





## Historic, archived document

Do not assume content reflects current scientific knowledge, policies, or practices.



# UNITED STATES DEPARTMENT OF AGRICULTURE



## DEPARTMENT BULLETIN No. 1351



Washington, D. C.



September, 1925

### WHAT MAKES THE PRICE OF OATS<sup>1</sup>

By HUGH B. KILLOUGH

*Assistant Agricultural Statistician, Bureau of Agricultural Economics*

#### CONTENTS

	Page		Page
Statement of the problem.....	1	Discussion of method.....	19
Sources of data and scope of analysis.....	3	Future prices and condition reports as price indicators.....	20
Factors affecting annual price of oats.....	4	Relation of future prices to cash prices.....	20
Production and the area of the market.....	4	Relation between crop condition estimates and production.....	21
Trend of prices of oats.....	6	Conclusions regarding future prices.....	22
Comparison of the values of large and small crops.....	9	A study of wheat prices.....	23
Application to a cooperative marketing problem.....	10	Extent of wheat market and its influence on Chicago price.....	23
Multiple correlation of price factors.....	11	Notes on method.....	25
Seasonal variations in oat prices.....	13	Appendix A.—Statistical tables.....	27
Application of seasonal trend in estimating price.....	17	Appendix B.—Selected references.....	38
September price as a basis for estimating.....	18		

#### STATEMENT OF THE PROBLEM

The market price of oats, like the prices of all other farm products, changes from year to year, season to season, month to month, and day to day. It is commonly recognized that these price changes are caused by changes in supply or demand, or both; but the general phrase "supply and demand" furnishes no precise explanation of given variations in price. A detailed study is necessary to define the supply, allocate the demand, and furnish quantitative measures of the influence of the various price-making forces.

In recent years the application of statistical methods to economic problems has had remarkable development. Most of this development, however, in so far as it has been applied to forecasting the future, has been in the field of business statistics, and the problem of forecasting the prices of the various agricultural products has been comparatively neglected. The value of any method which would offer to the farmer or to the student of agricultural prices even an approximate means of estimating future prices is evident. To the farmer, its most important use would be in determining the best time to sell his

<sup>1</sup> E. M. Daggitt, Associate Agricultural Economist, gave valuable assistance in the preparation of this bulletin.

crop, and in settling his age-old problem of whether to store or not to store. Good business management on the part of the present-day farmer requires, in addition to a knowledge of the best methods of production, as thorough an understanding as possible of the forces which determine the prices of his product; low profits may result as well from poor judgment in selling as from poor judgment in growing the crop.

It may be objected that the average farmer can not make use of scientific price studies because of their unavoidable technicality; but the development in recent years of trained intermediaries in the dissemination of market and other information among farmers and the growing up of a more scientific-minded farming class have brought to them more and more of the benefits of scientific studies of all kinds.

To the student of agricultural prices any thorough price study brings to light new methods of attacking the problem and new uses of old methods. It helps to point out the strength and weakness of the various statistical methods, to the end that their fields of usefulness may be more clearly defined for those who continue the work. For this purpose the mention of trials which have been found to give no worth-while results should be of considerable value.

In any price analysis it is first necessary to determine the area of the market, for upon that depends in large measure the selection of methods that may be used. The difference in the scope of the market for wheat and oats, for example, makes a great difference in the characteristics of demand and supply for the two crops. Upon the area of the market and the characteristics of production depend the answer to the question as to whether, for the given crop, one may assume that there is a normal annual price—an average price at which the annual supply will be moved from the market. It is difficult to assume a normal annual price in the case of wheat, because of the influence of foreign production. On the other hand, such an annual price may be assumed for a crop which is grown and consumed almost entirely within the country for which the study is made and of which there is a single annual supply which becomes available for the market within a short period of the year.

The oat crop of the United States is found to come within this classification. Normally the supply of oats in this country is produced and mostly consumed within the borders of the Nation. From 1909 to 1913, and since the war, our exports have averaged not more than 2 per cent of the crop and our imports have been still less. During the war, however, the demand for oats was abnormal and a considerable quantity was exported, causing a temporary widening of the market.

Having decided that oats is a crop that may be treated in a price analysis by assuming a normal annual price, the problem is to discover the factors which determine this annual price and to measure their influence. This requires the bringing together of such factors as may be expected to influence the price of oats, the study of each by the application of various statistical methods, and the selection of those which are found to have a measurable influence for use in developing an estimating formula, by means of which the most probable average annual price may be estimated from given values of the selected factors.

Obtaining a method for estimating the annual price, however, is but one important step in the price analysis. The problem remains of explaining the variations in price during the year, in order that by using the annual price as a basis one may calculate the probable seasonal movement. It is this seasonal movement of prices, rather than the average annual price, which is of great importance to the farmer who has oats to sell. The statistical methods used in this part of the study differ from those used in the earlier part, and on the whole the analysis offers greater difficulty.

The most difficult part of the crop year for those attempting to forecast prices is during the growing season. Once the supply is fairly well known the principal factor to be considered by the forecaster is changes in demand; but during the growing season not only demand, but supply as well, must be estimated. The problem is to discover some indicator of the volume of production which anticipates the harvest. Two of these, the price of September futures and the monthly condition of the crop as reported by the United States Department of Agriculture, are available. These must be compared by the use of statistical methods as to their relative accuracy in predicting the future prices, and the variations in accuracy as the growing season progresses must be measured.

Finally, to make this study of greater value to those who may continue the work of analyzing grain prices, it has been thought advisable to apply to wheat some of the methods used in the analysis of oat prices, to emphasize the difference in the method of treatment necessary for this crop. The market for oats is a domestic market, and the price is determined very largely by the domestic supply. The demand for wheat, on the other hand, is a world demand, the supply a world supply. The price is determined in the world market and is affected comparatively little by the size of the crop of the United States, as is shown in this study. The application of the theory of an average annual price resulting from an annual supply is more difficult.

#### SOURCES OF DATA AND SCOPE OF ANALYSIS

The data on oat prices used in this study were obtained from the annual reports of the Chicago Board of Trade. The weekly high and low prices for No. 2 oats at Chicago were averaged to give monthly prices, and the monthly prices thus obtained were averaged for the crop years July to June, to obtain the yearly figures. In the study of the relation between cash and future prices a more accurate series was needed. For this purpose monthly cash and future prices were obtained by averaging the daily high and low prices.

Data on production were obtained, for the most part, from the Yearbooks of the United States Department of Agriculture. No figures were available regarding the world carry-over of wheat, so these were calculated for the period 1891 to 1923 from data obtained from various sources, as explained in the Appendix.

The wheat prices were obtained from the annual reports of the Chicago Board of Trade. The monthly figures are averages of daily high and low prices. Owing to the changes in grades from time to time, there was some difficulty in getting comparable grades for the entire period. Those used were: July, 1890, to January, 1897, No. 2

Spring wheat; January, 1897, to January, 1898, No. 2 wheat; January, 1898, to March, 1903, regular No. 2; March, 1903, to June, 1922, No. 2 Red wheat.

That the Chicago price is representative of prices for the United States as a whole is shown by the fact that correlation coefficients of +0.98 for oats and of +0.99 for wheat are obtained when Chicago prices are correlated with the December farm prices as estimated by the Department of Agriculture.

The period covered in the major part of the study is that from 1896 to 1922, omitting the years 1916, 1917, 1918, 1919, and 1920. The omission of these years was considered necessary for accurate results, since the abnormal conditions of demand and abnormal changes in the price level during this period would tend to obscure the effect of forces that under normal conditions would be operative in the oats market.

An examination of exports of oats during the war period as compared with years before and after the war will show the extent to which the export demand was abnormal during the period omitted in this study.

TABLE 1.—Exports of oats from the United States, years ending June 30<sup>1</sup>

Year	Quantity
	<i>Bushels</i>
Av. 1910-1914.....	8,304,000
Av. 1915-1919.....	96,774,000
1920.....	33,945,000
1921.....	4,302,000

<sup>1</sup> U. S. Dept. Agr., Yearbook, 1921, p. 74.

## FACTORS AFFECTING ANNUAL PRICE OF OATS

### PRODUCTION AND THE AREA OF THE OAT MARKET

A study of the production of oats in the United States shows that for 30 years or more preceding the World War the trend of production has been steadily upward, reaching its peak in 1917 with a production of 1,593 million bushels. Since 1917 the production has slightly declined, the 1923 figure being 1,300 million bushels. Practically all of the oats produced in this country are consumed here. In the pre-war period, 1909 to 1913, the exports of oats from the United States averaged not quite 1 per cent of the crop, and imports were less.<sup>2</sup> During the war an abnormal foreign demand increased our exports, but since the war they have returned to their former low figures.

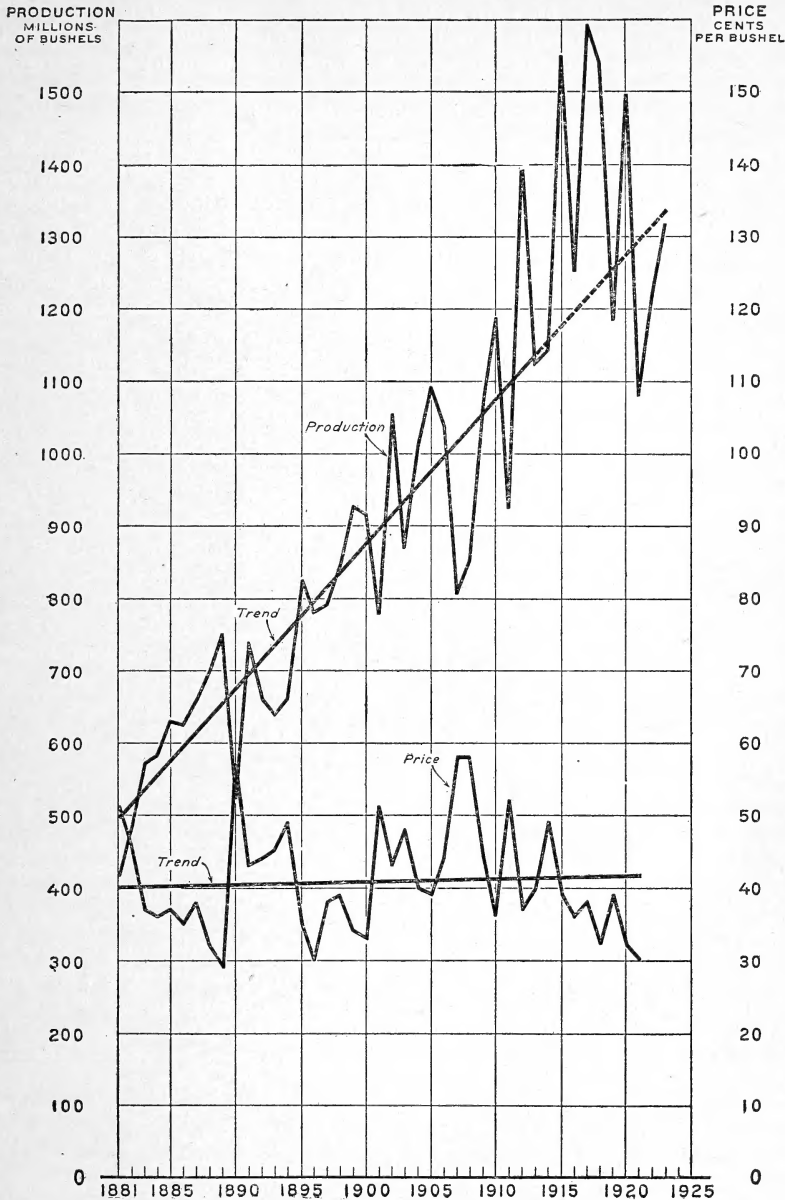
Figure 1 represents graphically the production of oats during the period 1881-1922. Superimposed upon the graph of production is a straight line representing the trend of production during that period.<sup>3</sup>

These figures regarding production, imports, and exports indicate at once that in spite of the enormous production of oats in this country the market area is limited to the United States, a fact which is of considerable importance in the selection of methods for the study of prices.

<sup>2</sup> U. S. Dept. Agr., Yearbook, 1921, p. 781, 551.

<sup>3</sup> The method of calculating this trend is explained in Table I, Appendix A, p. 27.





A comparison of the production and the price of oats

FIG. 1.—Although the production of oats has shown a downward tendency since 1917, the general trend since 1881 has been decidedly upward. The price of oats, corrected for changes in the general level of prices, has shown a slight upward trend for the period 1881-1922, though in recent years the tendency has been downward, in spite of the decrease in production. During most of the period the price was above the trend when production was below, and vice versa

Incidentally, an understanding of the size of the market is of considerable importance to farmers who are considering the shifting of acreage between wheat and oats, for a change in production which would cause a relatively small change in wheat prices might cause a great change in the price of oats, because oats are sold in a narrower market.

#### TREND OF PRICES OF OATS

The trend of oat prices for the period 1881 to 1922 has been less uniform than the trend of production. It tended downward from 1881 until the low point was reached during the nineties. After 1896 the trend was gradually upward until the close of the war period. The lack of uniformity in the trend of prices makes it impossible of representation by a straight line. There are actually two trends, one downward until 1896 and the other an irregular trend from 1896 to the present year in a general upward direction. Two methods of representing this trend are illustrated in Figure 2. One is the use of two straight lines to show the downward and upward trends; the other is the use of a third degree parabola curve.

A comparative examination of the graphs of production and price in Figure 1, where the prices have been corrected for changes in the price level, shows that during the period from 1881 to 1913 prices were usually below the normal, represented by the trend line, when production was above, and above when production was below. In fact, during much of the period one curve seems almost the exact opposite of the other, if allowance is made for the difference in trends. The closeness of the relationship as indicated by the two graphs shows clearly that the size of the oat crop in the United States has an important influence upon the year-to-year changes in price. Prices used in these two graphs are averages for crop years, September to August, instead of July to June, as used in the rest of the bulletin.

