frac{8}{15}\left(\frac{47}{60}-l\,2\right)$ V. T. 163. N°. 7.

5) $\int Arcsin.x.\,lx\,\frac{dx}{x} = -\frac{1}{4}\pi\left\{(l\,2)^2+\frac{1}{12}\pi^2\right\}$ V. T. 164. N°. 1.

6) $\int Arcsin.x.\left\{x\,l(1-p^2x^2)-\frac{p^2x^3}{1-p^2x^2}\right\}dx=\frac{1}{8}\pi\left\{2l\frac{2(1-p^2)}{1+\sqrt{(1-p^2)}}+\frac{1-\sqrt{(1-p^2)}}{1+\sqrt{(1-p^2)}}\right\}, p<1;$ V. T. 165. N°. 17.

7) $\int Arcsin.x.\left\{l(1+q^2x^2)+\frac{q^2x^2}{1+q^2x^2}\right\}x\,dx=\frac{\pi}{4}\left\{l\frac{2(1+q^2)}{1+\sqrt{(1+q^2)}}-\frac{1-\sqrt{(1+q^2)}}{q^2}-\frac{1}{2}\right\}, q<1;$ V. T. 165. N°. 8.

8) $\int Arcsin.x.\left\{l(px+1)-\frac{px}{px+1}\right\}\frac{dx}{x}=\frac{1}{8}\pi^2-\frac{1}{2}(Arccos.p)^2-\frac{1}{2}\pi\,l(1+p), p^2<1;$ V. T. 165. N°. 3.

9) $\int Arcsin.x.\left\{l\frac{1+qx}{1-qx}-\frac{2qx}{1-q^2x^2}\right\}\frac{dx}{x^2}=\frac{\pi}{2}l\frac{1-q}{1+q}+\pi\,Arcsin.q$ V. T. 166. N°. 6.

10) $\int Arcsin.x.\left\{l\frac{1+x\,Sin.\lambda}{1-x\,Sin.\lambda}-\frac{2x\,Sin.\lambda}{1-x^2\,Sin.^2\lambda}\right\}\frac{dx}{x^2}=\pi\,l\,Cot.\left(\frac{\pi}{4}-\frac{1}{2}\lambda\right)+\pi\lambda$ V. T. 166. N°. 7.

11) $\int Arcsin.x.\left\{\frac{1}{x}l\frac{1+x}{1-x}+\frac{2}{1-x^2}\right\}\frac{dx}{x}=\infty$ V. T. 166. N°. 5.

12) $\int Arcsin.x.\left\{\frac{1+qx^2}{(1-qx^2)^2}l\frac{1+px}{1-px}+\frac{2p}{1-qx^2}\,\frac{x}{1-p^2x^2}\right\}dx=\frac{\pi}{2(1-q)}l\frac{1+p}{1-p}+$

$+\frac{\pi}{\sqrt{q}(1-q)}l\frac{p\sqrt{q}-\{1-\sqrt{(1-q)}\}\{1-\sqrt{(1-p^2)}\}}{p\sqrt{q}+\{1-\sqrt{(1-q)}\}\{1-\sqrt{(1-p^2)}\}}$ V. T. 166. N°. 17.

13) $\int Arccos.x.\left\{\frac{1+qx^2}{(1-qx^2)^2}l\frac{1+px}{1-px}+\frac{2p}{1-qx^2}\,\frac{x}{1-p^2x^2}\right\}dx=$

$\frac{\pi}{\sqrt{q}(1-q)}l\frac{p\sqrt{q}+\{1-\sqrt{(1-q)}\}\{1-\sqrt{(1-p^2)}\}}{p\sqrt{q}-\{1-\sqrt{(1-q)}\}\{1-\sqrt{(1-p^2)}\}}$ V. T. 166. N°. 17.

14) $\int Arccos.x.\{1+l(x^2)\}\,x\,dx = \frac{\pi}{4}\left(\frac{1}{2}-l\,2\right)$ V. T. 163. N°. 4.

15) $\int Arccos.x.\{1+l(x^3)\}\,x^2\,dx = -\frac{2}{3}\left(\frac{5}{6}-l\,2\right)$ V. T. 163. N°. 5.

16) $\int Arccos.x.\{1+l(x^4)\}\,x^3\,dx = -\frac{3\pi}{16}\left(l\,2-\frac{7}{12}\right)$ V. T. 163. N°. 6.

17) $\int Arccos.x.\{1+l(x^5)\}\,x^4\,dx = -\frac{8}{15}\left(\frac{47}{60}-l\,2\right)$ V. T. 163. N°. 7.

18) $\int Arccos.x.\left\{l(1+q^2x^2)+\frac{q^2x^2}{1+q^2x^2}\right\}x\,dx = \frac{\pi}{4}\left\{l\frac{1+\sqrt{(1+q^2)}}{2}+\frac{1-\sqrt{(1+q^2)}}{q^2}+\frac{1}{2}\right\}, q<1;$ V. T. 165. N°. 8.

19) $\int Arccos.x.\left\{x\,l(1-p^2x^2)-\frac{p^2x^3}{1-p^2x^2}\right\}dx = \frac{1}{8}\pi\left\{2l\frac{1+\sqrt{(1-p^2)}}{2}-\frac{1-\sqrt{(1-p^2)}}{1+\sqrt{(1-p^2)}}\right\}, p<1;$ V. T. 165. N°. 17.

20) $\int Arccos.x.\left\{l(px+1)-\frac{px}{px+1}\right\}\frac{dx}{x^2} = \frac{1}{2}(Arccos.p)^2-\frac{1}{8}\pi^2+\frac{1}{2}p\pi, p^2<1;$ V. T. 165. N°. 3.

21) $\int Arccos.x.\left\{l\frac{1+x}{1-x}-\frac{2x}{1-x^2}\right\}\frac{dx}{x^2} = -\pi\frac{2+\pi}{2}$ V. T. 166. N°. 5.

22) $\int Arccos.x.\left\{l\frac{1+qx}{1-qx}-\frac{2qx}{1-q^2x^2}\right\}\frac{dx}{x^2} = -\pi\{q+Arcsin.q\}$ V. T. 166. N°. 6.

23) $\int Arccos.x.\left\{l\frac{1+x\,Sin.\lambda}{1-x\,Sin.\lambda}-\frac{2x\,Sin.\lambda}{1-x^2\,Sin.^2\lambda}\right\}\frac{dx}{x^2} = -\pi\{\lambda+Sin.\lambda\}$ V. T. 166. N°. 7.

24) $\int Arctang.x.\,l\,x\frac{dx}{x} = -\frac{1}{32}\pi^3$ V. T. 154. N°. 1.

25) $\int Arctang.x.(lx)^3\frac{dx}{x} = -\frac{5}{256}\pi^5$ V. T. 155. N°. 1.

26) $\int Arctang.x.(lx)^5\frac{dx}{x} = -\frac{61}{1536}\pi^7$ V. T. 155. N°. 8.

27) $\int Arctang.x.(lx)^{q-1}\frac{dx}{x} = \frac{(-1)^{q-1}}{q}\Gamma(q+1)\sum_1^\infty\frac{(-1)^n}{(2n+1)^{q+1}}$ V. T. 158. N°. 1.

28) $\int Arctang.x\frac{(5-3x^2)\,lx+1-x^2}{(1-x^2)^2}x^4\,dx = \frac{\pi^2-4\pi-8}{32}$ V. T. 152. N°. 13.

29) $\int Arctang.\, x \frac{(1+x^2)lx+1-x^2}{(1+x^2)^2} dx = \frac{1}{4} l2$ V. T. 153. N°. 15.

30) $\int Arctang.\, x \frac{(3-x^2)lx+1-x^2}{(1-x^2)^2} x^2 dx = \pi \frac{\pi-12}{96}$ V. T. 152. N°. 17.

31) $\int Arctang.\, x \frac{(1+x^2)lx-x^2+1}{(1-x^2)^2} dx = -\pi \frac{4-\pi}{32}$ V. T. 152. N°. 16.

32) $\int Arctang.\, (x^2) \frac{(1+x^2)lx+1-x^2}{(1-x^2)^2} dx = \frac{\pi^2}{8(2+\sqrt{2})} - \frac{1}{16}\pi$ V. T. 153. N°. 16.

33) $\int \frac{Arctang.\,(lx)}{x} \frac{dx}{1-x^{-2\pi}} = \frac{1}{4} l 2\pi - \frac{1}{2}$ V. T. 299. N°. 2.

34) $\int \frac{Arctang.\left(\frac{1}{p} lx\right)}{x} \frac{dx}{1-x^{-2\pi}} = l\left[\left(\frac{p}{e}\right)^p \frac{1}{\Gamma(p)} \sqrt{\frac{2\pi}{p}}\right]$ V. T. 299. N°. 3.

F. Alg. irrat.
Log. en num.
Circ. Inv.

TABLE 423.

Lim. 0 et 1.

1) $\int Arcsin.\, x \frac{1+x^2+lx}{\sqrt{(1+x^2)^3}} dx = \frac{\pi}{8} l2$ V. T. 163. N°. 12.

2) $\int Arcsin.\, x \frac{2-l(1-p^2x^2)}{\sqrt{(1-p^2x^2)^3}} x\, dx = \frac{1}{2p^2} l(1-p^2).\left\{F'(p) - \frac{\pi}{\sqrt{(1-p^2)}}\right\}, p<1;$ V. T. 165. N°. 18.

3) $\int Arcsin.\, x \frac{3l(1-p^2x^2)-2}{\sqrt{(1-p^2x^2)^5}} x\, dx = \frac{1}{2p^2(1-p^2)}\left[\frac{\pi}{\sqrt{(1-p^2)}} l(1-p^2) + \right.$

$\left. + (2-p^2)\, 2\, F'(p) - \{4+l(1-p^2)\}\, E'(p)\right]$ V. T. 165. N°. 20.

4) $\int Arcsin.\, x.\left(\frac{p^2x^2lx}{1-p^2x^2}+1\right)\frac{dx}{x\sqrt{(1-p^2x^2)}} = \frac{1}{2} lp.F'(p) + \frac{1}{4}\pi F'\{\sqrt{(1-p^2)}\}, p<1;$ V. T. 163. N°. 14.

5) $\int Arcsin.\, x.\left\{\frac{p\,l(1+px^2)}{1-p^2x^2} + \frac{2}{1+px^2}\right\} \frac{x}{\sqrt{(1-p^2x^2)}} dx = -\frac{1}{2p}\left[l\left\{\frac{2(1+p)}{\sqrt{p}}\right\}.F'(p) - \right.$

$\left. - \frac{1}{4}\pi F'\{\sqrt{(1-p^2)}\} - \frac{\pi}{\sqrt{(1-p^2)}} l(1+p)\right], p^2<1;$ V. T. 165. N°. 9.

6) $\int Arcsin.\, x.\left\{\frac{p\,l(1-px^2)}{1-p^2x^2}-\frac{2}{1-px^2}\right\}\frac{x}{\sqrt{(1-p^2x^2)}}dx=-\frac{1}{2p}\left[l\left\{\frac{2(1-p)}{\sqrt{p}}\right\}.F'(p)-\right.$

$\left.-\frac{1}{4}\pi F'\{\sqrt{(1-p^2)}\}-\frac{\pi}{\sqrt{(1-p^2)}}l(1-p)\right]$, $p<1$; V. T. 165. N°. 19.

7) $\int Arcsin.\, x.\left\{l(1-p^2x^2 Sin.^2\lambda)-\frac{1-p^2x^2}{1-p^2x^2 Sin.^2\lambda}2\,Sin.^2\lambda\right\}\frac{x\,dx}{\sqrt{(1-p^2x^2)^3}}=$

$=\frac{1}{p^2}\left[\frac{\pi}{2\sqrt{(1-p^2)}}l(1-p^2 Sin.^2\lambda)-E'(p).\{F(p,\lambda)\}^2+2F'(p).\Upsilon(p,\lambda)\right]$, $p<1$; V.T.165. N°. 13.

8) $\int Arcsin.\, x.\left\{\frac{l(1-p^2x^4)}{1-p^2x^2}-\frac{4x^2}{1-p^2x^4}\right\}\frac{x}{\sqrt{(1-p^2x^2)}}dx=$

$-\frac{1}{2p^2}\left[l\left\{\frac{4(1-p^2)}{p}\right\}.F'(p)-\frac{1}{2}\pi F'\{\sqrt{(1-p^2)}\}-\frac{\pi}{\sqrt{(1-p^2)}}l(1-p^2)\right]$, $p<1$; V. T. 165. N°. 24.

9) $\int Arcsin.\, x.\left\{\frac{1}{\sqrt{(1-p^2x^2)}}l\frac{1+q\sqrt{(1-p^2x^2)}}{1-q\sqrt{(1-p^2x^2)}}-\frac{2q}{1-q^2(1-p^2x^2)}\right\}\frac{x}{(1-p^2x^2)^2}dx=$

$=\frac{\pi}{p^2}\left[\frac{1}{2\sqrt{(1-p^2)}}l\frac{1+q\sqrt{(1-p^2)}}{1-q\sqrt{(1-p^2)}}-F'\{\sqrt{(1-p^2)},Arcsin.\,q\}\right]$ V. T. 166. N°. 11.

10) $\int Arccos.\, x\,\frac{1+x^2+lx}{\sqrt{(1+x^2)^3}}dx=\frac{\pi}{8}l\frac{1}{2}$ V. T. 163. N°. 12.

11) $\int Arccos.\, x\,\frac{2-l(1-p^2x^2)}{\sqrt{(1-p^2x^2)^3}}x\,dx=\frac{1}{2p^2}l(1-p^2).F'(p)$, $p<1$; V. T. 165. N°. 18.

12) $\int Arccos.x\,\frac{3l(1-p^2x^2)-2}{\sqrt{(1-p^2x^2)^5}}xdx=\frac{1}{p^2(1-p^2)}\left[(p^2-2)F'(p)+\left\{2+\frac{1}{2}l(1-p^2)\right\}E'(p)\right]$ V. T. 165. N°. 20.

13) $\int Arccos.x.\left\{\frac{p^2l(1-x^2)}{1-p^2x^2}-\frac{2}{1-x^2}\right\}\frac{x\,dx}{\sqrt{(1-p^2x^2)}}=\frac{1}{2}l\left\{\frac{1-p^2}{p^2}\right\}.F'(p)-\frac{1}{2}\pi F'\{\sqrt{(1-p^2)}\}$, $p<1$; V.T.165. N°. 12.

14) $\int Arccos.\, x.\left\{\frac{p\,l(1+px^2)}{1-p^2x^2}+\frac{2}{1+px^2}\right\}\frac{x}{\sqrt{(1-p^2x^2)}}dx=$

$-\frac{1}{2p}\left[\frac{1}{4}\pi F'\{\sqrt{(1-p^2)}\}-l\left\{\frac{2(1+p)}{\sqrt{p}}\right\}.F'(p)\right]$, $p<1$; V. T. 165. N°. 9.

15) $\int Arccos.\, x.\left\{\frac{p\,l(1-px^2)}{1-p^2x^2}-\frac{2}{1-px^2}\right\}\frac{x}{\sqrt{(1-p^2x^2)}}dx=$

$-\frac{1}{2p}\left[\frac{1}{4}\pi F'\{\sqrt{(1-p^2)}\}-l\left\{\frac{2(1-p)}{\sqrt{p}}\right\}.F'(p)\right]$, $p<1$; V. T. 165. N°. 19.