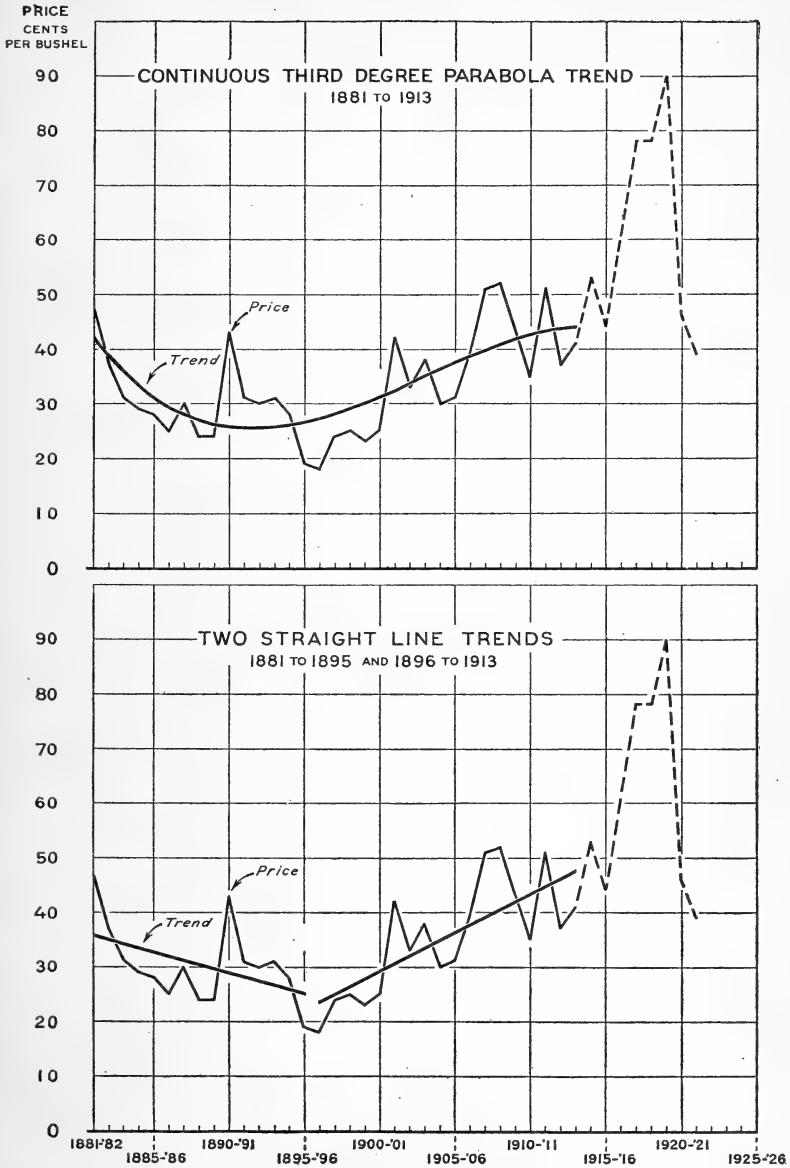
Graphs like these, useful as they are in showing the nature of the relationship between two factors, furnish no measure of the closeness of that relationship, nor do they provide a method of estimating one when the other is known. To obtain these two results it is necessary to make use of the statistical device of correlation.<sup>4</sup>

The relation between these two factors, when the ratio of production to the trend of production is correlated with the ratio of price to the trend of price, is expressed by a correlation coefficient of  $-0.82$ . The negative sign indicates that a change in one factor is accompanied by a change in the opposite direction in the other; and the size of the coefficient measures both the extent to which changes in one factor are associated with changes in the other and the accuracy with which values of one factor may be predicted from known values of the other during the period covered by the study, assuming that perfect correlation is represented by the coefficient  $\pm 1.00$ .

Not the actual value of the coefficient, however, but the square of the value represents the proportion of the change in price that is accounted for by the change in the other variable.

A coefficient of correlation, squared, measures between  $X$  and  $Y$  the proportion of variation in  $Y$  that can be accounted for by variations in  $X$ , provided one is defining variability as the standard

<sup>4</sup> The method of obtaining the coefficient of correlation is explained in Appendix A, Table II. p. 23.



Two methods of representing the trend of oat prices

FIG. 2.—It is impossible to represent the trend of actual oat prices by a single straight line. The trend of the price of oats was downward from the Civil War period until 1896, and upward from 1896 to 1919. A parabola and two straight lines are used in this figure to represent the trend of oat prices from 1881 to 1913

deviation squared. Ratios to trends are used here as a method of eliminating the influence of the upward tendency of production and the downward and upward trends of price which would partially obscure the relationship between the two factors and result in a lower coefficient of correlation.

To predict the average annual price of oats when the production is known, an "estimating equation" must be worked out from the results obtained in the correlation. This equation has the form  $y = a + bx$ , in which  $y$  is the price to be estimated,  $x$  is the production during the given year, and  $a$  and  $b$  are constant terms that must be

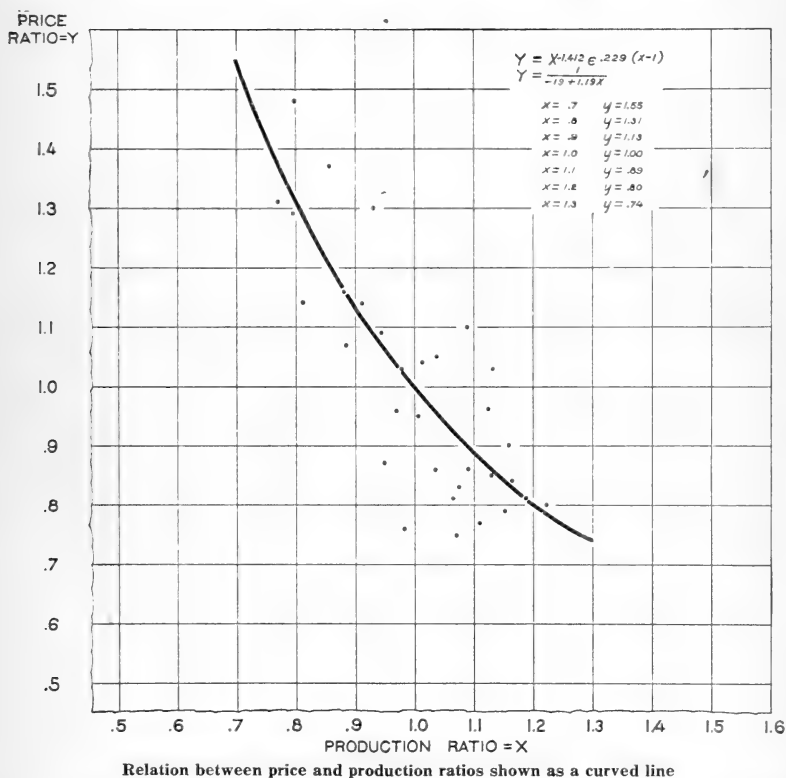


FIG. 3.—A curved line describes the relation between price and production better than does a straight line. If the relation were perfect, all of the dots in the scatter diagram would fall exactly on the line. The curve in this figure is described by two formulae, a reciprocal formula used by Working coinciding with an exponential formula used by Moore

calculated from the data. A coefficient of 0.82, however, is not large enough to give sufficiently accurate results in forecasting prices. Other factors must be considered in addition to production, so that more of the variation in price will be accounted for than that due to production alone.

The equation just given assumes that the relation between the two factors is expressed graphically by a straight line; that is, that regardless of the size of the factors, a given change in one is always associated with the same estimated change in the other. Consideration of the theory of elasticity of demand and the concept of diminishing utility suggests that a straight line may not represent most

accurately the relation between production and price, but that better results might be obtained through the use of curvilinear functions such as those employed by Moore<sup>5</sup> and Working.<sup>6</sup> A curvilinear relation suggests, that, for example, the addition of 50 million bushels of oats to a 1,400 million bushel crop may lower the price per bushel less than the addition of the same amount to a crop of only 800 million bushels.

In Figure 3 curves of the type referred to are fitted to the scatter diagram of production ratio and price ratio of oats. A curve of the type used by Working described by the equation  $Y = \frac{1}{-.19 + 1.19X}$  coincides approximately with a curve of the type used by Moore described by the equation  $Y = X^{-1.412} \epsilon^{.229(X-1)}$  where  $Y$  and  $X$  are price ratio and production ratio, respectively, and  $\epsilon$  is a constant.<sup>7</sup> A curve described by either formula fits the data somewhat more closely than a straight line; that is, the sum of the squares of the deviations or residuals from the curves is less than the sum of the squares of the deviations from a best fitting straight line fitted by the method of least squares. Several other curves were tried and found not to fit the data as well as those illustrated.<sup>8</sup>

#### COMPARISON OF THE VALUES OF LARGE AND OF SMALL CROPS

The fact that the relation between production and price is found to be represented by a curved line of the type illustrated in Figure 3 suggests an interesting problem regarding the values of oat crops of various sizes. It is commonly said that a large crop may often be worth less than a small crop. This idea is borne out by a study of Figure 3. Here it is found that a decrease of 10 per cent from normal, from 1.0 on the scale to 0.9, is accompanied by an increase of 13 per cent in price, whereas an increase of 10 per cent above normal is accompanied by a decrease of 11 per cent in price. The values of production multiplied by price in both cases are illustrated in Table 2.

TABLE 2.—*Product of price multiplied by production when production is below and when production is above normal*

Production in terms of normal	Corresponding price in terms of normal	Product of production and price
0.90	1.13	1.017
1.10	.89	.979

These conclusions may be applied to actual data by comparing the values of the large crops of 1902, 1904, 1905, and 1906, with the values of the small crops of 1901, 1903, 1907, and 1908. Table 3

<sup>5</sup> H. L. Moore. Elasticity of demand and flexibility of prices. In Jour. Amer. Statist. Assoc., March, 1922.

<sup>6</sup> Holbrook Working. Factors determining the price of potatoes in St. Paul and Minneapolis. Minn. Agr. Exp. Sta. Tech. Bul. 10. 1922.

<sup>7</sup> The value of  $\epsilon$ , the base of the Napierian system of logarithms, is 2.7182818. The common logarithm of  $\epsilon$  is 0.4343.

<sup>8</sup> A suggestion has been made that these coincident curves do not exactly correspond to the economic concept of a demand curve and that the terminology used here may not be of the best. See Appendix B, p. 39, for reference on the subject of demand curves.

shows that, after the prices were adjusted for changes in the general price level, the total value of the four large crops was \$69,000,000 less than the total value of the four small crops.

TABLE 3.—Value product of oats for large and small crop years

Large crop years				Small crop years			
Year	Production, United States	Price adjusted for changes in price level <sup>1</sup>	Value of product	Year	Production, United States	Price adjusted for changes in price level <sup>1</sup>	Value of product
	<i>Million bushels</i>	<i>Cents per bushel</i>	<i>Million dollars</i>		<i>Million bushels</i>	<i>Cents per bushel</i>	<i>Million dollars</i>
1902.....	1, 053	27	284	1901.....	778	36	280
1904.....	1, 009	27	272	1903.....	869	30	261
1905.....	1, 090	24	262	1907.....	805	35	282
1906.....	1, 036	25	259	1908.....	851	38	323
Total.....			1, 077	Total.....			1, 146

<sup>1</sup> Price divided by the Bureau of Labor Statistics index number of all commodities, base 1890-1899, converted to the crop year by averaging monthly relatives, July to June. See Wholesale Prices 1890 to 1913. U. S. Bur. Labor Statis. Bul. 149, 1914.

#### APPLICATION TO A COOPERATIVE MARKETING PROBLEM

The foregoing study suggests a possible method of stabilizing oat prices. For 50 years farmers' organizations in the United States have tried in a variety of ways, ranging from prohibition of future trading to monopoly control, to reduce the fluctuations in the prices of farm products. At the present time two of the chief purposes of the American Farm Bureau Federation are (1) to extend the cooperative marketing of farm products, and (2) so to estimate the effective world supply of any farm product and so to regulate the flow to market as to eliminate sharp and extreme price fluctuations.

During the period 1881 to 1913 the oat crops of the United States were above the trend of production 11 times, below the trend 14 times, and about normal 6 times. There was no regularity in the sequence of large and small crops. Under the existing marketing system limited quantities of oats are carried from one crop year to the next. Carry-over figures<sup>9</sup> show that quantities consumed annually from 1896 to 1913 closely followed production and were not uniform from year to year. The question arises as to what would be the effect upon the gross value of oats if the surpluses from bounteous years were carried over to years in which the crop was small.

The gross value of the oats consumed in the United States from 1895 to 1913, on the basis of the December 1 farm price, was \$5,964,000,000.<sup>10</sup> Assuming that price would have followed the trend of prices, 1895 to 1913, had the supply of oats put on the market been made to conform to the trend of production or consumption by carrying surpluses from years of overproduction to years of relative shortage, the most probable value of these crops is \$6,135,000,000. The difference, or gain to the producers, is \$171,000,000, or approximately 9 cents a bushel for the carry over from surplus years.

<sup>9</sup> See footnote 13, p. 11.

<sup>10</sup> See Table III, Appendix A, p. 29.

Using estimates for the period 1881 to 1913 and Chicago prices gave similar results. Of course, these findings are not conclusive, because at best they are estimates and because considerations such as local prices, differences due to grades, and cost of storage have not been taken into account. However, they do suggest that there is an economic basis for efforts to distribute the supply in a more orderly manner.

#### MULTIPLE CORRELATION OF PRICE FACTORS

Since a correlation high enough to furnish an accurate basis for forecasting prices can not be obtained by using production alone, it will be necessary to look for other factors that have an influence upon the price of oats and to measure their influence. Those factors which may be expected to have some effect include: (1) Changes in the general price level; (2) the year-to-year carry over of oats; (3) substitute crops, such as corn; and (4) production of oats in Canada.

The effects of long-time changes in the general price level were taken account of to some extent in the first correlation by eliminating the long-time price trends. This is not an accurate method for the trend corresponds only roughly with the actual year-to-year changes in the price level. By the use of multiple correlation the general price level may be treated as a separate factor or variable, along with production and price. The annual carry over of oats, which is, in effect, merely an addition to the annual production, may be taken care of by adding it to the production for each year.

A multiple correlation<sup>11</sup> using (1) the percentage change in the price per bushel over the price of the previous year, (2) the percentage change in the index number<sup>12</sup> and (3) the percentage change in the United States production of oats plus carry over<sup>13</sup> gave a coefficient of 0.86. The inclusion of two additional factors has thus raised the coefficient by four points. The significance of a coefficient of multiple correlation differs somewhat from that of a coefficient of simple correlation. It indicates the closeness of the relation between the dependent variable and the independent variables, but not the nature of the relationship, as does the coefficient of simple correlation. Accordingly, it is not accompanied by a positive or negative sign.

Additional correlations, using the other factors mentioned, show that corn added as a fourth variable does not increase the coefficient of correlation. This may be partly owing to the fact that the corn crop tends to be small when the oat crop is small and large when the oat crop is large. Using the index number of all commodities of the Bureau of Labor Statistics in place of the index number of farm products gives a correlation that is still 0.86. Adding Canadian production to the above, as a fourth variable, does not raise the coefficient. The period covered in each of the correlations was 1896 to 1922, with the omission of the crop years 1916, 1917, 1918, 1919, and 1920.

<sup>11</sup> The method of working out a correlation of three variables is explained in Table IV, Appendix A, p. 31.

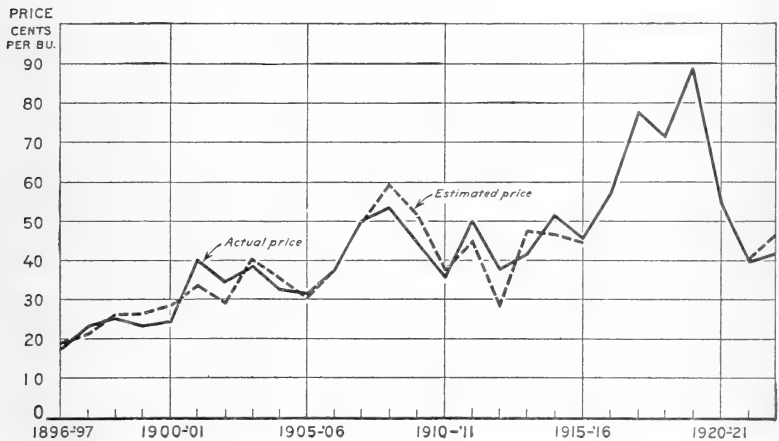
<sup>12</sup> The index of farm products of the United States Bureau of Labor Statistics converted to the crop year, July to June, by averaging the monthly index numbers back to 1913. Previous to 1913 the annual index numbers for two calendar years were averaged to give an index number for the crop year.

<sup>13</sup> Carry over includes old stocks of oats on farms August 1, 1895 to 1923, obtained from the publication Weather Crops and Markets, United States Department of Agriculture, August 11 and 18, 1923, and the visible supply of oats on August 1, as reported by the Yearbooks of the United States Department of Agriculture.

An estimating equation may now be calculated from the data which gives the highest correlation with price. The equation is  $x_1 = 4.20 + 2.03 x_2 - 1.16 x_3$ . Here  $x_1$  is the percentage change in price per bushel over that of the previous year;  $x_2$  is the percentage change in the index number; and  $x_3$  is the percentage change in the production of the United States plus carry over.<sup>14</sup>

Figure 4 is a comparison of the annual prices as estimated from the equation just given and the actual prices during that period. The average error for the 22 years is 3.6 cents, or 9.8 per cent of the average price.

In the correlation just described changes in the price level were taken account of by using the index number of prices of farm products as a separate variable. Approximately the same results would be



A comparison of the actual price of oats at Chicago with the price estimated by the use of the estimating formula

FIG. 4.—The accuracy with which oat prices may be estimated by use of the formula developed in this bulletin is indicated. The inaccuracy during certain years may have been due to unusual changes in demand, to inaccuracy in the estimate of production, or to other factors not accounted for in the estimating equation

secured by dividing the price of oats by the corresponding index number.

The variables were expressed as percentage changes over the preceding year because it was impossible to extend satisfactory trends through the war years. The difficulty will be recognized by an inspection of the price and production graphs in Figure 1. A first difference or percentage change over the previous year is more or less free from cumulative error, and may be used satisfactorily in such a situation. Percentage changes, however, are subject to a type of error similar to that described by Fisher in "The making of index numbers,"<sup>15</sup> that is, rising prices tend to augment the percentage changes, whereas falling prices have an opposite effect. It may be possible to correct this error by the use of percentage changes over the average of the figures for the current and the preceding years.

<sup>14</sup> See Table V, Appendix A, p. 33, for the method of estimating prices by the use of an estimating equation.

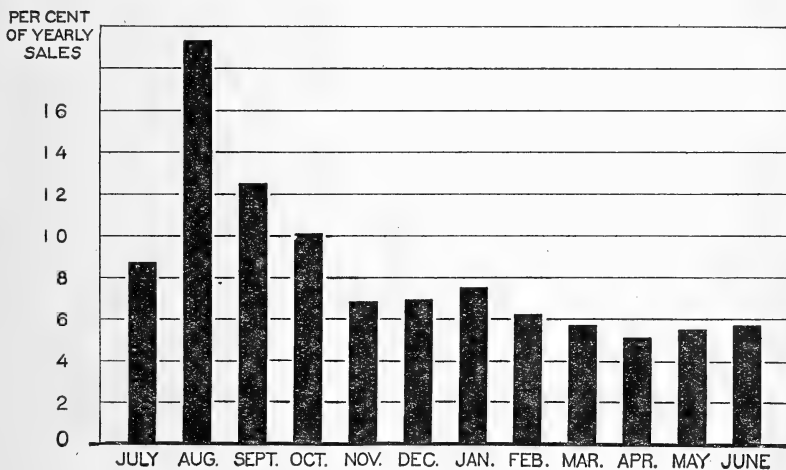
<sup>15</sup> Irving Fisher. The making of index numbers. 1922.



That part of the change in price which has not been accounted for by the factors included in this correlation may be due to the interaction of many factors, each in itself of minor importance. Unusually large or small substitute crops may affect the price of oats in some years; the estimates of production may not correspond to the actual production; industrial conditions may affect the price of oats in a way that is not accounted for by correcting for changes in the price level. Other methods of analysis may give more accurate results.

#### SEASONAL VARIATIONS IN OAT PRICES

The price of practically every farm product is subject to variations resulting from changes in the seasonal conditions of demand and supply. In most cases, as is especially true with those products which become marketable during a short period of the year, the influence of the supply conditions predominates. In the case of prod-



Monthly marketings of oats by farmers in terms of percentages of the year's sales

FIG. 5.—The heaviest marketings of oats come in August and September, immediately after the crop is threshed; the lightest marketings occur during seeding time in April. Prices show an opposite tendency, being lowest in August and September and highest in the spring.

ucts which are consumed more heavily at one time of the year than at another the conditions of demand may be of more importance. The supply of some products, such as meats, butter, and eggs, may be partially adjusted to meet seasonal changes in demand.

The price of oats is subject to seasonal variations resulting principally from the fact that the supply becomes available for market during a short period of the year and must be carried at some expense throughout the crop year to meet the demands of consumers. The size of the crop also has an important influence upon the seasonal trend, as will be shown in the following pages.

Table 4 gives the quantities of oats marketed monthly by farmers, with the percentage which each month is of the year's sales. Figure 5 represents this graphically. It will be seen that the heaviest marketing comes in August, September, and October, with August leading during each of the five years. Table 5 shows that the lowest

prices have come at about the time of the heaviest marketing, during August and September, and that the highest prices have come oftenest in May and July.<sup>16</sup> This indicates that there may be a relationship between the quantities marketed and the price. If there is such a relationship, that is, if a large crop causes an undue depression in the price during the heavy marketing period, there should be a correspondingly large rise in price after the marketing period is over. This may be studied by comparing the movement of prices after a large crop with the movement of prices after small and normal crops.

TABLE 4.—Oats: Monthly marketings by farmers, 1916-1921<sup>1</sup>

Month	Estimated quantity sold monthly by farmers of United States (millions of bushels)						Per cent of year's sales					
	1916-17	1917-18	1918-19	1919-20	1920-21	5-yr. aver.	1916-17	1917-18	1918-19	1919-20	1920-21	5-yr. aver.
July.....	31	24	34	47	36	34	8.3	4.7	8.0	14.4	8.3	8.7
August.....	87	82	82	60	80	78	23.3	16.4	19.6	18.4	18.7	19.3
September.....	51	67	50	33	59	52	13.5	13.5	11.9	10.1	13.8	12.5
October.....	40	56	42	30	41	42	10.7	11.1	9.9	9.2	9.5	10.1
November.....	30	38	30	19	24	28	8.0	7.7	7.2	5.8	5.5	6.8
December.....	21	39	28	27	25	28	5.7	7.8	6.7	8.3	5.8	6.9
January.....	28	42	28	26	28	30	7.5	8.3	6.7	8.2	6.6	7.5
February.....	20	40	19	21	28	26	5.3	8.0	4.5	6.6	6.6	6.2
March.....	20	35	23	16	26	24	5.2	7.1	5.5	4.9	6.0	5.7
April.....	14	33	27	14	20	22	3.8	6.5	6.3	4.3	4.6	5.1
May.....	17	20	29	17	29	22	4.4	4.0	7.0	5.2	6.8	5.5
June.....	16	24	28	15	34	23	4.3	4.9	6.7	4.6	7.8	5.7
Season.....	375	500	420	325	430	409	100.0	100.0	100.0	100.0	100.0	100.0

<sup>1</sup> U. S. Dept. Agr., Yearbook; 1921, p. 545.

TABLE 5.—Frequency in occurrence of lowest average monthly prices and of highest average monthly prices of oats, July to following June, for 33 years, 1881 to 1914

	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
Times lowest....	4	7	12	2	0	2	0	1	3	1	0	1
Times highest....	14	1	1	0	1	0	2	1	0	2	6	5

Table 6 compares the September with the May prices during years (1) when production was about normal, as measured by the straight-line trend of production, (2) when production was 5 per cent or more below normal, and (3) when production was 5 per cent or more above normal. In the first group, including the 8 years about normal, the average rise from September to May was 5.9 cents, or 23.6 per cent over the September price. In the second group, including 11 years with production below normal, the average rise was 6.3 cents, or 16.5 per cent. In the third group, including 14 years when production was above normal, the average rise was 3.8 cents, or 14.9 per cent. These results would indicate that a large crop does not unduly depress the price during the heavy marketing season below a price fixed by the interaction of demand and supply throughout the 12 months' period of consumption.

<sup>16</sup> The high showing of prices in July can not, in most instances, be considered as due to the crop of the season in which it is included in this table, but to a relatively smaller crop of the preceding year.

The seasonal trend may be calculated by averaging the monthly prices for the period 1881 to 1913 and correcting for the trend in prices. This will give a trend expressed in average monthly prices. This is not so useful for estimating purposes, however, as a seasonal

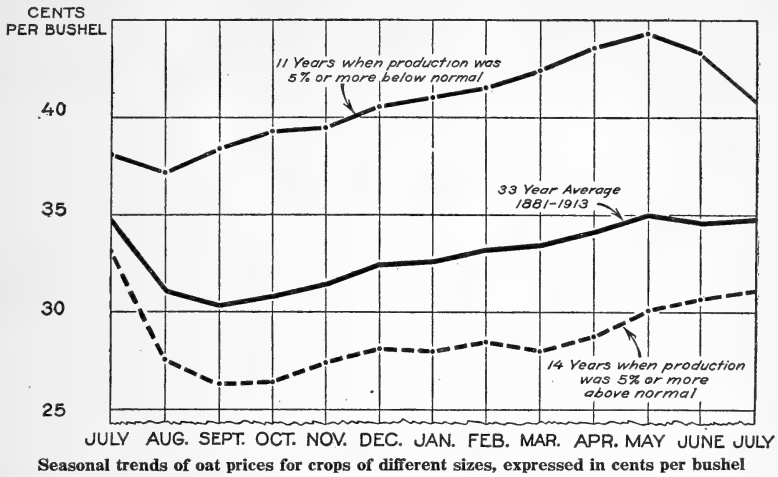


FIG. 6.—Although the seasonal trends in oat prices after August or September are very nearly the same, regardless of the size of the crop, the levels of prices are very different. Furthermore, the smaller the crop the sooner the price tends to rise after the decline in July and August due to heavy marketings

index expressed in percentages. Two methods of calculating such an index are illustrated in Table 7.

Since it was found that there was considerable difference in the seasonal movement of prices during years of large crops as compared

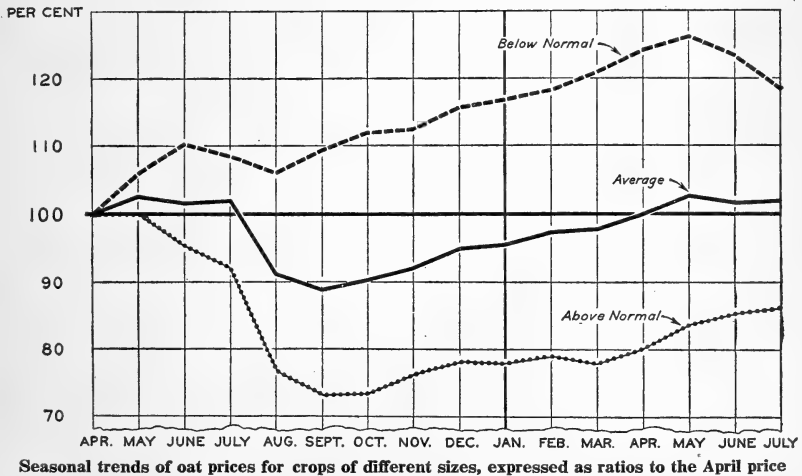


FIG. 7.—Nearly all of the difference in the seasonal trends of oat prices as between crops of different sizes occurs during the growing and harvesting period

with years of small crops, three seasonal trends were calculated, as given in Table 8. One is the trend for the entire period, the second for 14 years when production was above normal, and the third for 11 years when production was below normal. These are illustrated

in Figure 6. Figure 7 represents the average trends of prices for 16 months during the growing and marketing period. Values are expressed as ratios to the beginning April price on the assumption that this price is not influenced by the crop of that year.

TABLE 6.—Variations in oat prices, per bushel, September to the following May, 1881-1913

Year	When production was about normal				Year	When production was 5 per cent or more below normal				Year	When production was 5 per cent or more above normal			
	September price <sup>1</sup>	Following May price <sup>1</sup>	Spread between September and May			September price <sup>1</sup>	Following May price <sup>1</sup>	Spread between September and May			September price <sup>1</sup>	Following May price <sup>1</sup>	Spread between September and May	
1882-83	Cts. 33	Cts. 41	Cts. +8	P. ct. +24. 2	1881-82	Cts. 39	Cts. 53	Cts. +14	P. ct. +35. 9	1883-84	Cts. 27	Cts. 32	Cts. +5	P. ct. +18. 5
1896-97	16	18	+2	+12. 5	1890-91	37	51	+14	+37. 8	1884-85	25	35	+10	+40. 0
1897-98	20	30	+10	+50. 0	1892-93	34	31	-3	-8. 8	1885-86	25	29	+4	+16. 0
1898-99	21	27	+6	+28. 6	1893-94	26	35	+9	+34. 6	1886-87	25	26	+1	+4. 0
1900-01	22	29	+7	+31. 8	1894-95	30	29	-1	-3. 3	1887-88	25	35	+10	+40. 0
1904-05	31	31	0	0	1901-02	35	43	+8	+22. 9	1888-89	24	23	-1	-4. 2
1906-07	32	43	+11	+34. 4	1903-04	35	40	+5	+14. 3	1889-90	20	28	+8	+40. 0
1909-10	40	43	+3	+7. 5	1907-08	49	53	+4	+8. 2	1891-92	32	31	-1	-3. 1
Av			+5. 9	+23. 6	1908-09	50	60	+10	+20. 0	1895-96	20	19	-1	-5. 0
					1911-12	45	56	+11	+24. 4	1899-1900	22	23	+1	+4. 5
					1913-14	43	41	-2	-4. 7	1902-03	29	33	+4	+13. 8
					Av			+6. 3	+16. 5	1905-06	26	33	+7	+26. 9
										1910-11	34	35	+1	+2. 9
										1912-13	34	39	+5	+14. 7
										Av			+3. 8	+14. 9

<sup>1</sup> Average monthly price.