16) $\int Arccos.\,x.\left\{l(1-p^2x^2\,Sin.^2\lambda)-\frac{1-p^2x^2}{1-p^2x^2\,Sin.^2\lambda}2\,Sin.^2\lambda\right\}\frac{x\,dx}{\sqrt{(1-p^2x^2)^3}}=$

$=\frac{1}{p^2}\left[E'(p).\{F(p,\lambda)\}^2-2F'(p).\Upsilon(p,\lambda)\right],\ p<1;$ V. T. 165. N°. 13.

17) $\int Arccos.\,x.\left\{\frac{l(1-p^2x^4)}{1-p^2x^2}-\frac{4x^2}{1-p^2x^4}\right\}\frac{x}{\sqrt{(1-p^2x^2)}}dx=$

$-\frac{1}{2p^2}\left[\frac{1}{2}\pi F'\{\sqrt{(1-p^2)}\}-l\left\{\frac{4(1-p^2)}{p}\right\}.F'(p)\right],\ p<1;$ V. T. 165. N°. 23.

18) $\int Arccos.\,x.\left\{\frac{1}{\sqrt{(1-p^2x^2)}}l\frac{1+q\sqrt{(1-p^2x^2)}}{1-q\sqrt{(1-p^2x^2)}}-\frac{2q}{1-q^2(1-p^2x^2)}\right\}\frac{x}{(1-p^2x^2)^2}dx=$

$=\frac{\pi}{p^2}\left[\frac{1}{2}l\frac{1-q}{1+q}+F\{\sqrt{(1-p^2)},\ Arcsin.\,q\}\right]$ V. T. 166. N°. 11.

19) $\int Arcsin.\,x.\,lx\frac{x\,dx}{\sqrt{(1-x^2)^3}}=\frac{1}{8}\pi^3-2\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 261. N°. 6 et T. 152. N°. 13.

20) $\int (Arcsin.\,x)^{p-1}.\,lx\frac{dx}{\sqrt{(1-x^2)}}=-\frac{1}{p}\left(\frac{1}{2}\pi\right)^p\left\{1-\sum_1^\infty\frac{2}{p+2m}\sum_1^\infty\frac{1}{(2n)^{2m}}\right\}$ V. T. 257. N°. 2.

21) $\int (Arccos.\,x)^{p-1}.\,l(1+x)\frac{dx}{\sqrt{(1-x^2)}}=\frac{1}{p}\left(\frac{\pi}{2}\right)^p\sum_1^\infty\left\{\frac{2^{2m}-1}{4^{m-1}}\frac{1}{p+2m}\sum_1^\infty\frac{1}{(2n)^{2m}}\right\}$ V. T. 259. N°. 3.

22) $\int (Arccos.\,x)^{p-1}.\,l(1-x)\frac{dx}{\sqrt{(1-x^2)}}=\frac{1}{p}\left(\frac{\pi}{2}\right)^p\left[-2+\sum_1^\infty\left\{\frac{1}{4^{m-1}}\frac{1}{p+2m}\sum_1^\infty\frac{1}{(2n)^{2m}}\right\}\right]$ V. T. 259. N°. 4.

23) $\int (Arccos.\,x)^{p-1}.\,l(1-x^2)\frac{dx}{\sqrt{(1-x^2)}}=\frac{2}{p}\left(\frac{1}{2}\pi\right)^p\left\{-1+\sum_1^\infty\frac{2}{p+2m}\sum_1^\infty\frac{1}{(2n)^{2m}}\right\}$ V. T. 259. N°. 8.

24) $\int lx\left\{\frac{x}{\sqrt{(1-x^2)}}Arcsin.x+p\right\}(Arcsin.x)^{p-1}\frac{dx}{1-x^2}=\left(\frac{\pi}{2}\right)^p\left[1+\sum_1^\infty\left\{\frac{1}{4^{m-1}}\frac{2^{2m-1}-1}{p+2m}\sum_1^\infty\frac{1}{(2n)^{2m}}\right\}\right]$ V. T. 261. N°. 24.

25) $\int l(1+x^2).\left\{\frac{x}{\sqrt{(1+x^2)^3}}Arcsin.x-\frac{1}{\sqrt{(1-x^4)}}\right\}dx=-\frac{\pi}{2}\sqrt{2}-\frac{\pi}{2\sqrt{2}}l2+\sqrt{2}.F'\left(Sin.\frac{\pi}{4}\right)$ V. T. 261. N°. 7.

26) $\int l(1+x^2).\left\{2\,Arctang.\,x-\frac{x}{1+x^2}\right\}\frac{dx}{x^3}=\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 260. N°. 4.

27) $\int l(1+x^2).\left\{2\,Arctg.x-\frac{px}{1+x^2}\right\}(Arctgx)^{p-1}\frac{dx}{x^3}=\left(\frac{\pi}{4}\right)^p\left[2-l2-\sum_1^\infty\frac{4}{p+2m}\sum_1^\infty\frac{1}{(4n)^{2m}}\right]$ V. T. 260. N°. 20.

28) $\int l(1-x^2).\left\{Arccos.x+\frac{x}{\sqrt{(1-x^2)}}\right\}\frac{dx}{x^2} = -4\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 258. N°. 13.

29) $\int l(Sin.^2\lambda-x^2).\left\{Arccos.x+\frac{x}{\sqrt{(1-x^2)}}\right\}\frac{dx}{x^2} = -4\,Cosec.\lambda\sum_0^\infty\frac{Sin.\{(2n+1)\lambda\}}{(2n+1)^2}$ V. T. 258. N°. 15.

30) $\int l(1-x^2\,Sin.^2\mu)\frac{1-x^2\,Sin.^2\lambda-2x\,Sin.^2\lambda.\,Arccos.x.\sqrt{(1-x^2)}}{(1-x^2\,Cos.^2\lambda)^2}\frac{dx}{\sqrt{(1-x^2)}} =$

$= \frac{2\pi}{Cos.^2\lambda-Cos.^2\mu}\,l\,Cos.\frac{1}{2}\mu.Sec.\frac{1}{2}\lambda$ V. T. 260. N°. 11.

31) $\int l(Cos.^2\mu+x^2\,Sin.^2\mu)\frac{Cos.^2\lambda+x^2\,Sin.^2\lambda-2x\,Sin.^2\lambda.\,Arcsin.x.\sqrt{(1-x^2)}}{(Cos.^2\lambda+x^2\,Sin.^2\lambda)^2}\frac{dx}{\sqrt{(1-x^2)}} =$

$= \frac{2\pi\,Sin.^2\mu}{Sin.(\lambda+\mu).\,Sin.(\lambda-\mu)}\,l\,Cos.\frac{1}{2}\lambda.Sec.\frac{1}{2}\mu$ V. T. 260. N°. 6.

F. Alg.
Log. en dén.
Circ. Inv.

TABLE 424.

Lim. 0 et 1.

1) $\int Arctang.x\,\frac{1-x}{x}\,\frac{2x\,lx-x+1}{(lx)^2}\,dx = l\frac{4}{\pi}$ V. T. 172. N°. 1.

2) $\int Arctang.x\,\frac{x^q-x^{-q}-q(x^q+x^{-q})\,lx}{x(lx)^2}\,dx = l\,Tang.\left\{\frac{1+q}{4}\pi\right\}$ V. T. 174. N°. 5.

3) $\int Arctang.x\,\frac{lx}{\{\pi^2+(lx)^2\}^2}\,\frac{dx}{x} = \frac{3-\pi}{8\pi}$ V. T. 173. N°. 7.

4) $\int Arctang.x\,\frac{lx}{\{\pi^2+4(lx)^2\}^2}\,\frac{dx}{x} = \frac{l2-1}{32\pi}$ V. T. 173. N°. 8.

5) $\int Arctang.x\,\frac{lx}{\{q^2+(lx)^2\}^2}\,\frac{dx}{x} = \frac{1}{8q}\left\{-\frac{\pi}{q}+Z'\left(\frac{2q+3\pi}{4\pi}\right)-Z'\left(\frac{2q+\pi}{4\pi}\right)\right\}$ V. T. 173. N°. 9.

6) $\int Arccot.x\,\frac{lx}{\{\pi^2+(lx)^2\}^2}\,\frac{dx}{x} = \frac{\pi-5}{8\pi}$ V. T. 173. N°. 7.

7) $\int Arccot.x\,\frac{lx}{\{\pi^2+(lx^2)^2\}^2}\,\frac{dx}{x} = -\frac{1+l2}{32\pi}$ V. T. 173. N°. 8.

8) $\int Arccot.x\,\frac{lx}{\{q^2+(lx)^2\}^2}\,\frac{dx}{x} = \frac{1}{8q}\left\{-\frac{\pi}{q}+Z'\left(\frac{2q+\pi}{4\pi}\right)-Z'\left(\frac{2q+\pi}{4\pi}\right)\right\}$ V. T. 173. N°. 9

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9) $\int \frac{Arccos. x}{(Arccos. x)^2 + (l x)^2} \frac{dx}{x} = \frac{\pi}{2 l 2}$ V. T. 412. N°. 1.

10) $\int \frac{Arccos. x}{(Arccos. x)^2 + (l x)^2} \frac{x}{1-x^2} dx = \infty$ V. T. 412. N°. 8.

11) $\int \frac{l x}{(Arccos. x)^2 + (l x)^2} \frac{l x}{(1-p)^2 - 4 p x^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2} \frac{\pi}{p^2-1} \left\{ \frac{1}{l 2 - l(1+p)} - \frac{1+p}{1-p} \right\}, p^2 \leqq 1;$ V. T. 412. N°. 9.

12) $\int \frac{Arccos. x}{(Arccos. x)^2 + (l x)^2} \frac{x}{(1-p)^2 - 4 p x^2} dx = \frac{\pi}{8 p} \left\{ \frac{1}{l 2 - l(1+p)} - \frac{1}{l 2} \right\}$ V. T. 412. N°. 10.

13) $\int \frac{l x}{(Arccos. x)^2 + (l x)^2} \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2} \pi \left(1 - \frac{1}{l 2}\right)$ V. T. 412. N°. 2.

14) $\int \frac{l(1+2 x^2)}{(Arccos. x)^2 + (l x)^2} \frac{dx}{\sqrt{(1-x^2)^3}} = \frac{1}{2} \pi$ V. T. 412. N°. 7.

15) $\int \frac{l x}{(Arccos. x)^2 + (l x)^2} \frac{dx}{x^2 \sqrt{(1-x^2)}} = \infty$ V. T. 412. N°. 6.

16) $\int \frac{(Arccos. x)^2 + 2 Arccos. x . l x \frac{x}{\sqrt{(1-x^2)}} - (l x)^2}{\{(Arccos. x)^2 + (l x)^2\}^2} \frac{Arccos. x}{x} dx = \pi \frac{l 2 - 1}{2 l 2}$ V. T. 424. N°. 13.

17) $\int \frac{(Arccos. x)^2 - 2 Arccos. x . l x \frac{\sqrt{(1-x^2)}}{x} - (l x)^2}{\{(Arccos. x)^2 + (l x)^2\}^2} \frac{l x}{\sqrt{(1-x^2)}} dx = -\frac{\pi}{2 l 2}$ V. T. 424. N°. 9.

F. Alg.
Log.
Circ. Inv.

TABLE 425.

Lim. 0 et ∞.

1) $\int Arctang. x . (l x)^{2a-1} \frac{dx}{x} = \infty$ V. T. 180. N°. 5.

2) $\int Arctang. x . l x . \left\{ Arctang. x - \frac{2 x}{1+x^2} \right\} \frac{dx}{x^2} = \pi l 2$ V. T. 264. N°. 4.

3) $\int Arctang. x . \left\{ l x + \frac{1+x^2}{1-x^2} \right\} \frac{1-x^2}{(1+x^2)^2} dx = 0$ V. T. 182. N°. 2.

4) $\int Arctang. \frac{x}{q} . \left\{ \frac{q^2-x^2}{q^2+x^2} l(1+x) + \frac{x}{1+x} \right\} \frac{dx}{q^2+x^2} = -\frac{1}{2(1+q^2)} \left\{ \frac{\pi}{2} + q l q \right\}$ V. T. 182. N°. 8.

5) $\int Arctang.\frac{x}{q}.\left\{\frac{q^2-x^2}{q^2+x^2}l(1-x)^2-\frac{2x}{1-x}\right\}\frac{dx}{q^2+x^2}=\frac{1}{1+q^2}\left\{\frac{\pi}{2}-q\,l\,q\right\}$ V. T. 182. N°. 9.

6) $\int Arctang.\frac{x}{q}.\left\{\frac{q^2-x^2}{q^2+x^2}l(1-x^2)^2-\frac{4x^2}{1-x^2}\right\}\frac{dx}{q^2+x^2}=-\frac{2q}{1+q^2}lq$ V. T. 182. N°. 20.

7) $\int Arctg.x\frac{\{(p-1)x^p-(q-1)x^q\}lx-x^p+x^q}{x^2(lx)^2}dx=\pi l\left(Tg.\frac{1}{4}p\pi.\ Cot.\frac{1}{4}q\pi\right), p<1, q<1;$ V.T. 180. N°. 7.

8) $\int l\,x.\ Arctang.\,x\frac{x}{(1+x^2)^2}dx=\frac{\pi}{4}l\,2$ V. T. 266. N°. 1 et T. 182. N°. 2.

9) $\int lx.\left\{2x\,Arctang.\,x-\frac{q^2+x^2}{1+x^2}\right\}\frac{dx}{(q^2+x^2)^2}=\frac{\pi}{2q^2}l(1+q)$ V. T. 266. N°. 2.

10) $\int lx.\left\{2x\,Arctang.\frac{x}{p}-p\frac{q^2+x^2}{p^2+x^2}\right\}\frac{dx}{(q^2+x^2)^2}=\frac{1}{2q^2}\pi l\frac{p+q}{p}$ V. T. 266. N°. 5.

11) $\int lx.\left\{2x\,Arctang.\frac{x}{p}+p\frac{q^2-x^2}{p^2+x^2}\right\}\frac{dx}{(q^2-x^2)^2}=\frac{\pi}{4q^2}l\frac{p^2+q^2}{p^2}$ V. T. 266. N°. 6.

12) $\int lx.\left\{\frac{Arctg.\frac{x}{a}.Arctg.\frac{x}{b}}{x^2}-\frac{a}{x(a^2+x^2)}Arctg.\frac{x}{b}-\frac{b}{x(b^2+x^2)}Arctg.\frac{x}{a}\right\}dx=\frac{1}{2}\pi\left\{\frac{1}{a}l\frac{a+b}{b}+\frac{1}{b}l\frac{a+b}{a}\right\}$ V.T. 264. N°. 14.

13) $\int lx.(1-2x\,Arctang.\,x)\frac{dx}{\sqrt{(1+x^2)^3}}=2\sum_0^\infty\frac{(-1)^{n-1}}{(2n+1)^2}$ V. T. 268. N°. 2.

14) $\int l\,x.\ Arctang.\left(x\sqrt{\frac{p^2-1}{1+x^2}}\right)\frac{x}{\sqrt{(1+x^2)^3}}dx=\frac{\pi}{2}l\{p+\sqrt{(p^2-1)}\}-$

$-\frac{\pi\sqrt{(p^2-1)}}{2p}lp\ ,\ p\geqq 1;$ V. T. 269. N°. 10 et T. 180. N°. 10.

15) $\int l(1+x).\left\{Arctang.\,x-\frac{2x}{1+x^2}\right\}\frac{dx}{x\sqrt{x}}=\frac{1}{2}\pi^2$ V. T. 268. N°. 1.