TABLE 7.—Seasonal variations in prices of oats figured by two methods, 1895-96 to 1912-13

Month	Average monthly price per bushel not corrected for trend	Correc-tion for trend <sup>1</sup>	Average monthly price corrected for trend	Average of monthly prices per bushel, each corrected separately for trend	Trend	Seasonal indices		Column IV divided by its average
						Method of averages <sup>2</sup>	Method of link relatives <sup>3</sup>	
	I	II	III	IV	V	VI	VII	VIII
September	Cents 31.83	Cents +.715	Cents 32.54	Cents 32.53	Cents 33.99	94	94	92
October	32.53	+.585	33.12	33.11	34.12	95	96	94
November	33.00	+.455	33.46	33.57	34.25	96	96	95
December	34.23	+.325	34.56	34.62	34.38	100	98	98
January	55.18	+.195	35.38	35.37	34.51	102	100	100
February	35.88	+.065	35.94	35.94	34.64	104	101	102
March	36.35	-.065	36.28	36.06	34.76	105	102	102
April	36.65	-.195	36.46	36.45	34.89	105	104	103
May	37.41	-.325	37.08	37.09	35.02	107	106	105
June	37.65	-.455	37.20	37.19	35.15	107	106	106
July	37.59	-.585	37.00	37.12	35.28	107	105	105
August	33.88	-.715	33.16	33.76	35.41	96	95	96

<sup>1</sup> An average price would ordinarily be expressed to the nearest whole number, therefore, the correction for trend is of minor significance for practical uses.

<sup>2</sup> Column I divided by Column V. For method of averages see Introduction to Economic Statistics, by G. R. Davies, 1922.

<sup>3</sup> For method of link relatives see The Review of Economic Statistics, preliminary vol. 1, 1919.

TABLE 8.—Seasonal price trends of oats, 1881 to 1913

Month	Average seasonal prices 1881-1914				Seasonal prices when production was 5 per cent or more above normal trend		Seasonal prices when production was 5 per cent or more below normal trend			
	Average monthly price	Correc- tion for trend <sup>1</sup>	Correc- ted price	Ratios to sea- sonal aver- age	Average monthly price <sup>2</sup>	Ratios to sea- sonal aver- age	Average monthly price	Correc- tion for trend <sup>1</sup>	Correc- ted price	Ratios to sea- sonal aver- age
July.....	Cents 34.8	Cents 0.06	Cents 34.7	105.8	Cents 33.1	115.8	Cents 38.1	Cents 0.04	Cents 38.1	93.5
Aug.....	31.2	.11	31.1	94.8	27.6	96.6	37.3	.07	37.2	91.3
Sept.....	30.5	.17	30.3	92.4	26.3	92.1	38.5	.11	38.4	94.2
Oct.....	31.0	.22	30.8	93.9	26.4	92.4	39.4	.14	39.3	96.4
Nov.....	31.7	.27	31.4	95.7	27.4	95.9	39.7	.18	39.5	96.9
Dec.....	32.7	.33	32.4	98.8	28.1	98.3	40.8	.22	40.6	99.6
Jan.....	33.0	.38	32.6	99.4	28.0	98.0	41.2	.25	41.0	100.6
Feb.....	33.6	.44	33.2	101.2	28.4	99.4	41.8	.29	41.5	101.8
Mar.....	33.9	.50	33.4	101.8	28.0	98.0	42.7	.32	42.4	104.0
Apr.....	34.7	.55	34.1	104.0	28.8	100.8	44.0	.36	43.6	106.9
May.....	35.6	.61	35.0	106.7	30.1	105.3	44.7	.40	44.3	108.7
June.....	35.3	.66	34.6	105.5	30.7	107.4	43.7	.43	43.3	106.2
Average.....			32.8		28.6				40.8	

<sup>1</sup> The correction for trend is little greater than the probable error of the price arrived at by averaging weekly high and low quotations.

<sup>2</sup> The correction for trend is negligible—only 0.12 cents per year.

In order to see how closely the 33-year seasonal average reflects price movements for any one month, the standard deviations of the monthly averages have been calculated. (See Table 9.) These show that the September price is most accurately measured by the average price for that month, for the deviations of prices during that month from the average price have been least. The May price has the greatest deviation from its average.

#### APPLICATION OF SEASONAL TREND IN ESTIMATING PRICE

After having estimated the annual average price an index of seasonal price movements may be used for two purposes: To determine whether the price for any given month is above or below the normal level of seasonal prices and to indicate the probable trend of prices during the following months of the crop year. For these purposes the ratios of monthly average prices to the yearly average are most useful. (See Table 8.) These should be used with reference to whether the crop is about normal, below normal, or above normal, the normal production being represented by a trend which indicates as accurately as possible the general direction in which production is going. A price at the beginning of the crop year, then, which seems considerably above the normal seasonal price as indicated by the ratio for the given month, may be expected to fall below the estimated seasonal price before the end of the crop year.

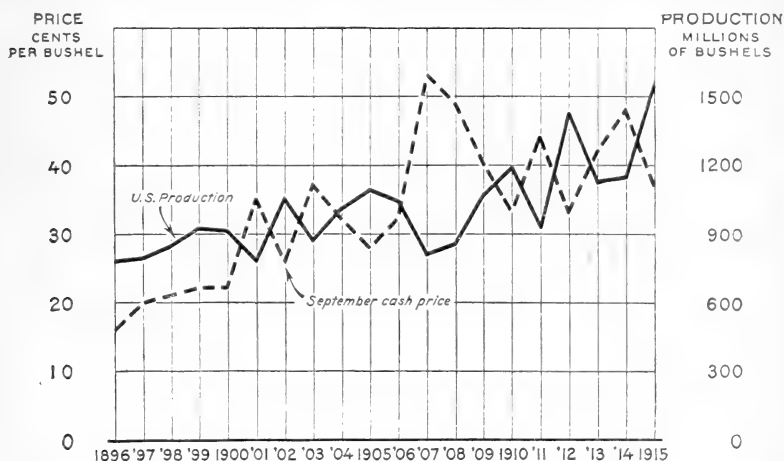
TABLE 9.—Average monthly prices of oats, 1881-1913, and their standard deviations

Month	Price	Standard deviation	Month	Price	Standard deviation
	<i>Cents</i>			<i>Cents</i>	
July.....	34.8	10.1	February.....	33.5	9.6
August.....	31.2	8.6	March.....	33.9	10.0
September.....	30.5	8.4	April.....	34.6	10.2
October.....	31.0	8.7	May.....	35.6	10.3
November.....	31.7	8.5	June.....	35.3	9.7
December.....	32.7	9.1			
January.....	33.0	9.3	Entire period.....	33.0	8.8

With reference to this subject, Working says:<sup>17</sup>

A properly adjusted price would remain the same throughout the season, except for a gradual advance to cover cost of storage, and would maintain a fairly uniform consumption throughout the season. But since an abnormally high price early in the season causes small consumption, it must be compensated by an abnormally low price during the remainder of the season or not all the crop can be sold.

Similarly, he says that if the price is abnormally low early in the season the supply will be exhausted too rapidly, and for the resulting



A comparison of the United States production of oats and the September price

FIG. 8.—The inverse relation between the production of oats and the September price is clearly shown.

small supply later in the year abnormally high prices will be paid, the result being that, although the price at any one time may differ from the normal price justified by demand and supply conditions, the average price for a season will come very close to the normal.<sup>18</sup> This statement regarding potato prices may likewise be applied to oat prices, with the modification that variations in the carry over of oats may cause the influence of one crop to extend into the following season.

#### SEPTEMBER PRICE AS A BASIS FOR ESTIMATING SEASONAL PRICES

The fact that the standard deviation of the September price from the average for the period studied is less than that of any other month and less than that of the average annual price suggests that it may

<sup>17</sup> See footnote 5.

<sup>18</sup> Another method of taking account of seasonal variation is to correlate prices of each month with the price-making factors.

be more accurate as a basis for estimating the seasonal trend of prices than is the average annual price heretofore used. That the September price can be predicted with greater accuracy than the annual price is shown by the fact that a correlation coefficient of  $-0.85$  is obtained when the year-to-year changes in the September cash price are correlated with the changes in the final estimate of oat production of the Department of Agriculture as compared with a coefficient of  $-0.82$  when the annual price is used. If to the production figure is added the carry over from the previous year, and changes in this new figure representing supply are correlated with changes in the September price, a coefficient of  $-0.91$  is obtained.<sup>19</sup> Figure 8 represents graphically the close relationship between production and the September price.

### DISCUSSION OF METHOD

As indication of the relative effectiveness of using percentage changes as compared with absolute changes in correlating the September price with production plus carry over, four correlations were carried out, using different combinations of the two methods. The resulting coefficients are given in Table 10.

TABLE 10.—*Correlation of changes in September price and production plus carry over of oats, 1896 to 1913*

Variables	Coefficient	Standard error
(a) Price: Percentage change over that of the preceding year.....	-0.74	Cents 16.1
(b) Production plus carry over: Percentage change over that of the preceding year.....		
(a) Price: Change in cents per bushel over that of the preceding year.....	-.90	3.8
(b) Production and carry over: Percentage change over that of the preceding year.....		
(a) Price: Change in cents per bushel over that of the preceding year.....	-.90	3.8
(b) Production plus carry over: Percentage change over the average figure of the preceding and current years.....		
(a) Price: Change in cents per bushel.....	-.91	3.5
(b) Production plus carry over: Change in tens of millions of bushels.....		

<sup>1</sup> Approximate standard error 18.6+per cent of average price, 33 cents.

A correlation using absolute changes (first differences) does not require the elimination of a straight-line trend which may be present, for this method of correlation is itself a method of eliminating such trends, owing to the fact that the trend affects the change from year to year by a constant amount. Adding a constant to a series does not affect the deviations from the average of the series, since each item is increased as much as any other and the relationship between them remains the same.<sup>20</sup> It is necessary, however, in first difference correlations, to make corrections for nonlinear trends which may disturb the price series, either by dividing each price item by its

<sup>19</sup> A correlation for the period 1895 to 1915 gives a coefficient of 0.94 when the September cash price, the Bureau of Labor Statistics all-commodities index number for September, and the final estimates of production plus carry over were correlated as changes over the values for the preceding years. Expressing changes in the average September price in cents per bushel as  $x_1$ ; changes in the index number as  $x_2$ ; and changes in the estimated production, plus old stocks on farms August 1, plus visible supply July 1, in 10,000,000 of bushels, as  $x_3$ , the estimating equation is  $x_1 = 0.388 x_2 - 0.446 x_3$ . Applying this equation to the 20 years covered by the correlations, an average error of 3 cents per bushel in predicting the September price is found. This may be compared with the average error of 3.6 cents in predicting the annual price. (See p. 12.) The error is less than 3 cents for 13 of the 20 years.

<sup>20</sup> B. B. Smith. The use of punched card tabulating equipment in multiple correlation problems; collected and prepared for the use of statisticians of the Bureau of Agricultural Economics, U. S. Department of Agriculture. 1923. Mimeographed.

corresponding index number or by using first differences of the index number as a separate variable in the correlation with first differences of price uncorrected. This was done in all the correlations mentioned, using either an all-commodity index or a commodity group index.

With no correction for the nonlinear trend in the series of September prices, 1901 to 1921, the correlation with production plus carry over resulted in a coefficient of only  $-0.43$ . Dividing the price by the Bureau of Labor Statistics index number of all commodities for September resulted in a correlation of  $-0.73$ . Using first differences of this index number as a separate variable, instead of dividing the price by the index number, raised the correlation to  $-0.86$ . In this case the latter method gave better results.

#### FUTURE PRICES AND CONDITION REPORTS AS PRICE INDICATORS

During the growing season, when the price forecaster must estimate supply as well as demand and when the price of oats is being influenced by the past year's crop as well as by the crop which is maturing, the movement of prices is most difficult to predict. Two indicators of price movements are available to farmers during this period: (1) The condition reports for the oat crop issued periodically by the United States Department of Agriculture; and (2) the price of September futures, which represents the best opinion of the grain trade as to the probable September cash price.

The Department of Agriculture issues in the spring a preliminary estimate of acreage planted and early in June issues the first of a series of monthly condition reports and production forecasts. Early in March an estimate of stocks of oats on farms is published. A final estimate of acreage, yield, and production is made in December. These data are among those used by dealers in oats to estimate the probable future prices.

Future prices are quoted throughout the year for oats to be delivered during specified delivery months, usually September, December, May, and July. It may thus be said that the oat crop is bargained for on the grain exchanges before it is planted, and at the time of harvest sales and purchases are made for delivery eight or nine months later. The fact that in making these sales for future delivery grain men must look ahead and estimate the probable price conditions at the date of delivery results in making the prices of oat futures an indicator of future cash prices. Just how good an indicator they are can be established by correlating future prices with cash prices during the corresponding delivery months.

#### RELATION OF FUTURE PRICES TO SUPPLY AND TO CASH PRICES

The quantity of new oats harvested ordinarily determines in large measure not only the September cash price of oats but also the September price of May futures. As compared with the  $-0.91$  correlation between September cash price and the new supply of oats, the correlation between the September price of May futures and the new supply of oats is  $-0.93$ ; that is, prices of May futures are strongly influenced by the supply of oats available in September. By May, however, the closeness of agreement between cash prices and the



supply of oats apparent in September has become diminished somewhat, for it is measured by a coefficient of  $-0.87$ .

The correlation between visible supply plus stocks on farms March 1 and the May cash price is  $-0.85$ , whereas consumption to March 1 seems to maintain a practically constant relationship to production—the correlation between final estimate of production plus carry over and the stocks on farms plus visible supply March 1 is  $+0.97$ . Thus the September price of May futures, although closely in line with production and carry over at the end of the harvest, may be somewhat out of line with the cash price which has come to prevail in May.

As between May and the following September a somewhat similar but more complex condition exists. For a period of 18 typical years, 1896 to 1913, inclusive, the May prices of September futures were in line neither with the quantity of new oats yet to be matured and harvested nor with the September cash prices. The correlation between May prices of September futures and final estimates of the new crop is measured by a coefficient of  $+0.31$ .<sup>21</sup> Instead of showing the negative relationship usually assumed between production and price, this coefficient indicates a slight tendency for May prices of September futures to be high when the new crop is large, and vice versa.<sup>22</sup> It suggests, therefore, that the size of the new crop has no significant effect as early as May upon prices of contracts for future delivery. May prices of September futures appear in fact to be influenced more by prevailing cash prices than by any other factor.

As the season progresses, however, and as forecasts of the new crop become more and more accurate, prices of September futures are gradually adjusted toward the average September cash price, until in August the average price of September futures corresponds very closely with the September cash price. The closeness of agreement at this time is measured by a coefficient of  $+0.95$ . The relation between the price of September futures and the August cash price is measured by a smaller coefficient,  $+0.86$ , showing that the September cash price can be predicted more accurately from the August price of September futures than from the August cash price.

Table 11 shows the gradually increasing closeness of the relationship between cash and future prices as the harvest season approaches. The variables in these correlations are expressed in terms of changes in cents per bushel over the corresponding months of preceding years. These coefficients indicate the increasing accuracy with which September cash prices can be estimated from the prices of September futures.

#### RELATION BETWEEN CROP CONDITION ESTIMATES AND PRODUCTION

In recent years the United States Department of Agriculture has published monthly forecasts of production for the principal grains, including oats, during the latter part of the growing season. These are available for only a part of the period covered in this study, but

<sup>21</sup> The correlation between corresponding changes in the May price of September futures and the September cash price is measured by a coefficient of  $-0.03$ .

<sup>22</sup> This positive correlation is probably due to some tendency for a large crop to be followed by a small one, and vice versa, and not to any real relation between the May price of September futures and the size of the new crop. Since the May price of September futures is influenced more by the size of the preceding crop than by any other factor, the actual relation measured by the correlation coefficient is that between the crop of one year and a price based on the crop of the preceding year.

condition estimates were available from 1890 to date. A study of the relation between the condition estimates and the final estimate of production for the years 1896-1913 showed that the condition estimates came more and more to agree with the final estimate of production as the harvest season approached. Since there is close relationship between the final estimate of production plus carry over and the September price, as indicated by a correlation coefficient of +0.91, the coefficients can be taken as approximately measuring the increasing accuracy with which the September price could be estimated from the monthly condition estimates, using condition estimate in place of production as one factor in the estimating equation.

The monthly forecasts of production during the growing season, which are available since 1912, make it unnecessary to take account of condition estimates in predicting the September price, for the forecasts of production can be used directly in the estimating equation. These production estimates would undoubtedly be more accurate than the condition estimates, though their relative accuracy as the harvest season approached would approximately compare with the coefficients in Table 11. The variables are expressed in terms of changes over preceding years. The unit employed for condition estimates is that used by the Department of Agriculture.

TABLE 11.—*Correlation of September future prices of oats during the growing season with cash prices in September*

September cash price correlated with—	Coefficient
May prices of September futures.....	-0.03
June prices of September futures.....	+ .25
July prices of September futures.....	+ .79
August prices of September futures.....	+ .95

TABLE 12.—*Correlation coefficients of preliminary condition and final production estimates of oats, 1896-1913<sup>1</sup>*

Final production estimate correlated with—	Coefficient
June 1 condition estimate.....	+0.45
July 1 condition estimate.....	+ .77
August 1 condition estimate.....	+ .86
September 1 condition estimate.....	+ .94

<sup>1</sup> In these correlations changes in acreage are not taken into account. When preliminary estimates of acreage are multiplied by September estimates of condition for the period 1896 to 1922, inclusive, and the result correlated with the final estimates of production for the same period, a higher coefficient is obtained, +0.97. All variables were expressed as changes from the corresponding months of preceding years.

#### CONCLUSIONS REGARDING FUTURE PRICES

The conclusions from this part of the study may be summarized in two brief statements:

Both cash and future prices of oats are highly sensitive to changes in the supply of oats when the supply is known.

Future prices, unlike cash prices, anticipate supply. Consequently, future prices which span the gap between one crop year and the next vary, as a rule, as widely from cash prices at the time of delivery as forecasts of production and carry over at the time contracts are made vary from the final measure of supply.

Three significant relationships between cash and future prices have been brought out:

(1) May prices of September futures are strongly influenced by prevailing May cash prices.

(2) May cash prices of oats conform to supply of oats as measured by the production plus carry over of the previous fall.

(3) September cash prices conform to supply as measured by the size of the new crop and carry over.

By virtue of these facts, cash prices in May following a large crop and carry over of the previous fall will tend to be relatively low, and consequently the May prices of September futures will be lower than the cash prices in September, unless the new crop also is unusually large.

However, the fact that in years when the supply of oats is above normal, as indicated by the straight-line trend of production plus carry over, May prices of September futures are below cash prices of the following September, does not necessarily mean that in these years it will always pay to hold oats from May to September. May prices are normally above September cash prices because of the expense of carrying oats from September to May.

## A COMPARATIVE STUDY OF WHEAT PRICES

### EXTENT OF WHEAT MARKET, AND ITS INFLUENCE ON CHICAGO PRICE

The price of wheat, unlike that of oats, is determined in large measure in the world market by the world crop. This fact is brought out by a study of the extent of the market and by correlations showing the degree of relationship between the Chicago price of wheat and the production of wheat in the United States and in the world.

Table 13 gives an idea of the extent of the market by showing the principal countries which import and export wheat, with average imports and exports of wheat and flour for the period 1909-10 to 1913-14. The United States during this period was the second most important exporting country, exceeded only by Russia in the volume of exports. The fact that the United States must seek a foreign market for a large part of its wheat crop, where its wheat comes into competition with wheat from other parts of the world, would suggest that the production in other countries should have considerable influence upon the price of wheat in this country.

Correlations given in Table 14 show that the influence of the crop of the United States upon the price of wheat at Chicago is measured by a coefficient of only  $-0.32$ , whereas the influence of the crop of the entire world is measured by a coefficient of  $-0.71$ . The table shows also that the price of wheat is influenced to some extent by the production of rye. The Chicago price is very closely related to the price at Liverpool, as indicated by a coefficient of  $+0.93$  when the prices at these two markets are correlated during the period 1890 to 1921.

TABLE 13.—Wheat, including flour: International trade, average of years 1909–10 to 1913–14

Principal importing countries			Principal exporting countries		
Country	Imports	Exports	Country	Imports	Exports
	<i>1,000 bushels</i>	<i>1,000 bushels</i>		<i>1,000 bushels</i>	<i>1,000 bushels</i>
United Kingdom.....	220, 570	3, 768	Russia.....	556	164, 862
Germany.....	91, 338	23, 264	United States.....	1, 607	110, 075
Netherlands.....	80, 702	58, 435	Canada.....	448	95, 828
Belgium.....	73, 398	23, 045	Argentina <sup>1</sup> .....	3	95, 243
Italy.....	56, 784	3, 682	Rumania.....	196	54, 630
France.....	44, 822	1, 203	British India.....	208	49, 889
Brazil <sup>1</sup> .....	20, 495		Australia <sup>2</sup> .....	7	49, 732
Switzerland.....	16, 937	14	Hungary.....	7, 214	49, 116
Austria.....	11, 402	871	Bulgaria.....		11, 182
Egypt.....	8, 244	59	Algeria.....	639	5, 936
Denmark.....	7, 155	597	Chile <sup>1</sup> .....	170	2, 593
Sweden.....	7, 080	23			
Union of South Africa <sup>1</sup> .....	6, 274	253			
Spain.....	6, 262	70			

<sup>1</sup> Calendar years, 1909 to 1922.<sup>2</sup> Years ended June 30.

International Institute of Agriculture, except figures with footnotes 1 and 2, which are compiled from official sources.

TABLE 14.—Correlation coefficients relating to the price of wheat<sup>1</sup>

Item	Coefficient
Ratio of price of wheat correlated with: <sup>2</sup>	
Period 1891–1913, inclusive—	
Ratio of United States production of wheat.....	–0.32
Ratio of world production of wheat.....	–.71
Ratio of world production plus world carry over.....	–.80
Multiple correlation—	
(a) Ratio of world production plus world carry over.....	.86
(b) Ratio of world production of rye.....	
Period 1900–1914— <sup>3</sup>	
Ratio of world production of barley.....	–.44
Ratio of world production of potatoes.....	–.29

<sup>1</sup> All variables are expressed as ratios to their straight-line trends.<sup>2</sup> Ratio of Chicago average crop-year price per bushel of wheat, divided by the Bureau of Labor Statistics farm products index, to the straight-line trend of price so corrected.<sup>3</sup> World production statistics of barley are available only from 1894, and of potatoes only from 1900.

A coefficient of net correlation shows the effect of one independent variable in a multiple correlation upon the dependent variable when the other independent variables are held constant. The coefficient of net correlation between the ratio of price and the ratio of United States production is  $-0.47$ , as compared with a coefficient of  $-0.66$  for the net correlation between the ratio of price and the ratio of production outside of the United States. This shows that the wheat crop of the remainder of the world has a greater net effect upon the price of wheat at Chicago than has the production of wheat in the United States.

The meaning of these coefficients is suggested by the fact that prices of wheat declined following the short United States crop of 1893 when world production was large, whereas they rose after the short crops of the United States and of the world, 1907, 1908, and 1911, and declined again in 1913 when the crop of the United States was normal and the world crop was large.

Table 15 shows the coefficients of correlation resulting from the use of different methods of treating the variables, as in the study of prices of oats. It will be seen that with wheat the results were approximately the same in the three cases.

## NOTES ON METHOD

As a study of wheat and oat prices has expanded, so also has the statistical technique employed been adapted to new uses. At the beginning, for example, variables were expressed as ratios to trends. Later, variables expressed as percentage changes over the preceding year were used for comparisons extending through and beyond the period of the World War, because straight-line trends could not be satisfactorily extended through that period.

TABLE 15.—Table of coefficients of multiple correlation relative to wheat prices, period 1895 to 1914

Dependent variable	Independent variables	Coefficient	
(a) Ratio of price divided by farm products relative to its straight-line trend.	(b) Ratio of world production plus carry over of wheat to its straight-line trend. (c) Ratio of world production of barley to its straight-line trend. (d) Ratio of world production of rye to its straight-line trend.	0.83	
(a) Ratio of price divided by farm products relative to the same of the preceding year.	(b) Ratio of world production plus carry over of wheat to the same of preceding year. (c) Ratio of world production of barley to that of preceding year. (d) Ratio of world production of rye to that of the preceding year.		.82
(a) Ratio of price, uncorrected, to that of preceding year.	(b) Ratio of farm products index number to that of preceding year. (c) Ratio of world production plus carry over of wheat to the same of the preceding year. (d) Ratio of world production of barley to that of the preceding year. (e) Ratio of world production of rye to that of the preceding year.		

On page 12 it was suggested that variables be expressed as percentage changes over the average of the figures for the preceding and current years. Finally variables expressed as simple changes in cents, tens of millions of bushels, and points of an index have come to be used. Table 10 shows that for the purpose at hand variables expressed in this way were relatively accurate as well as simple, direct, and convenient.

No generalizations are made to show the superiority of any method. In one case it may be desirable to sacrifice exactness for the sake of simplicity; in another inaccuracy may be too high a price to pay for simplicity; whereas in some instances simplicity may accord with the greatest accuracy. In the first part of the bulletin straight-line trends were used because they best described the data. Linear comparisons were used in relating oat prices to production because, for the data at hand, curves suggested but slightly closer agreement and involved considerably more work. For expressing seasonal variation the methods of link relatives and averages gave closely comparable results.

Higher coefficients of correlation were obtained when a farm-products index was used to correct for changes in the price level, than when an all-commodities index was used.

"Index numbers made from the prices of raw materials or of raw materials and slightly manufactured products must be expected to show wider oscillations than index numbers including a liberal representation of finished commodities," says Mitchell.<sup>23</sup> Thus the farm products relative would be expected to vary more than the index number of all commodities, which is desirable for the purposes of a price study.

The fact that the farm-products relative is more susceptible to change caused by changes in the demand for farm products as a group is an argument in favor of this relative for purposes of estimating the price of wheat or oats. On the other hand, the fact that wheat and oats have more influence to change the index number of farm products than to change the index of all commodities is an argument against using the farm-products index for price estimating purposes.

The combination of an index of manufactured products to reflect changes in the purchasing power of money and some index to reflect changes in demand for the product under consideration should be more accurate for purposes of price estimating than either the farm-products or the all-commodities index.

There was little difference in the correlation coefficients when the index number was run as a separate variable and when prices corrected by the index number were correlated directly. In most cases the index number was run as a separate variable because this method was more direct.

---

<sup>23</sup> W. C. Mitchell. Index numbers of wholesale prices in the United States and foreign countries. 1921. U. S. Bur. Labor Statis. Bul. 284.

## APPENDIX A

## OAT AND WHEAT STATISTICS AND CALCULATIONS

TABLE I.—Trend of oat production in the United States 1881 to 1913

Year	<i>y</i> Production <sup>1</sup>	<i>x</i> Origin 1897	<i>x</i> <sup>2</sup>	<i>xy</i>	Straight- line trend 1881 to 1913 <sup>2</sup>
	<i>Million bushels</i>				<i>Million bushels</i>
1881.....	416	-16	256	-6,656	496
1882.....	488	-15	225	-7,320	516
1883.....	571	-14	196	-7,994	536
1884.....	583	-13	169	-7,579	556
1885.....	629	-12	144	-7,548	576
1886.....	624	-11	121	-6,864	596
1887.....	659	-10	100	-6,590	616
1888.....	701	-9	81	-6,309	636
1889.....	751	-8	64	-6,008	656
1890.....	523	-7	49	-3,661	676
1891.....	738	-6	36	-4,428	696
1892.....	661	-5	25	-3,305	716
1893.....	639	-4	16	-2,556	736
1894.....	662	-3	9	-1,986	756
1895.....	824	-2	4	-1,648	776
1896.....	780	-1	1	-780	796
1897.....	791	0	0	0	<sup>3</sup> 816
1898.....	843	+1	1	+843	836
1899.....	926	+2	4	+1,852	856
1900.....	914	+3	9	+2,742	876
1901.....	778	+4	16	+3,112	896
1902.....	1,053	+5	25	+5,265	916
1903.....	869	+6	36	+5,214	936
1904.....	1,009	+7	49	+7,063	956
1905.....	1,090	+8	64	+8,720	976
1906.....	1,036	+9	81	+9,324	996
1907.....	805	+10	100	+8,050	1,016
1908.....	851	+11	121	+9,361	1,036
1909.....	1,068	+12	144	+12,816	1,056
1910.....	1,186	+13	169	+15,418	1,076
1911.....	922	+14	196	+12,908	1,096
1912.....	1,418	+15	225	+21,270	1,116
1913.....	1,122	+16	256	+17,952	1,136
Total.....	26,930		2,992	+141,910	
Average.....	816			-81,232	
Sum.....				+60,678	
Slope (60,678 divided by 2,992).....				= +20	

<sup>1</sup> U. S. Dept. Agr. Yearbooks.<sup>2</sup> For discussion of method see A Study of Statistical Method, by W. M. Persons in Review of Economic Statistics, preliminary vol. 1, 1919; Graphical and Mechanical Computation, by Joseph Lipka, 1918; Introduction to Economic Statistics, by G. R. Davies, 1922; or other standard texts dealing with the method of least squares.<sup>3</sup> Average.