16) $\int l(1+x).\left\{Arccot.\,x+\frac{2x}{1+x^2}\right\}\frac{dx}{x\sqrt{x}}=\frac{1}{2}\pi^2$ V. T. 268. N°. 9.

17) $\int l(1+x^2).\ Arccot.\,x\frac{x}{(1+x^2)^2}dx=\frac{\pi}{4}(1-l2)$ V. T. 267. N°. 17 et T. 182. N°. 12.

18) $\int l(1+x^2).\ Arctang.\,x\frac{dx}{x^2}=\frac{1}{3}\pi^2$ V. T. 265. N°. 1 et T. 184. N°. 14.

F. Alg. Log. Circ. Inv.	TABLE 425 suite.	Lim. 0 et ∞.

19) $\int l(1+x^2).\,Arctang.\,x\,\frac{x}{(1+x^2)^2}\,dx = \frac{1}{4}\pi l2$ V. T. 267. N°. 2 et T. 182. N°. 12.

20) $\int l(1+x^2).\,Arccot.\,x\,\frac{dx}{x^2} = \frac{1}{6}\pi^2$ V. T. 265. N°. 9 et T. 184. N°. 14.

21) $\int l(1+x^2).\left\{2\,Arctang.\,x - \frac{x}{1+x^2}\right\}\frac{dx}{x^3} = \pi l2$ V. T. 266. N°. 1.

22) $\int l(1+x^2).\left\{Arccot.\,x + \frac{px}{1+x^2}\right\}(Arccot.\,x)^{p-1}\frac{dx}{x^2} = \frac{\pi^{p+1}}{2^p(p+1)}$ V. T. 265. N°. 27.

23) $\int l(q^2+x^2).\left\{2\,Arctang.\,x - \frac{x}{1+x^2}\right\}\frac{dx}{x^3} = \frac{\pi}{q^2}l(1+q)$ V. T. 266. N°. 2.

24) $\int l(q^2+x^2).\left\{2\,Arctang.\frac{x}{p} - \frac{px}{p^2+x^2}\right\}\frac{dx}{x^3} = \frac{\pi}{q^2}l\frac{p+q}{p}$ V. T. 266. N°. 5.

25) $\int l(q^2+x^2).\left\{2x\,Arctang.\,x - \frac{q^2+x^2}{1+x^2}\right\}\frac{dx}{(q^2+x^2)^2} = \frac{\pi}{2q(q+1)}$ V. T. 267. N°. 3.

26) $\int l(q^2+x^2).\left\{2x\,Arctang.\frac{x}{p} - p\frac{q^2+x^2}{p^2+x^2}\right\}\frac{dx}{(q^2+x^2)^2} = \frac{\pi}{2q(p+q)}$ V. T. 267. N°. 5.

27) $\int l(q^2+x^2).\left\{\frac{2x}{x^2-q^2}Arctang.\,px - \frac{p}{1+p^2x^2}\right\}\frac{dx}{x^2-q^2} = \frac{\pi}{4q^2}l\frac{(pq+1)^2}{p^2q^2+1}$ V. T. 265. N°. 4.

28) $\int l(1+x^2).\{2Arctg.x - px\}(Arctg.x)^{p-1}\frac{dx}{(p^2+x^2)x^3} = \left(\frac{\pi}{2}\right)^p\left[1-\sum_1^\infty\frac{2}{p+2m}\sum_1^\infty\frac{1}{(2n)^{2m}}\right]$ V. T. 266. N°. 10.

29) $\int l(q^2-x^2).\left\{2\,Arctang.\,px - \frac{px}{1+p^2x^2}\right\}\frac{dx}{x^3} = \frac{\pi}{2q^2}l(p^2q^2+1)$ V. T. 266. N°. 6.

F. Alg. Log. Circ. Inv.	TABLE 426.	Lim. 1 et ∞.

1) $\int Arctang.x.(lx)^p\frac{dx}{x} = \infty$ V. T. 187. N°. 6.

2) $\int Arctang.\,x.\left(\frac{1-x^2}{1+x^2}lx+1\right)\frac{dx}{1+x^2} = -\frac{1}{4}l2$ V. T. 187. N°. 4.

3) $\int Arctang.(lx)\frac{1}{x^{2\pi}-1}\frac{dx}{x} = \frac{1}{2}-\frac{1}{4}l2\pi$ V. T. 299. N°. 2.

4) $\int Arctang.(p\,l\,x)\frac{1}{x^{2\pi}-1}\frac{dx}{x} = \frac{1}{2}\,l\left[\left(\frac{e}{p}\right)^p\Gamma(p)\sqrt{\frac{p}{2\pi}}\right]$ V. T. 299. N°. 3.

5) $\int Arccot.x.\,lx\frac{dx}{x^2} = \frac{1}{4}\pi-\frac{1}{2}(l\,2)^2-\frac{1}{24}\pi^2-\frac{1}{2}l\,2$ V. T. 270. N°. 5 et T. 187. N°. 8.

6) $\int Arccot.x.\left\{\frac{1-x^2}{1+x^2}lx+1\right\}\frac{dx}{1+x^2} = \frac{1}{4}l\,2$ V. T. 187. N°. 4.

7) $\int lx.\{2\,x\,Arctang.x-1\}\frac{dx}{(1+x^2)^2} = \frac{3}{8}\pi\,l\,2-\frac{1}{2}\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 2.

8) $\int lx.\left\{Arccot.\frac{x}{p}+\frac{px}{p^2+x^2}\right\}\frac{dx}{x^2} = Arctang.p-\frac{1}{2p}l(1+p^2)$ V. T. 270. N°. 6.

9) $\int lx.\,Arccosec.x\frac{dx}{x^2} = l2+\frac{\pi}{2}-2$ V. T. 270. N°. 9 et T. 187. N°. 5.

10) $\int l(1+x^2).\left\{2\,Arctang.x-\frac{x}{1+x^2}\right\}\frac{dx}{x^3} = \frac{3\pi}{8}l\,2-\frac{1}{2}\sum_0^\infty\frac{(-1)^n}{(2n+1)^2}$ V. T. 270. N°. 2.

F. Alg.
Log.
Circ. Inv.

TABLE 427.

Lim. diverses.

1) $\int_0^{\sqrt{\frac{1}{2}}}(Arcsin.x)^{p-1}.lx\frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2p}\left(\frac{\pi}{4}\right)^p\left[-l2-2+\sum_1^\infty\frac{4}{p+2m}\sum_1^\infty\frac{1}{(4n)^{2m}}\right]$ V. T. 271. N°. 5.

2) $\int_{\sqrt{\frac{1}{2}}}^1(Arccos.x)^{p-1}.l(1-x^2)\frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{p}\left(\frac{\pi}{4}\right)^p\left[l2-2+\sum_1^\infty\frac{4}{p+2m}\sum_1^\infty\frac{1}{(4n)^{2m}}\right]$ V. T. 271. N°. 7.

F. Alg.
Log.
Autres Fonctions.

TABLE 428.

Lim. diverses.

1) $\int_0^1 li.(x).\left(l\frac{1}{x}\right)^{p-1}\frac{dx}{x} = -\frac{1}{p}\Gamma(p),\ 1\geqq p\geqq 0;$ V. T. 402. N°. 3.

2) $\int_0^1 li.(x).\left(l\frac{1}{x}\right)^{p-1}\frac{dx}{x^2} = -\pi\,Cosec.p\pi.\Gamma(p),\ 1\geqq p\geqq 0;$ V. T. 402. N°. 2.

3) $\int_0^1 li.(x)\frac{x^{p-1}}{\sqrt{l\frac{1}{x}}}dx = -2\sqrt{\frac{\pi}{p}}.\,l\,\{\sqrt{p}+\sqrt{(1+p)}\},\ p<1;$ V. T. 300. N°. 4.

4) $\int_0^1 li.(x)\frac{dx}{x^{p+1}\sqrt{l\frac{1}{x}}} = -2\sqrt{\frac{\pi}{p}}.Arcsin.(\sqrt{p}),\ p<1;$ V. T. 800. N°. 5.

5) $\int_1^\infty li.(x).(lx)^{p-1}\frac{dx}{x^2} = -\pi\, Cot.p\pi.\Gamma(p)$ V. T. 402. N°. 1.

1) $\int\frac{x}{Sin.x.Tang.x}\left\{Arctg.(pSin.x)-\frac{p\,Sin.x}{1+p^2Sin.^2x}\right\}dx = \frac{\pi}{2}\left[-Arctg.p+l\{p+\sqrt{(1+p^2)}\}\right]$ V. T. 369. N°. 3.

2) $\int\frac{x}{Cos.x}\left\{Arctang.(p\,Cos.x)-\frac{p\,Cos.x}{1+p^2\,Cos.^2x}\right\}Tang.x\,dx = \frac{\pi}{2}\left[p-l\{p+\sqrt{(1+p^2)}\}\right]$ V. T. 369. N°. 6.

3) $\int\left[Arctang.\left\{\frac{Cot.\lambda}{\sqrt{(1-p^2\,Sin.^2x)}}\right\}-\frac{1}{2}\frac{Sin.2\lambda.\sqrt{(1-p^2\,Sin.^2x)}}{1-p^2\,Sin.^2\lambda.Sin.^2x}\right]\frac{x\,Sin.2x}{\sqrt{(1-p^2\,Sin.^2x)}}dx =$

$= \frac{\pi}{p^2}\left[E(p,\varphi)-\frac{Cot.\lambda}{\sqrt{(1-p^2)}}\{\sqrt{(1-p^2\,Sin.^2\varphi)}-\sqrt{(1-p^2)}\}-\varphi\sqrt{(1-p^2)}\right]$ V. T. 368. N°. 13.

4) $\int\left[Arctang.\left\{\frac{Cot.\lambda}{\sqrt{(1-p^2\,Sin.^2x)}}\right\}+\frac{1}{2}\frac{Sin.2\lambda.\sqrt{(1-p^2\,Sin.^2x)}}{1-p^2\,Sin.^2\lambda.Sin.^2x}\right]\frac{x\,Sin.2x}{\sqrt{(1-p^2\,Sin.^2x)^3}}dx =$

$= \frac{\pi}{p^2}\left[\frac{\varphi}{\sqrt{(1-p^2)}}-F(p,\varphi)\right]$ V. T. 369. N°. 15.

5) $\int\left[Arctang.\left\{\frac{Cot.\lambda}{\sqrt{(1-p^2\,Sin.^2x)}}\right\}+\frac{1}{6}\frac{Sin.2\lambda.\sqrt{(1-p^2\,Sin.^2x)}}{1-p^2\,Sin.^2\lambda.Sin.^2x}\right]\frac{x\,Sin.2x}{\sqrt{(1-p^2\,Sin.^2x)^5}}dx =$

$= \frac{\pi}{3p^2}\left[\frac{\varphi}{\sqrt{(1-p^2)^3}}-\frac{1}{1-p^2}E(p,\varphi)+\frac{Tang.\lambda}{\sqrt{(1-p^2)}}\{1-\sqrt{(1-p^2\,Sin.^2\varphi)}\}\right]$ V. T. 369. N°. 17.

Dans les formules 3) à 5) on a $Cot.\varphi = Tang.\lambda.\sqrt{(1-p^2)}$.

6) $\int\left[Arctang.\{Tang.\lambda.\sqrt{(1-p^2\,Sin.^2x)}\}+\frac{1}{2}\frac{Sin.2\lambda.\sqrt{(1-p^2Sin.^2x)}}{1-p^2\,Sin.^2\lambda.Sin.^2x}\right]\frac{x\,Sin.2x}{\sqrt{(1-p^2\,Sin.^2x)}}dx =$

$= \frac{\pi}{p^2}\left[E(p,\lambda)-Cot.\lambda.\{1-\sqrt{(1-p^2Sin.^2\lambda)}\}-\sqrt{(1-p^2)}.Arctg.\{Tang.\lambda.\sqrt{(1-p^2)}\}\right]$ V. T. 368. N°. 11.

7) $\int\left[Arctang.\left\{Tang.\lambda.\sqrt{(1-p^2 Sin.^2 x)}\right\}-\frac{1}{2}\frac{Sin.2\lambda.\sqrt{(1-p^2 Sin.^2 x)}}{1-p^2 Sin.^2\lambda. Sin.^2 x}\right]\frac{x\, Sin.2x}{\sqrt{(1-p^2 Sin.^2 x)^3}}dx =$

$= \frac{\pi}{p^2}\left[\frac{1}{\sqrt{(1-p^2)}}Arctang.\left\{Tang.\lambda.\sqrt{(1-p^2)}\right\}-F(p,\lambda)\right]$ V. T. 369. N°. 14.

8) $\int\left[Arctang.\left\{Tang.\lambda.\sqrt{(1-p^2 Sin.^2 x)}\right\}-\frac{1}{6}\frac{Sin.2\lambda.\sqrt{(1-p^2 Sin.^2 x)}}{1-p^2 Sin.^2\lambda. Sin.^2 x}\right]\frac{x\, Sin.2x}{\sqrt{(1-p^2 Sin.^2 x)^5}}dx =$

$= \frac{\pi}{3p^2}\left[\frac{1}{\sqrt{(1-p^2)^3}}Arctg.\left\{Tg.\lambda.\sqrt{(1-p^2)}\right\}-\frac{1}{1-p^2}E(p,\lambda)+\frac{Tg.\lambda}{1-p^2}\left\{\sqrt{(1-p^2 Sin.^2\lambda)}-\sqrt{(1-p^2)}\right\}\right]$ V.T. 369. N°. 16.

F. Alg.
Circ. Dir.
Circ. Inv.

TABLE 430.

Lim. 0 et π.

1) $\int Arctg.\left\{\frac{p\, Sin.x}{1-p\, Cos.x}\right\}.Sin.ax.x^{2b}\,dx = \frac{(-1)^b\pi p^a}{2a^{2b+1}}1^{2b/1}\sum_0^{2b}\frac{(-alp)^n}{1^{n/1}}$

2) $\int Arctg.\left\{\frac{p\, Sin.x}{1-p\, Cos.x}\right\}.Cos.ax.x^{2b+1}\,dx = \frac{(-1)^{b+1}\pi p^a}{2a^{2b+2}}1^{2b+1/1}\sum_0^{2b+1}\frac{(-alp)^n}{1^{n,1}}$

3) $\int Arctg.\left\{\frac{2p\, Sin.x}{1-p^2}\right\}.Sin.2ax.x^{\pm 2b}\,dx = 0$

4) $\int Arctg.\left\{\frac{2p\, Sin.x}{1-p^2}\right\}.Sin.\{(2a-1)x\}.x^{2b}\,dx = \frac{(-1)^b\pi p^{2a-1}}{2^{2b}(2a-1)^{2b+1}}1^{2b/1}\sum_0^{2b}\frac{\{-(2a-1)lp\}^n}{1^{n/1}}$

5) $\int Arctg.\left\{\frac{2p\, Sin.x}{1-p^2}\right\}.Cos.2ax.x^{\pm 2b+1}\,dx = 0$

6) $\int Arctg.\left\{\frac{2p\, Sin.x}{1-p^2}\right\}.Cos.\{(2a-1)x\}.x^{2b+1}dx = \frac{(-1)^b\pi p^{2a-1}}{2^{2b+1}(2a-1)^{2b+2}}1^{2b+1/1}\sum_0^{2b+1}\frac{\{-(2a-1)lp\}^n}{1^{n/1}}$

7) $\int Arctg.\left\{\frac{2p\, Sin.x}{1-p^2}\right\}.Sin.\{(2a-1)x\}.Sin.x.x^{\pm 2b+1}\,dx = 0$

8) $\int Arctg.\left\{\frac{2p\, Sin.x}{1-p^2}\right\}.Sin.\{(2a-1)x\}.Cos.x.x^{\pm 2b}\,dx = 0$

9) $\int Arctg.\left\{\frac{2p\, Sin.x}{1-p^2}\right\}.Cos.\{(2a-1)x\}.Sin.x.x^{\pm 2b}\,dx = 0$

10) $\int Arctg.\left\{\frac{2p\, Sin.x}{1-p^2}\right\}.Cos.\{(2a-1)x\}.Cos.x.x^{\pm 2b+1}\,dx = 0$

(1–10): $p^2 < 1,\ 0 \leqq q < 1$

Bierens de Haan Gr. 13. 193.