TABLE II.—Correlation of Chicago price and United States production of oats

Year	United States production			Chicago price per bushel			Variation from mean		$x^2$	$y^2$	$xy$
	Total	Straight line trend	Ratio to trend	Actual	Straight line trend	Ratio to trend	Production ratios $x$	Price ratios $y$			
	Million bushels	Million bushels		Cents	Cents						
1881	416	496	0.84	47	36.0	1.30	-0.16	+0.30	0.0256	0.0900	-0.0480
1882	488	516	.95	37	35.3	1.05	-.05	+0.05	.0025	.0025	-.0025
1883	571	536	1.07	31	34.6	.90	+0.07	-.10	.0049	.0100	-.0070
1884	583	556	1.05	29	34.0	.85	+0.05	-.15	.0025	.0225	-.0075
1885	629	576	1.09	28	33.2	.77	+0.09	-.16	.0081	.0256	-.0144
1886	624	596	1.05	25	32.5	.84	+0.05	-.23	.0025	.0529	-.0115
1887	659	616	1.07	30	31.2	.96	+0.07	-.04	.0049	.0016	-.0028
1888	701	636	1.10	24	30.5	.79	+0.10	-.21	.0100	.0441	-.0210
1889	751	656	1.14	24	29.8	.81	+0.14	-.19	.0196	.0361	-.0236
1890	523	676	.77	43	29.0	1.48	-.23	+0.48	.0529	.2304	-.1104
1891	738	696	1.06	31	28.3	1.10	+0.06	+0.10	.0036	.0100	+0.0060
1892	661	716	.92	30	27.5	1.09	-.08	+0.09	.0064	.0081	-.0072
1893	639	736	.87	31	26.8	1.16	-.13	+0.16	.0169	.0256	-.0208
1894	662	756	.88	28	26.1	1.07	-.12	+0.07	.0144	.0049	-.0084
1895	824	776	1.06	19	25.3	.75	+0.06	-.25	.0036	.0625	-.0150
1896	780	796	.98	18	23.6	.76	-.02	-.24	.0004	.0576	+0.0048
1897	791	816	.97	24	25.0	.96	-.03	-.04	.0009	.0016	+0.0012
1898	843	836	1.01	25	26.4	.95	+0.01	-.05	.0001	.0025	-.0005
1899	926	856	1.08	23	27.8	.83	+0.08	-.17	.0064	.0289	-.0136
1900	914	876	1.04	25	29.2	.86	+0.04	-.14	.0016	.0196	-.0056
1901	778	896	.87	42	30.6	1.37	-.13	+0.37	.0169	.1369	-.0481
1902	1,053	916	1.15	33	32.0	1.03	+0.15	+0.03	.0225	.0009	+0.0045
1903	869	936	.93	38	33.4	1.14	-.07	+0.14	.0049	.0196	-.0098
1904	1,009	956	1.06	30	34.8	.86	+0.06	-.14	.0036	.0196	-.0084
1905	1,090	976	1.12	31	36.2	.86	+0.12	-.14	.0144	.0196	-.0168
1906	1,036	996	1.04	39	37.6	1.04	+0.04	+0.04	.0016	.0016	+0.0016
1907	805	1,016	.79	51	39.0	1.31	-.21	+0.31	.0441	.0961	-.0651
1908	851	1,036	.82	52	40.4	1.29	-.18	+0.29	.0324	.0841	-.0522
1909	1,065	1,056	1.01	43	41.8	1.03	+0.01	+0.03	.0001	.0009	+0.0003
1910	1,186	1,076	1.10	35	43.2	.81	+0.10	-.19	.0100	.0361	-.0190
1911	922	1,096	.84	51	44.6	1.14	-.16	+0.14	.0256	.0196	-.0224
1912	1,418	1,116	1.27	37	46.0	.80	+0.27	-.20	.0729	.0400	-.0540
1913	1,122	1,136	.99	41	47.4	.87	-.01	-.13	.0001	.0169	+0.0013
Total			32.99			32.83			.4369	1.2289	-.5989
Mean			1.00			.994					

$$\sigma x = \sqrt{\frac{0.4369}{33}} = 0.115. \quad y \sqrt{\frac{1.2289}{33}} = 0.193. \quad r = \frac{-0.5989}{33 \times 0.115 \times 0.193} = \frac{-0.5989}{0.7324} = -0.82.$$



TABLE III.—Value of oats produced and consumed in the United States, 1895–1913

Year	Production United States		Under actual conditions of consumption			With a uniform increase in consumption			
	Actual	Trend	Annual consumption <sup>1</sup>	December farm price per bushel <sup>2</sup>	Total value	Annual consumption <sup>3</sup>	Estimated December farm price per bushel <sup>4</sup>	Estimated total value	Necessary addition to carry over <sup>5</sup>
	Million bushels	Million bushels	Million bushels	Cents	Million dollars	Million bushels	Cents	Million dollars	Million bushels
1895.....	824	776	746	20	149	776	22.1	171	0
1896.....	780	796	820	19	156	796	23.2	185	24
1897.....	791	816	822	21	173	816	24.4	199	30
1898.....	843	836	835	26	217	836	25.5	213	29
1899.....	926	856	920	25	230	856	26.7	229	93
1900.....	914	876	919	26	239	876	27.8	244	136
1901.....	778	896	810	40	324	896	29.0	260	50
1902.....	1,053	916	1,004	31	311	916	30.1	276	138
1903.....	869	936	902	34	307	936	31.3	293	104
1904.....	1,009	956	989	31	307	956	32.4	310	137
1905.....	1,090	976	1,076	29	312	976	33.6	328	237
1906.....	1,036	996	1,040	32	333	996	34.7	346	281
1907.....	805	1,016	840	44	370	1,016	35.9	365	105
1908.....	851	1,036	863	47	406	968	40.0	387	6 68
1909.....	1,068	1,056	1,030	40	412	1,030	39.0	402	6 26
1910.....	1,186	1,076	1,179	34	401	1,076	39.3	423	103
1911.....	922	1,096	961	45	432	1,064	42.0	447	6 32
1912.....	1,418	1,116	1,338	32	428	1,116	41.6	464	222
1913.....	1,122	1,136	1,172	39	457	1,136	42.8	486	258
Value of carry-over								7 107	
Total.....					5,964			6,135 —5,964	1,947
								171	

<sup>1</sup> Production plus carry over from previous year less carry over to following year.

<sup>2</sup> From U. S. Dept. Agr. Yearbooks.

<sup>3</sup> Same as trend of production except for the deficit years 1908, 1909, and 1911, when the sum of carry over plus production was less than the trend of production.

<sup>4</sup> Straight-line trend of price except for the deficit years, 1908, 1909, and 1911, when price was estimated on the basis of percentage changes.

<sup>5</sup> Obtained by subtracting the production trend figure from the actual consumption figure and adding the necessary additional carry over from the preceding year. For example the 1897 carry over figure of 30 is obtained by subtracting the trend figure (816) from consumption (822) and adding the addition to carry over (24) to the remainder. This figure, therefore, signifies hypothetical addition to actual carry over.

<sup>6</sup> Consumption for year computed is greater than production.

<sup>7</sup> The figure 107 represents the value of 258 million bushels carried from 1913 to 1914 at the 1914 price (44 cents) less the value of 30 million bushels carried from 1894 to 1895 at the 1895 price.

TABLE IV A.—Statistics of price and price determining factors of oats

Year beginning July 1	Chicago price of oats <sup>1</sup>	Change over preceding year	Bureau of labor statistics index of farm products <sup>2</sup>	Change over preceding year	Production of oats in the United States <sup>3</sup>	Old stocks on farms Aug. 1, in the United States <sup>4</sup>	United States visible supply of oats July 1 <sup>5</sup>	United States production of oats plus old stocks on farms plus visible supply	Change over preceding year
	<i>Cents</i>	<i>Per cent</i>		<i>Per cent</i>	<i>Million bushels</i>	<i>Million bushels</i>	<i>Million bushels</i>	<i>Million bushels</i>	<i>Per cent</i>
1895.....	20		58		824	43	7	874	
1896.....	17	-15	57	-2	780	119	9	908	+4
1897.....	23	+35	61	+7	791	80	8	879	-3
1898.....	25	+9	64	+5	843	51	6	900	+2
1899.....	23	-8	67	+5	926	59	6	991	+10
1900.....	24	+4	72	+7	914	64	7	985	-1
1901.....	40	+67	78	+8	778	55	11	844	-14
1902.....	34	-15	79	+1	1,053	32	2	1,087	+29
1903.....	38	+12	79	0	869	79	4	952	-12
1904.....	32	-16	80	+1	1,009	46	4	1,059	+11
1905.....	31	-3	80	0	1,090	63	7	1,160	+10
1906.....	37	+19	84	+5	1,036	78	6	1,120	-3
1907.....	50	+35	86	+2	805	73	7	885	-21
1908.....	53	+6	92	+7	851	41	4	896	+1
1909.....	44	-17	100	+9	1,068	27	6	1,101	+23
1910.....	35	-20	98	-2	1,186	67	4	1,257	+14
1911.....	50	+43	97	-1	922	68	10	1,000	-20
1912.....	37	-26	100	+3	1,418	35	4	1,457	+46
1913.....	41	+11	102	+2	1,122	104	15	1,241	-15
1914.....	51	+24	104	+2	1,141	62	7	1,210	-3
1915.....	45	-12	114	+10	1,549	56	4	1,609	+33
1916.....	56	+24	156	+37	1,252	114	12	1,378	-14
1917.....	77	+38	204	+31	1,593	48	10	1,651	+20
1918.....	71	-8	224	+10	1,538	81	13	1,632	-1
1919.....	88	+24	224	0	1,184	93	18	1,295	-21
1920.....	54	-39	171	-24	1,496	55	4	1,555	+20
1921.....	39	-28	128	-25	1,078	161	34	1,273	-18
1922.....	41	+5	139	+9	1,215	75	43	1,333	+5
1923.....					1,316	70			

<sup>1</sup> Average of monthly prices of No. 2 oats July to following June. Monthly prices obtained by averaging weekly high and low prices as quoted in the Chicago Board of Trade Annual Reports.

<sup>2</sup> From 1913 to 1922 monthly index numbers, July to June, were averaged, to convert to a crop year basis. Previous to 1913 the annual index numbers for each two consecutive calendar years were averaged for the July to June crop year index.

<sup>3</sup> U. S. Dept. of Agr. Yearbooks.

<sup>4</sup> U. S. Dept. Agr., Weather, Crops and Markets, Aug. 11, 18, 1923.

<sup>5</sup> U. S. Dept. Agr., Yearbooks, or Chicago Board of Trade Annual Reports.

TABLE IV B.—Oats: Multiple correlation of price of oats, index of prices of farm products, and United States production plus carryover of oats, in terms of percentage changes over preceding years

Year beginning July 1	Percentage changes			$x_1^2$	$x_{12}$	$x_{13}$	$x_2^2$	$x_{23}$	$x_3^2$
	Price, $x_1$	Index, $x_2$	U. S. prod. oats plus carry-over, $x_3$						
1896	-15	-2	+4	225	+30	-60	4	-8	16
1897	+35	+7	-3	1,225	+245	-105	49	-21	9
1898	+9	+5	+2	81	-45	+18	25	+10	4
1899	-8	+5	+10	64	-40	-80	25	+50	100
1900	+4	+7	-1	16	+28	-4	49	-7	1
1901	+67	+8	-14	4,489	+536	-938	64	-112	196
1902	-15	+1	+29	225	-15	-435	1	+29	841
1903	+12	0	-12	144	0	-144	0	0	144
1904	-16	+1	+11	256	-16	-176	1	+11	121
1905	-3	0	+10	9	0	-30	0	0	100
1906	+19	+5	-3	361	+95	-57	25	-15	9
1907	+35	+2	-21	1,225	+70	-735	4	-42	441
1908	+6	+7	+1	36	+42	+6	49	+7	1
1909	-17	+9	+23	289	-153	-391	81	+207	529
1910	-20	-2	+14	400	+40	-280	4	-28	196
1911	+43	-1	-20	1,849	-43	-860	1	+20	400
1912	-26	+3	+46	676	-78	-1,196	9	+138	2,116
1913	+11	+2	-15	121	+22	-165	4	-30	225
1914	+24	+2	-3	576	+48	-72	4	-6	9
1915	-12	+10	+33	144	-120	-396	100	+330	1,089
(1)									
1921	-28	-25	-18	784	+700	+504	625	+450	324
1922	+5	+9	+5	25	+45	+25	81	+45	25
	+270	+83	+188		+1,946	-6,124		+1,297	
	-160	-30	-110		-465	+553		-269	
Total	+110	+53	+78	13,220	+1,481	-5,571	1,205	+1,028	6,896
Average	+5.00	+2.41	+3.54	600.9091	+67.3182	-253.2273	+54.7727	+46.7273	+313.4545
Squares	+25.0000	+5.8038	+12.5706						
Subtract				-25.000	+12.0455	+17.7275	+5.8038	+8.5415	+12.5706
				575.9091	+55.2727	-270.9548	+48.9689	+38.1858	+300.8839
				$\sigma x_1^2$	$\rho x_{12}$	$\rho x_{13}$	$\sigma x_2^2$	$\rho x_{23}$	$\sigma x_3^2$

<sup>1</sup> War years omitted.<sup>2</sup> Product of 5×2.41.<sup>3</sup> Product of 5×3.54.<sup>4</sup> Product of 2.41×3.54.

TABLE IV C.—*Solution of simultaneous equations involving ( $x_1$ ) price, ( $x_2$ ) index, and ( $x_3$ ) United States production plus carryover of oats<sup>1</sup>*

Equations:

1.  $\sigma x_2^2 b_{12} + \rho x_{23} b_{13} = \rho x_{12}$

2.  $(\rho x_{23} b_{12}) + \sigma x_3^2 b_{13} = \rho x_{13}$

Substituting actual values, as calculated in IV B.

First equation,  $+48.97 b_{12} + 38.19 b_{13} = + 55.27$ .

Second equation,  $(+38.19 b_{12}) + 300.88 b_{13} = -270.95$ .

	$b_{12}$	$b_{13}$	$p$	Check sum
1. Bring down first equation.....	+48.97	+38.19 *	=+55.27	+142.43
2. Divide by first term with sign changed, or -48.97=...	-1.0000	- .7799	=-1.1287	-2.9086
3. Bring down second equation.....		+300.88	=-270.95	+68.12
4. Multiply line (2) by +38.19=.....		-29.78	=-43.11	-111.08
5. Add above 2 lines.....		+271.10	=-314.06	-42.96
6. Divide by first term with sign changed, or -271.10=...		-1.0000	=+1.1585	+ .1585

Change signs of values in column ( $p$ ), lines (6) and (2), then

$$b_{13} = -1.1585$$

$$b_{12} = +1.1287 + (-1.1585 \times -0.7799) = +2.0322.$$

Product moments:

$$b_{13}x_{13} = -1.1585 \times +270.95 = 313.90$$

$$b_{12}x_{12} = +2.0322 \times + 55.27 = 112.32$$

$$\text{Adding, P. M.} = 426.22$$

$$\begin{aligned} \text{The coefficient of multiple correlation, } R &= \sqrt{\frac{P. M.}{\sigma x_1^2}} \\ &= \sqrt{\frac{426.22}{575.91}} \\ &= 0.86 \end{aligned}$$

The estimating equation is  $x_1 = a + b_{12} x_2 + b_{13} x_3$ Solve for "a" by substitution as follows:  $+5.00 = a + (2.0322 \times 2.41) + (-1.1585 \times 3.54)$ .  $a = +4.20$ .Inserting known values for the constants:  $a$ ,  $b_{12}$ , and  $b_{13}$ , the estimating equation becomes  $x_1 = 4.20 + 2.03x_2 - 1.16x_3$ .