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11) $$\int Arctg.\left\{\frac{q\,Sin.2x}{1-q\,Cos.2x}\right\}.Sin.2ax.x^{2b}\,dx=\frac{(-1)^b\pi q^a}{2^{2b}a^{2b+1}}1^{2b/1}\sum_0^{2b}\frac{(-a\,l\,q)^n}{1^{n/1}}$$

12) $$\int Arctg.\left\{\frac{q\,Sin.2x}{1-q\,Cos.2x}\right\}.Sin.\{(2a-1)x\}.x^{\pm 2b}\,dx=0$$

13) $$\int Arctg.\left\{\frac{q\,Sin.2x}{1-q\,Cos.2x}\right\}.Cos.2ax.x^{2b+1}dx=\frac{(-1)^{b+1}\pi q^a}{2^{2b+1}a^{2b+2}}1^{2b+1/1}\sum_0^{2b+1}\frac{(-alq)^n}{1^{n/1}}$$

14) $$\int Arctg.\left\{\frac{q\,Sin.2x}{1-q\,Cos.2x}\right\}.Cos.\{(2a-1)x\}.x^{\pm 2b+1}\,dx=0$$

15) $$\int Arctg.\left\{\frac{q\,Sin.2x}{1-q\,Cos.2x}\right\}.Sin.2ax.Sin.x.x^{\pm 2b+1}\,dx=0$$

16) $$\int Arctg.\left\{\frac{q\,Sin.2x}{1-q\,Cos.2x}\right\}.Sin.2ax.Cos.x.x^{\pm 2b}\,dx=0$$

17) $$\int Arctg.\left\{\frac{q\,Sin.2x}{1-q\,Cos.2x}\right\}.Cos.2ax.Sin.x.x^{\pm 2b}\,dx=0$$

18) $$\int Arctg.\left\{\frac{q\,Sin.2x}{1-q\,Cos.2x}\right\}.Cos.2ax.Cos.x.x^{\pm 2b+1}\,dx=0$$

(11–18) $,p^2<1, 0<q<1;$ Bierens de Haan, Gr. 13. 193.

19) $$\int(a^2+2abCos.x+b^2)^{\frac{1}{2}p}Cos.\left\{cx-pArctg.\left(\frac{aSin.bx}{b+aCos.x}\right)\right\}dx=\frac{\pi a^c b^{p-c}\Gamma(1+p)}{\Gamma(1+c)\Gamma(1+p-c)},\ b>a;$$ Hill, Cr. 7. 102. (faut.) — Clausen, Cr. 7. 309.

F. Alg. rat. fract.
Circ. Dir.
Circ. Inv.

TABLE 451.

Lim. 0 et ∞.

1) $$\int Arctang.x.Cos.ax\frac{dx}{x}=-\frac{1}{2}\pi\,l\,i.(e^{-a})$$ Schlömilch, Beitr. III. § 8.

2) $$\int Arctang.\frac{x}{a}.Cos.x\frac{dx}{x}=-\frac{1}{2}\pi\,li.(e^{-a})$$

3) $$\int Arctang.\left(\frac{q-\frac{1}{q}}{1+x^2}x\right).Cos.x\frac{dx}{x}=\frac{1}{2}\pi\left\{l\,i.(e^{-q})-l\,i.(e^{-\frac{1}{q}})\right\}$$

(2–3) Schlömilch, Gr. 9. 307.

4) $$\int Arctg.(cx).Sin.px\frac{dx}{x^2+q^2}=\frac{\pi}{4q}e^{-pq}\left\{\frac{1}{2}l\left(\frac{1+qc}{1-qc}\right)^2+Ei.\left(-\frac{p}{c}+pq\right)\right\}-\frac{\pi}{4q}e^{pq}Ei.\left(-\frac{p}{c}-pq\right)$$

5) $$\int Arctang.\left(\frac{x}{q}\right).Sin.px\frac{dx}{x^2+q^2}=\frac{\pi}{4q}e^{-pq}\{A+l(2pq)\}-\frac{\pi}{4q}e^{pq}Ei.(-2pq)$$

(4–5) Arndt, Gr. 11. 70.

6) $\int Arctg.(cx).Cos.px\frac{x}{x^2+q^2}dx=-\frac{1}{4}\pi e^{-pq}\left\{\frac{1}{2}l\left(\frac{1+qc}{1-qc}\right)^2+Ei.\left(-\frac{p}{c}+pq\right)\right\}-\frac{1}{4}\pi e^{pq}Ei.\left(-\frac{p}{c}-pq\right)$

7) $\int Arctg.\left(\frac{x}{q}\right).Cos.px\frac{x}{x^2+q^2}dx=-\frac{1}{4}\pi e^{-pq}\{A+l(2pq)\}-\frac{1}{4}\pi e^{pq}Ei.(-2pq)$

Arndt, Gr.11. 70.

8) $\int Arctang.\left(\frac{p\,Sin.x}{1+p\,Cos.x}\right)\frac{x}{x^2+q^2}dx=\frac{1}{2}\pi l(1+pe^{-q}),\ p^2\leqq 1;$

9) $\int Arctang.\left(Tang.\frac{1}{2}x\right)\frac{x}{x^2+q^2}dx=\frac{1}{2}\pi l\frac{e^q+1}{e^q}$

10) $\int Arctang.\left(Cot.\frac{1}{2}x\right)\frac{x}{x^2+q^2}dx=\frac{1}{2}\pi l\frac{e^q}{e^q-1}$

Schlömilch, Stud. II. 18.

11) $\int Arctang.\left(\frac{p\,Sin.rx}{1+p\,Cos.rx}\right)\frac{x}{x^2+q^2}dx=\frac{\pi}{2}l(1+pe^{-qr})$ Boncompagni, Cr. 25. 74; où il y a faut. qdx.

F. Alg. irrat. fract. à dén. binôme.
Circ. Dir. TABLE 432. Lim. 0 et ∞.
Circ. Inv.

1) $\int Sin.\left\{q\,Arctg.\left(\frac{a\,Sin.cx}{1+aCos.cx}\right)\right\}.(1+2aCos.cx+a^2)^{\frac{1}{2}q}\frac{x}{p^2+x^2}dx=\frac{1}{2}\pi(1+ae^{-cp})^q-\frac{1}{2}\pi$

2) $\int Cos.\left\{q\,Arctg.\left(\frac{a\,Sin.cx}{1+a\,Cos.cx}\right)\right\}.(1+2a\,Cos.cx+a^2)^{\frac{1}{2}q}\frac{dx}{p^2+x^2}=\frac{\pi}{2p}(1+ae^{-cp})^q$

3) $\int Sin.\left\{q\,Arctg.\left(\frac{aSin.cx}{1+aCos.cx}\right)\right\}.\frac{(1+2aCos.cx+a^2)^{\frac{1}{2}q}+(1+2aCos.cx+a^2)^{-\frac{1}{2}q}}{p^2+x^2}xdx=$

$=\frac{1}{2}\pi\{(1+ae^{-cp})^q-(1+ae^{-cp})^{-q}\}$

4) $\int Sin.\left\{q\,Arctg.\left(\frac{a\,Sin.cx}{1+a\,Cos.cx}\right)\right\}\frac{(1+2a\,Cos.cx+a^2)^{\frac{1}{2}q}-(1+2a\,Cos.cx+a^2)^{-\frac{1}{2}q}}{p^2+x^2}xdx=$

$=\frac{1}{2}\pi\{(1+ae^{-cp})^q+(1+ae^{-cp})^{-q}-2\}$

5) $\int Cos.\left\{q\,Arctg.\left(\frac{a\,Sin.cx}{1+a\,Cos.cx}\right)\right\}\frac{(1+2a\,Cos.cx+a^2)^{\frac{1}{2}q}+(1+2a\,Cos.cx+a^2)^{-\frac{1}{2}q}}{p^2+x^2}dx=$

$=\frac{\pi}{2p}\{(1+ae^{-cp})^q+(1+ae^{-cp})^{-q}\}$

Boncompagni, Cr. 25. 74; où 3, 4, 5 sont fautives.

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6) $\int Cos.\left\{q Arctg.\left(\frac{a Sin.cx}{1+a Cos.cx}\right)\right\}\frac{(1+2a Cos.cx+a^2)^{\frac{1}{2}q}-(1+2a Cos.cx+a^2)^{-\frac{1}{2}q}}{p^2+x^2}dx =$ Boncompagni, Cr. 25. 74. qui est fautive.

$$= \frac{\pi}{2p}\{(1+ae^{-cp})^q-(1+ae^{-cp})^{-q}\}$$

7) $\int Sin.\left\{(a+1) Arctang.\frac{x}{b}\right\}.Sin.cx\frac{dx}{(b^2+x^2)^{\frac{a+1}{2}}} = \frac{\pi e^{-bc}c^a}{2\Gamma(a+1)}$

8) $\int Cos.\left\{(a+1) Arctang.\frac{x}{b}\right\}.Cos.cx\frac{dx}{(b^2+x^2)^{\frac{a+1}{2}}} = \frac{\pi e^{-bc}c^a}{2\Gamma(a+1)}$

9) $\int Sin.\left\{(a+1)Arctg.\frac{x}{b}\right\}.Sin.\left\{x\lambda+c Arctg.\left(\frac{Sin.x}{Cos.x-e^{-b}}\right)\right\}.(e^{2b}-2e^b Cos.x+1)^{\frac{1}{2}c}\frac{dx}{(b^2+x^2)^{\frac{1}{2}(a+1)}} =$

$$= \frac{\pi}{2e^{b\lambda}\Gamma(a+1)}\Delta^b.\lambda^a$$

10) $\int Cos.\left\{(a+1)Arctg.\frac{x}{b}\right\}.Cos.\left\{x\lambda+c Arctg.\left(\frac{Sin.x}{Cos.x-e^{-b}}\right)\right\}.(e^{2b}-2e^b Cos.x+1)^{\frac{1}{2}c}\frac{dx}{(b^2+x^2)^{\frac{1}{2}(a+1)}} =$

$$= \frac{\pi}{2e^{b\lambda}\Gamma(a+1)}\Delta^b.\lambda^a$$

11) $\int Cos.\left[x\lambda+c Arctg.\left(\frac{Sin.x}{Cos.x-e^{-b}}\right)-(a+1) Arctg.\frac{x}{b}\right].(e^{2b}-2e^b Cos.x+1)^{\frac{1}{2}c}\frac{dx}{(b^2+x^2)^{\frac{1}{2}(a+1)}} =$

$$= \frac{\pi}{e^{b\lambda}\Gamma(a+1)}\Delta^b.\lambda^a$$

12) $\int Cos.\left[x\lambda+c Arctg.\left(\frac{Sin.x}{Cos.x-e^{-b}}\right)+(a+1) Arctg.\frac{x}{b}\right].(e^{2b}-2e^b Cos.x+1)^{\frac{1}{2}c}\frac{dx}{(b^2+x^2)^{\frac{1}{2}(a+1)}} = 0$

Sur ces intégrales 7) à 12) voyez: Cauchy, P. 28. 147. P. III. § 2.

13) $\int Cos.(px-p Arctang.x)\frac{dx}{(1+x^2)^{\frac{1}{2}p}} = \frac{\pi}{\Gamma(p+1)}\left(\frac{p}{e}\right)^p$ V. T. 61. N°. 1.

14) $\int Cos.(qx-p Arctang.x)\frac{dx}{(1+x^2)^{\frac{1}{2}p}} = \frac{\pi q^{p-1}e^{-q}}{\Gamma(p)}$ V. T. 61. N°. 5.

15) $\int Cos.(qx+p Arctang.x)\frac{dx}{(1+x^2)^{\frac{1}{2}p}} = 0$ V. T. 61. N°. 3.

16) $\int Cos.(qx+p Arctang.x)\frac{dx}{(1+x^2)^{\frac{1}{2}p-1}} = \frac{\pi e^{-q}}{2^{p+1}}$ V. T. 61. N°. 4.

1) $\int Sin.\left(b\, Arctang.\frac{x}{a}\right)\frac{dx}{x(x^2+a^2)^{\frac{1}{2}b}} = \frac{1}{2}\pi a^{-b}$ Liouville, Cr. 13. 209.

2) $\int Sin.(p\, Arctang.x)\frac{dx}{x^q(1+x^2)^{\frac{1}{2}p}} = \frac{1}{2}\pi\, Cosec.\frac{1}{2}q\pi\frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)}, 2>q>0;$

3) $\int Cos.(p\, Arctang.x)\frac{dx}{x^q(1+x^2)^{\frac{1}{2}p}} = \frac{1}{2}\pi\, Sec.\frac{1}{2}q\pi\frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)}, 1>q>0;$

Schlömilch, Stud. I. 15. — Id, Cr. 33. 353.

4) $\int Sin.\left(p\, Arctg.\frac{x}{r}\right)\frac{dx}{x^q(r^2+x^2)^{\frac{1}{2}p}} = \frac{\pi}{2r^{p+q-1}} Cosec.\frac{1}{2}q\pi\frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)}, 2>q>0;$

5) $\int Cos.\left(p\, Arctg.\frac{x}{r}\right)\frac{dx}{x^q(r^2+x^2)^{\frac{1}{2}p}} = \frac{\pi}{2r^{p+q-1}} Sec.\frac{1}{2}q\pi\frac{\Gamma(p+q-1)}{\Gamma(p)\Gamma(q)}, 1>q>0;$

Schlömilch, Gr. 6 200.

6) $\int Sin.(ax+p\, Arctang.x)\frac{dx}{x(1+x^2)^{\frac{1}{2}p}} = \frac{1}{2}\pi$ V. T. 70. N°. 22.

7) $\int\frac{Sin.(p\, Arctang.x)+Sin.(bx-p\, Arctang.x)}{x(1+x^2)^{\frac{1}{2}p}}dx = \frac{1}{2}\pi$ V. T. 70. N°. 23.

8) $\int\frac{Sin.(p\, Arctang.x)}{(1+x^2)^{\frac{1}{2}p}}\frac{x^{2b+1}}{c^2+x^2}dx = (-1)^{\frac{1}{2}b}\frac{c^{2b}}{(1+c)^p}\frac{\pi}{2}$

9) $\int\frac{Cos.(p\, Arctang.x)}{(1+x^2)^{\frac{1}{2}p}}\frac{x^{2b}}{c^2+x^2}dx = (-1)^{\frac{1}{2}b}\frac{c^{2b-1}}{(1+c)^p}\frac{\pi}{2}$

Schlömilch, Cr. 33. 353. ne valent que pour $b=0$.

10) $\int\frac{Sin.(p\, Arctang.x)}{(1+x^2)^{\frac{1}{2}p}} Sin.\left\{(q+1)\, Arctang.\frac{x}{r}\right\}\frac{x^a}{(r^2+x^2)^{\frac{1}{2}(q+1)}}dx =$

$= (-1)^{\frac{1}{2}a}\frac{\pi}{2}\frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q+1)}\frac{r^a}{(1+r)^{p+q}}\sum_0^\infty(-1)^n\binom{a}{n}\frac{q^{n/-1}}{(p+q-1)^{n/-1}}\left(\frac{1+r}{r}\right)^n$

11) $\int\frac{Sin.(p\, Arctang.x)}{(1+x^2)^{\frac{1}{2}p}} Cos.\left\{(q+1)\, Arctang.\frac{x}{r}\right\}\frac{x^{a-1}}{(r^2+x^2)^{\frac{1}{2}(q+1)}}dx =$

$= (-1)^{\frac{1}{2}a+1}\frac{\pi}{2}\frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q+1)}\frac{r^{a-1}}{(1+r)^{p+q}}\sum_0^\infty(-1)^n\binom{a-1}{n}\frac{q^{n/-1}}{(p+q-1)^{n/-1}}\left(\frac{1+r}{r}\right)^n$

12) $\int\frac{Cos.(p\, Arctang.x)}{(1+x^2)^{\frac{1}{2}p}} Sin.\left\{(q+1)\, Arctang.\frac{x}{r}\right\}\frac{x^{a-1}}{(r^2+x^2)^{\frac{1}{2}(q+1)}}dx =$

$= (-1)^{\frac{1}{2}a}\frac{\pi}{2}\frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q+1)}\frac{r^{a-1}}{(1+r)^{p+q}}\sum_0^\infty(-1)^n\binom{a-1}{n}\frac{q^{n/-1}}{(p+q-1)^{n/-1}}\left(\frac{1+r}{r}\right)^n$

Schlömilch, Cr. 33. 353. ne valent que pour $a=0$.

13) $\int \frac{Cos.(p\, Arctang.\, x)}{(1+x^2)^{\frac{1}{2}p}} Cos.\left\{(q+1)\, Arctang.\frac{x}{r}\right\} \frac{x^a}{(r^2+x^2)^{\frac{1}{2}(q+1)}}\, dx =$

$= (-1)^{\frac{1}{2}a}\frac{\pi}{2}\frac{\Gamma(p+q)}{\Gamma(p)\,\Gamma(q+1)}\frac{r^a}{(1+r)^{p+q}}\sum_0^\infty (-1)^n \binom{a}{n}\frac{q^{n/-1}}{(p+q-1)^{n/-1}}\left(\frac{1+r}{r}\right)^n$

14) $\int \frac{Sin.(p\, Arctg.\, x)}{(1+x^2)^{\frac{1}{2}p}} \frac{x}{(r^2+x^2)^{a+1}}\, dx = \frac{\pi\,\Gamma(p+a)}{\Gamma(p)\Gamma(a+1)(2r)^{a+1}(1+r)^{p+a}}\sum_0^\infty \frac{(a+n-1)^{2n/-1}}{2^{n/2}(p+a-1)^{n/-1}}\left(\frac{1+r}{r}\right)^n$

15) $\int \frac{Cos.(p\, Arctg.\, x)}{(1+x^2)^{\frac{1}{2}p}} \frac{dx}{(r^2+x^2)^{a+1}} = \frac{\pi\,\Gamma(p+a)}{\Gamma(p)\,\Gamma(a+1)(2r)^{a+1}(1+r)^{p+a}}\sum_0^\infty \frac{(a+n)^{2n/-1}}{2^{n/2}(p+a-1)^{n/-1}}\left(\frac{1+r}{r}\right)^n$

Schlömilch, Cr. 33. 353. 13) ne vaut que pour $a=0$.

16) $\int \frac{Sin.\left\{p\, Arctang.\, x+(q+1)\, Arctang.\frac{x}{r}\right\}}{(1+x^2)^{\frac{1}{2}p}} \frac{x^{a-1}}{(r^2+x^2)^{\frac{1}{2}(q+1)}}\, dx = 0$ V. T. 433. N°. 11, 12. ne vaut que pour $a=0$.

17) $\int \frac{Cos.\left\{p\, Arctang.\, x+(q+1)\, Arctang.\frac{x}{r}\right\}}{(1+x^2)^{\frac{1}{2}p}} \frac{x^{a-1}}{(r^2+x^2)^{\frac{1}{2}(q+1)}}\, dx = 0$ V. T. 433. N. 10, 13. ne vaut que pour $a=1$.

18) $\int Sin.\left\{p\, Arctg.\left(\frac{q\, Sin.\, ax}{1+q\, Cos.\, ax}\right)\right\}\frac{x}{r^2+x^2}\frac{dx}{(1+2q\, Cos.\, ax+q^2)^{\frac{1}{2}p}} = \frac{1}{2}\pi - \frac{1}{2}\pi(1+qe^{-ar})^{-p}$

19) $\int Cos.\left\{p\, Arctg.\left(\frac{q\, Sin.\, ax}{1+q\, Cos.\, ax}\right)\right\}\frac{1}{r^2+x^2}\frac{dx}{(1+2q\, Cos.\, ax+q^2)^{\frac{1}{2}p}} = \frac{1}{2}\pi\left\{1+qe^{-ar}\right\}^{-p}$

Boncompagni, Cr. 25. 74. (fautives).

20) $\int \left[Cos.\, x - \frac{Cos.(p\, Arctang.\, x)}{(1+x^2)^{\frac{1}{2}p}}\right]\frac{dx}{x} = Z'(p)$ Arndt, Gr. 10. 225.

F. Alg.
Circ. Dir.
Circ. Inv.

TABLE 434.

Lim. diverses.

1) $\int_{-\infty}^{\infty} \frac{Cos.(p\, Arctang.\, qx)}{1+x^2}\frac{dx}{(1+q^2x^2)^{\frac{1}{2}p}} = \frac{\pi}{(1+q)^p}$ Cauchy, C. R. 11. 1008.

2) $\int_0^{\frac{\pi}{2}} Arctang\left(\frac{1}{q}\, Tang.\, x\right)\frac{x}{Sin.^2 x}\, dx = \frac{\pi}{2q}\left\{l(1+q)+q\, l\frac{1+q}{q}\right\}$ V. T. 264. N°. 13.

1) $\int Sin.\,ax.\,Si.\,(cx)\frac{dx}{b^2+x^2}=\frac{\pi}{4b}e^{-ab}\{li.\,(e^{bc})-li.\,(e^{-bc})\}$, $a>c$;

2) $=\frac{\pi}{4b}\{e^{-ab}li.(e^{ab})-e^{ab}li.(e^{-ab})+(e^{ab}-e^{-ab})li.(e^{-bc})\}$, $a<c$;

Schlömilch, Gr. 11.174.

3) $\int Sin.\,ax.\,Si.\,(cx)\frac{dx}{b^2+x^2}=\frac{\pi}{4b}e^{-ab}\{Ei.\,(bc)-Ei.\,(-bc)\}$, $a\geqq c$;

4) $=\frac{\pi}{4b}\left[e^{-ab}\{Ei.(ab)-Ei.(-bc)\}-e^{ab}\{Ei.(-ab)-Ei.(-bc)\}\right]$, $a\leqq c$;

5) $\int Cos.\,ax.\,Ci.\,(cx)\frac{dx}{b^2+x^2}=\frac{\pi}{4b}(e^{ab}+e^{-ab})\,Ei.\,(-bc)$, $a\geqq c$;

6) $=\frac{\pi}{4b}\left[e^{-ab}\{Ei.\,(bc)+Ei.\,(-bc)-Ei.\,(ab)\}+e^{ab}Ei.\,(-ab)\right]$, $a\leqq c$;

7) $\int Sin.\,ax.\,Ci.\,(cx)\frac{x\,dx}{b^2+x^2}=-\frac{1}{4}\pi(e^{ab}-e^{-ab})\,Ei.\,(-bc)$, $a\geqq c$;

8) $=\frac{1}{4}\pi\left[e^{-ab}\{Ei.\,(bc)+Ei.\,(-bc)-Ei.\,(ab)\}-e^{ab}Ei.\,(-ab)\right]$, $a\leqq c$;

9) $\int Cos.\,ax.\,Si.\,(cx)\frac{x\,dx}{b^2+x^2}=-\frac{1}{4}\pi e^{-ab}\{Ei.\,(bc)-Ei.\,(-bc)\}$, $a\geqq c$;

10) $=-\frac{1}{4}\pi\left[e^{-ab}\{Ei.(ab)-Ei.(-bc)\}-e^{ab}\{Ei.\,(-ab)-Ei.(-bc)\}\right]$, $a\leqq c$;

11) $\int\left[Cos.ax.\{Ci.(cx)-Ci.(ax)\}+Sin.ax.\{Si.(cx)-Si.(ax)\}\right]\frac{dx}{b^2+x^2}=\frac{\pi}{2b}e^{-ab}\{Ei.(bc)-Ei\,(ab)\}$, $a\geqq c$;

12) $\int\left[Sin.ax.\{Ci.(cx)-Ci.(ax)\}-Cos.ax.\{Si.(cx)-Si.(ax)\}\right]\frac{x\,dx}{b^2+x^2}=\frac{\pi}{2}e^{-ab}\{Ei\,(bc)-Ei.(ab)\}$, $a\geqq c$;

13) $\int\left[Sin.\,ax.\,Ci.\,(ax)+Cos.\,ax.\left\{\frac{1}{2}\pi-Si.\,(ax)\right\}\right]\frac{x\,dx}{b^2+x^2}=-\frac{1}{2}\pi e^{ab}Ei\,(-ab)$

14) $\int\left[Sin.\,ax.\,Ci.\,(cx)-Cos.\,ax.\left\{\frac{1}{2}\pi-Si.\,(cx)\right\}\right]\frac{x\,dx}{b^2+x^2}=\frac{1}{2}\pi e^{-ab}Ei.\,(-bc)$

15) $\int\left[Cos.\,ax.\,Ci.\,(ax)-Sin.\,ax.\left\{\frac{1}{2}\pi-Si.\,(ax)\right\}\right]\frac{dx}{b^2+x^2}=\frac{\pi}{2b}e^{ab}Ei.\,(-ab)$

16) $\int\left[Cos.\,ax.\,Ci.\,(cx)+Sin.\,ax.\left\{\frac{1}{2}\pi-Si.\,(cx)\right\}\right]\frac{dx}{b^2+x^2}=\frac{\pi}{2b}e^{-ab}Ei.\,(-bc)$

Sur ces intégrales (3) à (16) voyez: Arndt, Gr. 11. 70.

F. Exp. monôme.
Log.
Circ. Dir. ent.

TABLE 436.

Lim. 0 et $\frac{\pi}{2}$.

1) $\int dx\, l(1-e^{-2a\pi\, Tang. x}) = \pi\left\{a(la-1)+\frac{1}{2}l\,2a\pi - l\,\Gamma(a+1)\right\}$ V. T. 378. N°. 4.

2) $\int e^{-2a Sec. x}\, l(2\, Sec. x - 1).\, Tang. x\, dx = \frac{1}{2}\{li.(e^{-a})\}^2$ V. T. 383. N°. 3.

3) $\int e^{p Cos. 2x}\, l\, Sin. x.\, Cos.(p\, Sin. 2x + 2x)\, dx = \pi\frac{1-e^{-p}}{4p}$ V. T. 296. N°. 9.

4) $\int e^{p Cos. 2x}\, l\, Cos. x.\, Cos.(p\, Sin. 2x + 2x)\, dx = \pi\frac{1-e^{p}}{4p}$ V. T. 296. N°. 10.

5) $\int e^{p Cos. 2x}\, l\, Tang. x.\, Cos.(p\, Sin. 2x + 2x)\, dx = \pi\frac{e^{-p}-e^{p}}{4p}$ V. T. 296. N°. 11.

6) $\int e^{p Cos. 2x}\, l\, Tang.^2\left(\frac{\pi}{4}\pm x\right).\, Sin.(p\, Sin. 2x + 2x)\, dx = \pm\infty$ V. T. 296. N°. 12.

F. Exp. monôme.
Log.
Circ. Dir. fract.

TABLE 437.

Lim. 0 et $\frac{\pi}{2}$.

1) $\int e^{-q\, Cot. x}\, l\, Sin. x\, \frac{dx}{Sin.^2 x} = \frac{1}{q}\left[Ci.(q).\, Cos. q - Sin. q.\left\{\frac{1}{2}\pi - Si.(q)\right\}\right]$ V. T. 290. N°. 10.

2) $\int e^{-p\, Tang.^2 x}\, Tang.^{2a} x.\, l\, Tang. x\, \frac{2p\, Sin.^2 x - (2a-1)\, Cos.^2 x}{Sin.^2 2x}\, dx = \frac{1}{8(2p)^{a-1}}\, 1^{a-1/2}\sqrt{\frac{\pi}{p}}$ V. T. 289. N°. 5.

3) $\int e^{-p\, Tang.^2 x}\, Tang.^{2a+1} x.\, l\, Tang. x\, \frac{p\, Sin.^2 x - a\, Cos.^2 x}{Sin.^2 2x}\, dx = \frac{1}{2^{a+3} p^a}\, 1^{a-1/1}$ V. T. 289. N°. 4.

4) $\int e^{-q(Tang.^2 x + Cot.^2 x)}\, l\, Tang. x.\, Tang.^{2a+1} x\, \frac{(2a+1)\, Sin. 2x + 2q\, Cos. 2x}{Sin.^3 2x}\, dx =$

$-\frac{1}{32}e^{-2q}\sqrt{\frac{\pi}{q}}.\sum_0^a \frac{1}{(2q)^n}\frac{(a-n+1)^{2n/1}}{2^n 1^{n/1}}$ V. T. 289. N°. 11.

5) $\int e^{-Tang.^{2p} x}\, l\, Tang. x.\, Tang.^{2p} x\, \frac{2\, Sin.^{2p} x - Cos.^{2p} x}{Sin.^{p+1} 2x}\, dx = \frac{1}{2^{p+2} p^2}\sqrt{\pi}$ V. T. 289. N°. 7.

6) $\int e^{-q\, Tang. x}\, l\, Cos. x\, \frac{dx}{Cos.^2 x} = \frac{1}{q}\left[Ci.(q).\, Cos. q - Sin. q.\left\{\frac{1}{2}\pi - Si.(q)\right\}\right]$ V. T. 288. N°. 5.

7) $\int e^{-p\, Tang. x}\, l\, Tang.^2\left(\frac{\pi}{4}\pm x\right)\frac{dx}{Cos.^2 x} = \pm\frac{2}{p}\{e^p\, Ei.(-p) - e^{-p}\, Ei.(p)\}$ V. T. 290. N°. 12.

8) $\int e^{-p\,Tang.x}\, l\, Tang.^2\left(\frac{\pi}{4}\pm x\right)\frac{p\,Sin.x - Cos.x}{Cos.^3 x}\,dx = \mp 2\{e^{-p}Ei.(p) + e^{p}Ei.(-p)\}$ V. T. 290. N°. 13.

9) $\int e^{-p\,Tang.x}\, l\, Tang.^2\left(\frac{\pi}{4}\pm x\right)\frac{Tang.x}{Cos.^2 x}\,dx = \mp \{(1+p)e^{-p}Ei.(p) - (1-p)e^{p}Ei.(-p)\}$ V. T. 437. N°. 7, 8.

10) $\int l\, Tang.x.(p\,e^{-p\,Tang.x} - q\,e^{-q\,Tang.x})\frac{dx}{Cos.^2 x} = l\frac{q}{p}$ V. T. 289. N°. 14.