<sup>1</sup> The method used in solving the simultaneous equations is "The Doolittle method." See Geodesy—Application of Theory of Least Squares to the Adjustment of Triangulation, by O. S. Adams, 1915; also A Method of Handling Multiple Correlation Problems, by H. R. Tolley and M. J. B. Ezekiel, in Journal American Statistical Association, December, 1923.

TABLE V.—Chicago price of oats estimated from changes in the United States production plus carry over of oats and the index number<sup>1</sup>

Year beginning July 1	Percentage change over preceding year		2.03x <sub>2</sub>	-1.16x <sub>3</sub>	Estimated change in price	Same expressed in round numbers	Actual price	Estimated price	Residuals
	Farm products index x <sub>2</sub>	United States production plus carry over x <sub>3</sub>							
					Per cent	Per cent	Cents	Cents	Cents
1895							20		
1896	-2	+4	-4.06	-4.64	-4.50	-5	17	19	+2
1897	+7	-3	+14.21	+3.48	+21.89	+22	23	21	-2
1898	+5	+2	+10.15	-2.32	+12.03	+12	25	26	+1
1899	+5	+10	+10.15	-11.60	+2.75	+3	23	26	+3
1900	+7	-1	+14.21	+1.16	+19.57	+20	24	28	+4
1901	+8	-14	+16.24	+16.24	+36.68	+37	40	33	-7
1902	+1	+29	+2.03	-33.64	-27.41	-27	34	29	-5
1903	0	-12	0	+13.92	+18.12	+18	38	40	+2
1904	+1	+11	+2.03	-12.76	-6.53	-7	32	35	+3
1905	0	+10	0	-11.60	-7.40	-7	31	30	-1
1906	+5	-3	+10.15	+3.48	+17.83	+18	37	37	0
1907	+2	-21	+4.06	+24.36	+32.62	+33	50	49	-1
1908	+7	+1	+14.21	-1.16	+17.25	+17	53	59	+6
1909	+9	+23	+18.27	-26.68	-4.21	-4	44	51	+7
1910	-2	+14	-4.06	-16.24	-16.10	-16	35	37	+2
1911	-1	-20	-2.03	+23.20	+25.37	+25	50	44	-6
1912	+3	+46	+6.09	-53.36	-43.07	-43	37	28	-9
1913	+2	-15	+4.06	+17.40	+25.66	+26	41	47	+6
1914	+2	-3	+4.06	+3.48	+11.74	+12	51	46	-5
1915	+10	+33	+20.30	-38.28	-13.78	-14	45	44	-1
1916	+37	-14					56		
1917	+31	+20					77		
1918	+10	-1					71		
1919	0	-21					88		
1920	-24	+20					54		
1921	-25	-18	-50.75	+20.88	-25.67	-26	39	40	+1
1922	+9	+5	+18.27	-5.80	+16.67	+17	41	46	+5

<sup>1</sup> The equation is  $x_1 = 4.20 + 2.03x_2 - 1.16x_3$

TABLE VI.—Monthly average cash prices of oats, Chicago, July, 1881, to June, 1923<sup>1</sup>

Year	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	Average
1881-82...	Cents 41	Cents 36	Cents 39	Cents 45	Cents 44	Cents 45	Cents 44	Cents 42	Cents 43	Cents 49	Cents 53	Cents 51	Cents 44
1882-83...	56	48	33	34	35	38	36	39	41	42	41	38	40
1883-84...	34	27	27	28	29	33	33	33	32	30	32	33	31
1884-85...	30	27	25	26	26	24	27	29	28	31	35	33	28
1885-86...	32	26	25	26	28	28	29	30	30	29	29	27	28
1886-87...	29	27	25	25	26	27	23	25	24	27	26	26	26
1887-88...	26	25	25	26	27	31	31	29	30	31	35	32	29
1888-89...	31	26	24	24	26	26	25	26	25	23	23	23	25
1889-90...	22	20	20	19	20	21	21	20	21	24	28	28	22
1890-91...	30	37	37	41	41	43	43	46	51	55	51	41	43
1891-92...	36	29	32	28	32	33	30	30	29	29	34	33	31
1892-93...	32	33	34	31	32	31	32	32	31	29	31	30	31
1893-94...	28	24	26	28	28	29	28	28	30	32	35	41	30
1894-95...	39	30	30	29	29	30	30	28	29	30	29	29	30
1895-96...	24	21	20	19	19	17	18	20	20	20	19	18	20
1896-97...	17	17	16	18	19	18	17	16	17	18	18	18	17
1897-98...	18	19	20	19	21	23	23	26	27	27	30	28	23
1898-99...	23	22	21	23	26	27	27	28	27	27	27	25	25
1899-1900...	25	21	22	23	23	23	23	23	24	25	23	23	23
1900-01...	24	22	22	22	22	23	24	24	26	26	29	29	24
1901-02...	29	36	35	35	39	46	46	44	44	43	43	44	40
1902-03...	50	35	29	30	30	32	34	35	34	33	33	37	34
1903-04...	36	33	35	37	35	35	38	41	41	40	40	40	38
1904-05...	38	34	31	30	30	30	31	31	32	30	31	31	32
1905-06...	32	26	26	29	30	31	31	30	30	32	33	37	31
1906-07...	37	30	32	33	33	34	36	39	42	42	34	45	37
1907-08...	44	49	49	50	48	49	51	51	51	52	53	52	50
1908-09...	53	48	50	48	49	50	51	54	55	56	60	58	53
1909-10...	52	41	40	41	41	45	49	48	46	45	43	39	44
1910-11...	42	37	34	33	33	33	33	32	31	33	35	39	35
1911-12...	45	41	45	48	49	49	50	53	54	58	56	54	50
1912-13...	51	39	34	34	34	34	34	35	34	36	39	41	37
1913-14...	42	43	43	41	41	42	40	41	41	40	41	41	41
1914-15...	38	42	50	48	50	49	54	59	58	58	54	50	51
1915-16...	54	54	40	39	39	44	49	49	43	46	47	41	45
1916-17...	42	45	47	51	57	53	57	57	61	70	71	67	56
1917-18...	78	67	61	61	66	77	83	89	94	91	79	78	77
1918-19...	79	72	72	70	74	74	68	61	65	70	72	71	71
1919-20...	77	76	70	73	75	84	87	87	93	103	112	117	88
1920-21...	102	76	65	56	52	50	45	43	44	39	39	39	54
1921-22...	38	37	39	36	36	39	40	41	40	41	42	40	39
1922-23...	39	35	39	43	45	46							39

<sup>1</sup>Averages of weekly high and low prices of No. 2 oats, Chicago Board of Trade. Annual reports.

TABLE VII.—Oats: Cash and future prices, per bushel, in cents, 1892-1921<sup>1</sup>

Year when delivery is due	Septem- ber prices of May oats	May cash price of oats	May prices of Septem- ber oats	June prices of Septem- ber oats	July prices of Septem- ber oats	August prices of Septem- ber oats	Septem- ber cash prices of oats
	Cents	Cents	Cents	Cents	Cents	Cents	Cents
1892	32	31	29	31	31	34	33
1893	37	30	27	26	25	24	26
1894	31	35	26	30	29	30	29
1895	35	29	28	29	23	20	19
1896	21	19	( <sup>2</sup> )	18	17	17	16
1897	19	18	18	18	18	18	20
1898	23	30	24	21	20	21	21
1899	22	26	21	21	20	20	22
1900	23	22	( <sup>2</sup> )	( <sup>2</sup> )	24	22	22
1901	( <sup>2</sup> )	29	26	26	32	35	35
1902	38	44	29	29	30	27	26
1903	32	36	30	33	34	34	37
1904	39	42	31	32	33	33	32
1905	35	31	28	30	30	26	28
1906	30	33	30	35	34	30	32
1907	34	46	38	37	39	47	53
1908	54	55	38	38	43	48	49
1909	52	59	44	44	41	37	40
1910	42	41	38	37	39	36	33
1911	38	34	33	40	44	42	44
1912	49	55	43	41	34	32	33
1913	34	38	36	40	41	42	42
1914	47	39	37	38	36	43	48
1915	54	53	46	40	38	39	37
1916	38	47	40	39	40	45	46
1917	51	68	55	53	57	56	59
1918	61	76	( <sup>2</sup> )	( <sup>2</sup> )	70	70	72
1919	( <sup>2</sup> )	70	66	68	78	73	70
1920	73	112	76	83	77	69	63
1921	66	39	41	40	40	35	39

<sup>1</sup> Arithmetic average of daily high and low quotations of No. 2 oats, 1892 to 1902; standard oats, 1903; contract grade, 1904 to 1921 (standard and No. 2 oats on contract grades). The quotations are taken from the Chicago Board of Trade Annual Reports. They are not available for earlier years.

<sup>2</sup> No quotations.

TABLE VIII.—Oats: United States Department of Agriculture estimates of acreage and of condition, 1895-1923

Year	Acreage, preliminary estimate, June 1 <sup>1</sup>	Estimates of condition by months <sup>2</sup>			
		June	July	Aug.	Sept
	Thousand acres	Per cent	Per cent	Per cent	Per cent
1895	27, 878	84. 3	83. 2	84. 5	86. 0
1896	27, 566	98. 0	96. 3	87. 3	74. 0
1897	25, 730	89. 0	87. 5	86. 0	84. 6
1898	25, 321	98. 0	92. 8	84. 2	79. 0
1899	25, 608	88. 7	90. 0	90. 8	87. 2
1900	27, 365	91. 7	85. 5	85. 0	82. 9
1901	26, 315	85. 3	83. 7	73. 6	72. 1
1902	28, 653	90. 6	92. 1	89. 4	87. 2
1903	27, 732	85. 5	84. 3	79. 5	75. 7
1904	27, 646	89. 2	89. 8	86. 6	85. 6
1905	27, 688	92. 9	92. 1	90. 8	90. 3
1906	27, 678	85. 9	84. 0	82. 8	81. 9
1907	31, 491	81. 6	81. 0	84. 5	65. 5
1908	31, 837	92. 9	85. 7	76. 8	69. 7
1909	32, 422	88. 7	88. 3	85. 5	83. 8
1910	34, 380	91. 0	82. 2	81. 5	83. 3
1911	35, 250	85. 7	68. 8	65. 7	64. 5
1912	37, 844	91. 1	89. 2	90. 3	92. 3
1913	38, 341	87. 0	76. 3	73. 8	74. 0
1914	38, 383	89. 5	84. 3	79. 4	75. 8
1915	40, 193	92. 2	93. 9	91. 6	91. 1
1916	40, 780	86. 9	86. 3	81. 5	78. 0
1917	43, 161	88. 8	89. 4	87. 2	90. 4
1918	44, 475	93. 2	85. 5	82. 8	84. 4
1919	42, 169	93. 2	87. 0	76. 5	73. 0
1920	41, 032	87. 8	84. 7	87. 2	88. 3
1921	44, 829	85. 7	77. 6	64. 5	61. 1
1922	41, 822	85. 5	74. 4	75. 6	74. 9
1923	40, 768	85. 6	83. 5	81. 9	80. 3

<sup>1</sup> The June 1 estimate of acreage is the only preliminary estimate of acreage made.

<sup>2</sup> Percentage of normal. See The Use of "Pars" and "Normals" in Forecasting Crop Production, by W. F. Callander and J. A. Becker, in Jour. Farm Economics, Oct., 1923.

TABLE IX.—*Production statistics of wheat and rye*

Year	Wheat				Rye, world production <sup>4</sup>
	United States produc- tion <sup>1</sup>	World produc- tion <sup>2</sup>	World carry over <sup>3</sup>	World produc- tion plus carry over	
	Million bushels	Million bushels	Million bushels	Million bushels	Million bushels
1890.....	378	1,964	.....	.....	.....
1891.....	585	2,033	132	2,165	1,006
1892.....	528	2,242	193	2,417	1,238
1893.....	428	2,338	241	2,579	1,450
1894.....	516	2,420	219	2,639	1,530
1895.....	569	2,376	192	2,568	1,414
1896.....	544	2,303	181	2,484	1,450
1897.....	610	2,050	116	2,166	1,240
1898.....	772	2,821	112	2,933	1,407
1899.....	636	2,591	218	2,809	1,557
1900.....	603	2,463	210	2,673	1,546
1901.....	789	2,714	176	2,890	1,394
1902.....	725	2,902	152	3,054	1,593
1903.....	664	3,014	144	3,158	1,611
1904.....	597	2,919	165	3,084	1,699
1905.....	727	3,047	146	3,193	1,462
1906.....	757	3,150	180	3,330	1,390
1907.....	638	2,852	220	3,072	1,494
1908.....	645	2,865	133	2,998	1,554
1909.....	700	3,320	106	3,426	1,712
1910.....	635	3,261	157	3,418	1,622
1911.....	621	3,245	185	3,430	1,531
1912.....	730	3,500	175	3,675	1,834
1913.....	763	3,695	193	3,888	1,808
1914.....	891	3,226	163	3,389	1,526
1915.....	1,026	.....	147	.....	.....
1916.....	636	.....	355	.....	.....
1917.....	637	.....	330	.....	.....
1918.....	921	.....	261	.....	.....
1919.....	968	.....	307	.....	.....
1920.....	833	<sup>5</sup> 2,884	242	3,126	<sup>6</sup> 609
1921.....	815	3,079	220	3,299	848
1922.....	862	3,096	174	3,270	839
1923.....	<sup>1</sup> 793	3,434	172	3,606	970

<sup>1</sup> U. S. Dept. Agr. Yearbooks. The 1923 figure is the August forecast.

<sup>2</sup> The countries included in the total, 1890-1914, are United Kingdom, Sweden, Denmark, Netherlands, France, Spain, Belgium, Canada, United States, British India, Japan, Argentina, Australia, New Zealand, Algeria, Italy, Germany, Austria, Hungary, Rumania, Russia. Source: U. S. Dept. of Agr., Bur. Agr. Econ., Div. Statistical and Historical Research.

<sup>3</sup> See Table XI, Appendix.

<sup>4</sup> Production of 14 countries, 1890 to 1914, including Sweden, Denmark, Netherlands, France, Spain, Belgium, Finland, Algeria, United States, Germany, Austria, Hungary, Rumania, Russia. Source: U. S. Dept. Agr. Bureau of Agricultural Economics, Division of Statistical and Historical Research.