11) $\int e^{-Tang.^{ac}x}\,Tang.^{ab-1}x.\,l\,Tang.x\,\frac{c\,Sin.^{ac}x - b\,Cos.^{ac}x}{Cos.^{ac+2}x}\,dx = \frac{1}{a^2 b}1^{\frac{b}{c}/1}$ V. T. 289. N°. 9.

12) $\int e^{-2a\,Cosec.x}\,l(2\,Cosec.x - 1)\frac{dx}{Tang.x} = \frac{1}{2}\{li.(e^{-a})\}^2$ V. T. 383. N°. 3.

13) $\int e^{-p\,Cot\,x}\,l\,Tang.^2\left(\frac{\pi}{4}\pm x\right)\frac{p\,Cot.x - 1}{Sin.^2 x}\,dx = \pm 2\{e^{-p}Ei.(p) + e^{p}Ei.(-p)\}$ V. T. 291. N°. 1.

14) $\int e^{-Tg.^2 x}\,l\,Tang.x\,\frac{1 - Cos.2x.Sin.^2 x}{Cos.^2 x.Sin.^2 2x}\,dx = \frac{3}{8}\sqrt{\pi}$ V. T. 289. N°. 15.

15) $\int e^{-Tg.^2 x}\,l\,Sin.2x\,\frac{1 - Cos.2x.Sin.^2 x}{Cos.^2 x.Sin.^2 2x}\,dx = \frac{1}{8}\sqrt{\pi}$ V. T. 289. N°. 16.

16) $\int e^{-q\,Tang.x}\,l\,Tang.x\,\frac{q\,Sin.x - p\,Cos.x}{Sin.2x}\,\frac{Tang.^p x}{Cos.x}\,dx = \frac{1}{2q^p}\Gamma(p)$ V. T. 289. N°. 3.

17) $\int e^{-p\,Tang.x}\,l\,Cos.x\,\frac{2\,Tang.2x.Cos.^2 x - p}{Cos.2x.Cos.^2 x}\,dx = \frac{1}{2}\{e^{-p}Ei.(p) + e^{p}Ei.(-p)\}$ V. T. 290. N°. 13.

18) $\int e^{-Cot.^{2p}x}\,l\,Tang.x.(Sin.^{2p}x - 2\,Cos.^{2p}x)\frac{dx}{Sin.^{3p-1}x.Cos.^{1-p}x} = \frac{1}{2p^2}\sqrt{\pi}$ V. T. 291. N°. 7.

19) $\int e^{-Cot.^{ac}x}\,l\,Tang.x\,\frac{b\,Sin.^{ac}x - c\,Cos.^{ac}x}{Sin.^{ab+ac+1}x.\,Cos.^{1-ab}x}\,dx = \frac{1}{a^2 b}1^{\frac{b}{c}/1}$ V. T. 291. N°. 10.

20) $\int e^{-p\,Cot.^2 x}\,l\,Tang.x\,\frac{(2a+1)Sin.^2 x - 2p\,Cos.^2 x}{Sin.^2 2x.Tang.^{2a+2}x}\,dx = \frac{1}{8(2p)^a}1^{a/2}\sqrt{\frac{\pi}{p}}$ V. T. 291. N°. 5.

21) $\int e^{-Cot.^2 x}\,l\,Tang.x\,\frac{a\,Sin.^2 x - Cos.^2 x}{Sin.^4 x.\,Tang.^{2a-1}x}\,dx = \frac{1}{2^{a+1}}1^{a-1/1}$ V. T. 291. N°. 4.

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F. Exp. monôme.
Log.
Circ. Dir. fract.

TABLE 457 suite.

Lim. 0 et $\frac{\pi}{2}$.

22) $\int e^{-q(Tang.^2 x + Cot.^2 x)}\, l\, Tang.\, x \frac{(2a+1)\, Sin.^2 x.\, Cos.^2 x - 2q\, Cos.\, 2x}{Tang.^{2a+1} x.\, Sin.^3\, 2x} dx =$

$= -\frac{1}{32} e^{-2q} \sqrt{\frac{\pi}{q}}.\sum_0^a \frac{1}{(2q)^n} \frac{(a-n+1)^{2n/1}}{2^n\, 1^{n/1}}$ V. T. 291. N°. 12.

23) $\int e^{-q\, Cot.\, x}\, l\, Tang.\, x \frac{p\, Sin.\, x - q\, Cos.\, x}{Sin.\, 2x.\, Sin.\, x.\, Tang.^p x} dx = -\frac{1}{2q^p}\Gamma(p)$ V. T. 291. N°. 8.

24) $\int e^{-a\, Tang.\, x}\, l\, Tang.\, x \frac{dx}{Cos.\, x.\sqrt{Sin.\, 2x}} = -\{lq + 2\, l\, 2 + A\}\sqrt{\frac{\pi}{2a}}$ V. T. 381. N°. 6.

25) $\int e^{-p\, Tang.\, x}\, l(q\, Cos.\, x) \frac{pq\, l(q\, Cos.\, x) + 2\, Cos.^2 x}{Cos.^2 x} dx = -\frac{1}{16q}(lq^4)^2$ V. T. 378. N°. 7.

26) $\int e^{-p\, Tang.\, x}\, l\left(q^2 \frac{Cos.\, 2x}{Cos.^2 x}\right) \frac{r\, q\, Cos.\, 2x.\, l(q^2\, Cos.\, 2x.\, Sec.^2 x) - 4\, Cos.^2 x}{Cos.\, 2x.\, Cos.^2 x} dx = \frac{1}{4q}(lq^4)^2$ V. T. 378. N°. 8.

F. Exp. en dén. binôme.
Log.
Circ. Dir. fract.

TABLE 458.

Lim. 0 et $\frac{\pi}{2}$.

1) $\int \frac{l\, Cos.\, x}{(e^{2\pi\, Tang\, x} - 1)^2} e^{2\pi\, Tang.\, x} \frac{dx}{Cos.^2 x} = \frac{1}{8\pi}(1 - 2A)$ V. T. 292. N°. 13.

2) $\int \frac{l\, Cos.\, x}{(e^{q\, Tang.\, x} - 1)^2} e^{q\, Tang.\, x} \frac{dx}{Cos.^2 x} = -\frac{1}{2q}\left\{-l\frac{2\pi}{q} + \frac{\pi}{q} - Z'\left(\frac{q+2\pi}{2\pi}\right)\right\}$ V. T. 292. N°. 14.

3) $\int \frac{e^{\frac{1}{2}\pi\, Tang.\, x} + e^{-\frac{1}{2}\pi\, Tang.\, x}}{(e^{\frac{1}{2}\pi\, Tang.\, x} - e^{-\frac{1}{2}\pi\, Tang\, x})^2} \frac{l\, Cos.\, x}{Cos.^2 x} dx = \frac{1}{2\pi}(2 - \pi)$ V. T. 292. N°. 6.

4) $\int \frac{e^{\frac{1}{4}\pi\, Tang.\, x} + e^{-\frac{1}{4}\pi\, Tang.\, x}}{(e^{\frac{1}{4}\pi\, Tang.\, x} - e^{-\frac{1}{4}\pi\, Tang.\, x})^2} \frac{l\, Cos.\, x}{Cos.^2 x} dx = \frac{4}{\pi}\left\{1 - \frac{\pi}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\, l\frac{\sqrt{2}-1}{\sqrt{2}+1}\right\}$ V. T. 292. N°. 5.

5) $\int \frac{(\pi+p)(e^{(\pi-p)Tang.\, x} + e^{(p-\pi)Tang.\, x}) + (\pi-p)(e^{(p+\pi)Tang\, x} + e^{-(p+\pi)Tang.\, x})}{(e^{\pi\, Tang.\, x} - e^{-\pi\, Tang\, x})^2} \frac{l\, Cos.\, x}{Cos.^2 x} dx =$

$= \frac{1 - p\, Sin.\, p}{2} - \frac{1}{2} Cos.\, p.\, l\{2(1 + Cos.\, p)\}, 0 < p < \pi;$ V. T. 292. N°. 9.

6) $\int \frac{(p-q)(e^{(p+q)Tg\, x} + e^{-(p+q)Tg.\, x}) - (p+q)(e^{(p-q)Tg.\, x} + e^{(q-p)Tg.\, x})}{(e^{p\, Tang.\, x} + e^{-p\, Tang\, x})^2}\, l\, Tg.\, x \frac{dx}{Cos.^2 x} = l\, Tg.\left(\frac{p+q}{4p}\pi\right)$ V. T. 293. N°. 15.

7) $\int \frac{q(e^{p\, Tg.\, x} - e^{-p\, Tg\, x})(e^{q\, Tg.\, x} - e^{-q\, Tg.\, x}) - p(e^{q\, Tg.\, x} + e^{-q\, Tg\, x} - 2)(e^{p\, Tg\, x} + e^{-p\, Tg.\, x})}{(e^{p\, Tang.\, x} - e^{-p\, Tang.\, x})^2}\, l\, Tg.\, x \frac{dx}{Cos.^2 x} = l\, Cos.\frac{q\pi}{2p}$ V. T. 293. N°. 16.

1) $\int e^{-x}\, l\, x.\, Sin.\, x\, dx = \frac{1}{2}\left(\frac{1}{4}\pi - \frac{1}{2}\, l\, 2 - A\right)$ Schlömilch, Cr. 33. 316. — Id., Cr. 33. 325. (faut.)

2) $\int e^{-ax}\, l\frac{1}{x}.\, Sin.\, b\, x\, dx = \frac{1}{a^2+b^2}\left\{\frac{1}{2}\, b\, l\,(a^2+b^2) - a\, Arctang.\frac{b}{a} + b\, A\right\}$

3) $\int e^{-ax}\, l\frac{1}{x}.\, Cos.\, b\, x\, dx = \frac{1}{a^2+b^2}\left\{\frac{1}{2}\, a\, l\,(a^2+b^2) + b\, Arctang.\frac{b}{a} + a\, A\right\}$

Schlömilch, Stud. I. 14.

4) $\int e^{-px}\, Sin.\, 2\, q\, x.\, l\, x.\, (p\, Tang.\, q\, x - q)\, dx = \frac{1}{2}\, l\frac{p^2+4\, q^2}{p^2}$ V. T. 392. N°. 10.

5) $\int e^{-px}\, Cos.\, q\, x.\, l\, x.\, (p\, Tang.\, q\, x - q)\, dx = Arctang.\frac{q}{p}$ V. T. 392. N°. 3.

6) $\int e^{-px}\, l\, Sin.\,(q\, x)\, dx = -\frac{1}{2\, p}\, l\, 2 - \frac{p}{2}\sum_1^\infty \frac{1}{n}\frac{1}{p^2+n^2\, q^2}$

7) $\int e^{-px}\, l\, Cos.\,(q\, x)\, dx = -\frac{1}{2\, p}\, l\, 2 - \frac{p}{2}\sum_1^\infty \frac{(-1)^n}{n}\frac{1}{p^2+n^2\, q^2}$

8) $\int e^{-px}\, l\, Tang.\,(q x)\, dx = -p\sum_1^\infty \frac{1}{2\, n-1}\frac{1}{p^2+(2\, n-1)^2\, q^2}$

Schlömilch, Beitr. II. 5.

9) $\int e^{-x^2}\, l\, Sin.\,(a\, x)\, dx = \frac{1}{2}\sqrt{\pi}.\left\{-l\, 2 + \sum_1^\infty \frac{e^{-(na)^2}}{n}\right\}$

10) $\int e^{-x^2}\, l\, Cos.\,(a\, x)\, dx = \frac{1}{2}\sqrt{\pi}.\left\{-l\, 2 + \sum_1^\infty (-1)^n \frac{e^{-(na)^2}}{n}\right\}$

11) $\int e^{-x^2}\, l\, Tang.\,(a x)\, dx = \sqrt{\pi}.\sum_1^\infty \frac{e^{-(2n-1)^2 a^2}}{2\, n-1}$

Bidone, Mém. Turin. 1812. 231. Art. 3. N°. 38.

12) $\int e^{-x^2}\, l\,(2\, Sin.\, a\, x)^2\, dx = \sqrt{\pi}.\sum_1^\infty \frac{e^{-a^2 n^2}}{n}$

13) $\int e^{-x^2}\, l\,(2\, Cos.\, a\, x)^2\, dx = \sqrt{\pi}.\sum_1^\infty (-1)^n \frac{e^{-a^2 n^2}}{n}$

14) $\int e^{-x^2}\, l\,(1 - 2\, p\, Cos.\, 2\, a\, x + p^2)\, dx = \sqrt{\pi}.\sum_1^\infty \frac{1}{n}\, p^n\, e^{-a^2 n^2}$

Schlömilch, Stud. I. 25.

15) $\int l x \frac{(e^{\pi x}+e^{-\pi x})\, p\, Cos.\, p\, \pi - (e^{\pi x} - e^{-\pi x})\, \pi\, Sin.\, p\, \pi}{(e^{\pi x}+e^{-\pi x})^2}\, dx = -Arctang.(e^{\frac{1}{2}p})$ V. T. 396. N°. 1.

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16) $\int l x \frac{(e^{\pi x}-e^{-\pi x})\, p\, Sin.\, p x+(e^{\pi x}+e^{-\pi x})\, \pi\, Cos.\, p x}{(e^{\pi x}-e^{-\pi x})^2}\, dx = \infty$ V. T. 396. N°. 2.

17) $\int l(1+x^2) \frac{(e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x})\, 4 q\, Cos.\, q x-(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})\, \pi\, Sin.\, q x}{(e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x})^2}\, dx =$

$= 2\pi e^{-q}\sqrt{2}+(e^q-e^{-q})\sqrt{2}.\, l\frac{e^q+\sqrt{2}+e^{-q}}{e^q-\sqrt{2}+e^{-q}}-2(e^q+e^{-q})\sqrt{2}.\, Arctg.\left(\frac{\sqrt{2}}{e^q-e^{-q}}\right)$ V. T. 396. N°. 5.

18) $\int l(1+x^2)\frac{(e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x})2q\, Cos.\, qx-(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})\pi\, Sin.\, qx}{(e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x})^2}dx=2qe^{-q}-(e^q-e^{-q})l(1+e^{-2q})$ V. T. 396. N°. 7.

19) $\int l(1+x^2) \frac{(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})\, 2 q\, Sin.\, q x+(e^{\frac{1}{2}\pi x}+e^{-\frac{1}{2}\pi x})\, \pi\, Cos.\, q x}{(e^{\frac{1}{2}\pi x}-e^{-\frac{1}{2}\pi x})^2}\, dx =$

$= 2\, (e^q-e^{-q})\, Arctang.\, (e^{-q})+\pi e^{-q}-2$ V. T. 396. N°. 22.

20) $\int l(1+x^2)\frac{(e^{\pi x}-e^{-\pi x})qSin.\, qx+(e^{\pi x}+e^{-\pi x})\pi\, Cos.\, qx}{(e^{\pi x}-e^{-\pi x})^2}dx=-\frac{1}{2}+\frac{1}{2}qe^{-q}+\frac{e^q+e^{-q}}{2}l(1+e^{-q})$ V. T. 396. N°. 26.

F. Exp.
Log.
Circ. Dir.

TABLE 440.

Lim. diverses.

1) $\int_0^{\pi} e^{2\pi a x i}\, l\, Sin.\, \pi x\, dx = -\frac{1}{2a}$ Schaar, Mém. Cour. Brux. T. 23.