<sup>5</sup> Total world countries reporting and estimated. See U. S. Dept. Agr. unnumbered report, "The Wheat Situation." Russia omitted, additional countries included, 1920-1923.



TABLE X.—Wheat prices

Year beginning Aug. 1	Chicago price, average of daily high and low <sup>1</sup>	Chicago price, average of monthly high and low <sup>2</sup>	Farm price, December <sup>3</sup>	Liverpool price, average of monthly high and low <sup>4</sup>	Bureau of Labor Statistics index of farm products <sup>5</sup>	Chicago price, adjusted by farm products index
	Cents	Cents	Cents	Cents		Cents
1890.....	97	97	84	111	75	129
1891.....	89	88	84	115	72	124
1892.....	73	73	62	86	70	104
1893.....	60	60	54	75	66	91
1894.....	57	58	49	68	61	93
1895.....	61	62	51	78	58	105
1896.....	70	74	72	88	57	123
1897.....	97	101	81	116	61	159
1898.....	70	71	58	86	64	109
1899.....	69	69	59	86	67	103
1900.....	73	74	62	87	72	101
1901.....	72	73	63	87	78	92
1902.....	75	75	63	89	79	95
1903.....	90	89	70	90	79	114
1904.....	110	106	92	<sup>6</sup> 95	80	138
1905.....	86	88	75	<sup>7</sup> 98	80	108
1906.....	77	79	67	93	84	92
1907.....	95	100	87	110	86	110
1908.....	114	113	99	120	92	124
1909.....	115	115	103	120	100	115
1910.....	95	101	90	107	98	97
1911.....	99	105	90	112	97	102
1912.....	105	100	83	114	100	105
1913.....	93	92	81	106	102	91
1914.....	127	129	109	157	104	122
1915.....	116	117	100	175	114	102
1916.....	193	194	162	224	156	124
1917.....	219	225	206	235	204	107
1918.....	235	237	210	240	224	105
1919.....	251	272	227	215	224	112
1920.....	200	199	172	223	171	117
1921.....	127	138	106	149	128	99
1922.....	122	124	101	-----	139	88

<sup>1</sup> No. 2 Spring wheat, cash July, 1890, to January, 1897; No. 2 wheat, January, 1897, to January, 1898; regular No. 2 January, 1898, to March, 1903; No. 2 red, March, 1903, to June, 1922. Average of daily high and low prices as quoted in the Chicago Board of Trade Annual Reports.

<sup>2</sup> Prices published in National Grange Monthly, July, 1921, p. 11. The grade is No. 2 Spring wheat. The prices were compiled from Chicago papers by a representative of the National Grange working in cooperation with the Bureau of Agricultural Economics, U. S. Department of Agriculture.

<sup>3</sup> U. S. Dept. Agr., Yearbooks. December 1, farm price, 1890-91 to 1908-9; average yearly price, 1908-9 to 1922-23.

<sup>4</sup> Compiled by Market Statistics Section, Bureau of Agricultural Economics, U. S. Department of Agriculture; 1890 to 1903, compiled from Broomhall's 1904 Corn Trade Year Book, p. 136; 1914 to 1920 from Broomhall's 1921 Corn Trade Year Book. Remainder of the table from Broomhall's Corn Trade News. Conversions at par, 1862 to 1912. Current exchange rate for remainder of period. Prices of red wheat supplemented with prices of American wheat for some months, the margin between which is practically negligible. See U. S. Dept. Agr., Yearbook, 1922.

<sup>5</sup> Two-year average of Bureau of Labor Statistics calendar year relative of prices of farm products from 1890-91 to 1913-14. Average of monthly relatives, July to following June after 1914.

<sup>6</sup> Five months' average.

<sup>7</sup> Ten months' average.

TABLE XI.—Wheat: Estimated world visible supply on July 1 for the years 1891–1923

Year	Bushels	Remarks
1891....	132,472,385	To the "Corn Trade" figures were added 3,214,285 bushels for flour in the United Kingdom, 25,000,000 United States farm stocks, 2,800,000 other Europe, 709,000 Argentina, and 1,735,000 Australia.
1892....	193,225,845	To the "Red Book" figures were added 55,000,000 United States farm stocks, 2,800,000 other Europe, 1,153,000 Argentina, and 2,213,000 Australia.
1893....	240,671,000	Added 50,000,000 United States farm stocks, 1,020,000 Argentina, and 2,507,000 Australia.
1894....	218,901,000	Added 40,000,000 United States farm stocks, 1,209,000 Argentina, and 1,880,000 Australia.
1895....	191,513,180	Added 914,000 Argentina and 1,233,000 Australia.
1896....	181,276,038	Added Argentina stocks on page 118, Corn Trade Yearbook 1901-02, and 1,410,000 Australia.
1897....	116,277,293	Argentina same as 1896; Australia, 1,906,000 added.
1898....	111,988,952	Argentina same as 1896; Australia, 2,796,000 added.
1899....	218,031,392	Argentina same as 1896; Australia, 2,884,000 added.
1900....	210,316,765	Argentina same as 1896; Australia, 2,784,000 added.
1901....	176,336,456	Argentina included in "Red Book"; Australia, 3,367,000 added.
1902....	151,708,884	Added Australia, 2,685,000.
1903....	143,932,669	Added Australia, 862,000.
1904....	165,123,889	Added Australia, 5,163,000; Australia included remainder of time.
1905....	145,967,184	
1906....	179,755,289	
1907....	219,701,516	
1908....	133,128,000	
1909....	105,532,000	
1910....	157,202,000	
1911....	185,185,000	
1912....	175,452,000	
1913....	192,531,000	
1914....	163,114,000	
1915....	147,018,000	Australia out.
1916....	355,192,000	France, Germany, Belgium, Holland, Russia, Danubian States, other Europe out for remainder of time.
1917....	329,579,000	Stocks afloat, Argentina, Australia, and American stocks and United Kingdom only.
1918....	260,953,000	Same as 1917.
1919....	306,539,000	
1920....	241,856,000	
1921....	219,769,000	
1922....	174,342,000	
1923....	171,966,000	Minneapolis Market Record. July 14, 1923.

U. S. Dept. Agr., Yearbook, 1922, p. 607.

Australian average carry over, 1905-1914, inclusive, 5,433,000.

Continent omitted 1916 to 1923; carry over averaged 13,584,000—1905-1914.

Table XI was compiled from data of Broomhall's Corn Trade News, Minneapolis Daily Market Record, and Chicago Daily Trade Bulletin, which is published in the Red Books of Howard, Bartels & Co., from 1892-1922 under heading of "Monthly Supply of Breadstuffs." The "visible" included stocks of wheat and flour afloat for United Kingdom and the Continent; stocks in store in United Kingdom, France, Germany, Belgium, Holland, Russia, Danubian States, other portions of Europe, Argentina, Australia, United States, and Canada. To the "Red Book" total has been added United States farm stocks on July 1. The data for 1891 taken from the 1901-1922 Broomhall's Corn Trade Year Book, pages 114, 115, 116, 117, and 122. The data from "Red Book" were checked with Broomhall's Corn Trade Year Book figures from 1892 to 1901. The changes from the original data are noted after each number. The farm stocks of 1891-1894 were estimated; the remainder of the years are official.

## APPENDIX B

### Some selected references on grain price studies and statistical methods

ADAMS, O. S.

Geodesy—Application of theory of least squares to the adjustment of triangulation. 1915. U. S. Coast and Geodetic Survey, Spec. Pub. 28.

BEVERIDGE, Sir W. H.

Wheat prices and rainfall in western Europe. *In Jour. Roy. Statis. Soc.*, vol. 85, pt. 3, May 1922, pp. 412-459.

- CALLANDER, W. F., and BECKER, J. A.  
The use of "pars" and "normals" in forecasting crop production. *In Jour. Farm Economics*, vol. 5, no. 4, Oct. 1923, pp. 185-197.
- DAVIES, G. R.  
Introduction to economic statistics. New York, 1922.  
Includes a treatment of the method of averages in computing index numbers and the method of least squares.
- EDGEWORTH, F. Y.  
On the mathematical representation of statistical data. *In Jour. Roy. Statis. Soc.*, vol. 79, pt. 4, July 1916, pp. 455-500; vol. 80, pt. 1, Jan. 1917, pp. 65-83; vol. 80, pt. 2, March 1917, pp. 266-288.
- FISHER, IRVING.  
The making of index numbers. Boston and New York, 1922.
- FLUX, A. W.  
The measurement of price changes. *In Jour. Roy. Statis. Soc.*, vol. 84, pt. 2, March 1921, pp. 167-199.
- HOOKE, R. H.  
On the correlation of successive observations; illustrated by corn prices. *In Jour. Roy. Statis. Soc.*, vol. 68, pt. 4, Dec. 1905, pp. 696-703.
- \_\_\_\_\_.  
The suspension of the Berlin Produce Exchange and its effect upon corn prices. *In Jour. Roy. Statis. Soc.*, vol. 64, pt. 4, Dec. 1901, pp. 574-604.
- LEHFELDT, R. A.  
The elasticity of demand for wheat. *In Econ. Jour.*, vol. 24, June 1914, pp. 212-217.
- LIPKA, JOSEPH.  
Graphical and mechanical computation. New York, 1918.  
Contains a treatment of the method of least squares and methods of curve fitting.
- MILLS, F. C.  
Statistical methods applied to economics and business. New York, 1924.  
Contains an extended treatment of demand curves. Other contributions on this subject have been made by Walter T. Hedden of the U. S. Department of Agriculture and Henry Schultz Institute of Economics, Washington, D. C.
- MITCHELL, W. C.  
Index numbers of wholesale prices in the United States and foreign countries. 1921. U. S. Bur. Labor Statis. Bul. 284.
- MOORE, H. L.  
Elasticity of demand and flexibility of prices. *In Jour. Amer. Statis. Assoc.*, vol. 18, new ser. 137, March 1922, pp. 8-19.
- \_\_\_\_\_.  
Forecasting the yield and the price of cotton. New York, 1917.
- PERSONS, W. M.  
Correlation of economic statistics. *In Jour. Amer. Statis. Assoc.*, vol. 12, new ser. 92, Dec. 1910, pp. 287-322.
- \_\_\_\_\_.  
A study of statistical method. *In The Review of Economic Statistics*, Preliminary vol. 1, pp. 1-48; published by Harvard university Committee on economic research, 1919.
- PERSONS, W. M., FOSTER, W. T., and HETTINGER, A. J., jr., ed.  
The problem of business forecasting. Boston and New York 1924.
- THE REVIEW OF ECONOMIC STATISTICS, preliminary vol. 1. Cambridge, Mass. Harvard university Committee on economic research, 1919. See Index.
- SMITH, B. B.  
The use of punched card tabulating equipment in multiple correlation problems; collected and prepared for the use of statisticians of the Bureau of Agricultural Economics, U. S. Depart. Agr. 1923.  
Mimeographed.
- TOLLEY, H. R., and EZEKIEL, M. J. B.  
A method of handling multiple correlation problems. *In Jour. Amer. Statis. Assoc.*, vol. 18, new ser. 144, Dec. 1923, pp. 993-1003.
- WILSON, SIR JAMES.  
The world's wheat. *In Jour. Roy. Statis. Soc.*, vol. 184, pt. 3, May 1921, pp. 329-378.
- WORKING HOLBROOK.  
Factors determining the price of potatoes in St. Paul and Minneapolis. 1922. Minn. Agr. Exp. Sta. Tech. Bul. 10.
- YULE, G. U.  
On the time-correlation problem, with especial reference to the variate-difference correlation method. *In Jour. Roy. Statis. Soc.*, vol. 84, pt. 4, July 1921, pp. 497-526.

**ORGANIZATION OF THE  
UNITED STATES DEPARTMENT OF AGRICULTURE**

AUGUST 25, 1925

---

<i>Secretary of Agriculture</i> .....	W. M. JARDINE.
<i>Assistant Secretary</i> .....	R. W. DUNLAP.
<i>Director of Scientific Work</i> .....	_____
<i>Director of Regulatory Work</i> .....	WALTER G. CAMPBELL.
<i>Director of Extension Work</i> .....	C. W. WARBURTON.
<i>Director of Information</i> .....	NELSON ANTRIM CRAWFORD.
<i>Director of Personnel and Business Administration</i> .....	W. W. STOCKBERGER.
<i>Solicitor</i> .....	R. W. WILLIAMS.
<i>Weather Bureau</i> .....	CHARLES F. MARVIN, <i>Chief</i> .
<i>Bureau of Agricultural Economics</i> .....	_____, <i>Chief</i> .
<i>Bureau of Animal Industry</i> .....	JOHN R. MOHLER, <i>Chief</i> .
<i>Bureau of Plant Industry</i> .....	WILLIAM A. TAYLOR, <i>Chief</i> .
<i>Forest Service</i> .....	W. B. GREELEY, <i>Chief</i> .
<i>Bureau of Chemistry</i> .....	C. A. BROWNE, <i>Chief</i> .
<i>Bureau of Soils</i> .....	MILTON WHITNEY, <i>Chief</i> .
<i>Bureau of Entomology</i> .....	L. O. HOWARD, <i>Chief</i> .
<i>Bureau of Biological Survey</i> .....	E. W. NELSON, <i>Chief</i> .
<i>Bureau of Public Roads</i> .....	THOMAS H. MACDONALD, <i>Chief</i> .
<i>Bureau of Home Economics</i> .....	LOUISE STANLEY, <i>Chief</i> .
<i>Bureau of Dairying</i> .....	C. W. LARSON, <i>Chief</i> .
<i>Fixed Nitrogen Research Laboratory</i> .....	F. G. COTTRELL, <i>Director</i> .
<i>Office of Experiment Stations</i> .....	E. W. ALLEN, <i>Chief</i> .
<i>Office of Cooperative Extension Work</i> .....	C. B. SMITH, <i>Chief</i> .
<i>Library</i> .....	CLARIBEL R. BARNETT, <i>Librarian</i> .
<i>Federal Horticultural Board</i> .....	C. L. MARLATT, <i>Chairman</i> .
<i>Insecticide and Fungicide Board</i> .....	J. K. HAYWOOD, <i>Chairman</i> .
<i>Packers and Stockyards Administration</i> .....	JOHN T. CAINE, <i>in Charge</i> .
<i>Grain Futures Administration</i> .....	J. W. T. DUVEL, <i>Acting in Charge</i> .

---

This bulletin is a contribution from

<i>Bureau of Agricultural Economics</i> .....	_____, <i>Chief</i> .
<i>Division of Statistical and Historical Research</i> .....	O. C. STINE, <i>in Charge</i> .

40

---

ADDITIONAL COPIES  
OF THIS PUBLICATION MAY BE PROCURED FROM  
THE SUPERINTENDENT OF DOCUMENTS  
GOVERNMENT PRINTING OFFICE  
WASHINGTON, D. C.  
AT  
10 CENTS PER COPY