2) $\int_0^{\frac{\pi}{4}} e^{-2a Cot. x}\, l(2\, Cot.\, x-1)\, \frac{dx}{Sin.\, 2x} = \frac{1}{4}\{l i.\, (e^{-q})\}^2$ V. T. 383. N°. 3.

3) $\int_0^{\pi} e^{p Cos. x}\, l\left(\frac{1}{2}\, Sin.\, x\right).\, Cos.\, (p\, Sin.\, x+x)\, dx = -\frac{\pi}{4p}(e^{\frac{1}{2}p}-e^{-\frac{1}{2}p})^2$ V. T. 436. N°. 3, 4.

4) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(e^{px}+e^{-px})\, Sin.\, (p\, l\, Cos.\, x)\, dx = -2\pi\, Sin.\, (p\, l\, 2)$ V. T. 447. N°. 15.

5) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(e^{px}+e^{-px})\, Cos.\, (p\, l\, Cos.\, x)\, dx = 2\pi\, Cos.\, (p\, l\, 2)$ V. T. 447. N°. 16.

F. Exp.
Circ. Inv.
Circ. Dir.

TABLE 441.

Lim. 0 et ∞.

1) $\int Arctang.\, x \frac{(e^{\frac{1}{4}\pi x} + e^{-\frac{1}{4}\pi x})\, 4\, q\, Sin.\, q\, x + (e^{\frac{1}{4}\pi x} - e^{-\frac{1}{4}\pi x})\, \pi\, Cos.\, q\, x}{(e^{\frac{1}{4}\pi x} + e^{-\frac{1}{4}\pi x})^2}\, d\, x = \pi\, e^{-q} \sqrt{2} +$

$+ \frac{e^q + e^{-q}}{\sqrt{2}}\, l \frac{e^q - \sqrt{2} + e^{-q}}{e^q + \sqrt{2} + e^{-q}} + \frac{e^q - e^{-q}}{\sqrt{2}}\, 2\, Arctang. \left(\frac{\sqrt{2}}{e^q - e^{-q}}\right)$ V. T. 396. N°. 19.

2) $\int Arctg.x \frac{(e^{\frac{1}{4}\pi x} + e^{-\frac{1}{4}\pi x}) 2 q Sin.qx + (e^{\frac{1}{4}\pi x} - e^{-\frac{1}{4}\pi x}) \pi Cos.qx}{(e^{\frac{1}{4}\pi x} + e^{-\frac{1}{4}\pi x})^2}\, dx = q e^{-q} + \frac{e^q + e^{-q}}{2}\, l(1 + e^{-2q})$ V. T. 396. N°. 20.

3) $\int Arctang.\, x \frac{(e^{\frac{1}{4}\pi x} - e^{-\frac{1}{4}\pi x})\, 4\, q\, Cos.\, q\, x - (e^{\frac{1}{4}\pi x} + e^{-\frac{1}{4}\pi x})\, \pi\, Sin.\, q\, x}{(e^{\frac{1}{4}\pi x} - e^{-\frac{1}{4}\pi x})^2}\, d\, x = e^{-q} \sqrt{2} +$

$+ \frac{e^q - e^{-q}}{\sqrt{2}}\, l \frac{e^q - \sqrt{2} + e^{-q}}{e^q + \sqrt{2} + e^{-q}} - \frac{e^q + e^{-q}}{\sqrt{2}}\, 2\, Arctang. \left(\frac{\sqrt{2}}{e^q - e^{-q}}\right)$ V. T. 396. N°. 6.

4) $\int Arctg\, x \frac{(e^{\frac{1}{4}\pi x} - e^{-\frac{1}{4}\pi x}) 2 q\, Cos.qx - (e^{\frac{1}{4}\pi x} + e^{-\frac{1}{4}\pi x}) \pi Sin.qx}{(e^{\frac{1}{4}\pi x} - e^{-\frac{1}{4}\pi x})^2}\, dx = \frac{\pi}{2} e^{-q} - (e^q + e^{-q}) Arctg.(e^{-q})$ V. T. 396. N°. 9.

5) $\int Arctg.x \frac{(e^{\pi x} - e^{-\pi x}) q Cos.qx - (e^{\pi x} + e^{-\pi x}) \pi Sin.qx}{(e^{\pi x} - e^{-\pi x})^2}\, dx = \frac{1}{4} q e^{-q} - \frac{e^q - e^{-q}}{4}\, l(1 + e^{-q})$ V. T. 396. N°. 12.

F. Exp.
Circ. Dir.
Autres Fonctions.

TABLE 442.

Lim. diverses.

1) $\int_0^{\frac{\pi}{2}} li.(e^{-Tang.x}).\, Tang^p x \frac{d\, x}{Sin.\, 2\, x} = -\frac{1}{2\, p} \Gamma(p)$ V. T. 402. N°. 3.

2) $\int_0^{\infty} li.(e^{-x}).\, Sin.\, q\, x\, d\, x = -\frac{1}{2\, q}\, l\, (1 + q^2)$

3) $\int_0^{\infty} li.(e^{-x}).\, Cos.\, q\, x\, d\, x = -\frac{1}{q}\, Arctang.\, q$

(2, 3) Schlömilch, Beitr. III. § 8.

4) $\int_0^{\infty} li.(e^{-x}).\, e^x\, Sin.\, q\, x\, d\, x = -\frac{1}{1 + q^2} \left(\frac{1}{2}\pi + q\, l\, q\right)$

5) $\int_0^{\infty} li.(e^x).\, e^{-x}\, Sin.\, q\, x\, d\, x = \frac{1}{1 + q^2} \left(\frac{1}{2}\pi - q\, l\, q\right)$

6) $\int_0^{\infty} li.(e^{-x}).\, e^x\, Cos.\, q\, x\, d\, x = -\frac{1}{1 + q^2} \left(\frac{1}{2} q\, \pi - l\, q\right)$

(4, 5, 6) Schlömilch, Gr. 5. 204.

7) $\int_0^\infty li.(e^x).e^{-x}\,Cos.\,q\,x\,dx = -\frac{1}{1+q^2}\left(\frac{1}{2}q\pi + l\,q\right)$

8) $\int_0^\infty \{e^x\,li.(e^{-x}) + e^{-x}\,li.(e^x)\}\,Sin.\,q\,x\,dx = -\frac{2q}{1+q^2}\,l\,q$

9) $\int_0^\infty \{e^x\,li.(e^{-x}) - e^{-x}\,li.(e^x)\}\,Sin.\,q\,x\,dx = -\frac{\pi}{1+q^2}$

10) $\int_0^\infty \{e^x\,li.(e^{-x}) + e^{-x}\,li.(e^x)\}\,Cos.\,q\,x\,dx = -\frac{q\pi}{1+q^2}$

11) $\int_0^\infty \{e^x\,li.(e^{-x}) - e^{-x}\,li.(e^x)\}\,Cos.\,q\,x\,dx = \frac{2}{1+q^2}\,l\,q$

(7–11) Schlömilch, Gr. 5. 204.

12) $\int_0^\infty li.(e^{-x}).\,e^{-px}\,Sin.\,qx\,dx = -\frac{1}{p^2+q^2}\left[q\,l\sqrt{\{(1+p)^2+q^2\}} - p\,Arctg.\left(\frac{q}{1+p}\right)\right]$

13) $\int_0^\infty li.(e^{-x}).\,e^{-px}\,Cos.\,qx\,dx = -\frac{1}{p^2+q^2}\left[p\,l\sqrt{\{(1+p)^2+q^2\}} + q\,Arctg.\left(\frac{q}{1+p}\right)\right]$

(12–13) Schlömilch, Beitr. III. § 8.

F. Log.
Circ. Dir.
Circ. Inv.

TABLE 443.

Lim. diverses.

1) $\int_0^{\frac{\pi}{2}} l\,Tang.\,x.\left\{Cos.\,x.\,Arctg.\,(p\,Cos.\,x) - \frac{p\,Sin.^2\,x}{1+p^2\,Cos.^2\,x}\right\}dx = -\frac{1}{2}\pi\,l\,\{p+\sqrt{(1+p^2)}\}$ V. T. 369. N°. 8

2) $\int_0^{\frac{\pi}{2}} l\,Tang.\,x.\left\{Sin.\,x.\,Arctg.\,(p\,Sin.\,x) - \frac{p\,Cos.^2\,x}{1+p^2\,Sin.^2\,x}\right\}dx = \frac{1}{2}\pi\,l\,\{p+\sqrt{(1+p^2)}\}$ V. T. 369. N°. 8.

3) $\int_0^{\pi}\left[Sin.\,x.\,l\,(1+2\,p\,Cos.\,x+p^2) + 2\,Cos.\,x.\,Arcsin.\left(\frac{p\,Sin.\,x}{\sqrt{(1+2p\,Cos.\,x+p^2)}}\right)\right]dx =$

$= \frac{2}{p}\,l\frac{1+p}{1-p} + 2\,l(1-p^2) - 4$

4) $\int_0^{\pi}\left[Cos.\,x.\,l\,(1+2\,p\,Cos.\,x+p^2) - 2\,Sin.\,x.\,Arcsin.\left(\frac{p\,Sin.\,x}{\sqrt{(1+2p\,Cos.\,x+p^2)}}\right)\right]dx = 0$

(3–4) Dienger, Cr. 37. 363. , $p^2 < 1$;

1) $\int_0^1 li.\left(\frac{1}{x}\right).Sin.(q\,l\,x)\,dx = \frac{1}{1+q^2}\left(q\,l\,q - \frac{1}{2}\pi\right)$ V. T. 442. N°. 5.

2) $\int_0^1 li.\left(\frac{1}{x}\right).Cos.(q\,l\,x)\,dx = \frac{-1}{1+q^2}\left(l\,q + \frac{1}{2}q\,\pi\right)$ V. T. 442. N°. 7.

3) $\int_0^1 l\,\Gamma(x).Sin.(2a\pi x)\,dx = \frac{1}{2a\pi}(l\,a + A + l\,2\,\pi)$

4) $\int_0^1 l\,\Gamma(x).Cos.(2a\pi x)\,dx = \frac{1}{4a}$

(3, 4) Kummer, Cr. 35. 1.

5) $\int_0^\infty li.\left(\frac{1}{x}\right).Sin.(q\,lx)\,dx = -\frac{\pi}{1+q^2}$ V. T. 444. N°. 1 et 7.

6) $\int_0^\infty li.\left(\frac{1}{x}\right).Cos.(q\,lx)\,dx = -\frac{q\,\pi}{1+q^2}$ V. T. 444. N°. 2 et 8.

7) $\int_1^\infty li.\left(\frac{1}{x}\right).Sin.(q\,l\,x)\,dx = -\frac{1}{1+q^2}\left\{\frac{1}{2}\pi + q\,l\,q\right\}$ V. T. 442. N°. 4.

8) $\int_1^\infty li.\left(\frac{1}{x}\right).Cos.(q\,l\,x)\,dx = \frac{1}{1+q^2}\left\{l\,q - \frac{1}{2}q\,\pi\right\}$ V. T. 442. N°. 6.

9) $\int_0^{\frac{\pi}{2}} l\,Sin.\,Amp.\left(\frac{2x\,F'(p)}{\pi}\right)dx = \frac{-\pi}{4\,F'(p)}\left[F'(p).\,lp + \frac{1}{2}\pi\,F'\{\sqrt{(1-p^2)}\}\right]$

10) $\int_0^{\frac{\pi}{2}} l\,Cos.\,Amp.\left(\frac{2x\,F'(p)}{\pi}\right)dx = \frac{\pi}{4\,F'(p)}\left[F'(p).\,l\frac{\sqrt{(1-p^2)}}{p} - \frac{1}{2}\pi\,F'\{\sqrt{(1-p^2)}\}\right]$

(9, 10) Roberts, L. 12. 449.

F. Alg. rat. ent.
Plusieurs Fonctions. TABLE 445. Lim. diverses.

1) $\int_0^1 li.(x).Sin.(q\,l\,x).x^{p-1}\,dx = \frac{-1}{p^2+q^2}\left[p\,Arctg.\left(\frac{q}{1+p}\right) - \frac{1}{2}ql\,\{(1+p)^2+q^2\}\right]$ V. T. 442. N°. 12.

2) $\int_0^1 li.(x).Cos.(q\,l\,x).x^{p-1}\,dx = \frac{-1}{p^2+q^2}\left[q\,Arctg.\left(\frac{q}{1+p}\right) + \frac{1}{2}pl\,\{(1+p)^2+q^2\}\right]$ V. T. 442. N°. 13.

3) $\int_0^1 Sin.(q\, Arccos.\, x).\, l\, x.\, x^{q-1}\, dx = \frac{\pi}{2^{q+2}}\left(A + Z'(q) - \frac{1}{q} - 2\, l2\right)$ V. T. 331. N°. 12.

4) $\int_0^\infty e^{-x} Sin.x.lx.x^{p-1}\, dx = \frac{1}{2^{\frac{1}{2}p}}\, \Gamma(p)\left[\frac{1}{4}\pi\, Cos.\frac{1}{4}p\pi - \frac{1}{2}Sin.\frac{1}{4}p\pi.\, l2 + Sin.\frac{1}{4}p\pi.\, Z'(p)\right]$ Schlömilch, Cr. 33. 316.

5) $\int_0^\infty e^{-ax} Sin.\, bx.\, l\, x.\, x^{p-1}\, dx = \frac{-\Gamma(p)}{\sqrt{(a^2+b^2)^p}}\left\{\frac{1}{2}l(a^2+b^2).\, Sin.\left(p\, Arctang.\frac{b}{a}\right) - \right.$

$\left. - Arctang.\frac{b}{a}.\, Cos.\left(p\, Arctang.\frac{b}{a}\right) - Sin.\left(p\, Arctang.\frac{b}{a}\right).\, Z'(p)\right\}$

6) $\int_0^\infty e^{-ax} Cos.\, bx.\, l\, x.\, x^{p-1}\, dx = \frac{-\Gamma(p)}{\sqrt{(a^2+b^2)^p}}\left\{\frac{1}{2}l(a^2+b^2).\, Cos.\left(p\, Arctang.\frac{b}{a}\right) + \right.$

$\left. + Arctang.\frac{b}{a}.\, Sin.\left(p\, Arctang.\frac{b}{a}\right) - Cos.\left(p\, Arctang.\frac{b}{a}\right).\, Z'(p)\right\}$

Sur les intégrales (5) à (6) voyez: Legendre, Exerc. 3. 56. — Cauchy, P. 28. 147. I. § 6. — Schlömilch, Stud. I. 14.

7) $\int_0^\infty e^{-px} Cos.qx.lx.\{px Tang.qx - qx - a Tang.qx\}\, x^{a-1}\, dx = \frac{1^{a-1/1}}{(p^2+q^2)^{\frac{1}{2}a}}\, Sin.\left(a\, Arctg.\frac{q}{p}\right)$ V. T. 386. N°. 12.

8) $\int_0^\infty e^{-px} Cos\, qx.lx.\{px + qx\, Tang.qx - a\}\, x^{a-1}\, dx = \frac{1^{a-1/1}}{(p^2+q^2)^{\frac{1}{2}a}}\, Cos.\left(a\, Arctg.\frac{q}{p}\right)$ V. T. 386. N°. 13.

9) $\int_0^\infty e^{-px}\{p\, l\, Sin.\, qx - q\, Cot.\, qx\}\, x\, dx = -\frac{1}{2p}l2 - \frac{1}{2}p\sum_1^\infty \frac{1}{n}\frac{1}{p^2+n^2q^2}$ V. T. 439. N°. 6.

10) $\int_0^\infty e^{-px}\{p\, l\, Cos.\, qx + q\, Tang.\, qx\}\, x\, dx = -\frac{1}{2p}l2 - \frac{1}{2}p\sum_1^\infty \frac{1}{n}\frac{(-1)^n}{p^2+n^2q^2}$ V. T. 439. N°. 7.

11) $\int_0^\infty e^{-px}\{p\, l\, Tang.qx - 2q\, Cosec.2qx\}\, x\, dx = -p\sum_1^\infty \frac{1}{2n-1}\frac{1}{p^2+(2n-1)^2q^2}$ V. T. 439. N°. 8.

12) $\int_0^\infty e^{-bx} Sin.\left(cx - Arctang.\frac{c}{b}\right).\, lx.\, dx = \frac{1}{\sqrt{(b^2+c^2)}}\, Arctang.\frac{c}{b}$

13) $\int_0^\infty e^{-bx} Sin.\left(cx - a Arctang.\frac{c}{b}\right).\, lx.\, x^{a-1}\, dx = \frac{1^{a-1/1}}{(b^2+c^2)^{\frac{1}{2}a}}\, Arctang.\frac{c}{b}$

(12)–(13): Legendre, Exerc. 3. 56.

14) $\int_0^\infty e^{-hcx}(e^{-2cx}-2e^{-cx}\,Cos.\,bx+1)^{\frac{1}{2}g}\,Sin.\left\{bhx+g\,Arctang.\left(\frac{e^{-cx}\,Sin.\,bx}{e^{-cx}\,Cos.\,bx-1}\right)\right\}x^{q-1}\,dx=$

$=\frac{\Gamma(q)}{(b^2+c^2)^{\frac{1}{2}g}}\,Sin.\left(q\,Arctang.\frac{b}{c}\right)\Delta^g.\,h^{-q}$

15) $\int_0^\infty e^{-hcx}(e^{-2cx}-2e^{-cx}\,Cos.\,bx+1)^{\frac{1}{2}g}\,Cos.\left\{bhx+g\,Arctang.\left(\frac{e^{-cx}\,Sin.\,bx}{e^{-cx}\,Cos.\,bx-1}\right)\right\}x^{q-1}\,dx=$

$=\frac{\Gamma(q)}{(b^2+c^2)^{\frac{1}{2}g}}\,Cos.\left(q\,Arctang.\frac{b}{c}\right)\Delta^g.\,h^{-q}$

Les intégrales (14), (15) se trouvent chez Cauchy, P. 28. 147. P. III. § 1.

16) $\int_0^\infty e^{-px}\{lx+Z'(q)\}x^{q-1}\,dx=-\Gamma(q)\frac{lp}{p^q}$

17) $\int_0^\infty e^{-px}(e^{-x}-1)^a\{lx+Z'(q)\}x^{q-1}\,dx=-\Gamma(q)\,\Delta^a.\frac{lp}{p^q}$

(16), (17): Cauchy, P. 28. 147. 1. § 7.

18) $\int_0^{\frac{1}{a}} lx.\,Sin.\,(b\,Arccos.\,ax).\,x^{b-1}\,dx=\frac{\pi}{2^{b+2}a^b}\left[A+Z'(a)-\frac{1}{a}-2l(2a)\right]$ Lindmann, Stockh. Handl. 1850. III.

F. Alg. rat. fract.
Plusieurs Fonctions. TABLE 446. Lim. diverses.

1) $\int_0 Arctang.\,x.\,Sin.(plx)\frac{dx}{x}=-\frac{\pi}{4p}\frac{(e^{\frac{1}{2}p\pi}-1)^2}{e^{p\pi}+1}$ V. T. 404. N°. 7.

2) $\int_0^1 Arctang.\,x\frac{pl\,x.\,Cos.\,(plx)-Sin.\,(plx)}{(lx)^2}\frac{dx}{x}=\frac{1}{2}p\pi-Arctang.\,(e^{\frac{1}{2}p\pi})$ V. T. 406. N°. 15.

3) $\int_0^1 li.(x).\,Sin.\,(q\,lx)\frac{dx}{x}=\frac{1}{2q}l(1+q^2)$ V. T. 442. N°. 2.

4) $\int_0^1 li.(x).\,Sin.\,(qlx)\frac{dx}{x^2}=\frac{1}{1+q^2}\left(q\,l\,q+\frac{1}{2}\pi\right)$ V. T. 442. N°. 4.

5) $\int_0^1 li.(x).\,Cos.\,(q\,lx)\frac{dx}{x}=-\frac{1}{q}Arctang.\,q$ V. T. 442. N°. 3.

6) $\int_0^1 li.(x).\,Cos.\,(qlx)\frac{dx}{x^2}=\frac{1}{1+q^2}\left(lq-\frac{1}{2}q\pi\right)$ V. T. 442. N°. 6.

7) $\int_0^1 \left\{li.(x) + x^2\, li.\left(\frac{1}{x}\right)\right\} Sin.(q\,l\,x)\frac{dx}{x^2} = \frac{2q}{1+q^2}\,lq$ V. T. 442. N°. 8.

8) $\int_0^1 \left\{li.(x) - x^2\, li.\left(\frac{1}{x}\right)\right\} Sin.(q\,l\,x)\frac{dx}{x^2} = \frac{\pi}{1+q^2}$ V. T. 442. N°. 9.

9) $\int_0^1 \left\{li.(x) + x^2\, li.\left(\frac{1}{x}\right)\right\} Cos.(q\,l\,x)\frac{dx}{x^2} = -\frac{q\pi}{1+q^2}$ V. T. 442. N°. 10.

10) $\int_0^1 \left\{li.(x) - x^2\, li.\left(\frac{1}{x}\right)\right\} Cos.(q\,l\,x)\frac{dx}{x^2} = \frac{2\,lq}{1+q^2}$ V. T. 442. N°. 11.

11) $\int_0^\infty e^{-x^2}(l Cos\,ax + ax\,Tang.\,ax)\frac{dx}{x^2} = \sqrt{\pi}.\left\{-l2 + \sum_1^\infty (-1)^{n-1}\frac{e^{-(na)^2}}{n}\right\}$ V. T. 431. N°. 10.

12) $\int_0^\infty Sin.\{p\,l(1 + 2q\,Cos.\,ax + q^2)\}.\left[e^{pArctg.\left(\frac{qSin.ax}{1+qCos.ax}\right)} + e^{-pArctg.\left(\frac{qSin.ax}{1+qCos.ax}\right)}\right]\frac{x}{c^2+x^2}dx =$

$= \pi\,Sin.\{p\,l\,(1 + q\,e^{-ac})\}$

13) $\int_0^\infty Sin.\{p\,l(1 + 2q\,Cos.\,ax + q^2)\}.\left[e^{pArctg.\left(\frac{qSin.ax}{1+qCos.ax}\right)} - e^{-pArctg.\left(\frac{qSin.ax}{1+qCos.ax}\right)}\right]\frac{x}{c^2+x^2}dx =$

$= -\pi\,Sin.\{p\,l(1 + q\,e^{-ac})\}$

14) $\int_0^\infty Cos.\{p\,l(1 + 2q\,Cos.\,ax + q^2)\}.\left[e^{pArctg.\left(\frac{qSin.ax}{1+qCos.ax}\right)} + e^{-pArctg.\left(\frac{qSin.ax}{1+qCos.ax}\right)}\right]\frac{dx}{c^2+x^2} =$

$= \frac{\pi}{c}\,Cos.\{p\,l(1 + q\,e^{-2ac})\}$

15) $\int_0^\infty Cos.\{p\,l(1 + 2q\,Cos.\,ax + q^2)\}.\left[e^{pArctg.\left(\frac{qSin.ax}{1+qCos.ax}\right)} - e^{-pArctg.\left(\frac{qSin.ax}{1+qCos.ax}\right)}\right]\frac{dx}{c^2+x^2} =$

$-\frac{\pi}{2c} + \frac{\pi}{c}\,Cos.\{p\,l(1 + q\,e^{-2ac})\}$

Sur les intégrales (12) à (15) voyez: Boncompagni. Cr. 25. 74; elles sont fautives.

16) $\int_0^\infty \frac{Cos.(\frac{1}{2}a\pi - bx).\,l(1+x^2) + 2\,Sin.(\frac{1}{2}a\pi - bx).\,Arctg.\,x}{\{\frac{1}{2}l(1+x^2)\}^2 + (Arctg.\,x)^2}\,\frac{x^a}{x^2+c^2}dx = \frac{\pi c^{a-1}}{l(1+c)}e^{-bc}$

17) $\int_0^\infty \frac{Cos.\,bx.\,l(1+x^2) - 2\,Sin.\,bx.\,Arctg.\,x}{\{\frac{1}{2}l(1+x^2)\}^2 + (Arctg.\,x)^2}\,\frac{dx}{x^2+c^2} = \frac{\pi}{c}\left\{\frac{e^{-bc}}{l(1+c)} - \frac{1}{c}\right\}$

Cauchy, P. 19. 511. (16, 17)

18) $\int_0^\infty \frac{Sin.\,a\,x.\,l\,(1+p^2x^2)+2\,Cos.\,a\,x.\,Arctg.\,p\,x}{\{\frac{1}{2}l(1+p^2x^2)\}^2+(Arctg.\,p\,x)^2}\frac{x}{1+x^2}dx = \frac{\pi e^{-a}}{l(1+p)}$ Cauchy, Cours. Leç. 39.

19) $\int_0^\infty li.\,(x).\,Sin.\,(q l x)\frac{dx}{x^2} = \frac{\pi}{1+q^2}$ V. T. 446. N°. 4, 24.

20) $\int_0^\infty li.\,(x).\,Cos.\,(q\,l\,x)\frac{dx}{x^2} = -\frac{q\pi}{1+q^2}$ V. T. 446. N°. 6, 25.

21) $\int_0^\infty e^{-px}(e^{-x}-1)^a\frac{l\,x+Z'(a)}{x^{b+1}}dx = \frac{\pi}{1^{b/1}\,Sin.\,b\,\pi}\Delta^a.(p^b\,l\,p),\ b<a;$ Cauchy, P. 28. 147. I. § 7.

22) $\int_0^\infty e^{-px}(e^{-x}-1)^a\frac{lx-Z'(q+1)-\pi Cot.\{(q+1)\pi\}}{x^{q+1}}dx = -\frac{\pi}{\Gamma(q+1)}Cosec.\{(q+1)\pi\}.\,\Delta^a.(p^a\,lp),\ q<a;$

23) $= -\frac{\pi}{\Gamma(q+1)}Cosec.\{(q+1)\pi\}.\Delta^a.(p^a lp)$ valeur extraord. $q>a;$

Les intégrales 22) et 23) se trouvent chez Cauchy, Exerc. 1826. p. 58.

24) $\int_1^\infty li.\,(x).\,Sin.\,(q\,lx)\frac{dx}{x^2} = \frac{-1}{1+q^2}\left\{q\,l\,q-\frac{1}{2}\pi\right\}$ V. T. 442. N°. 5.

25) $\int_1^\infty li.\,(x).\,Cos.\,(qlx)\frac{dx}{x^2} = \frac{-1}{1+q^2}\left\{l\,q+\frac{1}{2}q\,\pi\right\}$ V. T. 442. N°. 7.

26) $\int_{-\infty}^\infty Cos.\,(p\ Arctang.\,a\,x)\frac{l(1+a^2x^2)}{(1+a^2x^2)^{\frac{1}{2}p}}\frac{dx}{1+x^2} = \frac{2\pi}{(1+a)^p}l(1+a)$

27) $\int_{-\infty}^\infty \frac{e^{pArctg.ax}+e^{-pArctg.ax}}{1+x^2}Sin.\left\{\frac{1}{2}p\,l(1+a^2x^2)\right\}dx = 2\pi\,Sin.\,\{p\,l\,(1+a)\}$

28) $\int_{-\infty}^\infty \frac{e^{pArctg.ax}-e^{-pArctg.ax}}{1+x^2}Cos.\left\{\frac{1}{2}p\,l\,(1+a^2x^2)\right\}dx = 2\pi\,Cos.\,\{p\,l\,(1+a)\}$

(26–28: Cauchy, C. R. 11. 1008. où 28) était fautive.)

F. Alg. irrat. fract. Plusieurs Fonctions. TABLE 447. Lim. diverses.

1) $\int_0^1 \{e^{q\sqrt{(1-x^2)}}-e^{-q\sqrt{(1-x^2)}}\}\,Sin.\,q\,x.\,Sin.\,(2\,c\,Arccos.\,x)\frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2}\pi\frac{(-1)^{c-1}q^{2c}}{1^{2c/1}}$ V. T. 288. N°. 6.

2) $\int_0^1 \{e^{q\sqrt{(1-x^2)}}+e^{-q\sqrt{(1-x^2)}}\}\,Sin.qx.Cos.\{(2c-1)\,Arccos.x\}\frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2}\pi\frac{(-1)^{c-1}q^{2c-1}}{1^{2c-1/1}}$ V. T. 288. N°. 8.

Page 571.

3) $\int_0^1 \left\{e^{q\sqrt{(1-x^2)}} - e^{-q\sqrt{(1-x^2)}}\right\} Cos.qx.Sin.\left\{(2c-1) Arccos.x\right\} \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2}\pi \frac{(-1)^{c-1} q^{2c-1}}{1^{2c-1/1}}$ V. T. 288. N°. 7.

4) $\int_0^1 \left\{e^{q\sqrt{(1-x^2)}} + e^{-q\sqrt{(1-x^2)}}\right\} Cos.q\,x.\,Cos.(2\,c\,Arccos.\,x) \frac{dx}{\sqrt{(1-x^2)}} = \frac{1}{2}\pi \frac{(-1)^c q^{2c}}{1^{2c/1}}$ V. T. 288. N°. 9.

5) $\int_0^\infty Sin.(q\,Arctg.\,x) \frac{lx}{x} \frac{dx}{(1+x^2)^{\frac{1}{2}q}} = -\frac{1}{2}\pi\left\{A + Z'(q)\right\}$ V. T. 333. N°. 9.

6) $\int_0^\infty Cos.(q Arctg.\,x).\,l\,x \frac{dx}{(1+x^2)^{\frac{1}{2}q}} = -\frac{\pi}{2(q-1)}$ V. T. 333. N°. 8.

7) $\int_0^\infty Sin.(q\,Arccot.\,x).\,l\,x \frac{x^{q-1}}{(1+x^2)^{\frac{1}{2}q}} dx = \frac{1}{2}\pi\left\{A + Z'(q)\right\}$ V. T. 447. N°. 5.

8) $\int_0^\infty Cos.(q Arccot.\,x).\,l\,x \frac{x^q}{(1+x^2)^{\frac{1}{2}q}} dx = \frac{\pi}{2(q-1)}$ V. T. 447. N°. 6.

9) $\int_0^\infty Sin.\left\{(c+1) Arctg.\frac{a}{bx}\right\}.lx \frac{x^c}{\sqrt{(a^2+b^2x^2)^{c+1}}} dx = \frac{\pi}{2b^{c+1}}\left\{l\frac{a}{b} + A + Z'(c+1)\right\}$

10) $\int_0^\infty Cos.\left\{(c+1) Arctg.\frac{a}{bx}\right\}.lx \frac{x^{c-1}}{\sqrt{(a^2+b^2x^2)^{c+1}}} dx = \frac{\pi}{2acb^c}$

(9, 10) Lindmann, Stockh. Handl. 1850. II.